

Theoretical and Experimental Studies of Manipulation of Elastic Waves in Helix Metamaterials

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Doctoral Dissertation
Doctoral Program in Mechanical Engineering (38th Cycle)

Theoretical and Experimental Studies of Manipulation of Elastic Waves in Helix Metamaterials

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* * * * *

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Politecnico di Torino
December, 2025

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Yumei Chen
December, 2025

Abstract

Elastic metamaterials exhibit mechanical properties unattainable in natural materials and demonstrate vast potential in vibration and noise reduction, acoustic functional devices, acoustic stealth, energy harvesting, and fault diagnosis. To fully leverage their advantages, a key challenge must be addressed: designing metamaterial structures with low-frequency and broadband bandgap characteristics. Currently, active control strategies for low-frequency elastic wave bandgaps remain limited, making large-range tunability within the low-frequency domain difficult. Therefore, widening the low-frequency bandgap, enhancing elastic wave attenuation, and achieving tunable control are critical for the practical application of elastic metamaterials. This dissertation investigates several novel helical metamaterial configurations, combining the Finite Element Method (FEM), the Carrera Unified Formulation (CUF), and laser Doppler vibrometry experiments. Theoretical analysis and experimental validation are conducted to elucidate the mechanisms of bandgap formation and tunability, achieving effective low-frequency broadband control. The main research outcomes are summarized as follows:

(1) Antisymmetric dual helix metamaterial beams, inspired by the nautilus shell, are proposed with and without attached mass spheres. The mechanisms of low-frequency flexural bandgap formation are analyzed using CUF, focusing on the effects of helix turns, mass sphere radius, and material composition. The results indicate that the low-frequency bandgaps originate from localized resonance, significantly enhanced by the coupling between the helical structure and the mass spheres. Adjusting the geometric parameters effectively tunes the bandgap range. Numerical and experimental results confirm the accuracy of CUF predictions.

(2) For two-dimensional dual helix metamaterial plates, numerical simulations reveal the dependence of flexural wave bandgaps on geometric and material parameters. The geometric configuration of the unit cell critically affects bandgap

formation. Increasing helix width shifts the bandgap to higher frequencies and broadens it, while increasing plate thickness raises both boundary frequencies with minimal effect on bandwidth. The bandgap frequency varies almost linearly with the mass sphere radius.

(3) A bandgap-tunable DNA dual helix metamaterial is proposed to address the challenge of fixed bandgaps. FEM analyses show that rearranging mass blocks enables programmable longitudinal wave bandgap control without altering unit geometry. Further adjustment of geometric parameters refines the bandgap range. Experimental results closely align with FEM predictions, particularly in the first longitudinal bandgap region.

(4) To achieve ultra-low and ultra-wide bandgaps, a DNA dual helix metamaterial with node masses is developed. Analyses of configurations with different node mass disks (steel, aluminum, and copper) reveal broad and deep longitudinal bandgaps in the 0–2000 Hz range. By tuning the helix diameter, disk thickness, and axial pitch, longitudinal bandgaps below 50 Hz are obtained, which are verified experimentally.

In conclusion, the study demonstrates that helical metamaterial structures are effective for achieving broad, low-frequency tunable bandgaps. The coupling of dual helix geometry and mass inclusions intensifies localized resonance, enabling ultra-low-frequency and wideband vibration attenuation. Adjusting geometric, material, and mass distribution parameters allows programmable bandgap control. Furthermore, the extended application of CUF in complex structural analyses has been validated through both COMSOL simulations and laser vibrometry experiments, confirming its precision and robustness. The findings provide a novel theoretical and experimental foundation for the design of wide, low-frequency tunable elastic metamaterials.

Key Words: Helix metamaterials, wide low-frequency bandgap, controllable design, laser vibration measurement, strong vibration attenuation, Carrera Unified Formulation

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Chapter 1

Introduction

1.1 Foreword

As a profoundly important wave phenomenon, sound waves significantly influence human life. Audible sound waves not only serve as the fundamental medium for interpersonal communication, but the complex interplay between their frequency (pitch) and temporal (rhythm) characteristics also forms the physical essence of musical art. Furthermore, ultrasonic and infrasonic waves, whose frequencies lie beyond the human hearing threshold, have demonstrated irreplaceable application value in fields such as medical diagnostics (e.g., ultrasound imaging) and industrial non-destructive testing, owing to their unique physical properties. However, the acoustic spectrum is extremely broad (spanning several Hz to tens of thousands of Hz), and its propagation behavior shows strong frequency dependence. Low-frequency sound waves, characterized by their long wavelengths, experience minimal energy attenuation and are challenging to absorb effectively in a medium. Conversely, high-frequency sound waves, with their short wavelengths, are susceptible to energy dissipation through scattering and reflection and are highly sensitive to the medium's microstructure. Simultaneously, wave propagation is highly dependent on the constitutive properties of the medium (such as gases, liquids, and solids). Its propagation speed and attenuation characteristics are directly governed by core physical parameters, such as medium density and elastic modulus, resulting in significant differences in wave propagation dynamics across media. These inherent physical properties, particularly the minimal attenuation of low-frequency sound in air and their strong penetrating capability through high-impedance barriers (such as walls), pose severe challenges to the effective control of sound waves. Although electronic technology can amplify or otherwise process sound after it is converted into electrical signals, this control is limited to the post-conversion stage. It cannot act directly on the original acoustic wave field itself.

In solids, elastic waves are mechanical disturbances that propagate through elastic media, with their physical nature being vibrations of medium particles about their equilibrium positions. Based on the relationship between the direction of particle vibration and the direction of wave propagation, elastic waves are primarily

categorized into two types: those in which the particle vibration direction aligns with the wave propagation direction are called compressional or longitudinal waves; those in which the particle vibration direction is perpendicular to the wave propagation direction are called shear or transverse waves. Effective control over elastic wave propagation is of significant scientific importance and engineering value, as evidenced by the following. Vibration and noise reduction: suppressing mechanical vibrations to minimize damage to equipment and structures, reducing environmental noise pollution, and enhancing the quality of living and working environments. Energy harvesting: converting elastic wave vibration energy from the environment or structures into electrical energy to achieve distributed energy capture. Non-destructive testing and imaging: utilizing elastic waves to detect internal material defects, assess structural health, and perform biomedical imaging and non-invasive treatment. Geological exploration: studying the Earth's internal structure, resource distribution, and geological activities by analyzing seismic wave propagation characteristics. Information processing: employing elastic waves (e.g., surface acoustic waves) to achieve signal filtering, delay, transmission, and other processing functions.

The human use of materials to control wave propagation dates back a long history. The field of optics serves as a prime example: by precisely designing lens geometry, optical wave manipulation, such as focusing and deflection, can be achieved. Nature also offers exquisite examples: the crystalline lens in animal eyes focuses light onto the retina to form vision; cetaceans (such as dolphins and sperm whales) utilize the graded acoustic impedance structure of their melon organs as biological acoustic lenses to focus sound beams for underwater echolocation. These biological systems employ relatively single material compositions to achieve effective control over light or sound waves through multi-scale structural designs (e.g., geometric configurations, gradient parameter distributions). By mimicking the periodic arrangements found in natural crystals, researchers have designed artificial periodic structures known as photonic crystals that exhibit bandgaps for electromagnetic and optical waves. Subsequently, by analogy to the concept of photonic crystals, the concept of phononic crystals was further proposed.

A phononic crystal is a type of functional material with an artificially periodic structure, possessing specific properties not found in natural substances. Its most characteristic feature is the acoustic/elastic wave bandgap, meaning that wave propagation is suppressed or completely blocked within specific frequency ranges. A phononic crystal consists of disconnected scatterers and a matrix that forms a continuous whole. Its fundamental characteristic lies in the periodic distribution of geometric or material parameters. Due to their bandgap properties, phononic crystals have attracted extensive attention and have been the subject of extensive research in recent years. In 1992, Sigalas et al. [1] first theoretically demonstrated the existence of elastic wave bandgaps by studying a matrix material embedded with spherical scatterers. In 1995, Martínez-Sala et al. [2] experimentally confirmed the existence of acoustic bandgaps for the first time through acoustic tests on a sculpture titled "Flowing Melody" in Madrid, Spain, with the findings published in *Nature*.

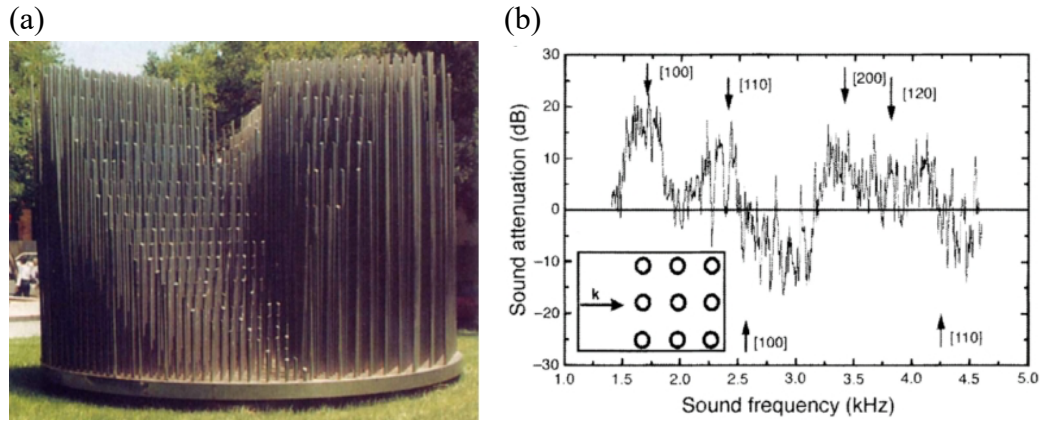


Figure 1.1 Experiment demonstrating an acoustic bandgap by Martinez-Sala et al. [2]: (a) the sculpture “Flowing Melody” in Madrid; (b) experimentally measured frequency response spectrum.

The bandgap formation mechanisms in phononic crystals are generally classified into two types: Bragg scattering mechanism [2] and local resonance mechanism [3]. Bragg scattering bandgaps can be formed using two different materials within the unit cell or a single material with embedded holes. Key parameters for generating Bragg bandgaps include the ratio of wave velocities between the scatterer and the host structure, as well as the volume fraction occupied by the scatterers. For elastic or acoustic waves, the density ratio is also significant. In Bragg scattering bandgaps, the wavelength is comparable to the spatial period of the unit cell. Therefore, generating low-frequency bandgaps typically requires a relatively large unit cell structure. On the other hand, bandgaps formed by locally resonant phononic crystals/elastic metamaterials exhibit lattice constants that are two orders of magnitude smaller than the relevant acoustic wavelength, enabling sub-wavelength wave control. In summary, the Bragg scattering mechanism emphasizes the influence of periodic structural dimensions or material properties, while the local resonance mechanism focuses on the resonant characteristics of individual resonant units. In 2000, Liu et al. [3] demonstrated a phononic crystal based on local resonance. Their structure, lead spheres in a rubber coating within an epoxy matrix, enabled low-frequency bandgaps at a sub-wavelength scale, directly catalyzing the emergence of acoustic/elastic metamaterials as a vibrant new field.

Research on acoustic/elastic metamaterials primarily focuses on areas such as bandgaps of acoustic/elastic waves [4-8], directional transmission [9], acoustic waveguiding [10, 11], acoustic filtering [12, 13], as well as defect states in phononic crystals [14], negative refraction [15-17], and acoustic focusing [18, 19]. Recent high-level studies [20-27] have significantly advanced the field and demonstrated its strong prospects. In engineering, the study of periodic structures has long attracted attention from scientists in traditional structural dynamics [28, 29]. Many

common civil engineering structures, such as high-rise buildings, multi-span bridges, and tower-mast trusses, are periodic or quasi-periodic structures. Acoustic/elastic metamaterials, possessing properties and functions not found in traditional materials, have demonstrated significant potential for applications in vibration reduction [30-32], noise control [33-35], acoustic functional devices [36, 37], acoustic cloaking [38-40], impact mitigation [41, 42], energy harvesting [43, 44], and fault diagnosis [45]. Through rational structural design, elastic metamaterials can exhibit extraordinary performance that surpasses their constituent materials, even achieving properties not found in nature or previously considered impossible. These characteristics not only overcome the limitations of traditional materials but also open entirely new possibilities for manipulating acoustic and elastic waves.

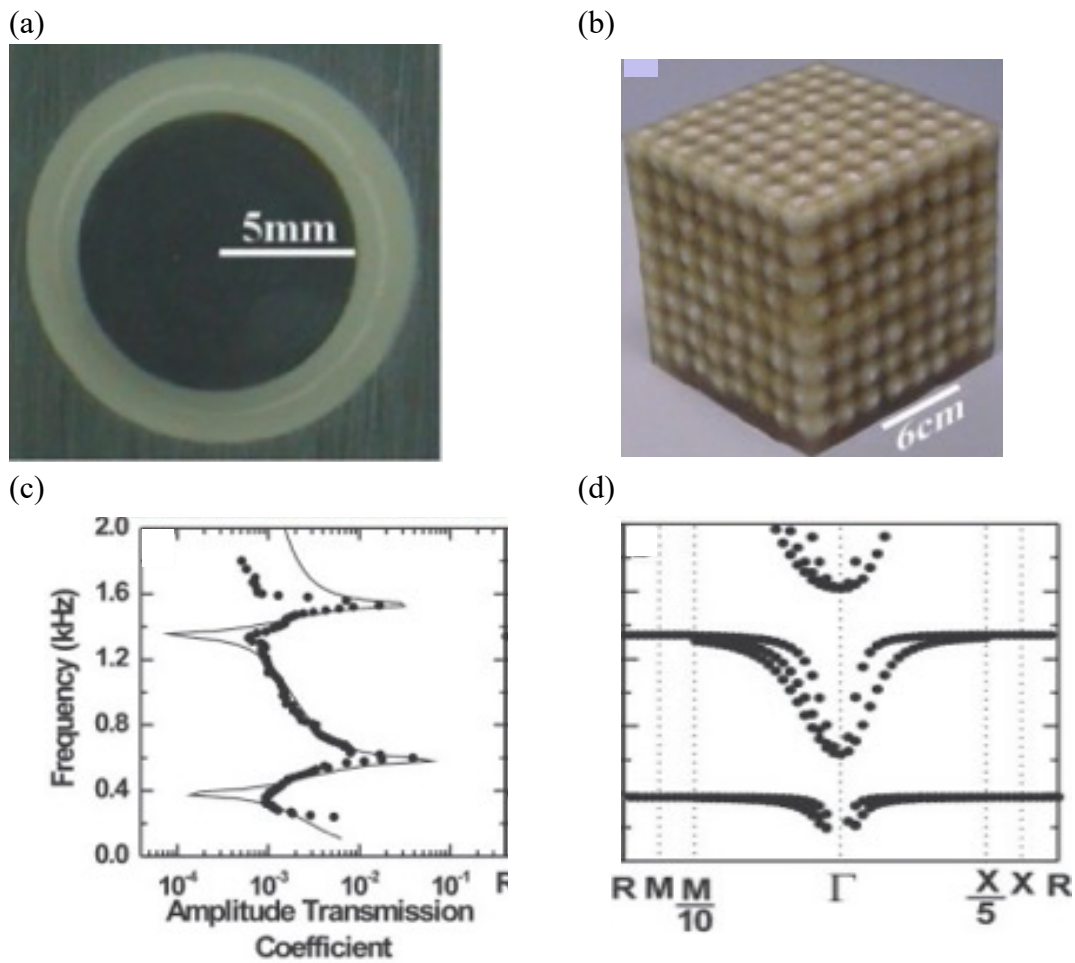


Figure 1.2 Locally resonant phononic crystal designed by Zhengyou Liu et al. and its bandgap characteristics [3]: (a) cross section of a unit cell; (b) photograph of the phononic crystal; (c) measured frequency response spectrum; (d) calculated band structure.

However, in both traditional and novel applications, elastic metamaterials typically require bandgap characteristics that are both lower in frequency and broader in bandwidth. Low-frequency noise, characterized by long wavelengths, intense penetration, concentrated energy, difficulty in shielding, and low perceptibility, poses serious risks to human health. Prolonged exposure to low-frequency noise, particularly in the 20–200 Hz range, can severely affect the cardiovascular, nervous, and internal organs, as well as mental well-being. Yin et al. [46] reviewed the development of mechanical metamaterials and the trends in their application, focusing on their extraordinary mechanical properties in fields such as equipment vibration and noise reduction, underwater acoustic control, and structural anti-vibration/impact design. They identified narrowband effects and intrinsic damping as major factors limiting the engineering application of mechanical metamaterials. Due to constraints on load-bearing capacity, physical dimensions, and local resonator structures, designing metamaterials with low-frequency broadband performance remains a significant challenge in current elastic metamaterial research. Moreover, existing methods for active control of low-frequency elastic wave bandgaps are limited, making it challenging to achieve wide-range dynamic tuning in the low-frequency domain. Therefore, broadening the low-frequency bandgap, enhancing elastic wave attenuation within the bandgap, and achieving active control over the bandgap range have become critical to extending the application of metamaterials.

1.2 Research progress on elastic metamaterials

1.2.1 Broad low-frequency bandgaps

Research into broadband low-frequency bandgaps currently focuses on developing bandgaps via various coupling mechanisms, acoustic black holes, quasi-zero-stiffness resonators, multi-polarization modes, and novel structural designs. The bandwidth of purely locally resonant bandgaps in the low-frequency range is typically narrow. Consequently, researchers are dedicated to developing elastic metamaterials that combine local resonance and Bragg scattering mechanisms [47, 48] to achieve both broadband and low-frequency bandgap. Xiao et al. [49] investigated the propagation of flexural waves in locally resonant (LR) plates, finding that both local resonance-type and Bragg-type bandgaps coexist in LR plates, and the resonances significantly influence their widths. Zhao et al. [50] proposed a low-frequency vibration-suppressing metastructure based on a honeycomb lattice as the host matrix. Tian et al. [51] designed a novel perforated elastic metamaterial plate that incorporates two bandgap-forming mechanisms (namely, Bragg scattering and local resonance). Their results revealed that the spiral waveguides introduced by the spiral holes enhanced wave scattering within the host material, while wave scattering from the orthogonal rectangular holes in the split host material coupled with the local resonance modes formed by the spiral holes. This coupled mode generated a coupled bandgap and improved wave attenuation. Zhang et al. [52] conducted theoretical and experimental studies of the low-

frequency sound-insulation mechanism in a perforated-plate-type acoustic metamaterial. Gao et al. [53] investigated a single-phase metamaterial plate for low-frequency broadband vibration suppression. Belle et al. [54] studied the influence of damping on the sound transmission loss of locally resonant metamaterial plates, finding that incorporating damping into the modelling of infinite periodic structures enables more accurate prediction of acoustic isolation performance. Wang et al. [55] theoretically and experimentally studied the acoustic transmission characteristics in an acoustic metamaterial sandwich plate with enhanced sound transmission loss. Poggetto et al. [56] investigated the flexural wave bandgap properties in ternary periodic metamaterial plates. Muhammad et al. [57] employed structural steel sections as seismic metamaterials in layered soil media to attenuate low-frequency broadband surface waves. Chen et al. [58] constructed a sound-absorbing material whose unit cell consisted of two axially coupled tubes in series. By carefully designing the geometric parameters of the coupled tubes, they achieved overlapping sound absorption coefficient curves, thereby broadening the absorption bandwidth. Rupin et al. [59] experimentally validated a multi-resonant-element metamaterial with a broadband gap for attenuating Lamb waves. Chen et al. [60] analysed the low-frequency bandgaps in a metamaterial plate using an antisymmetric dual helix structure as the basic local resonant unit. Lou et al. [61] designed a graded acoustic metamaterial rod capable of achieving ultra-wide vibration attenuation and rainbow trapping. This rod consists of resonators with different natural frequencies. Liu et al. [62] attempted to use resonators based on linear and geometrically nonlinear dampers in locally resonant acoustic metamaterials and evaluated their performance for low-frequency wave attenuation. Mei et al. [20] utilized a membrane-type acoustic metamaterial composed of asymmetrically arranged rigidly backed panels to achieve perfect absorption of low-frequency airborne sound within the specific resonance frequency range of 100–1000 Hz. Ruan et al. [6] proposed a novel chiral spiral metamaterial based on cochlear-inspired structures. This spiral metamaterial comprises four orthogonally oriented cochlear-inspired structures with identical geometric parameters, connected at their inner endpoints. Zhou et al. [63] reported a hybrid metamaterial that exhibits both a low-frequency bandgap and impact resistance.

By connecting multi-point resonators to the unit cell [64, 65], the bandgap can be opened by amplifying the resonators' effective inertia. Studies have shown that the application of Inerter-based Dynamic Vibration Absorbers (IDVA) in elastic metamaterials can expand the low-frequency bandgap [66]. Jamil et al. [67] conducted a comparative study among different configurations and identified the unique advantages of inerter-based elastic metamaterials. When the embedded inerter is connected to the matrix material at both ends, the metamaterial shows clear superiority, exhibiting a bandgap at ultra-low frequencies, i.e., the ultra-long wavelength regime, where the unit cell size can be four or more orders of magnitude smaller than the operating wavelength. Foehr et al. [68] created phononic materials containing Bragg scattering, local resonance, and inertial amplification by using the lattice of helical unit cells. Zhao et al. [69] proposed a nonlinear damped metamaterial with a nonlinear bandgap and a wideband attenuation based on a

rotational inertia amplifier. Fang et al. [70, 71] achieved the suppression of low-frequency vibrations by arranging and recombining IDVA arrays in metamaterials. To achieve a low-frequency bandgap and a smaller additional mass, Zhou et al. [5] periodically arranged dynamic vibration absorbers based on viscous dampers in the box girder, transforming the traditional box girder into a metamaterial girder with a local resonance bandgap. As absorbable metamaterials, acoustic black holes have long been a popular research field. Kafer et al. [72] conducted an experimental study on the vibration-absorption effect of multi-wedge black holes in beam structures by welding varying numbers of wedges via additive manufacturing. Deng et al. [31] absorbed the vibration of the plate by periodically adding a composite structure of acoustic black holes.

To achieve ultra-low frequency wave attenuation, Cai et al. [73] proposed a metamaterial plate with an elastic Quasi-Zero Stiffness (QZS) resonator, and an ultra-low frequency bent wave bandgap was realized in the QZS metamaterial plate. Xu et al. [74] designed a metamaterial shaft capable of effectively controlling low-frequency torsional vibration by periodically attaching the QZS torsional local resonator to a uniform shaft. Krushynska et al. [75] proposed lightweight metamaterials capable of uniformly attenuating elastic waves over extremely wide bandgaps and at low frequencies. Li et al. [76] discussed a temperature-controllable QZS, wide, low-frequency-tunable metamaterial beam using a memory alloy. Altering the temperature of the memory alloy can continuously tune the bandgap frequency from 26 Hz to 69 Hz. Chen et al. [77] utilized the large-deformation capacity and material hardening of soft waveguides to tune the topological protection state, achieving a mechanically tunable topological interface state for low-frequency longitudinal waves. In practical applications, vibration of engineering structures occurs across different frequency ranges and involves various polarization modes, such as bending, torsion, and expansion. At present, most reported elastic metamaterials are used only in a single frequency range or a single elastic wave mode. It is challenging to simultaneously suppress the propagation of vibration energy in multiple frequency ranges and various polarization modes. To address this issue, Rui et al. [78] designed a two-degree-of-freedom elastic metamaterial with multi-frequency range elastic wave bandgaps and multi-polarization mode elastic wave bandgaps. The vibration absorption units with periodic, gradient parameters and different polarization modes were integrated into the support structure of the actual equipment, thereby enhancing vibration attenuation, achieving broadband vibration absorption, and enabling multi-polarization-mode vibration absorption. Zheng et al. [79] designed a mechanical logic switch for chiral acoustic metamaterials inspired by DNA. By applying constraints at different positions, logical operations can be performed easily, and it has an ultra-wide bandgap in the low-frequency domain.

