Embodied representations in linear algebra: a study over linear dependence

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Introduction

Studies conducted at the beginning of the 2000s (e. g., Harel, 2000; Sierpinska, 2000), described the difficulties generally encountered by sophomore students in the learning of the first concepts of linear algebra. The so-called "obstacle of formalism" was blamed of being the main responsible for the emergence of these difficulties. The authors highlighted the fact that this obstacle does not represent just a general problem with formalism for students, but a difficulty in the use and understanding of formalism in the specific subject. Over the years, some experimental courses have been designed and carried out in order to try to overcome these obstacles (e.g., Dorier & Sierpinska, 2001; Harel, 2000). A few of them were based on the use of a geometrical entry approach into linear algebra, and aimed at providing a remedy to the obstacle of formalism by giving a more concrete meaning to linear algebra notions. Nevertheless, connections with geometry proved to be problematic. Notwithstanding the effectiveness of the emphasis on a geometric embodiment of abstract linear algebra concepts, scholars who experimented such teaching approaches concluded that it would be incorrect to implement a linear algebra course that starts with geometry and then builds algebraic notions through generalization from it (Dorier & Sierpinska, 2001; Harel, 2000).

In the last decade, with the increasing importance recognized to visualization as an epistemological learning tool (Presmeg, 2014), new studies have been conducted aiming at analyzing the potential of visual representations in linear algebra (Stewart et al., 2019). Results showed that the visual approaches presented in lectures, mainly involving the use of dynamic geometry software, allowed students to gain understanding by giving meaning to the symbolic inscriptions used.

Research problem

Despite their potential, in my opinion these attempts of overcoming the obstacles in linear algebra valuing visualization are insufficient to let students get the powerful structure of the subject, whose strength resides in being applicable to many more fields other than only geometry. As also Sierpinska (2000) points out, making the structural content more concrete through using visual images in low dimensions may lead to irrelevant interpretations, confusing what are only possible representations of examples of vector spaces with their whole structure. Thus, despite the fact that a visual geometric approach is valued by some researchers, its implication in teaching linear algebra, as well as the potential of different and more generic visual representations, deserve further investigation.

In order to try to fill this research gap, I am currently investigating a different approach to visualization in the teaching and learning of linear algebra, which emphasizes more the use of graphical semiotic resources as tools for diagrammatic reasoning, rather than focusing only on geometrical examples of concepts in R^2 or R^3. I am interested in understanding what possible

representations could convey, also in a dynamic and embodied way, and in helping students better understand basic linear algebra concepts, without losing the evidence of their applicability in fields other than geometry. This is the focus of my PhD project which aims at analyzing the visual epistemological potential of different semiotic representations in the subject, including also algebraic symbols themselves. According to the "symbol sense" notion developed by Arcavi (1994), also symbolic inscription can have indeed a high visual power.

Methodology

We started with a preliminary study, focused on the notion of linear dependence, aiming at investigating which semiotic resources students use and what embodied aspects they show when dealing with the concept. We conducted semi-structured interviews with university engineering students who had finished their linear algebra course. They were asked to solve different basic exercises, related to different vector spaces, involving the notion of linear dependence/independence. By use of the theory of the semiotic bundle (Arzarello, 2006), video recordings of these interviews, together with written answers, have been analyzed in order to recognize gestures, graphical signs or utterances which may suggest the presence of some sort of visual or embodied images of the concept of linear dependence/independence, which may appear when dealing with this concept in different examples of vector spaces.

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