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Hybrid High-Order Methods

A Primer with Applications to Solid Mechanics

Springer

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Preface

Hybrid High-Order (HHO) methods attach discrete unknowns to the cells and to the faces of the mesh. At the heart of their devising lie two intuitive ideas: (i) a local operator reconstructing in every mesh cell a gradient (and possibly a potential for the gradient) from the local cell and face unknowns and (ii) a local stabilization operator weakly enforcing in every mesh cell the matching of the trace of the cell unknowns with the face unknowns. These two local operators are then combined into a local discrete bilinear form, and the global problem is assembled cellwise as in standard finite element methods. HHO methods offer many attractive features: support of polyhedral meshes, optimal convergence rates, local conservation principles, a dimension-independent formulation, and robustness in various regimes (e.g., no volume-locking in linear elasticity). Moreover, their computational efficiency hinges on the possibility of locally eliminating the cell unknowns by static condensation, leading to a global transmission problem coupling only the face unknowns.

HHO methods were introduced in [77, 79] for linear diffusion and quasi-incompressible linear elasticity. A high-order method in mixed form sharing the same devising principles was introduced in [78], and shown in [6] to lead after hybridization to a HHO method with a slightly different, yet equivalent, writing of the stabilization. The realm of applications of HHO methods has been substantially expanded over the last few years. Developments in solid mechanics include nonlinear elasticity [26], hyperelasticity [1], plasticity [2, 3], poroelasticity [16, 27], Kirchhoff–Love plates [19], the Signorini [44], obstacle [59] and two-membrane contact [69] problems, Tresca friction [53], and acoustic and elastic wave propagation [33, 34]. Those related to fluid mechanics include convection-diffusion in various regimes [74], Stokes [6, 81], Navier–Stokes [23, 45, 82], Bingham [43], creeping non-Newtonian [24], and Brinkman [22] flows, flows in fractured porous media [47, 106], single-phase miscible flows [7], and elliptic [35] and Stokes [32] interface problems. Other interesting applications include the Cahn–Hilliard problem [49], Leray–Lions equations [72], elliptic multiscale problems [60], H^{-1} loads [95], spectral problems [38, 41], domains with curved boundary [21, 35, 36], and magnetostatics [48].

Bridges and unifying viewpoints emerged progressively between HHO methods and several other discretization methods which also attach unknowns to the mesh cells and faces. Already in the seminal work [79], a connection was established between the lowest-order HHO method and the hybrid finite volume method from [97] (and, thus, to the broader setting of hybrid mimetic mixed methods in [85]). Perhaps the most salient connection was made in [62] where HHO methods were embedded into the broad setting of Hybridizable Discontinuous Galerkin (HDG) methods [64]. One originality of equal-order HHO methods is the use of the (potential) reconstruction operator in the stabilization. Moreover, the analyses of HHO and HDG methods follow somewhat different paths, since the former relies on orthogonal projections, whereas the latter often invokes a more specific approximation operator [65]. We believe that the links between HHO and HDG methods are mutually beneficial, as, for instance, recent HHO developments can be transposed to the HDG setting. Weak Galerkin (WG) methods [148, 149], which were embedded into the HDG setting in [61 Sect. 6.6], are, thus, also closely related to HHO. WG and HHO were developed independently and share a common devising viewpoint combining reconstruction (called weak gradient in WG) and stabilization. Yet, the WG stabilization often relies on plain least-squares penalties, whereas the more sophisticated HHO stabilization is key to a higher-order consistency property. Furthermore, the work [62] also bridged HHO methods to the nonconforming virtual element method [10, 119]. Finally, the connection to the multiscale hybrid mixed method from [105] was uncovered in [46].

A detailed monograph on HHO methods appeared this year [73]. The present text is shorter and does not cover as many aspects of the analysis and applications of HHO methods. Its originality lies in targetting the material to computational mechanics without sacrificing mathematical rigor, while including on the one hand some mathematical results with their own specific twist and on the other hand numerical illustrations drawn from industrial examples. Moreover, several topics not covered in [73] are treated here: domains with curved boundary, hyperelasticity, plasticity, contact, friction, and wave propagation. The present material is organized into eight chapters: the first three gently introduce the basic principles of HHO methods on a linear diffusion problem, the following four present various challenging applications to solid mechanics, and the last one reviews implementation aspects.

This book is primarily intended for graduate students, researchers (in applied mathematics, numerical analysis, and computational mechanics), and engineers working in related fields of application. Basic knowledge of the devising and analysis of finite element methods is assumed. Special effort was made to streamline the presentation so as to pinpoint the essential ideas, address key mathematical aspects, present examples, and provide bibliographic pointers. This book can also be used as a support for lectures. As a matter of fact, its idea originated from a series of lectures given by one of the authors during the Workshop on Computational Modeling and Numerical Analysis (Petrópolis, Brasil, 2019).

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Contents