1.2.2 Tunable bandgaps

Phononic crystals and elastic metamaterials offer a powerful platform for controlling elastic waves. However, their practical application is hindered by the

fact that, once fabricated, their topological and material parameters are typically fixed and cannot be adjusted, severely limiting their versatility [80]. The significant potential of controllable/programmable elastic metamaterials for applications such as vibration/noise reduction, acoustic focusing, energy harvesting, and frequency control has spurred extensive research efforts. Currently, achieving a tunable bandgap primarily relies on altering a structure's boundary conditions, geometric configurations, or material properties in response to external stimuli. Gao et al. [81] investigated a local resonant phononic crystal with a tunable low-frequency bandgap, composed of a frame, Archimedean spiral beams, and an electromagnet. Goldsberry et al. [82] obtained a tunable elastic metamaterial by using a negative stiffness honeycomb composed of a doubly periodic array of curved beams as a tunable elastic medium. Origami metamaterials can control the propagation of elastic waves and energy transmission through the flexible design of microstructural topology, offering excellent prospects for structural impact resistance. Jiang et al. [83] combined Bloch's theorem with the local resonance mechanism and proposed a tunable hybrid origami metamaterial to simultaneously achieve low-frequency and broad bandgaps for realizing wave transmission and shock mitigation. Kim et al. [84] created a reconfigurable DNA origami metamaterial using the paper-folding mechanism. Liu et al. [85] applied compression-torsion coupled metamaterials to low-frequency vibration isolation based on the Kresling origami structure (composed of beams and plates). Tao et al. [86] considered an intelligent 4D-printed multi-stable metamaterial with reconfigurable shape, deployable function, mechanically adjustable features, and reusability. Chen et al. [87] reported complete tunable elastic wave bandgaps in periodic composites inspired by nacre-like, brick-and-mortar microstructure organisms. Chuang et al. [88] proposed a tunable-bandgap metamaterial beam with a periodic array of two-way shape-memory alloy. Chen et al. [89] proposed a new approach to designing programmable metamaterials with tunable crystal symmetry via a group-theory-based symmetry reduction, based on a structure composed of rotating rigid triangles.

External excitations, such as mechanical loads, temperature fields, electric fields, and magnetic fields, enable real-time bandgap tuning, with mechanical loads being the most practical approach. Tao et al. [90] constructed an acoustic metamaterial plate with an adjustable bandgap and negative Poisson's ratio, achieving lower frequencies and wider adjustable bandgaps by increasing compressive stress. Fang et al. [91] used compression-torsional buckling structures rather than bending structures as the basic elements of truss structures to design metamaterials, thereby breaking free of the constraints imposed by the base material's strength and toughness. Wang et al. [92] fabricated a metamaterial isolator based on magnetorheological elastomers and magnetic springs, with a tunable low-frequency bandgap. They successfully regulated the bandgap cut-off frequency by controlling the stiffness of the magnetorheological elastomers. Chen et al. [93] proposed a novel active elastic metamaterial with a negative capacitance piezoelectric shunt. By using piezoelectric patches with different negative capacitance values, the position and range of the induced bandgap can be effectively

adjusted. Wang et al. [94] conducted research on the structural design, manufacturing, and macroscopic property characterization of flexible mechanical metamaterials with adjustable elastic properties. Bilal et al. [95] proposed tunable phonon metamaterial plates with programmable deep subwavelength bandgaps by utilizing geometric and magnetic nonlinearity within the unit cell. Zhang et al. [96] combined topological optimization with a thermally driven strategy, using reverse engineering to 3D-print integrated, tunable components from thermosensitive materials. They numerically and experimentally characterized tunable metamaterial beams of three different heights, revealing that concentrated end-mass blocks are more effective in generating wide flexural wave bandgaps. Fang et al. [25] reported the realization of multifunctional programmable mechanical metamaterials using gear clusters composed of encoded stiffness gears. Roshdy et al. [97] applied external loads and polarization effects to calculate the static and dynamic behaviours of 3D dynamically tunable metamaterials, respectively. Cheng et al. [98] introduced a micro-lattice design that converts 2D films into programmable, curved, mesoscopic surfaces through mechanically guided assembly. Bae et al. [99] proposed a novel metamaterial that adjusts the resonant frequency by the length of the helical path. Ren et al. [100] employed 4D printing to fabricate a novel multi-material mechanical metamaterial that integrates curved-beam and foldable-straw structures. This customizable metamaterial enables independent tailoring of its geometric shape, stiffness, damping, and vibration-isolation properties, which were systematically investigated through both theoretical analysis and experimental validation. Gliozzi et al. [101] introduced elastic 3D-printed polymer metamaterials whose transmission spectra can be deterministically tuned by an optical field, an approach that offers distinct advantages over conventional methods (e.g., electric/magnetic fields, temperature, mechanical stimulation), including non-contact control, fully reversible switching, and rapid response.

1.2.3 Bio-inspired elastic metamaterials

The remarkable properties of biological structures, honed over millions of years of evolution, have long inspired and continue to drive extensive research into bio-inspired materials and structures across the scientific and engineering communities [102-107]. These studies reveal the relationship between the mechanical behavior and structural morphology of organisms, as well as how specific functions and performance can be achieved through micro- or macro-scale structures. By mimicking the dermis of sea cucumbers, Liu et al. [108] proposed a supramolecular polymeric material with exceptional impact protection performance, while its stiffness-switching characteristic is reversible. Inspired by the absorption model of the Papilio butterfly, Huang et al. [109] achieved an ideal stealth concept through the synergistic combination of microwave melanin and bio-inspired metastructures. By simulating the multi-vertebral structure of bird necks and their head-gaze stabilization mechanism, Deng et al. [110] proposed a multi-layer QZS nonlinear isolator. The results demonstrated that its high-static-low-dynamic-stiffness characteristics can resolve the traditional trade-off between isolation bandwidth and

load-bearing capacity, thereby reducing the reliance on soft stiffness. Nian et al. [111] reported a novel bio-inspired 3D-printed functionally graded metamaterial beam and conducted finite element simulations and experimental tests to investigate its deformation modes, failure mechanisms, and energy absorption. The 3D soft horseshoe-shaped network metamaterial constructed by Zhou et al. [112] exhibits tunable nonlinear mechanical responses and tension-torsion coupling, demonstrating extraordinary mechanical properties. Inspired by the load-bearing arched structures found in the human spine and turtle shells, Zhang et al. [113] proposed a shape-programmable, performance-tunable, and reusable multifunctional intelligent metamaterial.

Ma et al. [114, 115] designed bio-inspired acoustic metamaterials and acoustic functional devices based on the mammalian cochlea as a prototype. They pointed out that cochlear hair cells and stereociliary bundles constitute a natural biological acoustic metamaterial with negative stiffness properties, and that the overall cochlea, based on this structure, is a locally resonant system exhibiting negative dynamic effective mass characteristics. Rupin et al. [116] demonstrated experimentally that the biological cochlea can be modeled as an active one-dimensional acoustic metamaterial. However, while current cochlear implants restore functional hearing, they often fail to convey pitch and musical nuances due to their limited number of frequency channels. Inspired by this biological blueprint, Tang et al. [117] developed a soft, bio-inspired metamaterial that mimics the shape and key functions of the human cochlea, offering a “promising pathway” for advanced hearing restoration. Inspired by biological hierarchical structures, Chen et al. [118] developed a bilayer metamaterial mimicking the “brick-and-mortar” microstructure of biological composites to achieve broadband wave attenuation. While this work highlighted the potential of bio-inspired design, the fundamental influence of hierarchical organization on metamaterial dynamics remained underexplored. To address this gap, Pugno et al. [119] conducted a combined numerical and experimental study to evaluate how bio-inspired hierarchy and viscoelasticity affect wave attenuation in continuous elastic metamaterials. Ghavidelnia et al. [120] designed a mechanical metamaterial inspired by succulent plants that possesses self-healing capabilities, enabling functional recovery following structural damage. The non-self-similar hierarchical elastic metamaterial designed by Mazzotti et al. [121] can simultaneously generate bandgaps via Bragg scattering, local resonance, and inertial amplification.

Spider silk, a quintessential biological material, exhibits exceptional mechanical properties due to the hierarchical organization of its dragline and viscous silks, which confer unusual and tunable characteristics that have been extensively studied from a quasi-static perspective. Inspired by the orb-web structure of the wolf spider, Miniaci et al. [122] demonstrated the significant role of spider-web-inspired metamaterial structures in dynamic response and wave attenuation mechanisms through the design of periodically repeating mechanical metamaterials. Drawing from the construction of natural spider webs, Krushynska et al. [123] designed a labyrinthine acoustic metamaterial with a spider-web-like architecture for low-frequency acoustic control. This metamaterial leverages the

web's structure to achieve hybrid dispersion characteristics. Metamaterials exhibit significant vibration-damping effects on mechanical waves and have attracted attention for impact mitigation and protection. In this context, Hu et al. [124] investigated the vibration-damping performance of a double helix metamaterial inspired by DNA. Inspired by the chiral characteristics of the DNA double helix, Zheng et al. [125] proposed a helical structure with chirality and investigated its mechanical behavior through experimental characterization, theoretical analysis, and numerical simulation. These bio-inspired metamaterials show great promise for applications in performance and functional realization. Among them, DNA sequence specificity has also been utilized to guide the assembly of nanomaterials at the nanoscale [126].

Despite significant progress in broadband low-frequency bandgaps, tunability, and bio-inspired designs, several critical bottlenecks hinder the practical application of elastic metamaterials. These limitations can be summarized in three key aspects:

(1) A key challenge in achieving low-frequency broadband bandgaps lies in the inherent trade-off between bandwidth and structural compactness. Most current designs rely on heavy resonators or complex coupling mechanisms to open low-frequency bandgaps, but this often compromises lightweight construction and load-bearing capacity, which are critical for aerospace and precision instrumentation applications.

(2) The active tuning of low-frequency bandgaps remains significantly underdeveloped. Current reports are limited, and the achievable tuning range, speed, and flexibility fall short of practical needs. Moreover, most strategies rely on continuous external energy or preset fields, lacking the adaptability required for genuine “active” manipulation in response to changing environments, and systematic optimization approaches are still absent.

(3) How to simultaneously broaden low-frequency bandgaps, enhance their attenuation, and enable tunability remains an unresolved tripartite challenge, yet current research largely optimizes these objectives in isolation, overlooking their underlying coupling mechanisms and the need for co-design methodologies.

Addressing these challenges thus requires a deeper understanding of bandgap formation mechanisms and manipulation strategies, which the following sections will address through a systematic introduction to the relevant theoretical and experimental methods in elastic metamaterial research.

1.3 Research methodologies in elastic metamaterials

1.3.1 Bandgap calculation theory and numerical methods

The bandgap property is one of the most critical characteristics of elastic metamaterials. It is primarily represented in two ways: (1) the dispersion relation (or band structure) of elastic waves in infinite periodic structures, and (2) the displacement transmission curve, which indicates the attenuation degree of elastic waves in finite periodic structures. Current mainstream methods for calculating the bandgap properties of elastic metamaterials include the plane-wave expansion

method, transfer-matrix method, multiple-scattering theory, spectral-element method, finite-difference time-domain method, and finite-element method, among others. This section will also briefly introduce the Carrera Unified Formulation (CUF), a detailed exposition of which is provided in Chapter 2 of this thesis. In practical applications, each method presents its own advantages and disadvantages.

The Plane Wave Expansion Method (PWEM) is an early-developed and widely used computational approach for calculating the band structures of periodic systems. Its core concept is based on Bloch's theorem, employing a Fourier series expansion of plane waves to represent the wave field within a periodic medium, thereby transforming the governing partial differential equations into an algebraic eigenvalue problem in terms of frequency and wave vector. PWEM is an efficient and accurate numerical method, particularly well-suited for solving wave propagation problems in periodic structures. Sigalas [127] utilized PWEM to investigate the propagation of elastic waves in two-dimensional periodic composite materials. Khelif et al. [128] applied PWEM to study elastic wave propagation in two-dimensional phononic crystal bandgap materials. Based on Mindlin's piezoelectric plate theory and the plane wave expansion method, Hsu et al. [129] derived formulas for the frequency bandgaps and dispersion relations of low-order Lamb waves in two-dimensional piezoelectric phononic plates. Poggetto et al. [56, 130] employed PWEM to study the elastic wave bandgap properties in both two- and three-dimensional periodic metamaterials. Lu et al. [131] developed a plane-wave finite-element model to investigate the dynamic response of two-dimensional phononic crystals to point harmonic forces. A limitation of PWEM is its poor convergence when dealing with complex unit-cell geometries or significant differences in the elastic constants of constituent materials. Furthermore, because the method assumes an infinite structure or periodic boundary conditions, it cannot be used directly to calculate transmission or reflection coefficients.

The Transfer Matrix Method (TMM) offers high efficiency and strong analytical advantages for one-dimensional wave problems, making it particularly suitable for analyzing layered and periodic systems. Starting from the wave equations governing the structure, TMM considers continuity conditions for displacement, rotation angle, bending moment, and shear force at interfaces where material/structural properties change within a periodic configuration. Assuming identical, continuous states across adjacent unit cells, it derives the transfer matrix. Periodic boundary conditions are then introduced via Bloch's theorem, yielding the band structure of the periodic system. TMM is especially effective for addressing bandgap problems in one-dimensional periodic structures and can also handle issues like imperfect interfaces. Laly et al. [132] proposed a transfer-matrix method combined with finite-element computation to characterize homogeneous and heterogeneous sound-absorbing materials with in-plane periodicity. Lee [133] performed vibration analysis of one-dimensional structures using a spectral transfer matrix approach that integrates the spectral element method with TMM. Han et al. [134] introduced an extended transfer matrix method based on a specific combination of hyperbolic and trigonometric functions to compute the flexural vibration band structure of one-dimensional phononic crystal Euler beams. Lin et

al. [135] utilized a symmetry-enhanced transfer matrix method that leverages lattice symmetries to calculate the transmission spectra and band structures of two- and three-dimensional layer-by-layer photonic crystal slabs. However, the application of TMM is often limited in higher-dimensional problems, complex geometries, and high-frequency regimes.

The Multiple Scattering Theory (MST) [136, 137] treats each constituent element as an individual scatterer. Taking these scatterers as reference objects, it establishes characteristic equations relating the incident wave on each scatterer to the scattered waves from other scatterers. By employing the Mie scattering relationships among the scatterers, the coefficient relations are determined, ultimately yielding the band structure and transmission curves of the entire periodic system. Its advantages include fast convergence and the ability to handle scenarios where the elastic constants differ significantly, including disordered systems, which the PWEM struggles with. MST exhibits rapid convergence in calculating the band structures of two- or three-dimensional phononic crystals, even within ordered or disordered systems. The theory has also been successfully applied to three-dimensional phononic crystals [138, 139], enabling rigorous computation of the transmission and reflection coefficients of periodically distributed scatterers and facilitating direct comparison with experimental results. Pu et al. [140] validated a general framework based on MST and applied it to model space-time modulated elastic metamaterials. Furthermore, MST can handle phononic crystals with low elastic contrast, with fluid-solid coupling problems serving as typical examples. However, its limitations include difficulties in calculating overlapping scatterers, a restriction to spherical or cylindrical scatterer shapes, and a requirement for material isotropy.

The Spectral Element Method (SEM) [141] discretizes the overall structure by dividing it into multiple sub-elements. The global displacement field is obtained by calculating the displacements at each element's nodes. Typically, SEM describes the frequency-dependent dynamic characteristics of a structure through a spectral element formulation of the dynamic stiffness matrix. When calculating the bandgaps of elastic metamaterials, the dynamic model of a single periodic unit cell is first obtained, and its eigenvalues are computed to determine the band structure of the overall assembly. Subsequently, by sequentially assembling the unit cells, the displacement transmission curve of the finite structure under a given excitation can be determined. Lee and Kim et al. [142-144] applied SEM to address dynamic problems in single-layer beams, multi-layer beams, and plates. Zhang et al. [145] used SEM to investigate the flexural wave bandgap properties in two-dimensional periodic structures composed of locally resonant composite beams. Krawczuk et al. [146] studied the propagation of elastic waves in Timoshenko beams containing damage cracks using SEM. Shi et al. [147] employed SEM to solve the band structure of three-dimensional phononic crystals. Fang et al. [148] applied SEM to study unidirectional flexural waves in finite phononic crystal beams. Muhammad et al. [149] provided a comprehensive review of SEM applications across scientific, engineering, and mathematical fields. However, the use of high-order polynomial

basis functions also significantly increases computational cost per element, making SEM relatively expensive for large-scale periodic structures.

The Finite Difference Time Domain (FDTD) method discretizes time and space into finite grids, transforms partial differential equations into finite-difference equations, and solves them in the time domain. This approach is well-suited for analyzing transient phenomena and nonlinear effects. The method is not constrained by structural geometry; it can calculate both the band structure of infinite periodic systems and the transmission/reflection spectra of finite structures. Sigalas et al. [150] applied FDTD to theoretically investigate three-dimensional elastic bandgaps in inhomogeneous periodic media. Hsieh et al. [151] utilized a parallel FDTD algorithm and a cluster system to compute the band structure of three-dimensional phononic crystals. Pennec et al. [12] employed FDTD to study the transmission spectra of phononic crystals with hollow cylinders and liquid-filled inclusions for tunable filtering. Sun et al. [152] used FDTD to analyze the dispersion, transmission, and displacement fields of Lamb waves. While FDTD is convenient for handling inhomogeneous, defective, nonlinear, and anisotropic systems, its drawbacks include relatively poor convergence and slower computation speeds, especially for three-dimensional structures.

The Finite Element Method (FEM) is among the earliest approaches for investigating wave propagation and band structures in periodic systems [153-157]. Its application in the study of phononic crystals/elastic metamaterials has become increasingly widespread, particularly with the emergence of mature commercial software such as ANSYS® [158, 159], ABAQUS® [160, 161], and COMSOL Multiphysics® [162, 163]. The fundamental concept of FEM involves discretizing the solution domain into a finite number of elements, thereby transforming a continuous problem with an infinite number of degrees of freedom into a discrete one with a finite number. Within each element, the unknown field function at any point is approximated using shape functions and linear interpolation based on values at discrete grid nodes. The field over the entire domain is represented by assembling these element-level shape functions. By combining the elastic equilibrium equations with equivalent variational principles or weighted residual methods that incorporate boundary conditions, a system of algebraic or partial differential equations for the nodal field functions is established. When applying FEM to calculate band structures, the method is typically categorized into two approaches, depending on how Bloch's theorem is implemented. The first approach leverages the structure's periodicity to reduce the system's degrees of freedom, condensing the discrete eigenvalue equation for the entire structure into a single-unit-cell solution. The second approach involves meshing the unit cell, applying periodic boundary conditions at the corresponding nodes, and then solving the assembled discrete eigenvalue equations. The latter is more straightforward to implement within standard FEM frameworks and is therefore more commonly used. FEM offers advantages such as good convergence, strong adaptability, fast computation, and high accuracy. It can compute not only band structures but also related quantities, including eigenmodes, defect states, displacement transmission curves, and full-field displacement distributions. When using commercial FEM

software, COMSOL Multiphysics® can handle complex-number arithmetic, enabling the direct solution of complex-coefficient eigenvalue equations. This capability has enabled broader application in this field than in other software [164, 165]. In this work, the numerical results are primarily obtained using COMSOL, with partial results further validated by the Carrera Unified Formulation and experimental tests.

The Carrera Unified Formulation (CUF) is a methodology that employs hierarchically refined approximations to generate solutions of arbitrary accuracy for general elastic problems. It was first proposed in 1995 by Professor Erasmo Carrera of the Politecnico di Torino, Italy [166]. In his paper [167], Professor Carrera provided a detailed exposition of the theoretical foundations of the unified formulation, which subsequently served as the basis for its innovative applications and numerous extensions. The formulation's characteristics enable the inclusion of thermal loads [168], and it has been extended to the analysis of piezoelectric structures [169-172] and functionally graded materials [173, 174], among others. Carrera et al. [175] presented a comprehensive CUF framework for one-dimensional, two-dimensional, and three-dimensional structures in a dedicated monograph. In recent years, CUF has been widely applied to analyze beams [176, 177], plates [178, 179], shells [180-184], composite structures [185-187], aerospace structures [188], rotor dynamics [189], civil engineering structures [190], thermal stress analysis [187], and nonlinear dynamics [191]. Trombini et al. [192] investigated crack propagation in composite materials using a CUF-based high-order one-dimensional finite element model. Filippi et al. [193] combined CUF with the Wave Finite Element Method for wave propagation analysis in one-dimensional waveguides of arbitrary cross-section. Cinefra et al. [193] extended CUF to the vibroacoustic analysis of liquid-filled composite plate-cavity systems. Miguel et al. [194] integrated Bloch-Floquet theory with a CUF beam finite element model to validate the bandgap properties of elastic metamaterials with simple unit-cell geometries reported in the literature. Kiasat et al. [195] conducted a theoretical study of the modal damping behavior of layered viscoelastic structures using CUF. In Chapter 3, CUF is extended to compute the natural frequencies, vibration modes, displacement transmission curves, and full-field displacements of antisymmetric dual helix metamaterial beams with complex cross-sections, where the results are systematically compared with COMSOL simulations and experimental data.

Additionally, to investigate bandgap formation and tuning mechanisms, Shen et al. [196] improved the traditional PWEM by developing a rasterization-based variant that integrates computer image processing techniques and applied it to calculate the band structure of two-dimensional phononic crystals. To design phononic crystals with desired bandgap manipulation properties, Li et al. [197] proposed an image-based finite element analysis coupled with a data-driven deep learning approach. Dong et al. [198] employed an energy method to study the bandgap characteristics of membrane-type resonator metamaterial plates. Liu et al. [199] combined the dynamic stiffness method with the Wittrick-Williams algorithm to develop a precise wave-propagation analysis technique. Wu et al. [200] introduced a modular machine-learning-based design concept for metamaterials.

Grounded in MST, Hu et al. [201] developed a Willis dynamic homogenization method that accounts for spatial dispersion and successfully applied it to acoustic metamaterial systems.

1.3.2 Experimental investigation and testing methods

In phononic crystal/elastic metamaterial research, experimental measurements are essential for validating theoretically and numerically predicted bandgaps. Experimental methods fall into two main categories: excitation techniques and signal acquisition approaches. Excitation is typically achieved using impact hammers, shakers, or a piezoelectric stack, while signals are commonly acquired with accelerometers, Fiber Bragg grating sensors, or laser Doppler vibrometers.

The impact hammer, also known as a force hammer or excitation hammer, is a tool used for modal analysis and vibration testing. It excites the test structure by applying a transient impact force and simultaneously measures the impact force and the structural response to analyze the structure's dynamic characteristics, such as natural frequencies, damping ratios, and mode shapes. Zhou et al. [63] used an impact hammer with a plastic tip to assess the modal and impact-damping performance of their proposed metamaterial. In the modal test, reaction forces on the surface plate were recorded using a multi-component dynamometer, and in the impact test, displacement at the target location was captured by a high-speed camera operating at 20,000 Hz. The impact hammer can also be used to measure the impact response function of metamaterials [202].

A shaker is commonly used to generate vibration signals. Based on the principle of excitation force generation, shakers can be classified as mechanical or electrodynamic. There are two primary installation methods for shakers: rigid mounting and elastic mounting. Regardless of the installation method, a shaker introduces additional mass, stiffness, and damping to the vibrating structure. Therefore, under the premise of meeting operational requirements, it is advisable to select shakers with lower power, lighter weight, and reduced stiffness. Shakers are widely used in fields such as modal analysis and vibration testing and are a frequently employed excitation method in experiments involving phononic crystals/elastic metamaterials. Rui et al. [78] employed a shaker-based experimental system to validate the vibration-absorption effectiveness of a multi-band elastic metamaterial designed for low-frequency multi-polarization vibration absorption. Zhao et al. [50] conducted experimental research on the low-frequency vibration suppression performance of lightweight embedded metastructures under shaker excitation.

Piezoelectric stack excitation is a technique that utilizes the inverse piezoelectric effect of piezoelectric materials to generate vibrations. When subjected to an electric field, piezoelectric materials undergo minute deformation, a phenomenon known as the inverse piezoelectric effect. A piezoelectric stack exciter is typically composed of multiple layers of piezoelectric ceramic plates. When an alternating voltage is applied to the stack, the piezoelectric plates undergo periodic expansion and contraction due to the inverse piezoelectric effect, thereby

producing mechanical vibrations. These vibrations are then transmitted to the connected structure to achieve excitation. This excitation method offers advantages such as high precision, rapid response, high resolution, and absence of electromagnetic interference. Moreover, piezoelectric stacks generally feature simple structures, small size, light weight, and ease of installation and integration into various systems. Based on multi-layer piezoelectric stack excitation, Chuang et al. [14] experimentally demonstrated that the group velocity of flexural waves slows down at defect states in phononic crystal beams with defects.

Accelerometers are common instruments for signal measurement. Operating on the principle of Newton's second law, they measure inertial forces to indirectly determine acceleration. Their core components typically include a proof mass (inertial mass), a spring, and a damping system. When the sensor accelerates with the measured object, the proof mass undergoes relative displacement due to inertia. The sensor calculates acceleration by detecting this displacement or the associated force. Accelerometer technologies commonly include piezoelectric, capacitive, thermal convection, and Micro Electro Mechanical Systems (MEMS) types. Fujita et al. [203] drove a piezoelectric disk for excitation and used accelerometers to obtain the response spectra of a metamaterial beam designed to suppress coupled compression, shear, bending, and torsional modes. Zhang et al. [204] experimentally validated the force-acceleration transmission spectra of a metamaterial with curved-beam rotational symmetric lattices using an impact hammer and accelerometers.

Fiber Bragg Grating (FBG) displacement sensing systems detect displacement or strain by measuring shifts in the reflected wavelength of the fiber grating and have increasingly been applied to phononic crystals/elastic metamaterials in recent years. Chuang et al. [205] measured the point transient displacement response of a cantilevered phononic crystal beam impacted by a steel ball, thereby validating a self-demodulated FBG system for linear displacement sensing. This system was subsequently used to obtain the frequency response functions of a tunable metamaterial beam with planar curved shape memory alloy resonators [88] and a coupled compression-torsion origami resonator metamaterial beam [206]. Compared to traditional measurement methods, FBG sensing systems offer advantages such as immunity to electromagnetic interference, small size, and light weight. However, they also have drawbacks, including high costs for matching sensors and demodulation equipment, complex installation, and limited dynamic range.

The Laser Doppler Vibrometer (LDV) is a widely used instrument for measuring signals in metamaterial experiments. It precisely calculates the vibration velocity and displacement of an object's surface by detecting the frequency shift of a reflected laser beam. Currently, it is recognized as the vibration measurement method offering the best displacement and velocity resolution and has been extensively applied in fundamental scientific research [207-212]. Doppler vibrometry systems introduce no additional mass and offer excellent non-invasiveness, enabling measurements on extremely small and lightweight structures, such as spinning fibers [213]. Zouari et al. [4] experimentally validated the

vibration suppression of flexural wave bandgaps in a metamaterial plate by employing directional plane waves and point force excitation for stimulation and an LDV for signal measurement. The frequency range of the observed vibration attenuation matched the theoretically predicted directional bandgap for flexural waves. Zhao et al. [69] used a shaker for excitation and an LDV to measure and record the velocity field along arbitrary points, experimentally demonstrating broadband attenuation with nonlinear bandgaps and modal dissipation. In the experiment, the nonlinear metamaterial was vertically suspended, with one end clamped to a shaker to facilitate base excitation and create clamped-free boundary conditions; data were acquired by repeatedly collecting time-domain signals. Zhang et al. [96] experimentally tested the displacement transmission curves of an inversely designed flexural wave metamaterial beam with thermally induced tunability using an LDV.

It is worth noting that in the experiments presented in this work, a piezoelectric stack was consistently used as the excitation method, while vibration signals were acquired with a laser Doppler vibrometer. A detailed description of this experimental system is provided in Chapter 2.

1.4 Research objectives and thesis outline

1.4.1 Research objectives

As previously mentioned, elastic metamaterials possess exceptional mechanical properties not found in natural materials, offering significant potential for applications in vibration and noise reduction, acoustic functional devices, acoustic stealth, impact mitigation, energy harvesting, and fault diagnosis. In these applications, elastic metamaterials typically need to exhibit bandgap characteristics at lower frequencies and over broader bandwidths. However, designing metamaterial structures with low-frequency broadband bandgaps remains a major challenge in the current field of elastic metamaterials. Furthermore, reports on the active tuning of low-frequency elastic wave bandgaps remain relatively scarce, and achieving large-range tuning of elastic wave bandgaps in the low-frequency domain is generally difficult. Therefore, broadening the width of low-frequency bandgaps, enhancing the attenuation of elastic waves within the bandgap, and actively controlling the frequency range of low-frequency bandgaps are all pressing issues that must be addressed to expand the practical applications of metamaterials.

On the other hand, a wide variety of natural spiral structures exist, from the DNA within cells to galaxies in the cosmos. The regularity of these beautiful forms stands in stark contrast to the surrounding chaotic world. For example, sunflower seeds arrange themselves in spirals following strict mathematical rules (the Fibonacci sequence); the nautilus shell approximates a large logarithmic spiral; hurricanes organize into immense spirals hundreds of miles in diameter; and the tendrils of climbing plants exhibit cylindrical helices. The exceptional mechanical and wave manipulation properties of spiral structures fundamentally stem from their sophisticated geometric configurations. Through unique spatial pathway

design, they can amplify inherent material performance, achieving an excellent balance among strength, toughness, compliance, and compactness. Their core geometric advantage lies in the efficient coupling and mutual conversion between linear translation and rotational motion. They can transform axial force or displacement into bending or torsion around a central axis, and vice versa. This inherent motion-conversion capability allows them to reconcile stiffness with flexibility in mechanics and to precisely control wave modes and energy transport in wave propagation. Ultimately, this contributes to their remarkable performance across scales, from the microscopic to the macroscopic. Inspired by prior research and the geometric forms of natural spiral structures, this thesis aims to design several novel helix metamaterial configurations capable of achieving broadband, low-frequency, and tunable bandgaps. Furthermore, combining commercial finite element software, the CUF, and a laser vibrometry system, the proposed structures will undergo in-depth theoretical and experimental investigation into the mechanisms of elastic wave bandgap formation and manipulation.

1.4.2 Thesis structure

This study employs traditional FEM, the CUF method, and laser vibrometry-based experimental techniques as research tools. By integrating theoretical calculations with experimental validation, a comprehensive and systematic investigation of the elastic wave bandgap properties of various bio-inspired metamaterials featuring spiral unit cells is conducted. The aim is to provide novel insights and robust theoretical and experimental support for achieving broadband low-frequency bandgaps, tunable bandgaps, and strong vibration-attenuation bandgaps, all of which are required for potential engineering applications. The dissertation is organized into seven chapters, with the research content of each chapter summarized as follows:

Chapter 1: Introduction. This chapter introduces the concept, application prospects, and current research status of elastic metamaterials. It reviews the literature on the key issues addressed in this work: broadband low-frequency bandgaps, tunable bandgaps, and bio-inspired metastructure design. Furthermore, it summarizes existing theoretical methods for bandgap calculation and experimental techniques. Finally, the chapter presents the research objectives and outlines the dissertation's content.

Chapter 2: Research methods for bandgap characteristics of helix metamaterials. This chapter introduces the theoretical framework of the CUF and illustrates its application to structural analysis through detailed examples. It presents the theoretical foundation for calculating band structures using the commercial finite element software COMSOL. Furthermore, it explains the basic principles and experimental setup of the laser vibrometry system and describes the fundamental procedures for acquiring experimental data on the displacement transmission curves of finite metamaterial structures.

Chapter 3: Flexural wave bandgap characteristics of 1D dual helix metamaterial beams. This chapter proposes an antisymmetric dual helix

metamaterial beam and a variant incorporating concentrated mass spheres. The band structures, displacement transmission curves, vibration modes, and full-field displacement distributions of both beams are obtained through finite element calculations. The influence of parameters such as the number of helical turns, the radius of the attached mass sphere, and the unit-cell material properties on the flexural wave bandgap characteristics of both metamaterial beams is discussed. Furthermore, the CUF is applied to compute the natural frequencies of flexural vibration, vibration modes, displacement transmission curves, and full-field displacements for a rectangular cross-section beam, as well as for the two proposed metamaterial beams. An experimental platform based on a laser vibrometry system is designed and constructed to experimentally validate the displacement transmission characteristics curves obtained from both COMSOL simulations and CUF analyses.

Chapter 4: Flexural wave bandgap characteristics of 2D dual helix metamaterial plates. This chapter analyses the band structures, displacement transmission curves, band-edge vibration modes, and full-field displacement distributions of dual helix metamaterial plates, both with and without attached mass spheres. To explore the advantages of the dual helix configuration in forming low-frequency bandgaps, the band structures and edge modes of various metamaterial plates are analysed and compared, including those with square, circular, four-point square, four-point circular, and four-point dual helix unit cells. A detailed discussion of the influence of key parameters, such as the number of helical turns, helix width, helical orientation, plate thickness, and the radius of the mass spheres, on the formation and tuning of flexural wave bandgaps is provided.

Chapter 5: Longitudinal wave bandgap characteristics of DNA double helix metamaterials. This chapter proposes a double helix metamaterial structure inspired by B-form DNA, designed to exhibit tunable longitudinal-wave bandgap characteristics. Using COMSOL, a systematic study is conducted of the longitudinal wave bandgaps of double helix metamaterials with single-mass blocks and randomly arranged mass blocks, yielding their band structures, displacement transmission curves, vibration modes, and full-field displacement distributions. Taking a structure with attached nylon as an example, the influence of the hollow cylindrical end plate thickness and the minor diameter of the helix on the first longitudinal bandgap is investigated. Using a structure with attached steel as an example, the effect of the mass block length on the first longitudinal bandgap is studied. Finally, the displacement transmission curves of the various structures obtained from numerical simulations are experimentally validated using the laser vibrometry platform.

Chapter 6: Double helix metamaterials with ultra-broadband and ultra-low-frequency longitudinal wave attenuation. This chapter proposes a nodal-mass DNA double helix metamaterial exhibiting ultra-broadband, ultra-low-frequency longitudinal-wave bandgap properties. A comprehensive study is conducted on the band structures, band-edge modes, displacement transmission curves, and full-field displacement distributions of the metamaterial both without nodal mass disks and with nodal mass disks made of three different materials: steel,

aluminum, and copper. The influences of key parameters, namely, the minor diameter of the helix, the thickness of the mass disk, and the axial pitch of the helix, on the first complete bandgap and the first longitudinal bandgap are discussed. Finally, the displacement transmission characteristics of finite structures of the nodal-mass DNA double helix metamaterial under different configurations, as obtained from finite-element simulations, are experimentally validated.

Chapter 7: Conclusions and future work. This chapter summarizes the work presented in this dissertation and outlines directions for future research.

Chapter 2

Research methods for bandgap characteristics of helix metamaterials

2.1 Introduction

The essence of elastic metamaterials lies in their heterogeneous periodic structure, which requires precise and efficient computational tools for investigation. In engineering and scientific research, the FEM is widely adopted for its ability to efficiently predict the dynamic behavior of complex structures with recognized accuracy and economy. Implemented in commercial software such as ANSYS, ABAQUS, and COMSOL, FEM is routinely used to calculate band structures and displacement transmission curves of phononic crystals and elastic metamaterials. An alternative is the CUF, a methodology that employs hierarchically refined approximations to achieve solutions of arbitrary accuracy for general elastic problems. Numerous studies have demonstrated that CUF offers lower computational cost than traditional FEM while maintaining high accuracy.

On the other hand, experiments provide essential validation for theoretical calculations and serve as a crucial bridge between theory and practical application. In the sensing and measurement of mechanical, civil, and other engineering structures, the predominant measurement tools are mechanical or electronic sensors. However, electronic sensors are susceptible to electromagnetic interference, while mechanical sensors are prone to damage. The LDV experimental method achieves nanometer- and picometer-level amplitude resolution, offers high linearity, and maintains amplitude consistency even at extremely high frequencies. Utilizing coherent demodulation technology, its measurement effectiveness is unaffected by working distance. Compared to traditional vibration sensors, LDV offers significant advantages: it is non-contact, operates over long distances, provides extremely high precision and sensitivity, exhibits high frequency response, is suitable for high-temperature environments and various material types, and is immune to electromagnetic interference.

To investigate helix metamaterials with complex cross-sectional shapes, this study primarily employs the FEM, using COMSOL to compute key characteristics of the designed metamaterials, including their band structures, displacement transmission curves, wave modes, and full-field displacement distributions. To extend the application of the CUF to the study of bandgap properties in

metamaterials with intricate geometries, this work will, for the first time, utilize CUF to compute the natural frequencies, vibration modes, displacement transmission curves, and full-field displacements of antisymmetric dual helix metamaterial beams, both with and without attached mass spheres. All experimental validation is conducted on a self-constructed laser vibrometry platform. Therefore, this chapter will sequentially elaborate on the theoretical framework and numerical implementation of CUF, the methodology for calculating bandgap characteristics using COMSOL, and the working principles and platform construction for the laser vibrometry experiments.

2.2 Carrera Unified Formulation

2.2.1 Fundamental theoretical framework of CUF

Using the CUF, the three-dimensional displacement field of a structure can be represented in three-dimensional, two-dimensional, and one-dimensional forms, as shown in Figure 2.1. Based on the CUF beam model theory, the three-dimensional displacement field can be expanded as [176]:

$$\mathbf{u}(x, y, z) = F_m(x, z)\mathbf{u}_m(y), m = 1, 2, \dots, M \quad (2.1)$$

where $\mathbf{u}$, F_m , $\mathbf{u}_m$, and M denote the displacement vector, cross-sectional expansion function, generalized displacement vector, and number of expansion terms, respectively. Repeated subscript m implies summation. Here, both the cross-sectional expansion function F_m and the number of terms M are arbitrary. This allows the kinematic field of the structure to be represented using different basis functions to any desired order, a notable advantage of CUF.

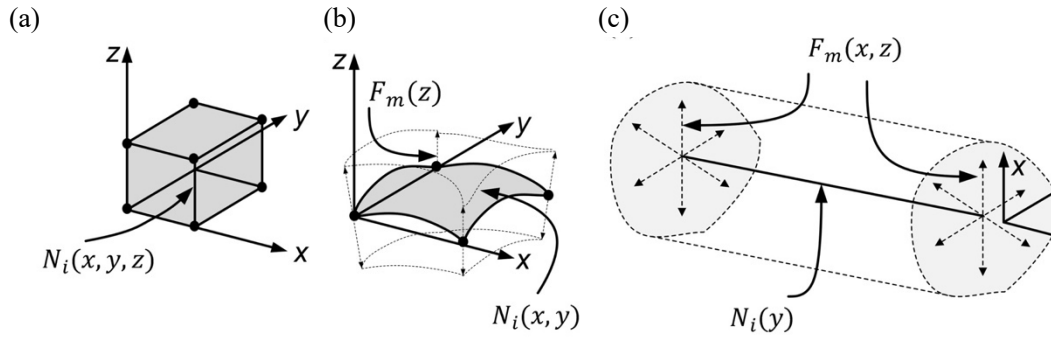


Figure 2.1 Structural models based on CUF theory: (a) 3D model; (b) 2D model; (c) 1D model.

In this work, Lagrange Expansions (LE) and Taylor Expansions (TE) are adopted as the cross-sectional expansion functions. The Lagrange model employs Lagrange-type polynomials to expand the generalized displacements over the beam cross-section, while the Taylor model utilizes Taylor-type polynomials for this expansion. The generalized coordinates of an LE element consist solely of

displacements, whereas those of a TE model also include rotations and higher-order terms. The Taylor expansion enriches the displacement field in a spectral sense simply by increasing the polynomial order. For instance, a second-order Taylor expansion can be expressed as:

$$\begin{aligned} u_x &= u_{x_1} + x u_{x_2} + z u_{x_3} + x^2 u_{x_4} + xz u_{x_5} + z^2 u_{x_6} \\ u_y &= u_{y_1} + x u_{y_2} + z u_{y_3} + x^2 u_{y_4} + xz u_{y_5} + z^2 u_{y_6} \\ u_z &= u_{z_1} + x u_{z_2} + z u_{z_3} + x^2 u_{z_4} + xz u_{z_5} + z^2 u_{z_6} \end{aligned} \quad (2.2)$$

For higher-order Taylor expansions, see Table 2.1.

Table 2.1 Taylor-like polynomials

N	M	F_m
0	1	$F_1 = 1$
1	3	$F_2 = x, F_3 = z$
2	6	$F_4 = x^2, F_5 = xz, F_6 = z^2$
3	10	$F_7 = x^3, F_8 = x^2z, F_9 = xz^2, F_{10} = z^3$
$\vdots$	$\vdots$	$\vdots$
N	$(N+1)(N+2)/2$	$F_{(N^2+N+2)/2} = x^N \cdots F_{(N+1)(N+2)/2} = z^N$

The mathematical model of a one-dimensional beam within the CUF framework is illustrated in Figure 2.1(c). This chapter will perform calculations using beam elements with 2 nodes (B2), 3 nodes (B3), and 4 nodes (B4). One form of their shape functions is given by:

$$\begin{aligned} N_1 &= \frac{1}{2}(1-r), N_2 = \frac{1}{2}(1+r) \\ N_1 &= \frac{1}{2}r(1-r), N_2 = \frac{1}{2}r(1+r), N_3 = -(1+r)(1-r) \\ N_1 &= -\frac{9}{16}\left(r + \frac{1}{3}\right)\left(r - \frac{1}{3}\right)(r-1), \quad N_2 = \frac{9}{16}\left(r + \frac{1}{3}\right)\left(r - \frac{1}{3}\right)(r+1) \\ N_3 &= +\frac{27}{16}(r+1)\left(r - \frac{1}{3}\right)(r-1), \quad N_4 = -\frac{27}{16}(r+1)\left(r + \frac{1}{3}\right)(r-1) \end{aligned} \quad (2.3)$$

Here, the natural coordinate (r) varies from -1 to +1. For a 2-node beam element: $r_1 = 1, r_2 = -1$. For a 3-node beam element: $r_1 = 1, r_2 = -1, r_3 = 1$. For a 4-node beam element: $r_1 = -1, r_2 = 1, r_3 = 1/3, r_4 = -1/3$.

The order of the beam model is defined by the expansion over its cross-section. The number of nodes per element is related to the approximation along the longitudinal axis. Therefore, an N th-order beam model is a theory that employs an N th-order Taylor polynomial to describe the displacement field across the cross-section. The choice of beam model order, beam element type, and mesh density (i.e., the number of beam elements) collectively determines the total number of degrees of freedom in the structural model.

According to the LE model, the cross-sectional geometry is subdivided into several sub-regions, upon which two-dimensional Lagrange functions are defined. Within the CUF framework, Lagrange expansion models have been implemented, including the 4-node bilinear element (LE4), the 9-node biquadratic element (LE9), and the 16-node bicubic element (LE16). One form of the interpolation functions for the 4-node bilinear element is given by:

$$F_m = \frac{1}{4}(1 + r r_m)(1 + s s_m), m = 1,2,3,4 \quad (2.4)$$

One form of the interpolation functions for the 9-node element (LE9) is given by:

$$\begin{aligned} F_m &= \frac{1}{4}(r^2 + r r_m)(s^2 + s s_m), m = 1,3,5,7 \\ F_m &= \frac{1}{2} s_m^2 (s^2 + s s_m)(1 - r^2) + \frac{1}{2} r_m^2 (r^2 + r r_m)(1 - s^2), m = 2,4,6,8 \\ F_m &= (1 - r^2)(1 - s^2), m = 9 \end{aligned} \quad (2.5)$$

where r and s are the normalized coordinates, and r_m and s_m are the actual coordinates of the nine nodes. The relationship between the actual geometric coordinates and the normalized coordinates is illustrated in Figure 2.2.

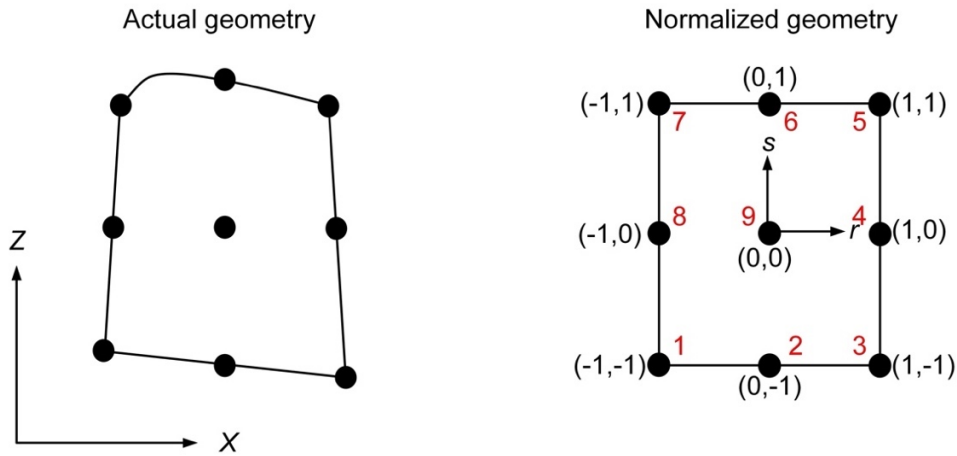


Figure 2.2 Nine-node Lagrange element (LE 9) in actual and normalized coordinates.

The displacement field of the LE9 element can be expressed as:

$$\begin{aligned} u_x(x, y, z) &= F_1(x, z)u_{x_1}(y) + F_2(x, z)u_{x_2}(y) + \cdots + F_9(x, z)u_{x_9}(y) \\ u_y(x, y, z) &= F_1(x, z)u_{y_1}(y) + F_2(x, z)u_{y_2}(y) + \cdots + F_9(x, z)u_{y_9}(y) \\ u_z(x, y, z) &= F_1(x, z)u_{z_1}(y) + F_2(x, z)u_{z_2}(y) + \cdots + F_9(x, z)u_{z_9}(y) \end{aligned} \quad (2.6)$$

The generalized displacement $\mathbf{u}_m$ in Equation (2.1) can be discretized as:

$$\mathbf{u}_m(y) = N_i(y)\mathbf{u}_{mi} \quad m = 1, 2, \dots, M, i = 1, 2, \dots, p + 1 \quad (2.7)$$

here, the vector $\mathbf{u}_{mi}$ represents the generalized displacement at the nodal positions. $N_i(y)$ is the finite element shape function, used to interpolate the displacement variables along the beam's longitudinal y -axis. The order p of the shape functions is determined by the number of structural nodes.

The stress and strain tensors in Voigt notation are:

$$\begin{aligned} \boldsymbol{\sigma} &= \{\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \tau_{yz} \quad \tau_{xz} \quad \tau_{xy}\}^T \\ \boldsymbol{\varepsilon} &= \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{zz} \quad \gamma_{yz} \quad \gamma_{xz} \quad \gamma_{xy}\}^T \end{aligned} \quad (2.8)$$

The relationship between strain and displacement is given by:

$$\boldsymbol{\varepsilon} = \mathbf{D}\mathbf{u} \quad (2.9)$$

Here, the linear differential operator $\mathbf{D}$ consists of two parts:

$$\mathbf{D} = \mathbf{D}_{xz} + \mathbf{D}_y \quad (2.10)$$

$$\mathbf{D}_{xz} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial x} & 0 \end{bmatrix}, \quad \mathbf{D}_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & 0 & 0 \\ \frac{\partial}{\partial y} & 0 & 0 \end{bmatrix} \quad (2.11)$$

The constitutive relationship can be written as:

$$\boldsymbol{\sigma} = \mathbf{C} \boldsymbol{\varepsilon} \quad (2.12)$$

Here, $\mathbf{C}$ is the material constant matrix. For isotropic materials, the elasticity tensor is:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} \quad (2.13)$$

Each of its components can be determined by the Lamé constants λ , Young's modulus E , and Poisson's ratio ν , where:

$$c_{11} = 2G + \lambda, c_{12} = \lambda, c_{44} = G, G = \frac{E}{2(1 + \nu)}, \lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \quad (2.14)$$

Applying the principle of virtual displacements yields the equations of motion. According to the principle of virtual displacements:

$$\delta L_{\text{int}} + \delta L_{\text{ine}} = \delta L_{\text{ext}} \quad (2.15)$$

where L_{int} is the internal elastic work, L_{ine} is the inertial work, L_{ext} represents the external virtual work, and δ denotes the variational operator. According to the unified formulation theory, the internal elastic work and the inertial work can be expressed as:

$$\delta L_{\text{int}} = \int_V \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV = \delta \mathbf{u}_{nj}^T \mathbf{K}^{mnij} \mathbf{u}_{mi} \quad (2.16)$$

$$\delta L_{\text{ine}} = \int_V \delta \mathbf{u} \rho \ddot{\mathbf{u}} dV = \delta \mathbf{u}_{nj}^T \mathbf{M}^{mnij} \ddot{\mathbf{u}}_{mi} \quad (2.17)$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are the strain and stress vectors, respectively, ρ is the material density, and $\ddot{\mathbf{u}}$ is the second-order time derivative of displacement; $\mathbf{K}^{mnij}$ and $\mathbf{M}^{mnij}$ are the stiffness and mass matrices, respectively, known as the Fundamental Nucleus (FN). Here, m and n are indices of the expansion functions, while i and j are indices of the shape functions [176]. As illustrated in Figure 2.3, the global mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ can be assembled by cycling through the indices m , n , i , and j .

The Fundamental Nucleus $\mathbf{K}^{mnij}$ is a 3×3 matrix.

$$\mathbf{K}^{mnij} = \begin{bmatrix} k_{xx}^{mnij} & k_{xy}^{mnij} & k_{xz}^{mnij} \\ k_{yx}^{mnij} & k_{yy}^{mnij} & k_{yz}^{mnij} \\ k_{zx}^{mnij} & k_{zy}^{mnij} & k_{zz}^{mnij} \end{bmatrix} \quad (2.18)$$

When the material under study is isotropic, the nine components of $\mathbf{K}^{mnij}$ are as follows:

$$\begin{aligned} k_{xx}^{mnij} &= c_{22} \int_l N_i N_j dy \int_A F_{m,x} F_{n,x} dA + c_{66} \int_l N_i N_j dy \int_A F_{m,z} F_{n,z} dz + \\ &\quad c_{44} \int_l N_{i,y} N_{j,y} dy \int_A F_m F_n dA \\ k_{xy}^{mnij} &= c_{23} \int_l N_{i,y} N_j dy \int_A F_m F_{n,x} dA + c_{44} \int_l N_i N_{j,y} dy \int_A F_{m,x} F_n dA \end{aligned}$$

$$\begin{aligned}
k_{xz}^{mnij} &= c_{12} \int_l N_i N_j dy \int_A F_{m,z} F_{n,x} dA + c_{66} \int_l N_i N_j dy \int_A F_{m,x} F_{n,z} dA \\
k_{yx}^{mnij} &= c_{44} \int_l N_{i,y} N_j dy \int_A F_m F_{n,x} dA + c_{23} \int_l N_i N_{j,y} dy \int_A F_{m,x} F_n dA \\
k_{yy}^{mnij} &= c_{55} \int_l N_i N_j dy \int_A F_{m,z} F_{n,z} dA + c_{44} \int_l N_i N_j dy \int_A F_{m,x} F_{n,x} dA + \\
&\quad c_{33} \int_l N_{i,y} N_{j,y} dy \int_A F_m F_n dA \\
k_{yz}^{m\bar{n}ij} &= c_{55} \int_l N_{i,y} N_j dy \int_A F_m F_{n,z} dA + c_{13} \int_l N_i N_{j,y} dy \int_A F_{m,z} F_n dA \quad (2.19) \\
k_{zx}^{mnij} &= c_{12} \int_l N_i N_j dy \int_A F_{m,x} F_{n,z} dA + c_{66} \int_l N_i N_j dA \int_A F_{m,z} F_{n,x} dz \\
k_{zy}^{mnij} &= c_{13} \int_l N_{i,y} N_j dy \int_A F_m F_{n,z} dA + c_{55} \int_l N_i N_{j,y} dy \int_A F_{m,x} F_n dA \\
k_{zz}^{mnij} &= c_{11} \int_l N_i N_j dy \int_A F_{m,z} F_{n,z} dA + c_{66} \int_l N_i N_j dy \int_A F_{m,x} F_{n,x} dA + \\
&\quad c_{55} \int_l N_{i,y} N_{j,y} dy \int_A F_m F_n dA
\end{aligned}$$

Similarly, the virtual work done by external forces can be expressed as:

$$\delta L_{\text{ext}} = \int_V \delta \mathbf{u}^T \mathbf{g} dV + \int_S \delta \mathbf{u}^T \mathbf{p} dS + \int_L \delta \mathbf{u}^T \mathbf{q} dy + \sum_l \delta \mathbf{u}^T \mathbf{P} \quad (2.20)$$

where $\mathbf{g}$, $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{P}$ denote body forces, surface tractions, line forces, and concentrated loads, respectively. Consequently, the differential equation of motion for the forced vibration of the structure can be written as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \check{\mathbf{P}} \quad (2.21)$$

where $\check{\mathbf{P}}$ is the sum of external forces and can be expressed in the following form:

$$\check{\mathbf{P}} = \int_V N_j \mathbf{g} dV + \int_S N_j \mathbf{p} dS + \int_L N_j \mathbf{q} dy + N_j \mathbf{P} \quad (2.22)$$

The differential equation for free vibration is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \quad (2.23)$$

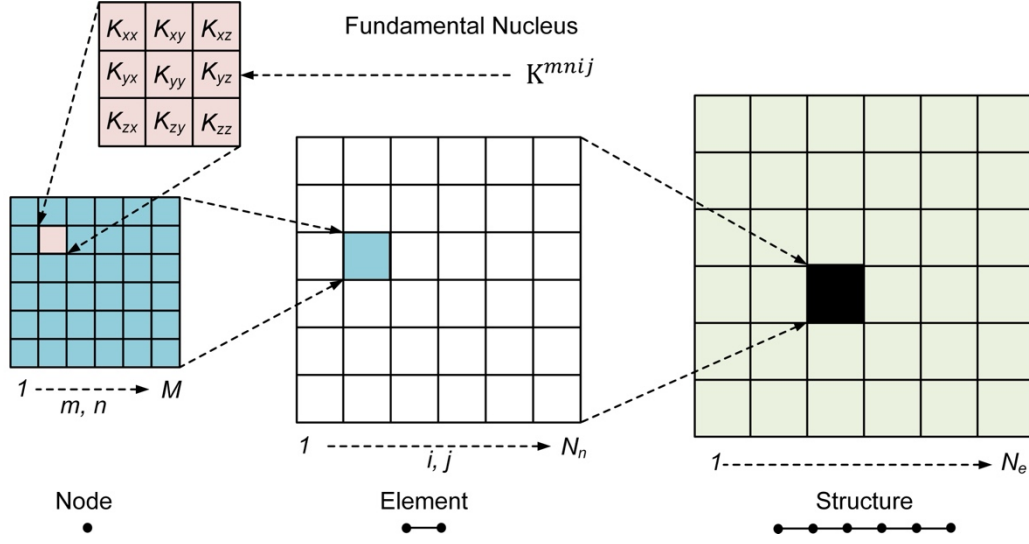


Figure 2.3 Stiffness matrix assembly procedure in CUF via Fundamental Nucleus.

Applying the harmonic assumption for displacement, $\mathbf{u} = \mathbf{u}_0 e^{i\omega t}$ (where $i = \sqrt{-1}$), and substituting it into Equation (2.23), to obtain a non-trivial solution, the corresponding determinant must be zero:

$$\det(-\omega^2 \mathbf{M} + \mathbf{K}) = 0 \quad (2.24)$$

This leads to determining the eigenfrequencies ω and the corresponding mode shapes $\mathbf{u}_0$. In solving various structural problems, CUF is implemented using the MUL2 (Multilayered Structures and Multifield Analyses, MUL2) program, a finite element analysis code developed from CUF. The next subsection will provide a detailed introduction, with practical examples, to applying CUF to determine the natural frequencies of structures.

2.2.2 CUF-based FE modelling and case studies

The MUL2 program, built upon the CUF theoretical framework, consists of the following input and configuration files: PATH_input.dat, VERSORS.dat, ANALYSIS.dat, NODES.dat, CONNECTIVITY.dat, MATERIAL.dat, LAMINATION.dat, EXP_MESH_NN.dat, EXP_CONN_NN.dat, BC.dat, and POST.dat.

PATH_input.dat: Specifies the directory path containing the model input files.

ANALYSIS.dat: Defines the type of analysis to be performed. For this study, the relevant codes are 103 for free vibration analysis and 106 for frequency response analysis.

NODES.dat: Contains the nodal coordinates along the axial direction (the y -coordinate for beam models).

EXP_MESH_NN.dat: Specifies the nodal coordinates on the cross-section (x and z coordinates for beam models). For beams with variable cross-sections,

multiple files (EXP_MESH_01.dat, EXP_MESH_02.dat, etc.) are required, where NN denotes the cross-section number.

EXP_CONN_NN.dat: Defines the connectivity between elements within each cross-section.

CONNECTIVITY.dat: Specifies the connectivity of beam elements along the axial direction.

MATERIAL.dat: Contains the material properties used in the model.

LAMINATION.dat: Defines the structural layup. For multilayer structures, information for each layer must be included.

BC.dat: Specifies the boundary conditions for the analysis.

POST.dat: Provides post-processing settings, such as node locations for output and the post-processing software to use.

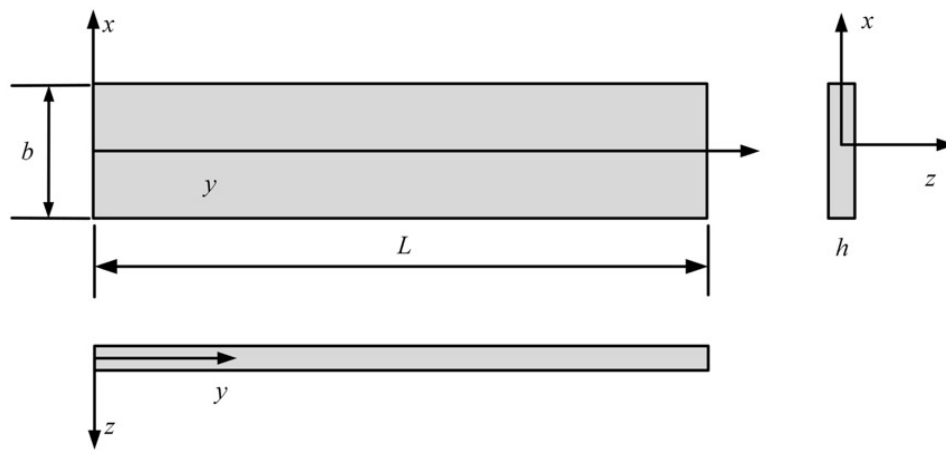


Figure 2.4 Coordinate system of finite element modeling of rectangular section beam based on the CUF frame.

Figure 2.4 illustrates a simple straight beam with free-free boundary conditions, modeled within the CUF framework. The following parameters are used for calculating its natural frequencies: length $L = 1.19$ m, width $b = 0.07$ m, height $h = 0.006$ m, Young's modulus $E = 72$ GPa, Poisson's ratio $\nu = 0.33$, and density $\rho = 2700$ kg/m³.

Table 2.2 First natural frequency error of the beam for different numbers of B4 elements.

Model	First-order natural frequency	Exact solution of the Euler-Bernoulli beam	Error (%)
5B4(TE-1)	22.489	22.491	0.0008
10B4(TE-1)	22.489	22.491	0.0008
12B4(TE-1)	22.496	22.491	0.02

A mesh convergence study is conducted by analyzing the required element types and their convergence behavior. Different numbers of beam elements, such as B4 elements, are tested to observe variations in the first natural frequency. Keeping the cross-sectional expansion (TE expansion) and the number of elements constant, for instance, using three Q9 elements (i.e., three 9-node elements), the discretization in both the cross-sectional and y-axial directions is illustrated in Figure 2.5. The corresponding computational results are presented in Table 2.2.

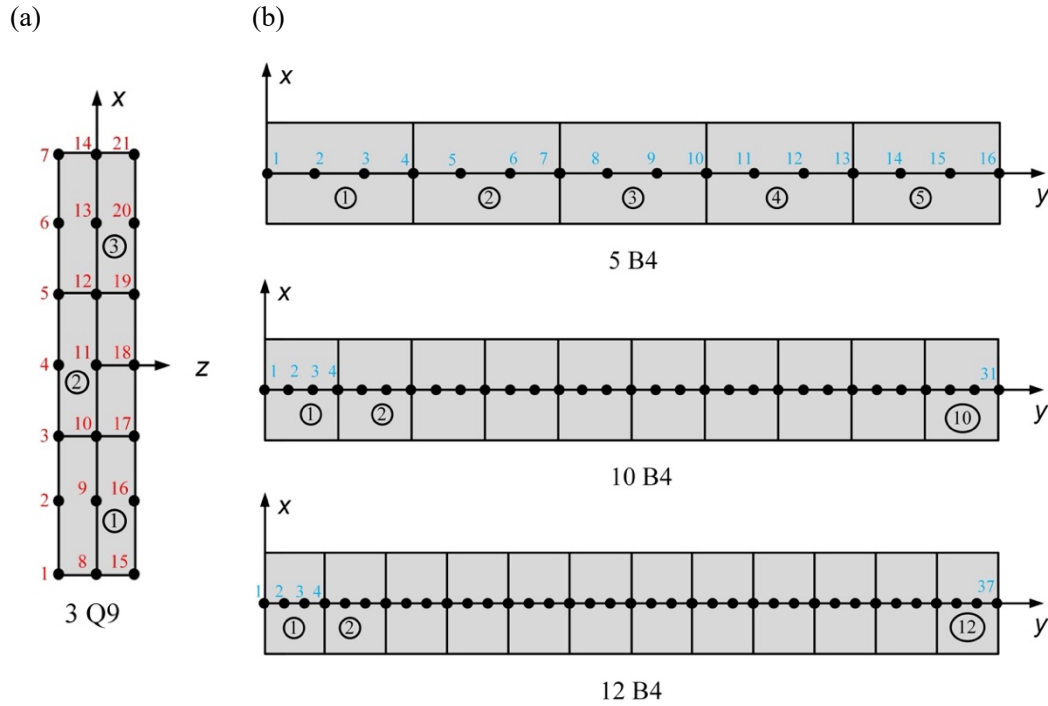


Figure 2.5 Discretization of a rectangular cross-section beam: (a) cross-section meshed with three 9-node elements; (b) y-axial direction discretized with varying numbers of B4 beam elements.

Keeping the number of B4 beam elements along the y-axis constant at 10, different numbers of Q9 elements are employed for expansion over the cross-section. The resulting structural discretization is shown in Figure 2.6, and the computational results are presented in Table 2.3.

Table 2.3 First natural frequency error for different numbers of Q9 elements in the cross-sectional direction

Model	First-order natural frequency	Exact solution of the Euler-Bernoulli beam	Error (%)
1Q9(TE-1)	22.48886415	22.491	0.0008
2Q9(TE-1)	22.48886415	22.491	0.0008
3Q9(TE-1)	22.48886415	22.491	0.0008
4Q9(TE-1)	22.48886415	22.491	0.0008

It can be observed that using one 9-node element in the cross-sectional direction with a first-order Taylor expansion (i.e., the Euler-Bernoulli beam model) and 10 B4 beam elements in the y -axial direction provides sufficient accuracy for calculating the first natural frequency. Therefore, this model configuration can be selected for computing the first natural frequency and mode shape. To obtain higher-order frequency solutions that meet the required accuracy, it is necessary to explore different models, beam element types, and cross-sectional expansion functions.

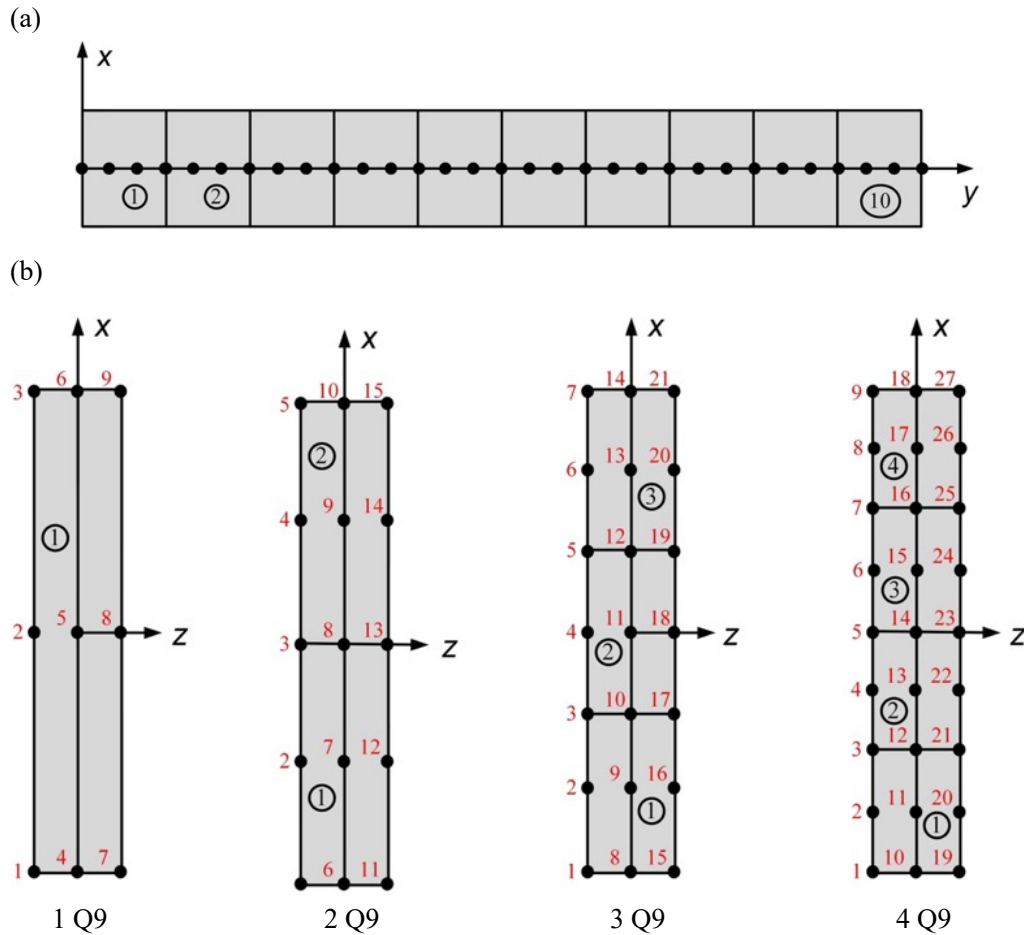


Figure 2.6 Discretization of a rectangular cross-section beam: (a) y -axial direction discretized with a fixed number of B4 elements (10B4); (b) cross-section meshed with a varying number of 9-node elements.

Table 2.4 presents the first eight natural frequencies obtained using the configuration of 1 Q9 and 10 B4 elements with different cross-sectional expansion functions. Rows 1 to 4 correspond to the use of minus first-order, zeroth-order, first-order, and second-order Taylor expansion functions, respectively. Row 5 employs a Lagrange expansion function. Row 6 shows the results from a COMSOL three-dimensional model, while the last row provides the exact solution calculated using Euler-Bernoulli Beam Theory (EBBT). In Table 2.4, the first column lists the computational models used, the second column indicates the number of Degrees of Freedom (DOF), and columns 3-10 present the results for the first eight frequencies.

As shown in the table, the first eight natural frequencies obtained with the 1LE9-10B4 model in the CUF framework are in close agreement with those from the three-dimensional COMSOL model.

Table 2.4 Natural frequencies of the flexural modes of the straight beam

Model	DOF	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
TE-1-10B4	279	22.489	61.989	121.52	200.87	300.10	419.34	558.84	719.09
TE0-10B4	279	22.489	61.981	121.48	200.77	299.88	418.89	558.04	717.75
TE1-10B4	279	22.489	61.981	121.48	200.77	299.88	418.89	558.04	717.75
TE2-10B4	558	22.530	62.366	122.90	204.26	306.71	430.45	575.75	743.05
1LE9-10B4	837	22.495	62.044	121.74	201.49	301.48	421.95	563.29	726.12
COMSOL	9492	22.499	62.082	121.89	201.91	302.35	423.46	565.47	728.61
EBBT	/	22.491	62.00	121.54	200.91	300.12	419.18	558.08	716.82

The detailed application of the CUF method for determining vibration modes, displacement transmission curves, and full-field displacements for both simple cross-section beams and complex cross-section antisymmetric dual helix metamaterial beams will be thoroughly presented in Chapter 3. Therefore, this subsection is limited to elucidating the procedure for calculating the natural frequencies of simple cross-section beams.

2.3 COMSOL simulations

Given that the majority of the research presented in this work is conducted using COMSOL, the following section introduces the theoretical foundation for calculating band structures with COMSOL.

2.3.1 Band structure calculation for infinite periodic structures

Due to the structural complexity of helix-based metamaterials, employing COMSOL to solve such problems is a highly suitable approach. The governing equations for elastic waves in helix metamaterial structures can be written as [51]:

$$\frac{E}{2(1+\nu)} \nabla^2 \mathbf{u} + \frac{E}{2(1+\nu)(1-\nu)} \nabla(\nabla \cdot \mathbf{u}) + \rho \omega^2 \mathbf{u} = \mathbf{0} \quad (2.25)$$

where $\mathbf{u}$ denotes the displacement vector, E is Young's modulus, ν is Poisson's ratio, ρ represents the density, and ω signifies the angular frequency. On the four sides of the unit cell, along the x - and y -axis directions respectively, Floquet-Bloch periodic boundary conditions are applied according to Bloch's theorem. Assuming a Bloch-wave solution of the form:

$$\mathbf{u}(\mathbf{r}, t) = \hat{\mathbf{u}}_{\mathbf{k}}(\mathbf{r})e^{i(\mathbf{k}\mathbf{r}-\omega t)} \quad (2.26)$$

where $\mathbf{k}$ is the wave vector and $\hat{\mathbf{u}}_{\mathbf{k}}(\mathbf{r})$ represents the amplitude of the lattice-periodic displacement field. According to the Floquet-Bloch theorem, the displacement field must satisfy the Bloch periodicity condition, namely:

$$\mathbf{u}(\mathbf{r} + \mathbf{a}, t) = e^{i\mathbf{k}\mathbf{a}}\mathbf{u}(\mathbf{r}, t) \quad (2.27)$$

where $\mathbf{r}$ is the position vector, and $\mathbf{a}$ is the lattice vector. Combining the boundary conditions with the discretized governing equations yields the following eigenvalue equation:

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{U} = \mathbf{0} \quad (2.28)$$

where $\mathbf{U}$ is the nodal displacement vector, and $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices assembled using the standard finite element procedure. For structures with a square lattice arrangement, all eigenfrequencies can be obtained by sweeping the wave vector across the first Brillouin zone, as illustrated in Figure 2.7. For the one-dimensional elastic metamaterials in this study, it is sufficient to scan along the ΓX direction. For two-dimensional elastic metamaterials, additional scanning along the XM and $M\Gamma$ directions is required.

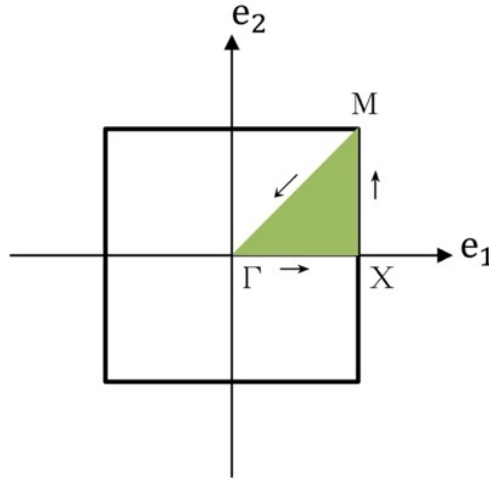


Figure 2.7 The first Brillouin zone and the irreducible Brillouin zone.

The first Brillouin zone is depicted in Figure 2.7, where $\mathbf{e}_i$ ($i = 1, 2$) represents the basis vectors of the lattice. The unit basis vectors of the lattice can be expressed in terms of the lattice constants a_i and the Cartesian basis vectors ($\mathbf{i}_1, \mathbf{i}_2$) [90]:

$$\mathbf{e}_1 = a_1\mathbf{i}_1 \quad (2.29a)$$

$$\mathbf{e}_2 = a_2\mathbf{i}_2 \quad (2.29b)$$

The position of a given point P in space can be expressed as:

$$\mathbf{r}_p(n_1, n_2) = \mathbf{r}_p + n_1 \mathbf{e}_1 + n_2 \mathbf{e}_2 \quad (2.30)$$

where n_1 and n_2 are integers. The basis vectors of the direct lattice are $\mathbf{e}_1$ and $\mathbf{e}_2$, and the corresponding reciprocal lattice vectors are $\mathbf{b}_1$ and $\mathbf{b}_2$. They satisfy the following relationship:

$$\mathbf{e}_i \mathbf{b}_j = 2\pi \delta_{ij} \quad (2.31)$$

where δ_{ij} is the Kronecker delta

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad (2.32)$$

2.3.2 Displacement transmission analysis for finite structures

The bandgap properties exhibited by elastic metamaterials in infinite periodic structures must be validated by calculating displacement transmission responses in their finite counterparts. In elasticity theory, the wave transmission characteristics of a structure or material are typically characterized by the frequency response function between excitation and response. Therefore, this work employs the frequency response function to describe the transmission behavior of finite metamaterial structures. Specifically, point or line excitations in the frequency domain are applied to metamaterial beams, plates, and lattice structures, and the corresponding displacement transmission curves are computed. The displacement transmission coefficient T is defined by the following equation [148]:

$$T = 20 \log_{10} \left(\frac{|U_{\text{out}}|}{|U_{\text{in}}|} \right) \quad (2.33)$$

where U_{in} and U_{out} represent the average displacements at specified input and output positions of the finite structure under harmonic excitation, respectively.

To accurately simulate wave propagation in open space, a Perfectly Matched Layer (PML) or Low Reflection Condition (LRC) is applied around the finite metamaterial structure. These boundary conditions effectively absorb outgoing wave energy, thereby more realistically modelling wave propagation in an unbounded domain and mitigating the influence of boundary reflections on transmission results. During the simulation, either a physics-controlled mesh or a user-defined mesh type is employed, with the mesh size ensured to meet numerical convergence requirements. This ensures the accuracy and reliability of the computational results.

2.4 Principles and setup of the Laser Vibrometer experimental platform

Laser Doppler vibrometry (LDV) traces its origins to the pioneering work of Yeh and Cummins at Columbia University in 1964 [214]. In their seminal paper, they demonstrated the use of laser Doppler shifts to measure fluid velocities as low as 0.007 cm/s, laying the foundation for modern LDV. The technique relies on the Doppler shift of light scattered from a moving surface, a frequency shift directly proportional to the surface vibration velocity, enabling a convenient, non-contact method for vibration measurement. Early implementations achieved signal discrimination by introducing a known frequency pre-shift into the reference beam [215, 216], while modern commercial instruments predominantly employ Bragg-cell-based frequency shifting.

The laser vibrometer employed in this study is the SOPTOP LV-S01 single-point Helium-Neon laser vibrometer, manufactured by Sunnyinnovation Optical Intelligence Co., Ltd. (Yuyao, China). The laser vibrometer operates based on the laser Doppler frequency shift effect and laser heterodyne interferometry. By detecting changes in laser frequency induced by object vibration, it enables precise measurement of parameters such as vibration velocity, displacement, and acceleration. Its core physical principle is the Doppler effect, which states that the observed frequency of a wave changes when there is relative motion between the wave source and the observer. As illustrated in Figure 2.8(a), for example, when the sound source and the observer move toward each other, the received sound wave frequency increases, raising the pitch; when they move apart, the frequency decreases, lowering the pitch. This change in frequency is proportional to the relative velocity between the two and inversely proportional to the wave speed, giving rise to the Doppler effect. This effect finds significant applications in numerous fields such as radar, astrophysics, and medical ultrasonography.

In laser vibrometry, when a laser beam is incident on a vibrating surface and reflected back,

$$f_D = \frac{2v}{\lambda} \quad (2.34)$$

where v is the velocity of the object (m/s), and λ is the wavelength of the incident laser light (m).

To accurately determine the target velocity, it is necessary to precisely measure the frequency shift under a known wavelength condition. This task is accomplished by the interferometric module within the laser vibrometer. Figure 2.8(b) illustrates the principle of heterodyne interferometry employed in the system. Essentially, a laser vibrometer consists of a high-precision laser interferometer and a signal processor. In the interferometer, polarized light (frequency f_0) emitted by an infrared laser is split by a beam splitter into two paths. One path serves as the measurement beam incident on the test object's surface; the object's vibration induces a frequency shift (f_d) in this beam. The other path acts as the reference beam,

which is given a fixed frequency shift (f_b) by an acousto-optic modulator. The two beams recombine and interfere at the sensor, generating an interference signal at the frequency $f_0 + f_b + f_d$. This signal carries the object's vibration information. After demodulation by the signal processor, it is converted into the object's velocity and displacement. The heterodyne interferometry scheme offers significant advantages: the system responds only to high-frequency AC signals, effectively suppressing interference from low-frequency noise such as power line hum; furthermore, the nonlinearity of photodetectors and signal preprocessing stages does not compromise the integrity of the Doppler modulation information.

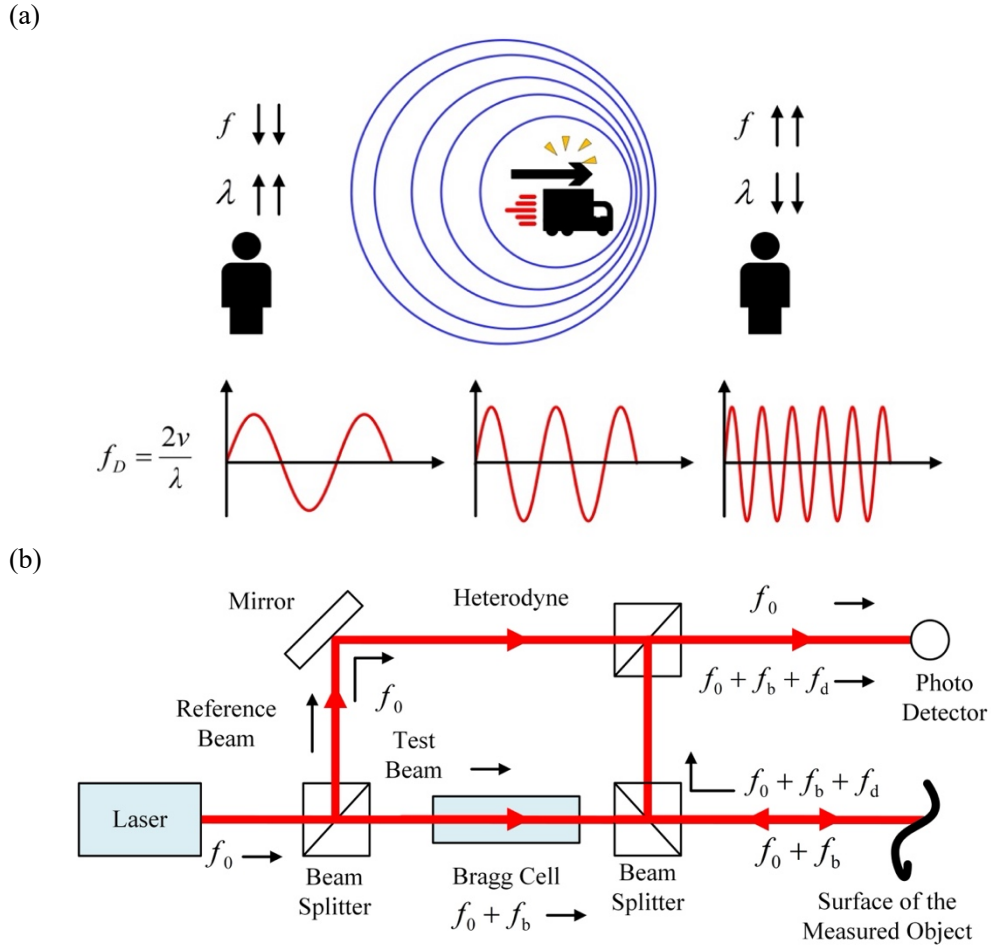


Figure 2.8 Principles of laser vibrometry: (a) Doppler effect; (b) Laser heterodyne interferometry.

The experimental investigation of elastic metamaterials aims to validate the bandgap properties of infinite periodic structures derived from theoretical or numerical methods by measuring the displacement transmission curves of their finite counterparts. In the following section, using the antisymmetric dual helix metamaterial beam with mass spheres as an example, the construction of the experimental platform based on the laser vibrometry system, as well as the procedures for measuring and processing experimental data, will be introduced.

Figure 2.9 shows the experimental setup for the antisymmetric dual helix metamaterial beam with attached concentrated mass spheres. The experimental system comprises a laser vibrometer with its control box, a signal generator (GW Instek, MFG-2220HM), a voltage amplifier (Core Tomorrow, XE50-C), a piezoelectric stack actuator (Thorlabs Inc., USA, model AE0505D16), and the metamaterial beam under test. Excitation is applied to the right end of the metamaterial beam via the piezoelectric stack, which is mounted in a custom fixture (Figure 2.10). The fixture features a rectangular through-slot measuring 8 mm wide by 3 mm deep (Figure 2.10(a)), designed to precisely position the piezoelectric stack. The right end of the beam rests gently on top of the stack. To ensure stable support, three evenly spaced soft rubber pads are placed beneath the beam. The laser vibrometer is positioned on the left side of the beam, with its beam aligned to the midpoint of the left end, to capture the displacement response at that point.

The test metamaterial beam is constructed by alternating 15 aluminum square plates, each containing antisymmetric dual helix perforations, with 15 steel spheres (radius: 8 mm). An end plate is attached at each terminus to implement low-reflection boundary conditions. All components are bonded together using AB adhesive. A single unit cell is illustrated in Figure 2.10(c). It is fabricated via wire electrical discharge machining (EDM), and the mass sphere is adhesively bonded at the center of the dual helix unit.

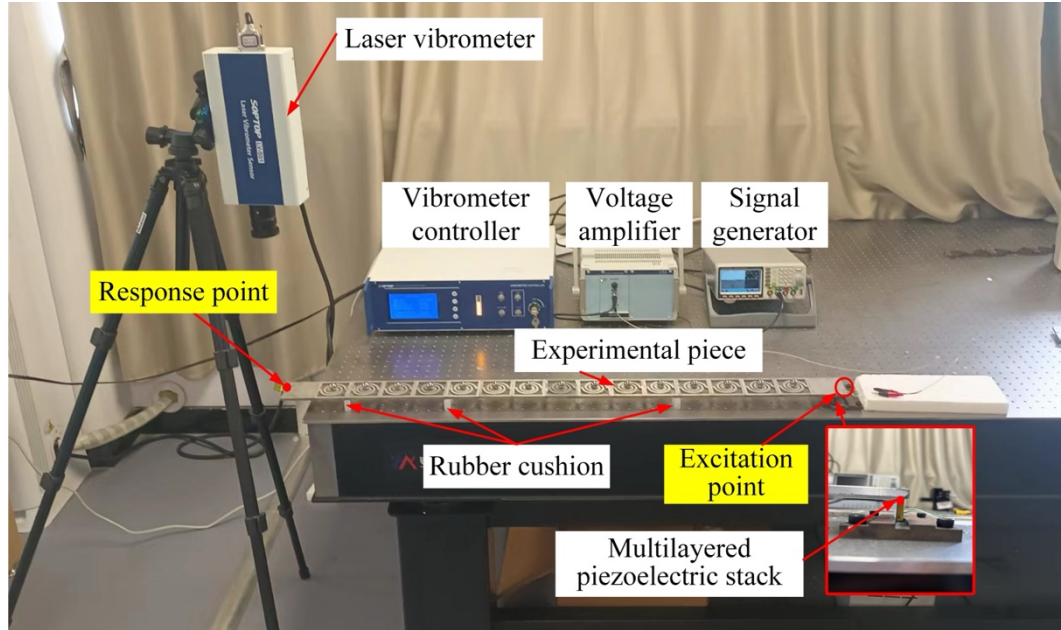


Figure 2.9 Photograph of the experimental setup of the dual helix metamaterial beam with mass spheres based on a laser vibrometer.

The material parameters are as follows: for the aluminum plate, $E_1 = 72$ GPa, $\nu_1 = 0.33$, and $\rho_1 = 2700$ kg/m³; for the steel sphere, $E_2 = 210$ GPa, $\nu_2 = 0.33$, and $\rho_2 = 7800$ kg/m³. The geometric parameters are: $a_1 = a_2 = 70$ mm, $h = 6$ mm, $w = 5$ mm, $r_0 = 8$ mm. The helical parameters are: $r_1 = 0$ mm, $r_2 = 24$ mm, $N = 3.0$. The total length of the finite structure is $L = 1190$ mm.

Prior to the experiment, the vibration testing and analysis software QuickSA 4.0, provided with the SOPTOP vibrometer, is used to configure the necessary measurement parameters. These include the bandpass filter range, recording duration, and signal source settings.

During the experiment, to obtain the displacement transmission curve, measurements are conducted according to the following procedure: First, an arbitrary waveform generator produces a frequency-sweep signal from 1 to 2000 Hz to excite flexural waves. This signal is then amplified by a voltage amplifier to drive the piezoelectric stack actuator, which applies a transverse flexural excitation at the center point on the lower side of the beam's right end. Subsequently, the laser vibrometry system captures the displacement response at the center point on the upper side of the beam's left end, continuously recording the time-domain vibration signal for a specified duration (typically 90 seconds).

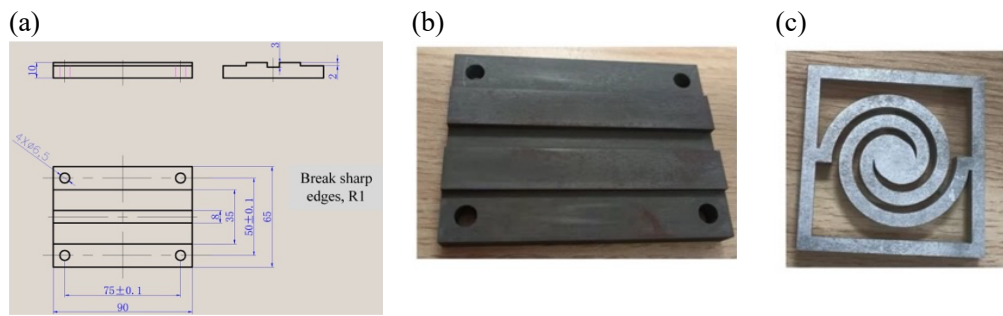


Figure 2.10 Experimental components of the anti-symmetric dual helix metamaterial beam: (a) schematic of the fixture; (b) photograph of the fixture; (c) photograph of a unit cell.

In the data processing stage, the displacement transmission coefficient is calculated according to the following equation:

$$T_d(\omega) = 20 \log_{10} \left| \frac{S_{out}(i\omega)}{S_{in}(i\omega)} \right| \quad (2.35)$$

where $S_{in}(i\omega)$ is the auto-spectral density function of the input sweep signal, and $S_{out}(i\omega)$ is the spectral density function of the output displacement. By processing the experimental data using this equation, the corresponding displacement transmission curve can be obtained [14].

2.5 Summary

This chapter introduced the CUF finite element theory based on the Carrera unified formulation. The core concept of CUF is to simplify three-dimensional problems using a layerwise theory, thereby yielding more compact, solvable formulations in elasticity. Through typical examples, the complete workflow of applying the CUF-based MUL2 program to structural mechanics problems was systematically illustrated. The results demonstrate that, while maintaining comparable

computational accuracy, the CUF method requires significantly fewer degrees of freedom than COMSOL when solving for the natural frequencies of free vibration in rectangular cross-section beams.

Furthermore, the chapter presented the theoretical foundation for calculating the band structures of infinite periodic systems and the displacement transmission characteristics of finite structures using COMSOL. Finally, it provided a detailed explanation of the basic principles of laser vibrometry, the setup of the experimental platform, and the measurement process.

The CUF theoretical framework, COMSOL numerical methodology, and laser vibrometry experimental platform presented in this chapter collectively form the core methodological system of this study, providing a solid theoretical and experimental foundation for the subsequent investigation of elastic wave manipulation in helix metamaterials.

Chapter 3

Flexural wave bandgap characteristics of 1D dual helix metamaterial beams

3.1 Introduction

In engineering, beams, as one of the most fundamental and widely used structural components, serve as the core mechanical component within structural systems. The essence of a beam is a straight or curved member that primarily resists internal forces induced by bending and shear, while carrying transverse loads. From primitive wooden logs spanning streams in ancient times to the intricate load-bearing frameworks of modern high-rise buildings, long-span bridges, and aerospace vehicles, beams have consistently served as the indispensable supportive “backbone” of engineering structures. They silently bear and transfer loads, forming the cornerstone of human engineering civilization.

Regarding wave behavior, beams can excite various elastic wave modes, including flexural, torsional, and longitudinal waves. Consequently, they serve as classic models for studying wave propagation, holding irreplaceable theoretical and experimental value in fields such as structural dynamics, vibration analysis, and wave control. When the cross-sectional dimensions of a beam are significantly smaller than its length, the effects of shear deformation and rotational inertia are often neglected, simplifying it to the Euler-Bernoulli beam model. Under more general conditions, however, accounting for shear and rotational inertia effects necessitates the use of the more accurate Timoshenko beam model to precisely capture its rich dynamic characteristics.

In recent years, bionic principles have provided abundant inspiration for the design of novel engineering materials and structures. Spiral structures, widely found in nature, have become ideal paradigms for wave-control design in artificial structures due to their efficient material distribution and exceptional mechanical properties. Among these, equiangular spirals (or logarithmic spirals), characterized by geometric self-similarity and scale invariance, exhibit a curvature that continuously evolves with the polar radius. This morphological feature opens new pathways for the design of high-performance elastic-wave metamaterials. Such structures demonstrate significant research value and application potential in wave manipulation: their gradient stiffness and continuous curvature variation enable effective control over the propagation paths of elastic waves and the distribution of

vibrational energy. This property not only facilitates vibration suppression over a broad frequency range but also provides a physical basis for designing low-scattering, low-loss elastic waveguides. By delving into the coupling between their geometric features and dynamic responses, it is anticipated that the practical application of such structures in fields such as flexible electronics, structural health monitoring, and vibration isolation can be further advanced. This also highlights the promising prospects of bio-inspired design for advanced wave-control materials.

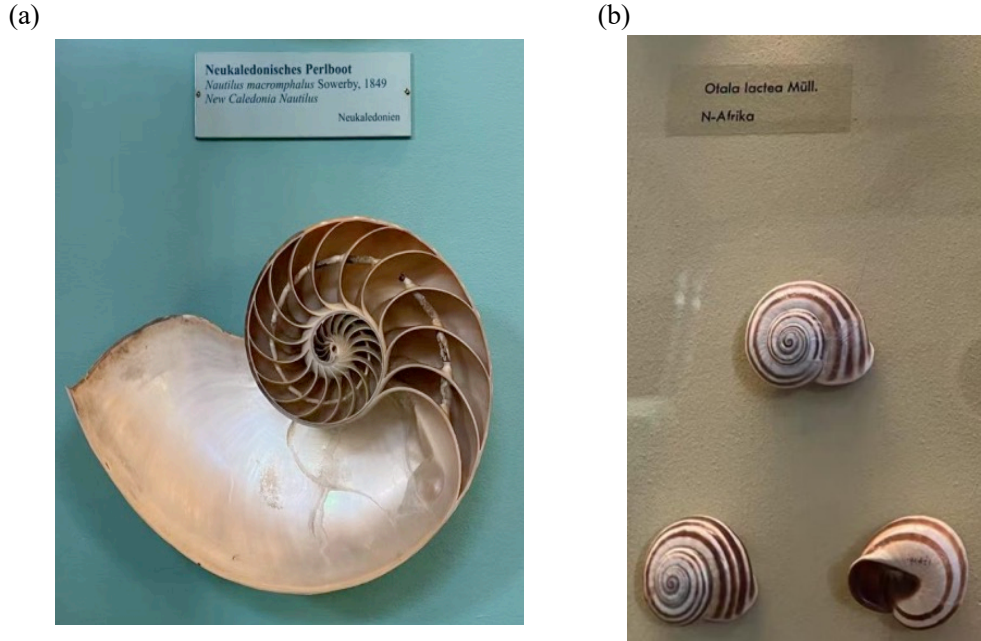


Figure 3.1 Logarithmic spirals in nature: (a) cross-section of a nautilus shell; (b) snail shell. (Photographed at the Naturhistorisches Museum, Vienna.)

As a quintessential natural example of a spiral structure, the nautilus shell exhibits an elegant equiangular spiral form. Its cross-sectional profile not only possesses the mathematical beauty conferred by the golden ratio but also demonstrates a thin, lightweight, and orderly coiled mechanical morphology, as shown in Figure 3.1(a). Similarly, snail shells exhibit the geometric properties of an equiangular spiral, as illustrated in Figure 3.1(b). Inspired by biological spirals, this chapter designs metamaterial beams featuring antisymmetric dual helix configurations as fundamental local resonance units, as well as metamaterial beams incorporating concentrated-mass spheres. Using COMSOL, the band structures, vibration modes, displacement transmission curves of finite structures, and full-field displacements are calculated for both the antisymmetric dual helix metamaterial beams with and without mass spheres. The mechanism underlying the formation of the flexural wave bandgap is subsequently investigated. Furthermore, by analyzing the influence of parameters such as the number of helical turns, the mass-sphere radius, and material properties on the flexural wave bandgap, the bandgap-tuning mechanism is studied. Additionally, the CUF theory is applied to compute the natural frequencies of flexural vibration, vibration modes, and

displacement transmission curves for rectangular cross-section beams and the two types of dual helix metamaterial beams. These results are then compared and analyzed against the numerical results obtained from COMSOL. Finally, based on the established laser vibrometry platform, their displacement transmission characteristics are experimentally validated.

3.2 Structural design and FE modelling

3.2.1 Unit cell design of antisymmetric dual helix metamaterials

The unit cell structure of the antisymmetric dual helix metamaterial designed in this chapter is illustrated in Figure 3.2. Its modelling process is as follows: First, a rectangular outer frame with dimensions $a_1 \times a_2$ is established. Within this frame, dual helix lines arranged antisymmetrically are constructed and connected to the outer frame via two small rectangular links. To maintain connectivity within the region enclosed by the helices, a circular structure with a radius r_0 is placed at the centre of the unit cell. Finally, all geometric parts are merged into a single entity. Extruding the merged two-dimensional profile by a thickness h in the out-of-plane direction yields the unit cell structure of the antisymmetric dual helix metamaterial without mass spheres (Figure 3.2(a)). Subsequently, by attaching a mass sphere at the central position, the unit cell structure with a mass sphere is formed (Figure 3.2(b)). The rectangular portions and the regions enclosed by the Archimedean spirals within the structure maintain a uniform width w . The spiral configuration is determined jointly by the spiral radius, ranging from a starting radius r_1 to an ending radius r_2 , and the number of helical turns N . Figures 3.2(c) and 3.2(d) depict schematic diagrams of antisymmetric dual helix metamaterial beams, each composed of 15-unit cells. For brevity, the antisymmetric dual helix metamaterial beams without and with mass spheres are hereafter referred to as HMB (Helix Metamaterial Beam) and HMBM (Helix Metamaterial Beam with Mass Sphere), respectively.

The equations for the four helical curves H_L^0, H_L^1, H_R^0 , and H_R^1 are:

$$x_L^0 = -s \cos\left(\frac{Ns\pi}{r_2}\right), y_L^0 = -s \sin\left(\frac{Ns\pi}{r_2}\right) \quad (3.1a)$$

$$x_L^1 = -(s+w) \cos\left(\frac{Ns\pi}{r_2}\right), y_L^1 = -(s+w) \sin\left(\frac{Ns\pi}{r_2}\right) \quad (3.1b)$$

$$x_R^0 = s \cos\left(\frac{Ns\pi}{r_2}\right), y_R^0 = s \sin\left(\frac{Ns\pi}{r_2}\right) \quad (3.1c)$$

$$x_R^1 = (s+w) \cos\left(\frac{Ns\pi}{r_2}\right), y_R^1 = (s+w) \sin\left(\frac{Ns\pi}{r_2}\right) \quad (3.1d)$$

The specific geometric parameters of the antisymmetric dual helix metamaterial were listed in Table 3.1. These parameters were optimized through parametric sweeps and multiple rounds of comparative simulations, targeting the desired bandgap frequency range while ensuring structural manufacturability. The

HMB and HMBM metamaterial beams used in the experimental tests presented in this chapter were fabricated from aluminum plates via wire electrical discharge machining (EDM), with the attached concentrated mass spheres being steel balls.

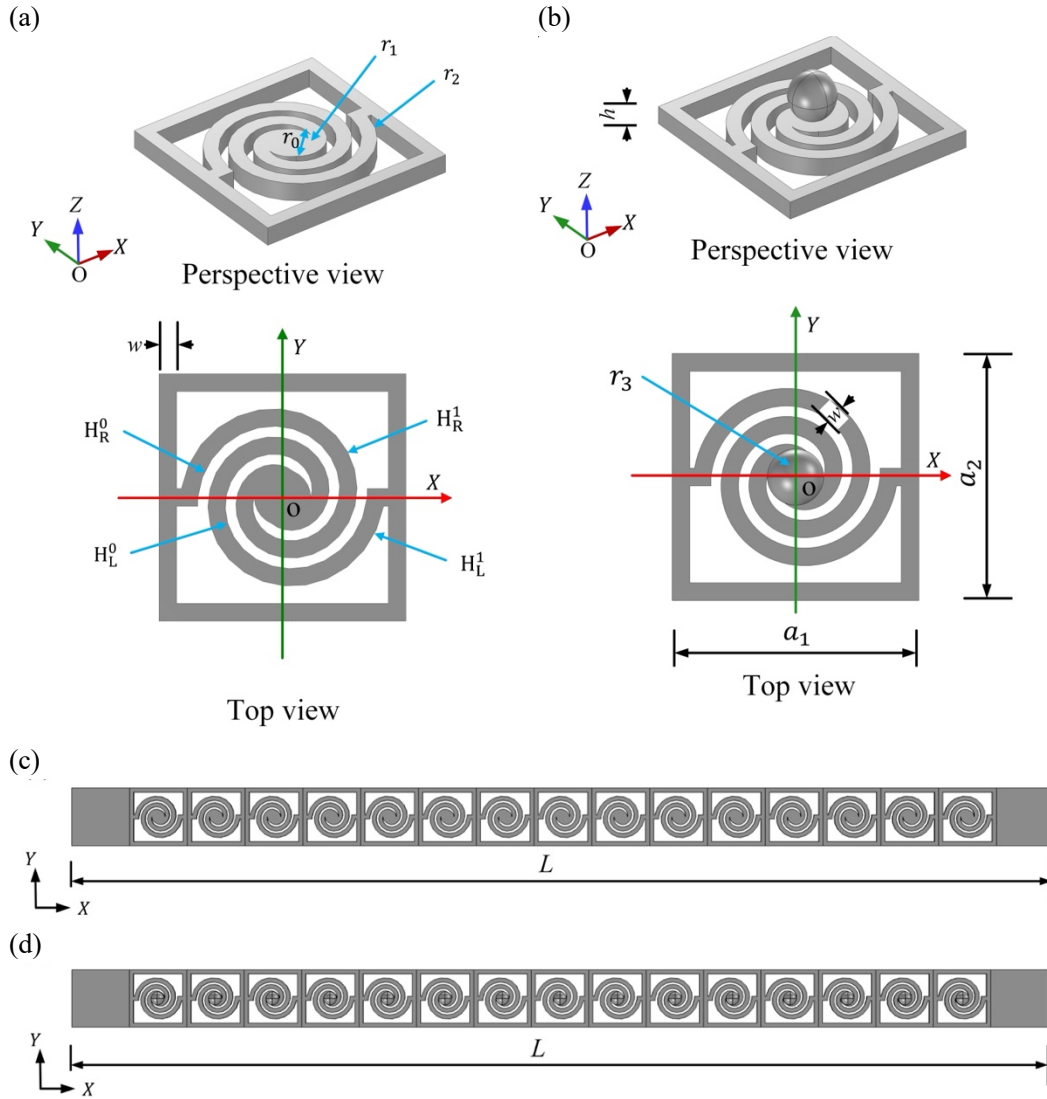


Figure 3.2 Configurations of the dual helix metamaterials. Unit cell structures: (a) HMB (without mass sphere); (b) HMBM (with mass sphere). Finite beam structures: (c) HMB beam; (d) HMBM beam.

Table 3.1 Geometrical parameters of HMB and HMBM

Parameters	Value	Parameters	Value
r_0	8 mm	w	5 mm
r_1	0 mm	a_1	70 mm
r_2	24 mm	a_2	70 mm
r_3	8 mm	h	6 mm
N	3	L	1190 mm

3.2.2 FE-Modelling and boundary condition implementation

In this chapter, COMSOL is used to conduct finite element numerical simulations and comparative analyses of the two designed antisymmetric dual helix metamaterial beams, both with and without concentrated mass spheres. The material parameters used in the calculations are listed in Table 3.2. The main body of the metamaterial beams is made of aluminium, steel, and copper, respectively, while the attached mass spheres are uniformly made of steel.

Table 3.2 Material parameters of the beams

Material	Young's modulus E (GPa)	Poisson's ratio ν	Density ρ (kg/m ³)
Aluminum	72	0.33	2700
Steel	210	0.33	7800
Copper	110	0.33	8900

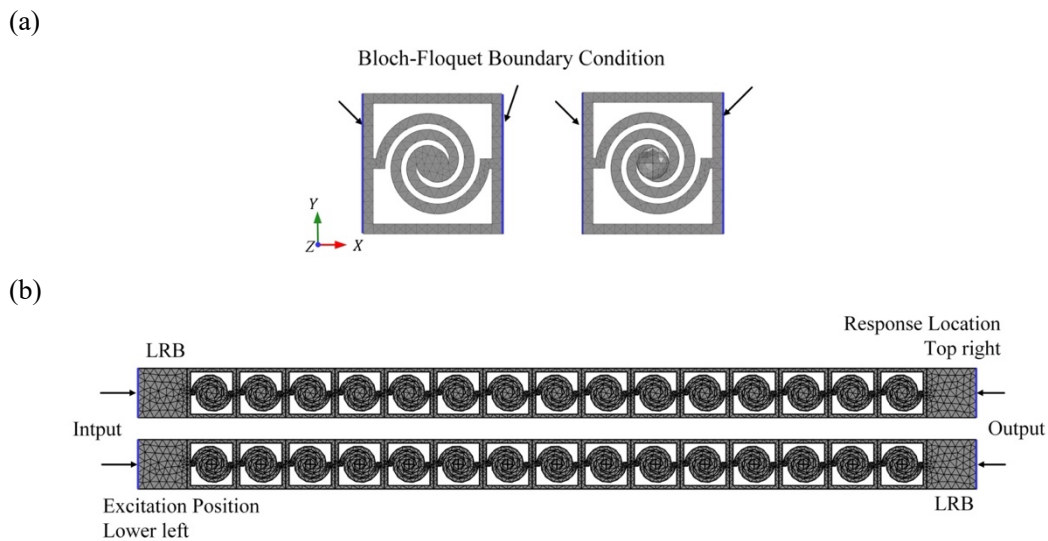


Figure 3.3 Finite element models used in simulations: (a) unit cell model with periodic Bloch-Floquet boundary conditions applied to calculate the band structure of the infinite periodic structure; (b) finite HMB and HMBM models, each composed of 15-unit cells, used to obtain the displacement transmission spectrum.

To characterize the bandgap properties of the two antisymmetric dual helix metamaterials, the bandgap positions are first determined through an analysis of their band structures. Floquet-Bloch periodic boundary conditions are applied to the two sides of the unit cell model along the x -axis, while the remaining boundaries are set as free. The corresponding finite element model is shown in Figure 3.3(a). Numerical calculations are performed using tetrahedral elements for discretization, with a mesh size set to a conventional level. By sweeping the dimensionless wave vector along the ΓX direction of the first Brillouin zone, as illustrated in Figure 2.7,

the eigenfrequencies are solved in the frequency domain, thereby obtaining the complete band structure.

To evaluate the vibration transmission characteristics of finite periodic structures, a metamaterial beam model comprising 15-unit cells was established. A frequency-domain line excitation perpendicular to the axial direction was applied at one end of the beam structure, and the displacement transmission curve was calculated based on Equation (2.33). Here, U_{in} and U_{out} represent the average input displacement at the specified input position and the average output displacement at the specified output position of the beam under harmonic excitation, respectively. To simulate open boundary conditions for the finite structure, a plain plate transition section with a length equivalent to one unit cell was attached at each end of the finite structure, serving as a Low Reflecting Boundary (LRB) condition. The excitation point was located at the lower edge of the left side of the beam, and the response measurement point was positioned at the upper edge of the right side of the beam, as illustrated in Figure 3.3(b). Tetrahedral meshing was also used for these calculations, with a mesh size set to “Extremely Fine” to ensure computational accuracy.

3.3 Numerical results and discussion

3.3.1 Dual helix metamaterial beam without mass spheres

The antisymmetric dual helix metamaterial beam used in the calculations of this subsection adopts the same unit cell geometric parameters as previously described (see Table 3.1), with the finite structure consisting of 15 periodically arranged unit cells. By sweeping the wave vector along the ΓX direction, the dispersion relation curve, i.e., the band structure, of the metamaterial unit cell is obtained. Figure 3.4(a) displays the band structure of the antisymmetric dual helix metamaterial beam without mass spheres (HMB) alongside the displacement transmission curve of its corresponding finite structure. The results indicate the presence of two flexural wave bandgaps (highlighted by red shaded regions in the figure) within the 0–2000 Hz frequency range, located at 429.86–670.18 Hz and 1387.4–1642.8 Hz, respectively. To investigate the bandgap formation mechanism, Figure 3.4(b) extracts the characteristic mode shapes at the bandgap boundary frequencies. These mode shapes clearly reveal the structural vibration modes associated with the opening and closing of bandgaps. Furthermore, Figure 3.4(c) presents the full-field displacement distributions at five selected characteristic frequency points, providing a more comprehensive view of the structure’s wave response.

A comparison between the band structure and the displacement transmission curve reveals good agreement for the frequency range of the first bandgap, whereas a certain discrepancy is observed for the second bandgap. This difference primarily arises because the band structure describes the intrinsic wave-propagation characteristics of an infinite periodic system, in which the passbands are continuous frequency intervals. In contrast, the displacement transmission curve characterizes the response of a finite structure under external excitation; vibrations within its passbands are dominated by a series of discrete natural frequencies and are

influenced by boundary reflections, thereby deviating from the idealized infinite model.

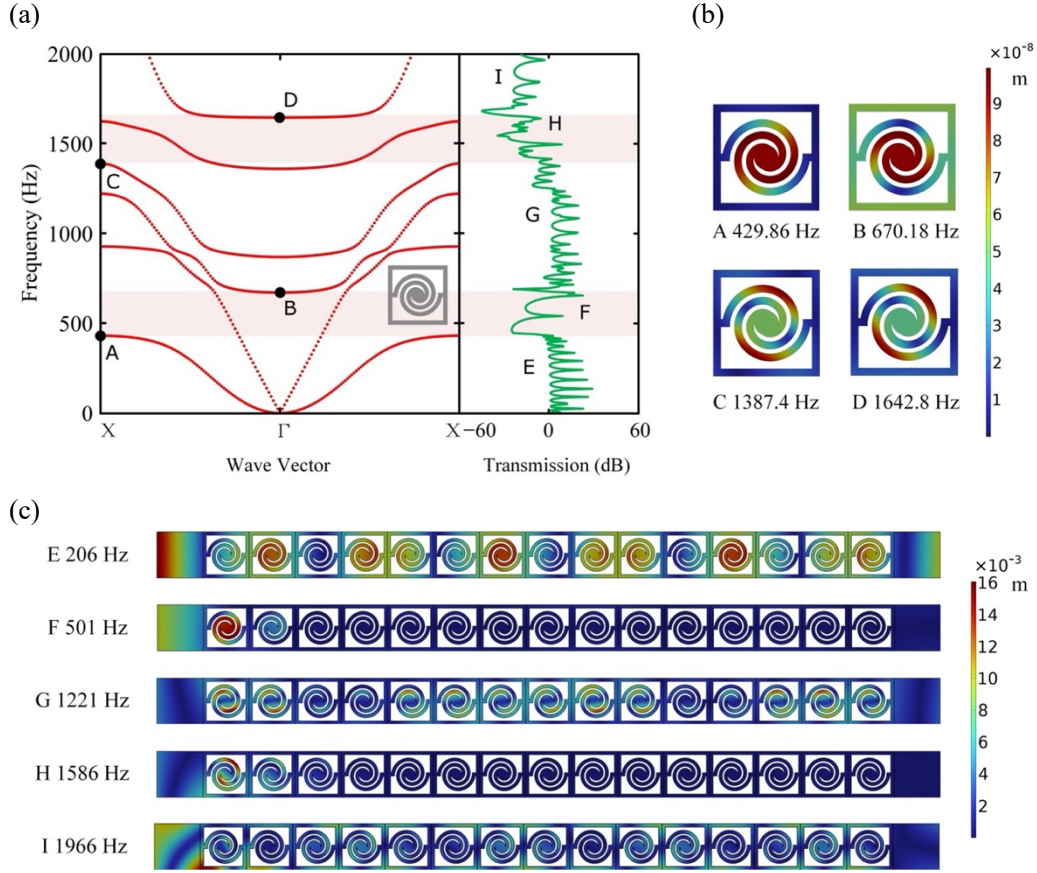


Figure 3.4 COMSOL results for the HMB with helix turns $N = 3.0$: (a) band structure and corresponding displacement transmission spectra; (b) vibration modes at the bandgap opening and closing frequencies; (c) full-field displacement of the finite structure at selected frequency points.

As shown in Figure 3.4(b), point A (429.86 Hz) corresponds to the vibration mode shape at the frequency marking the opening of the first flexural wave bandgap. It can be observed that the vibration of the unit cell is primarily concentrated in the central region of the dual helix, where the central circle and the helices appear to be pulled upward. Point B (670.18 Hz) represents the vibration mode shape at the frequency marking the closure of the first flexural wave bandgap. Here, besides the central circle and helices being pulled upward, a downward motion is also observable in the surrounding rectangular frame portion of the unit cell. Point C (1387.4 Hz) illustrates the vibration mode shape at the opening of the second flexural wave bandgap. Within the unit cell, parts of the dual helix are pulled upward while others are pushed downward; the entire helical section rotates symmetrically about the diagonal, exhibiting alternating forward and backward rotation. Point D (1642.8 Hz) shows the vibration mode shape at the closure of the second flexural wave bandgap. This mode shape is very similar to that at point C,

with the primary difference being a change in the direction of motion within the dual helix portion of the unit cell.

To further observe the bandgap characteristics of the dual helix metamaterial beam and reveal their formation mechanism, five characteristic frequency points (E, 206 Hz; F, 501 Hz; G, 1221 Hz; H, 1586 Hz; I, 1966 Hz) at different locations on the displacement transmission curve in Figure 3.4(a) were selected to visualize the full-field displacement of the finite structure. As shown in Figure 3.4(c), the frequency points E, G, and I lie outside the bandgap; their full-field displacement distributions indicate that flexural waves can propagate effectively from one end of the beam to the other. Point F is located within the first flexural wave bandgap, where the full-field displacement reveals complete suppression of the elastic wave. Closer observation also shows that the energy is primarily concentrated in the central dual helix region of the first unit cell, suggesting that the bandgap at point F is a locally resonant bandgap. Point H lies within the second flexural wave bandgap, where elastic wave propagation is significantly attenuated. The wave energy is mainly confined to the helical structure regions of the first two-unit cells, with the helix in the first unit cell exhibiting a characteristic rotational-tensile deformation mode. This energy-localization feature indicates that the bandgap arises from a local resonance mechanism.

3.3.2 Dual helix metamaterial beam with mass spheres

This subsection presents a numerical analysis of the HMBM. The geometric parameters of its unit cell are listed in Table 3.1, with a mass sphere radius of $r_3 = 8$ mm. The finite structure consists of 15-unit cells arranged periodically. The band structure and displacement transmission curve of the HMBM are shown in Figure 3.5(a); the vibration modes at the bandgap boundary frequencies are presented in Figure 3.5(b); and the full-field displacements at different characteristic frequency points are illustrated in Figure 3.5(c).

As shown in the band structure in Figure 3.5(a), within the frequency range 0–2000 Hz, the HMBM exhibits two flexural wave bandgaps (highlighted by light-blue shaded regions in the figure), with frequency ranges of 301.08–528.8 Hz and 1207.2–1484.1 Hz, respectively. The band structure and the displacement transmission curve agree well on the spectral positions and widths of the bandgaps. Compared to the HMB, both bandgaps of the HMBM are significantly shifted towards lower frequencies.

Specifically, the starting frequency of the first bandgap (BG1) of the HMBM is reduced by 127.78 Hz relative to the HMB, while the bandwidth narrows slightly from 240.32 Hz to 227.72 Hz. The starting frequency of the second bandgap (BG2) is lowered by 84.6 Hz, with the bandwidth decreasing from 351.0 Hz to 276.9 Hz. These results indicate that the introduction of concentrated mass spheres lowers the bandgap frequencies but is accompanied by a slight reduction in bandwidth, reflecting the synergistic tuning characteristics between frequency and bandwidth in locally resonant systems.

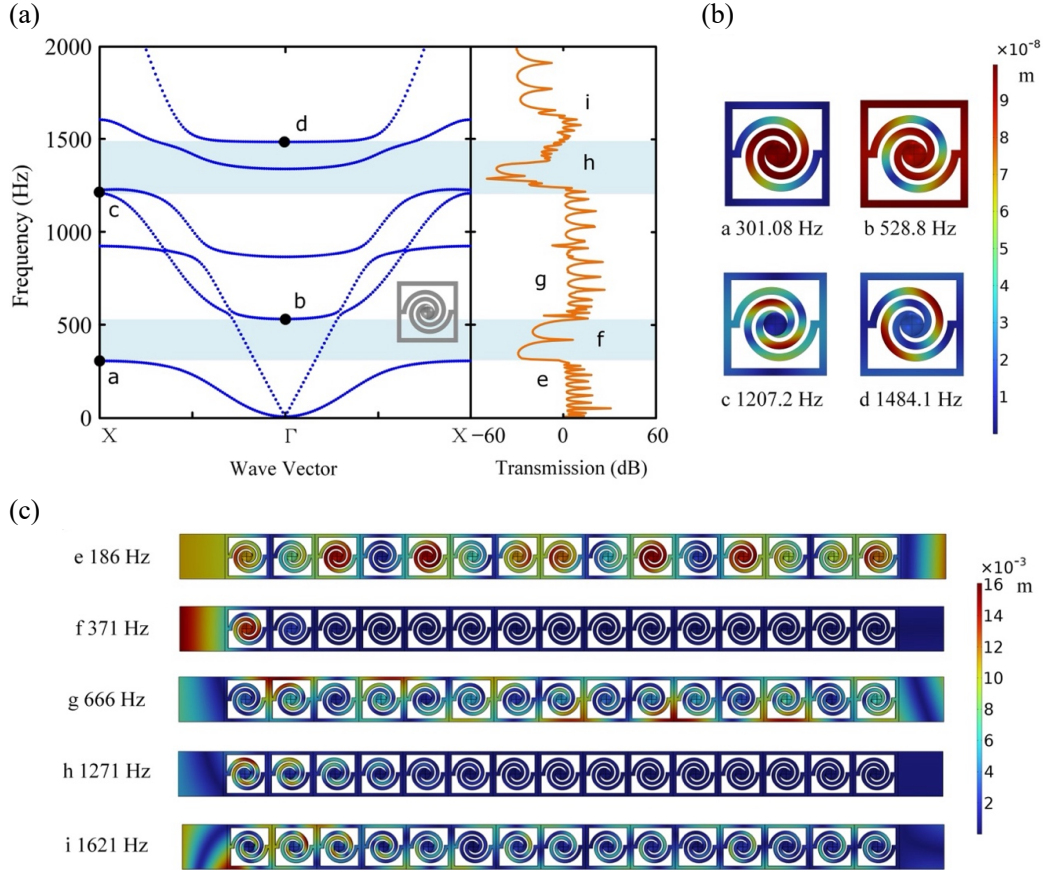


Figure 3.5 COMSOL results for the HMBM with helix turns $N = 3.0$: (a) band structure and corresponding displacement transmission spectra; (b) vibration modes at the bandgap opening and closing frequencies; (c) full field displacement of the finite structure at selected frequency points.

In Figure 3.5(b), point a (301.08 Hz) shows the vibration mode shape at the starting frequency of the first flexural wave bandgap. It can be observed that the vibration of the unit cell is primarily concentrated on the mass sphere and the central region of the dual helix, both appearing to be pulled upward as a whole. Point b (528.8 Hz) corresponds to the vibration mode shape at the cutoff frequency of the first flexural wave bandgap. Here, the central mass sphere and helices are observed to be pulled downward, while the surrounding rectangular frame portion is engaged in an upward motion. Point c (1207.2 Hz) illustrates the vibration mode shape at the starting frequency of the second flexural wave bandgap. It can be seen that the rectangular frame of the entire unit cell, along with the front part of the central dual helix, is pulled upward, whereas the rear part is pulled downward; the motion of the central mass sphere is not pronounced. Point d (1484.1 Hz) shows the vibration mode shape at the cutoff frequency of the second flexural wave bandgap. At this frequency, the vibration is mainly concentrated on the two helical frames located in the first and third quadrants, while the central mass sphere and the surrounding rectangular structure exhibit a slight downward movement.

Analysis of the full-field displacement distributions reveals the bandgap characteristics of the HMBM and their underlying formation mechanism. Based on

the five characteristic frequency points (e, 186 Hz; f, 371 Hz; g, 666 Hz; h, 1271 Hz; i, 1621 Hz) selected from the displacement transmission curve in Figure 3.5(a), the full-field displacement analysis presented in Figure 3.5(c) shows the following: Points e, g, and i, located in the passbands, all exhibit complete propagation characteristics of flexural waves. Point f, situated within the first bandgap, demonstrates complete wave suppression, with energy highly concentrated in the mass sphere and the central helical region of the first unit cell. Point h, located within the second bandgap, reveals wave energy localized in the helical regions of the first two-unit cells. This indicates that the combined action of the dual helix curved beam structure and the mass spheres result in the apparent mass of the spheres significantly exceeding their physical mass, thereby yielding an enhanced locally resonant bandgap.

3.3.3 Parametric analysis

(1) Number of helical turns

To systematically investigate how the number of helical turns, N , influences the bandgap characteristics and underlying mechanism of two antisymmetric dual helix metamaterial beams, a parametric study is conducted. For both beam configurations (without and with concentrated mass spheres), the band structures and bandgap edge modes are calculated for various N values, while other geometric parameters are held constant (see Table 3.1).

Figure 3.6 illustrates the evolution of the band structure for the dual helix metamaterial beam without mass spheres under different numbers of helical turns. By analyzing the characteristic vibration modes at the bandgap boundary frequencies, the underlying mechanism by which the number of helical turns influences the bandgap position and width can be further elucidated.

Figure 3.6(a) shows the band structure of the antisymmetric dual helix metamaterial beam when the number of helical turns is 0.5. The figure shows that within the frequency range 0–3000 Hz, the antisymmetric dual helix metamaterial beam exhibits a relatively wide flexural wave bandgap, spanning 1748.4–2508.5 Hz. It is worth noting that the cutoff frequency of this bandgap occurs at a wavenumber $k = 0.83$, where the wavenumber denotes the magnitude of the wave vector. As shown in Figure 3.6(b), when the number of helical turns $N = 1$, the HMB exhibits a very wide flexural wave bandgap within the frequency range of 0 to 2000 Hz, spanning from 1296.6 to 1966.2 Hz. The cutoff frequency occurs at a wavenumber $k = 0$. Moreover, as the number of helical turns continues to increase, the wave vector position of the cutoff frequency for the first bandgap remains fixed at zero. As illustrated in Figure 3.6(c), when $N = 1.5$, the HMB also displays a relatively wide flexural wave bandgap within the 0 to 2000 Hz frequency range, spanning 864.61 to 1360.7 Hz.

Figures 3.6(d)–3.6(h) sequentially illustrate the evolution of the band structure for the HMB at helical turn numbers $N = 2, 2.5, 3, 3.5$, and 4. Although each configuration exhibits two flexural wave bandgaps within the 0–2000 Hz range, this section focuses on the evolution of the low-frequency bandgap (i.e., the first

complete bandgap). As shown in the figures, the frequency ranges of their first flexural wave bandgaps are: 650.97–1021.6 Hz, 514.55–803.11 Hz, 429.86–670.18 Hz, 375.58–590.39 Hz, and 334.54–527.22 Hz, respectively. It is observed that as the number of helical turns increases, the first bandgap exhibits a systematic shift towards lower frequencies. This trend indicates that the densification of the helical topology effectively reduces the structure's equivalent stiffness, providing a key design parameter for achieving low-frequency vibration control.

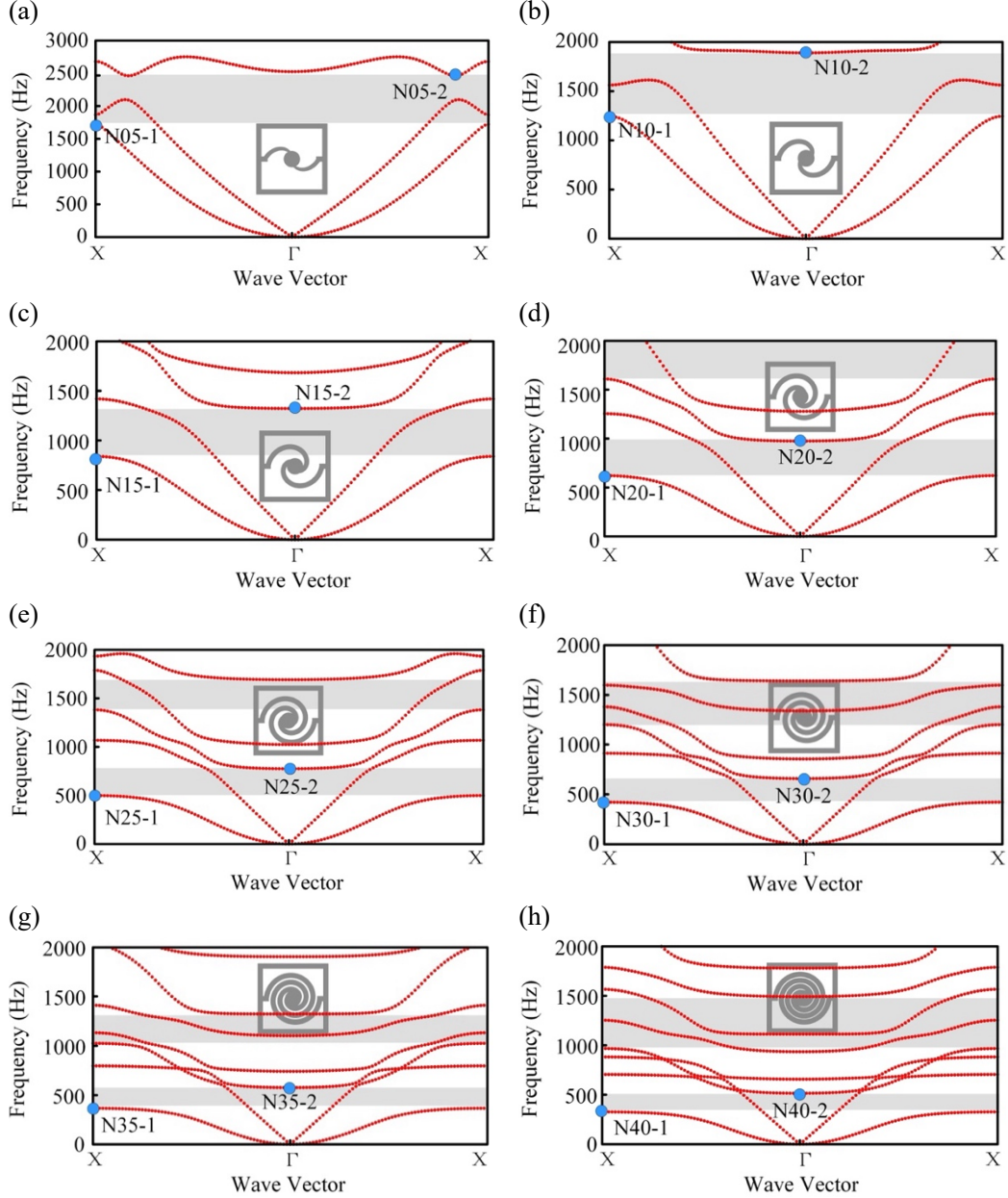


Figure 3.6 Band structure and first bandgap of HMB with various helical turns: (a) $N = 0.5$; (b) $N = 1$; (c) $N = 1.5$; (d) $N = 2$; (e) $N = 2.5$; (f) $N = 3$; (g) $N = 3.5$; (h) $N = 4$.

To gain a deeper understanding of the mechanism by which the number of helical turns influences the bandgap of the antisymmetric dual helix metamaterial,

16 characteristic frequency points were selected from Figure 3.6, designated as N05-1, N05-2, N10-1, N10-2, N15-1, N15-2, N20-1, N20-2, N25-1, N25-2, N30-1, N30-2, N35-1, N35-2, N40-1, and N40-2. Figure 3.7 displays the vibration modes at the boundary frequencies of the first bandgap for the eight different helical turns.

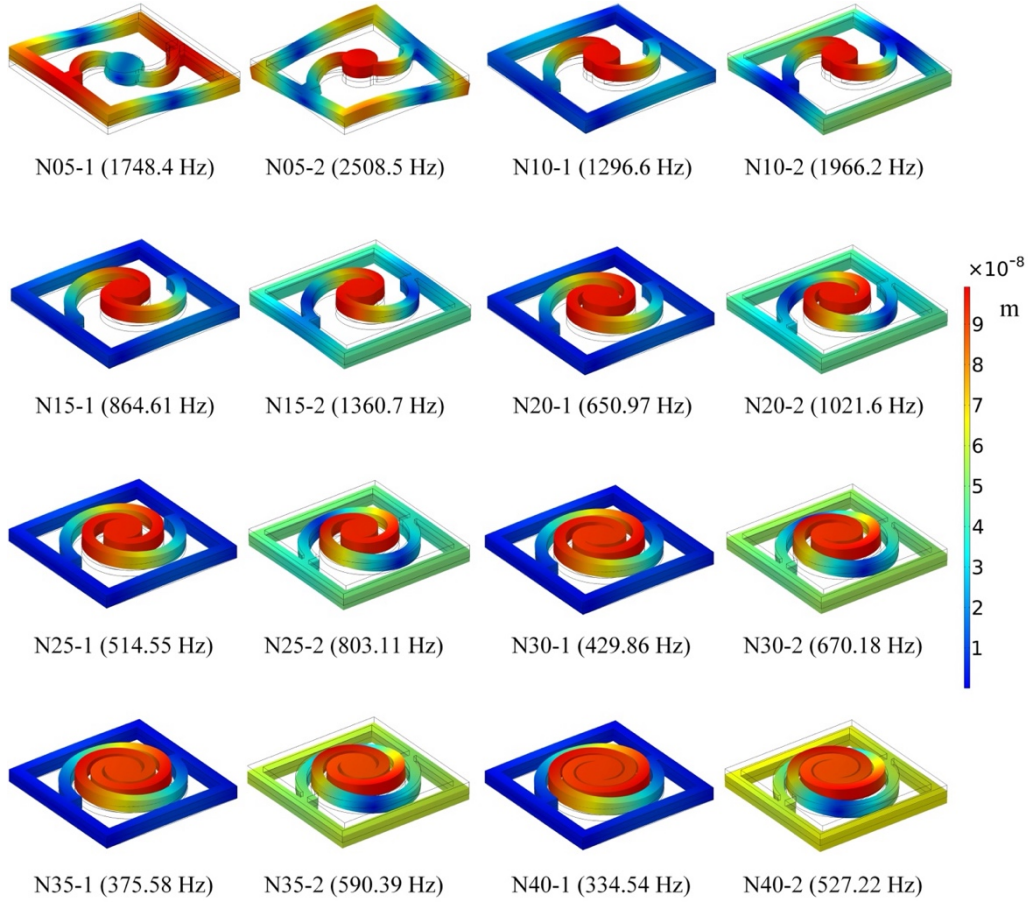


Figure 3.7 Vibration modes of the first bandgap of the HMB for various helical turns.

The modal analysis presented in Figure 3.7 indicates that, with the exception of the configuration with $N = 0.5$ turns, the starting frequency points of the first bandgap for all other helical turn numbers are characterized by a high concentration of energy in the central circular region of the unit cell and the adjacent dual helix areas. Further comparison of the boundary modes reveals that at the starting-frequency points, the peripheral frame remains stationary, whereas at the cutoff-frequency points, it exhibits varying degrees of relative motion. This observation suggests that the generation mechanism of the first bandgap in the HMB is attributable to locally resonant elements within the structure.

Figure 3.8 illustrates the band structures of the antisymmetric dual helix metamaterial beam with concentrated mass spheres for varying numbers of helical turns, N , while keeping the mass sphere radius constant at $r_3 = 8$ mm. The first flexural wave bandgap for each configuration is highlighted by the gray shaded regions, clearly demonstrating how the number of turns modulates both the position

and width of the bandgap. Specifically, as N increases from 0.5 to 4 in increments of 0.5, the corresponding first bandgap frequency ranges are: 965.58–1710.63 Hz, 705.67–1305.1 Hz, 506.23–939.36 Hz, 406.8–737.11 Hz, 342.01–606.33 Hz, 301.08–528.8 Hz, 274.42–481.48 Hz, and 251.23–440.94 Hz.

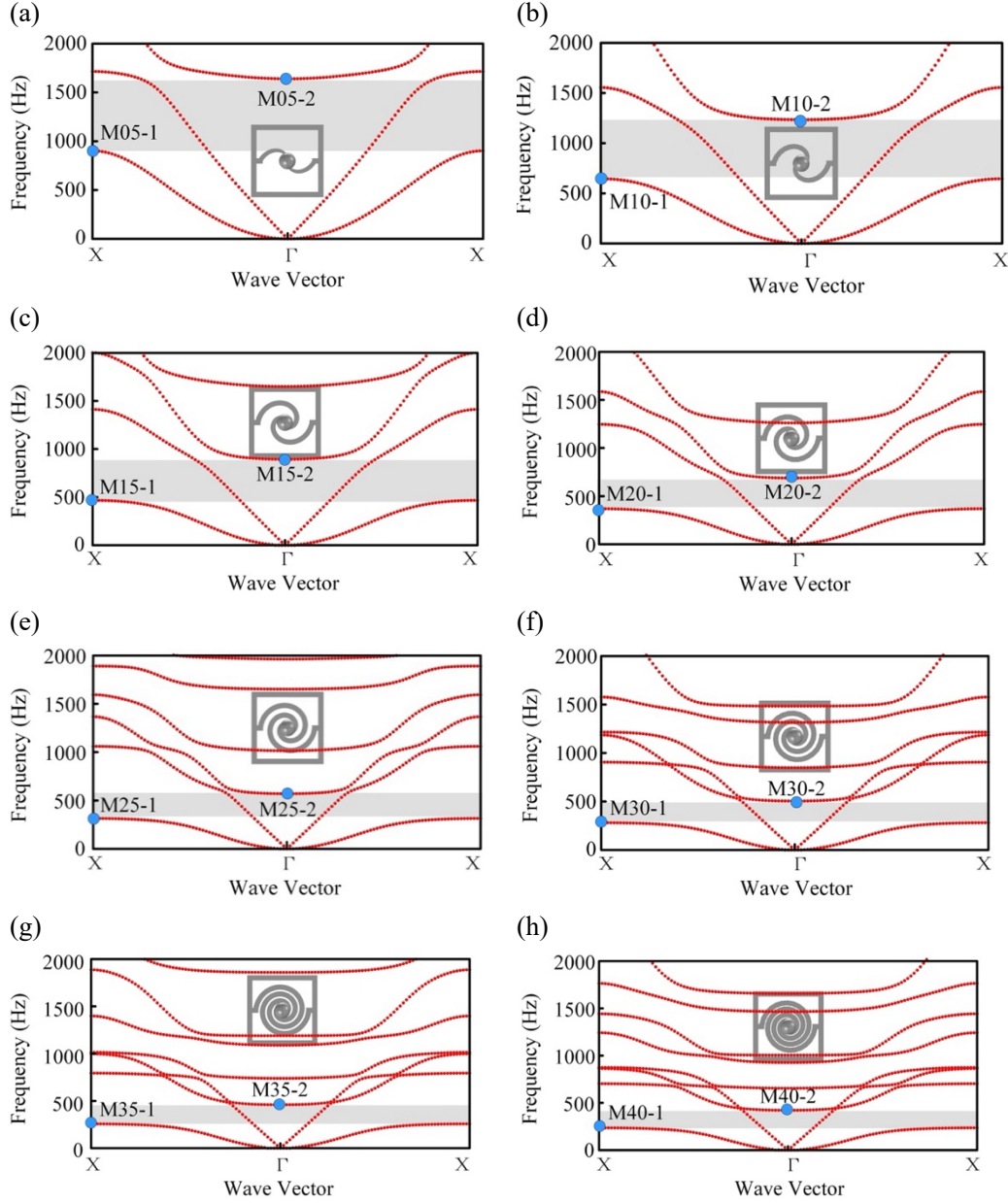


Figure 3.8 Band structure and first bandgap of HMBM with various helical turns: (a) $N = 0.5$; (b) $N = 1$; (c) $N = 1.5$; (d) $N = 2$; (e) $N = 2.5$; (f) $N = 3$; (g) $N = 3.5$; (h) $N = 4$.

To elucidate the influence of the number of helical turns on the formation mechanism of the flexural wave bandgap in the HMBM, 16 characteristic frequency points at the bandgap boundaries, designated as M05-1, M05-2, M10-1, M10-2, M15-1, M15-2, M20-1, M20-2, M25-1, M25-2, M30-1, M30-2, M35-1, M35-2,

M40-1, and M40-2, were selected from the band structures corresponding to the eight helical turn configurations shown in Figure 3.8 for vibration mode analysis.

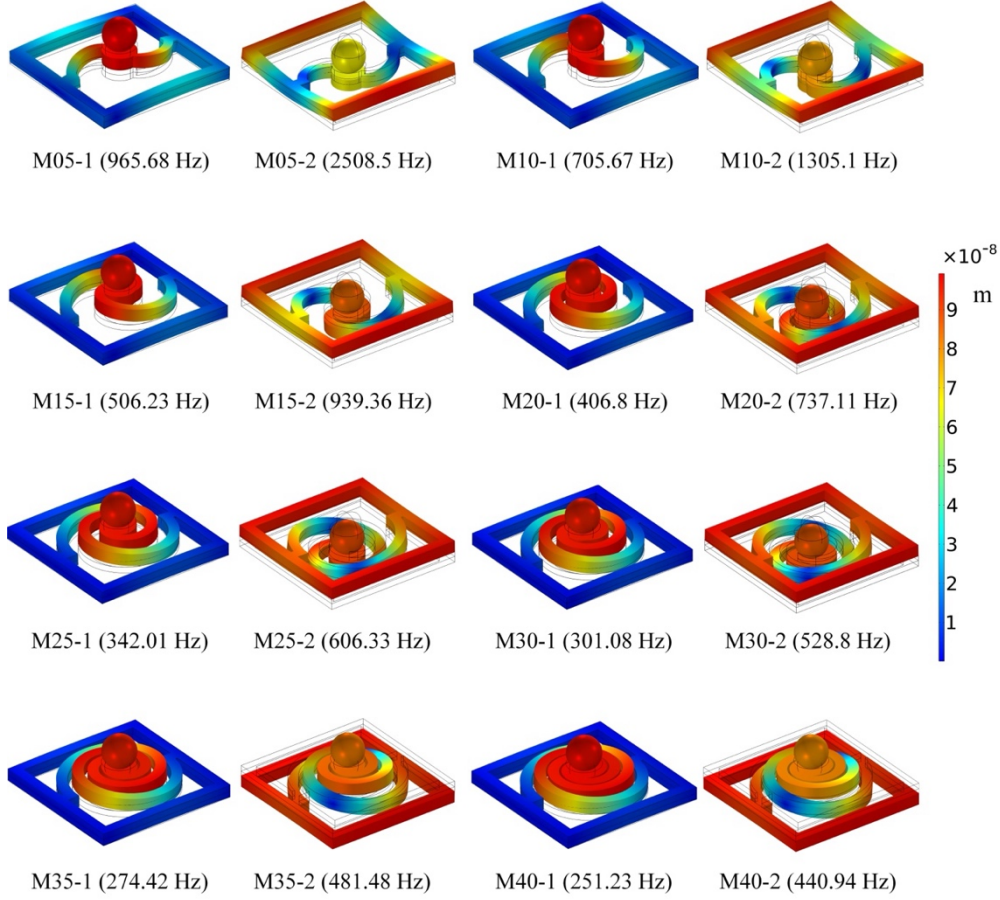


Figure 3.9 Vibration modes of the first bandgap of the HMBM for various helical turns.

As shown in Figure 3.9, for different numbers of helical turns, at the frequency points where the flexural wave bandgap opens in the HMBM, the unit-cell vibration is primarily concentrated in its central region and the attached mass sphere. In contrast, at the frequency points where the bandgap closes, a considerable portion of the vibration is also concentrated on the frame. Therefore, it can be concluded that the bandgap opening mechanism in the HMBM is local resonance, and the combined action of the concentrated mass sphere at the center of the unit cell structure and the helical curved beam significantly strengthens and amplifies this local resonance effect.

Figure 3.10 illustrates the influence of the number of helical turns on the first bandgap of the antisymmetric dual helix metamaterial beams, both without (HMB) and with (HMBM) concentrated mass spheres. The results indicate that for both configurations, the starting frequency of the bandgap decreases as the number of helical turns increases, while the bandgap width narrows with increasing turns. Notably, the rate of change in both the starting frequency and the bandwidth is considerably faster when the number of turns increases from 0.5 to 1.5 compared

to the increase from 1.5 to 4.0. When the number of helical turns $N \geq 3$, the variation in the bandgap frequency range becomes very small. This behavior suggests, on the one hand, that increasing the number of helical turns effectively reduces the system's equivalent stiffness, thereby lowering the bandgap frequency. On the other hand, it indicates that for $N \geq 3$, the change in the system's equivalent stiffness is no longer significant. Furthermore, the bandgap position of the HMBM is considerably lower than that of the HMB, while its bandwidth is slightly narrower. This is attributed to the combined action of the concentrated mass sphere and the dual helix curved beam, which significantly strengthens and enhances the local resonance effect.

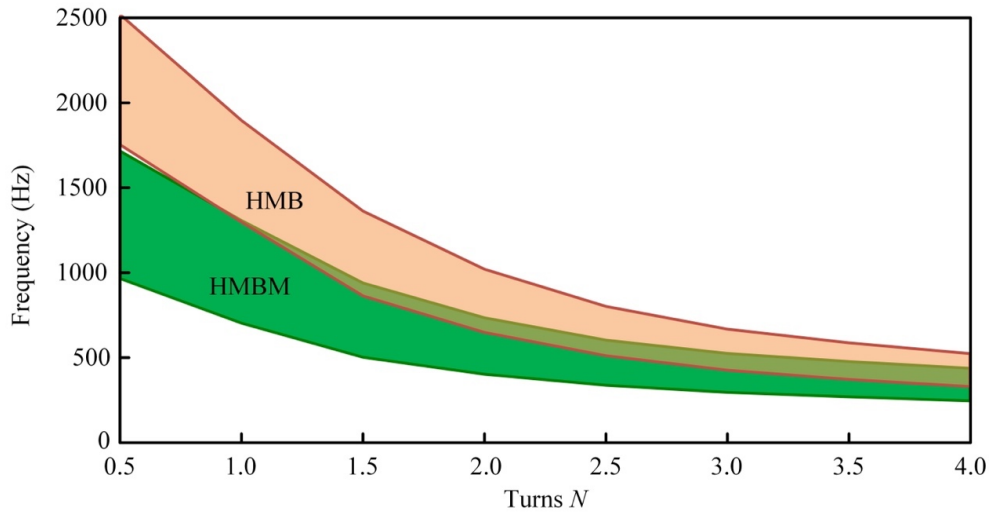


Figure 3.10 First bandgap frequency changes with the helical turns of the HMB and HMBM.

(2) Radius of the mass sphere

The influence of the radius of the concentrated mass sphere on the bandgap characteristics of the HMBM is systematically investigated in this section. While keeping other geometric parameters constant (see Table 3.1) and fixing the number of helical turns at $N = 3.5$, the tuning effect of the mass sphere radius r_3 on the bandgap position and width is analyzed by varying its value. Figure 3.11 shows the variation of the first flexural wave bandgap frequency range as the mass sphere radius increases from 5 mm to 10 mm. The results indicate that the first bandgap frequency decreases significantly, from 337.59–542.68 Hz down to 222.21–430.21 Hz; however, the change in bandgap width is minimal (with bandwidths of 205.09 Hz, 204.35 Hz, 202.12 Hz, 202.35 Hz, 203.88 Hz, and 208 Hz, respectively). The bandgap frequency decreases linearly with increasing mass sphere radius. It is evident that by adjusting the attached mass sphere radius of the HMBM, the bandgap frequency range can be conveniently tuned without affecting its bandgap width.

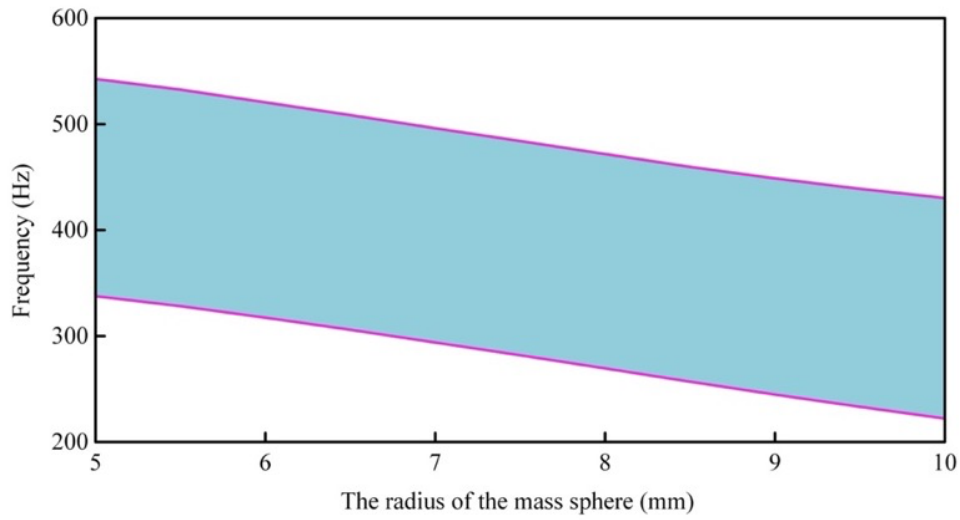


Figure 3.11 First bandgap frequency changes with the mass sphere radius of HMBM.

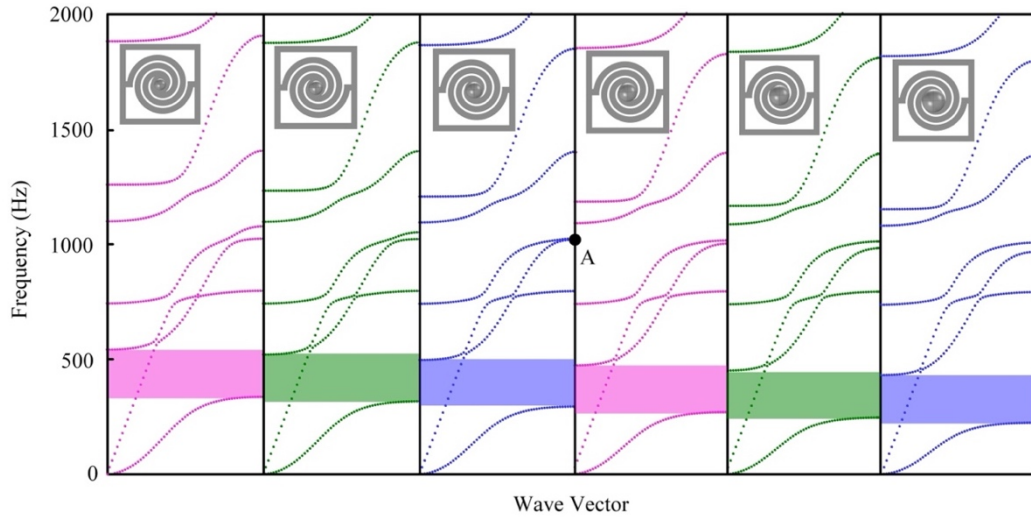


Figure 3.12 The band structure changes with the mass sphere radius of HMBM.

Figure 3.12 shows band diagrams over the 0–2000 Hz frequency range as the mass sphere radius varies from 5 mm to 10 mm. The band structures of the HMBM under various conditions are similar, with the bandgap gradually shifting to lower frequencies as the mass sphere radius increases. When the mass sphere radius is 7 mm, it can also be noted that the third and fourth bands nearly coincide at point A; however, as the mass sphere radius increases or decreases, they gradually separate again.

(3) Material properties

The bandgap is also influenced by the constituent materials of the metamaterial structure. Figure 3.13 presents the band structures of the two antisymmetric dual helix metamaterial beams when fabricated from aluminum, steel, and copper, respectively. The geometric parameters used in the calculations are based on those listed in Table 3.1, with $N = 3.5$.

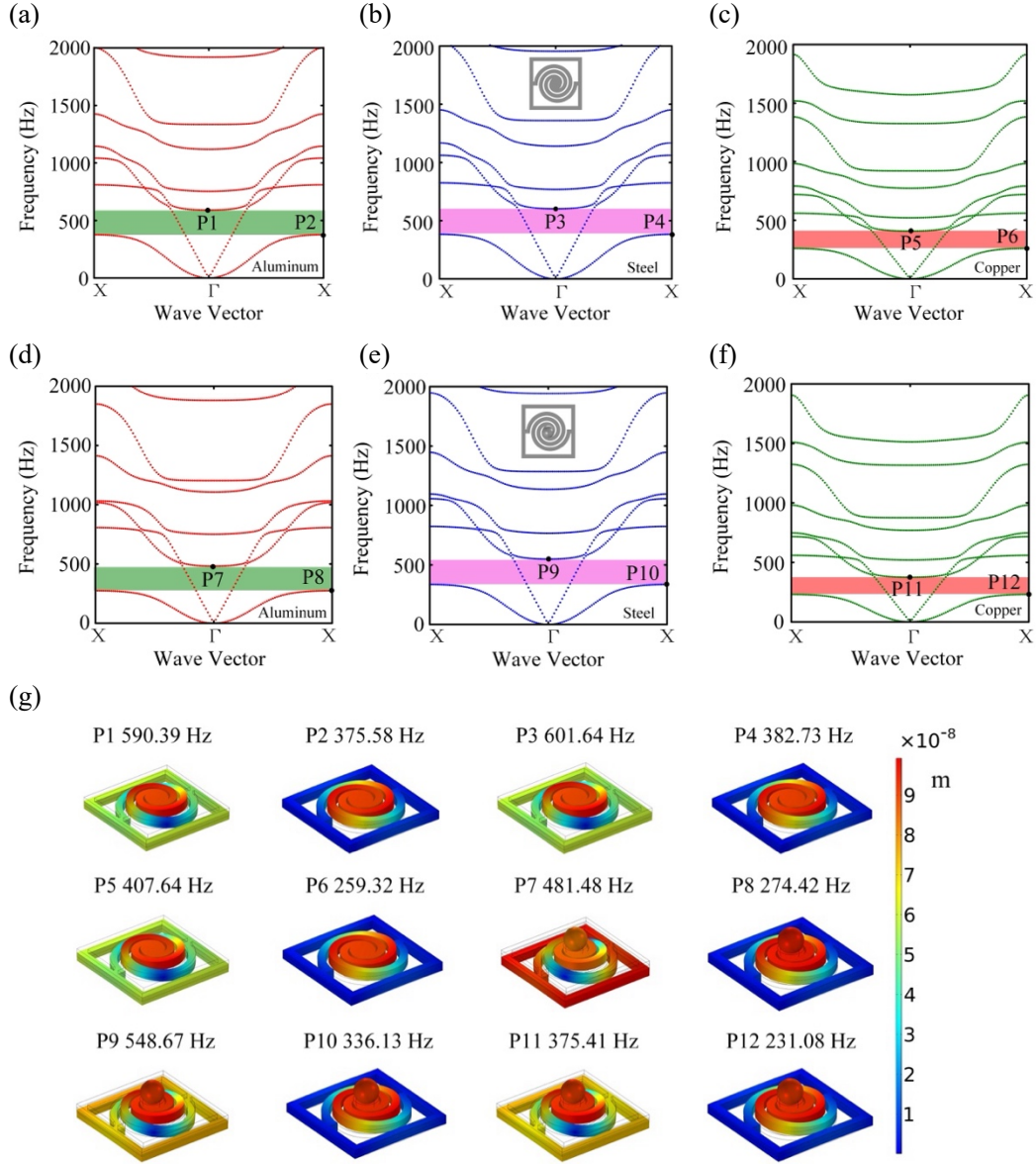


Figure 3.13 Band structures of dual helix metamaterial beams with different materials and corresponding bandgap edge modes. HMB (without mass spheres): (a) aluminum; (b) steel; (c) copper. HMBM (with steel spheres): (d) aluminum matrix; (e) steel matrix; (f) copper matrix. (g) Vibration modes at the band edges.

Figures 3.13(a)–3.13(c) present the calculated results for the HMB. From the figures, the first flexural wave bandgaps for the three material cases are: 375.58–590.39 Hz (bandwidth: 214.81 Hz); 382.73–601.64 Hz (bandwidth: 218.91 Hz); and 259.32–407.64 Hz (bandwidth: 148.32 Hz), respectively. Figures 3.13(d)–3.13(f) show the band structure results for the HMBM. Note that the mass spheres are uniformly made of steel in all cases. It can be seen from the figures that the first flexural wave bandgaps for the HMBM corresponding to the three base materials are: 274.42–481.48 Hz with a bandwidth of 207.06 Hz; 336.13–548.67 Hz with a bandwidth of 212.54 Hz; and 231.08–375.41 Hz with a bandwidth of 144.33 Hz, respectively.

Comparing the two structures, the bandgap frequency of the HMBM structure is consistently lower than that of the HMB structure under the same parameters. When the matrix material is aluminum (with steel mass spheres attached), the reduction in bandgap frequency brought about by the introduction of the mass sphere is the most significant; it is less pronounced when the matrix material is steel, and the reduction in bandgap frequency is the most limited when the matrix material is copper.

A closer examination of the bandgap edge modes shown in Figure 3.13(g) reveals that, for different material compositions, the primary vibrations at the first bandgap boundary frequencies of both types of antisymmetric dual helix metamaterial beams are concentrated in the central region of the unit cell. Notably, when the matrix material is aluminum (e.g., at position P7), some vibration is also distributed on the surrounding frame.

It is noteworthy that for the HMB structure, when the matrix material is aluminum, its bandgap frequency is slightly lower than that of the steel case but significantly higher than that of the copper case. In contrast, for the HMBM structure, the bandgap frequency with an aluminum matrix is even lower than in the steel case. This phenomenon indicates that, unlike the nearly linear influence of the mass sphere radius on the HMBM bandgap frequency, the bandgap characteristics of the antisymmetric dual helix metamaterial beams (including both HMB and HMBM) exhibit a distinctly nonlinear relationship with the density of their constituent materials. This reflects a coupling mechanism between material stiffness and density parameters in their effect on the bandgap characteristics.

3.4 Theoretical and experimental verification

The foregoing research results were all obtained based on the finite element software COMSOL. In this subsection, the CUF theory and laser vibrometry experimental techniques are applied to validate the displacement transmission curves of the rectangular cross-section beam and the two types of antisymmetric dual helix metamaterial beams (HMB and HMBM).

3.4.1 Theoretical verification based on CUF

To determine the appropriate CUF cross-sectional expansion model (TE or LE) and beam element type (B2, B3, or B4) for calculating the displacement transmission curves, the natural frequencies and vibration modes of flexural vibration are first computed for the rectangular cross-section beam, the HMB, and the HMBM. The materials used in the calculations are assumed to be linearly elastic and listed in Table 3.2. The models' boundary conditions are set uniformly to free-free. The specific geometric parameters for the HMB and HMBM are provided in Table 3.1.

Figure 3.14 illustrates the finite element modelling coordinate system and meshing within the CUF framework for the rectangular cross-section beam, the HMB, and the HMBM beam. Note that for the rectangular cross-section beam, the cross-section lies in the xz -plane and is rectangular, as shown in Figure 3.14(a). In contrast, for CUF calculations of the two dual helix metamaterial beams, the cross-

section is defined as a plane parallel to the dual helix structure, as illustrated in Figure 3.14(b). Figure 3.14(c) presents a schematic diagram of the CUF beam model. Figure 3.14(d) shows the mesh discretization of a unit cell for the HMB and HMBM, while Figure 3.14(e) displays the connection between the LE4 node elements and LE9 node elements within the area circled by the dashed line in Figure 3.14(d).

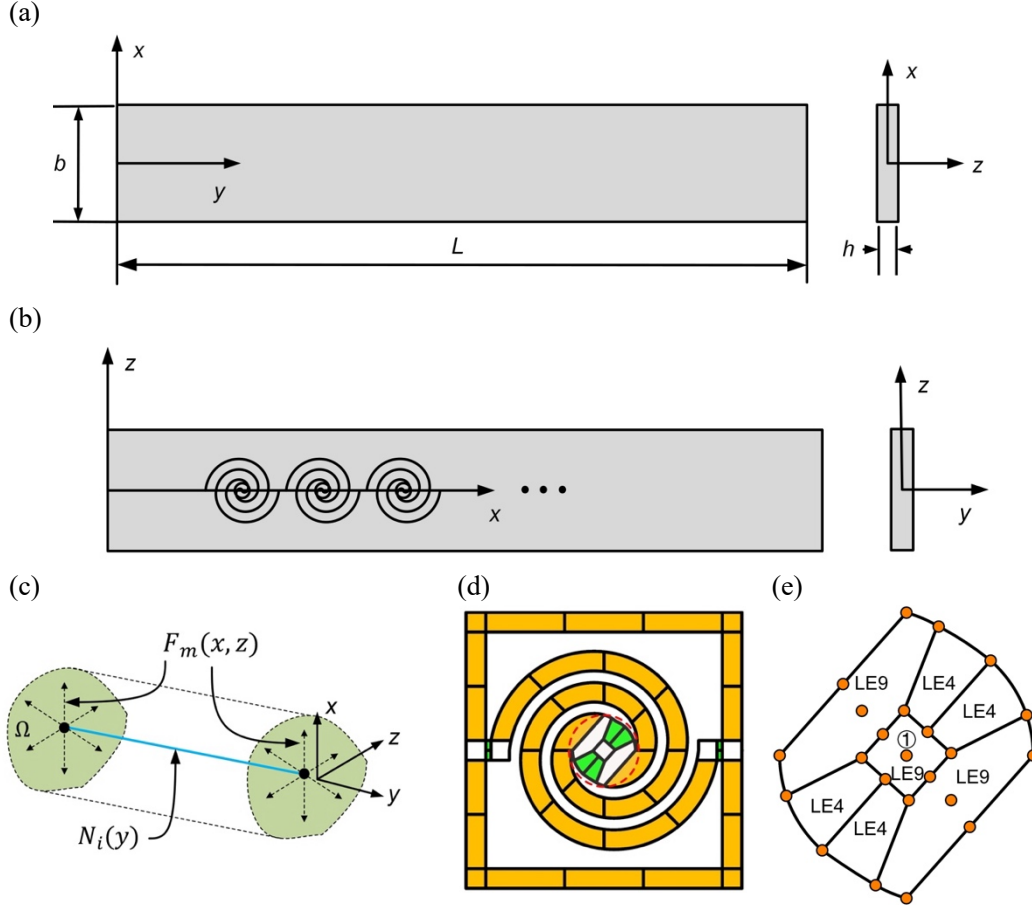


Figure 3.14 Coordinate systems and mathematical models within the CUF framework: (a) coordinate system of the rectangular section beam; (b) coordinate system of HMB and HMBM; (c) mathematical model for the one-dimensional beam; (d) cross-section discretization of the HMB and HMBM unit cells; (e) example of the connection between LE4 and LE9.

Based on the CUF theoretical framework, the differential equation of motion for the free vibration of a beam is given by Equation (2.23). The eigenfrequencies ω and the corresponding mode shapes $\mathbf{u}_0$ can be obtained by solving the eigenvalue problem in Equation (2.24). On the other hand, to calculate the displacement transmission curve of a finite metamaterial structure, a harmonic displacement excitation signal is applied at one edge of the structure (the midpoint of the lower left side), and the displacement response is measured at the other end (the midpoint of the upper right side), as illustrated in Figure 3.15. The structure's displacement transmissibility is given by Equation (2.33).

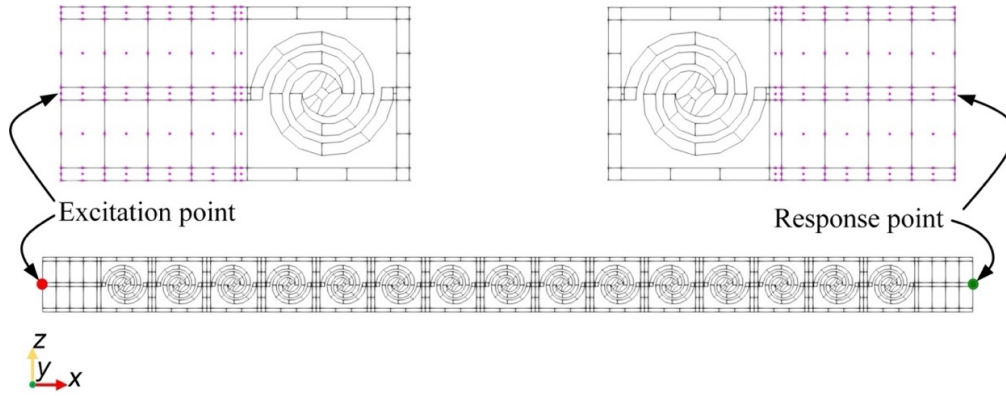


Figure 3.15 CUF model setup for calculating flexural wave displacement transmission in HMB and HMBM.

(1) Natural frequencies and vibration modes of flexural vibration

Example 1: Rectangular cross-section beam

Table 3.3 lists the natural frequencies of the first eight flexural vibration modes for the rectangular cross-section beam. The first column of the table presents the computational model used, the second column indicates the number of degrees of freedom (DOF) for the corresponding model, and columns 3 to 10 list the values of the first eight natural frequencies in sequence. The first ten rows of data display calculation results based on CUF theory, employing different cross-sectional expansion methods and beam element models. The final four rows present results from a COMSOL three-dimensional solid model (with physics-controlled mesh sizes Normal, Fine, and Finer, labeled COMSOL-A, B, and C, respectively) and from calculations based on the Euler-Bernoulli Beam Theory (EBBT).

Within the CUF theoretical framework, the EBBT and TBT expansions correspond to the classical Euler-Bernoulli beam theory and the Timoshenko beam theory (TBT), respectively. Rows 5 through 9 present calculation results obtained when the structural cross-sectional expansion is fixed at 1LE9, using different numbers of finite element discretization patterns. For instance, the 1LE9-20B3 calculation uses 1107 degrees of freedom, while the 1LE9-10B4 calculation uses 837 degrees of freedom, yet its computational accuracy surpasses that of the 20B3 case. It is evident that for a simple rectangular cross-section beam, employing four-node cubic elements (B4) yields better computational results than merely increasing the number of quadratic elements (B3) or linear elements (B2). Due to the simple geometry and mechanical characteristics of this example, the analysis results from both low- and high-order structural models are relatively close to those from the three-dimensional finite element model. The vibration modes obtained from the selected 1LE9-20B4 and COMSOL-C calculation models are displayed in Figures 3.16 and 3.17; it can be observed that the vibration modes calculated by the two methods are highly consistent. Therefore, selecting the 1LE9-20B4 theoretical model is appropriate for calculating the displacement transmission curve of the rectangular cross-section beam.

Table 3.3 Natural frequencies of the flexural modes of the rectangular cross-section beam

Model	DOF	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
EBBT-10B4	279	22.489	61.989	121.52	200.87	300.10	419.34	558.84	719.09
TBT-10B4	279	22.489	61.981	121.48	200.77	299.88	418.89	558.04	717.75
TE1-10B4	279	22.489	61.981	121.48	200.77	299.88	418.89	558.04	717.75
TE2-10B4	558	22.530	62.366	122.90	204.26	306.71	430.45	575.75	743.05
1LE9-10B4	837	22.495	62.044	121.74	201.49	301.48	421.95	563.29	726.12
1LE9-20B2	567	22.595	63.080	126.15	214.33	331.56	483.26	676.71	921.65
1LE9-20B3	1107	22.495	62.047	121.76	201.57	301.71	422.48	564.31	727.76
1LE9-20B4	1647	22.495	62.044	121.74	201.47	301.36	421.53	562.08	723.11
1LE9-40B4	3267	22.495	62.044	121.74	201.47	301.36	421.52	562.06	723.05
3LE9-40B4	7623	22.495	62.042	121.73	201.45	301.31	421.44	562.90	722.79
COMSOL-A	9492	22.499	62.082	121.89	201.91	302.35	423.46	565.47	728.61
COMSOL-B	28257	22.495	62.044	121.74	201.46	301.34	421.47	561.94	722.84
COMSOL-C	57966	22.494	62.04	121.72	201.42	301.24	421.28	561.62	722.32
EBBT	/	22.491	62.00	121.54	200.91	300.12	419.18	558.08	716.82

Figure 3.14(a) presents the finite element model and coordinate system for the rectangular cross-section beam within the CUF framework. The geometric parameters of the beam structure are: thickness $h = 0.006$ m, width $b = 0.07$ m, and length $L = 1.19$ m. The coordinate system is defined as follows: the cross-sectional plane is the xz -plane, and the longitudinal direction is along the y -axis.

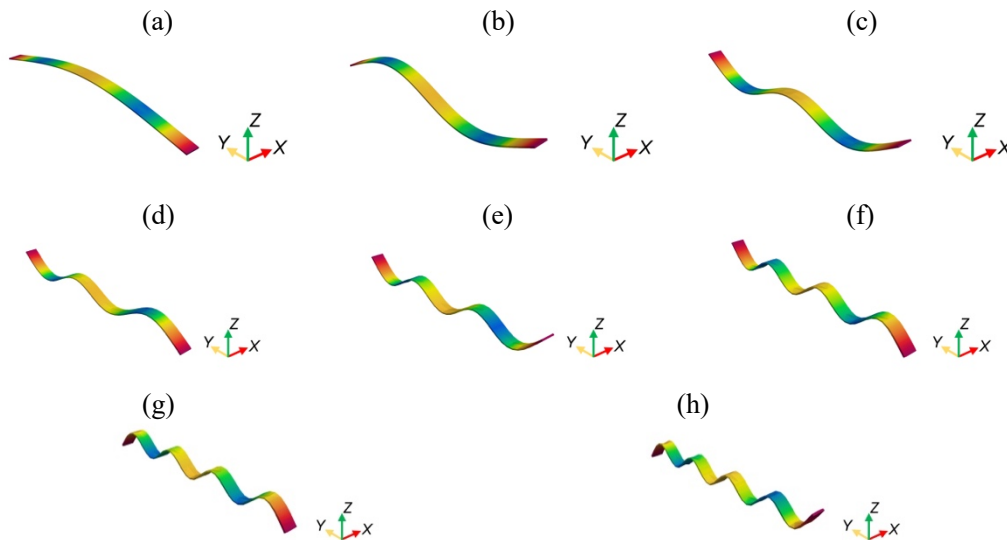


Figure 3.16 First eight flexural modes of straight beam by 1LE-20B4: (a) $f_1 = 22.495$ Hz; (b) $f_2 = 62.044$ Hz; (c) $f_3 = 121.74$ Hz; (d) $f_4 = 201.47$ Hz; (e) $f_5 = 301.36$ Hz; (f) $f_6 = 421.53$ Hz; (g) $f_7 = 562.08$ Hz; (h) $f_8 = 723.11$ Hz.

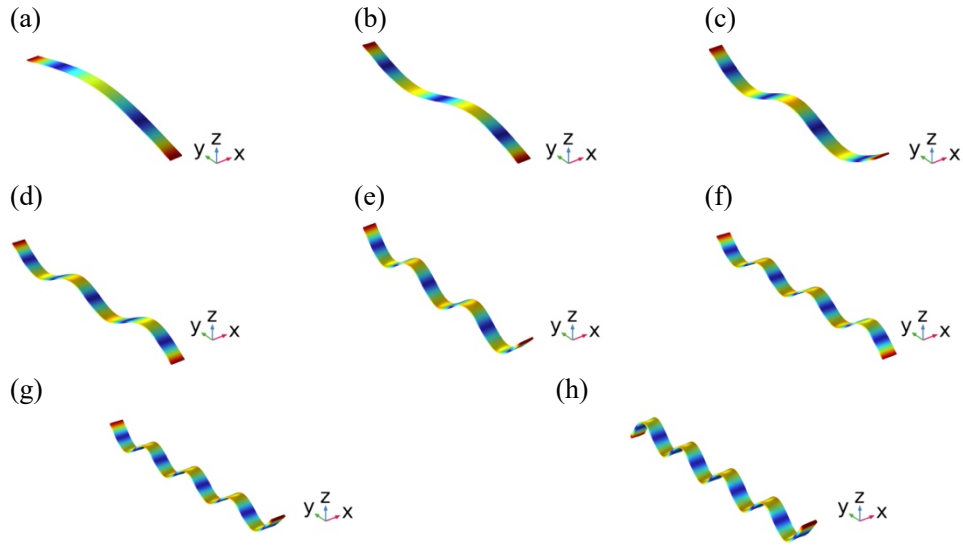


Figure 3.17 First eight flexural modes of rectangular cross-section beam by COMSOL: (a) $f_1 = 22.494$ Hz; (b) $f_2 = 62.04$ Hz; (c) $f_3 = 121.72$ Hz; (d) $f_4 = 201.42$ Hz; (e) $f_5 = 301.24$ Hz; (f) $f_6 = 421.28$ Hz; (g) $f_7 = 561.62$ Hz; (h) $f_8 = 722.32$ Hz.

Example 2: Two types of antisymmetric dual helix metamaterial beams

Table 3.4 presents the results of HMB calculations from various CUF and COMSOL models. The first four rows of results are obtained by Taylor-expanding the cross-section to the sixth and tenth orders. The results indicate that, due to the significant cross-sectional deformation of the dual helix metamaterial beam during bending, the convergence rate of the TE method is very low. The first four flexural frequencies calculated using the TE10-B4 model are nearly twice those obtained from the LE models and the COMSOL results.

Table 3.4 Natural frequencies of flexural modes of HMB

Model	DOF	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
TE6-B4	336	20.156	58.722	121.68	512.52	950.43			
TE10-B2	396	21.353	60.721	123.20	227.89	450.92	1267.6	2830.0	
TE10-B3	594	20.154	57.411	115.67	245.83	516.81	1764.8	2812.1	
TE10-B4	792	20.148	57.386	116.13	233.89	472.21	1430.8	2771.7	
LE9LE4-B2	23304	11.572	33.147	65.920	107.76	154.24	210.45	272.85	324.98
LE9LE4-B3	34956	10.949	31.389	62.495	102.11	146.41	199.97	260.07	311.80
LE9LE4-B4	46608	10.916	31.294	62.286	101.70	145.65	198.74	257.92	307.90
COMSOL-A	53223	11.038	31.135	61.704	102.09	150.96	206.31	264.99	323.22
COMSOL-B	84612	10.849	30.565	60.483	99.709	146.74	199.19	253.80	305.90
COMSOL-C	144138	10.780	30.365	60.039	98.890	145.35	196.93	250.07	300.44

Figure 3.14(d) illustrates the discretization scheme for the cross-section of a unit cell of the dual helix metamaterial beam. The cross-section of each unit cell

structure is divided into 43 LE9 elements and 8 LE4 elements. The connection between the LE9 and LE4 nodes is shown in Figure 3.14(e). Ultimately, the HMB cross-section consists of 681 LE9 and 120 LE4 elements.

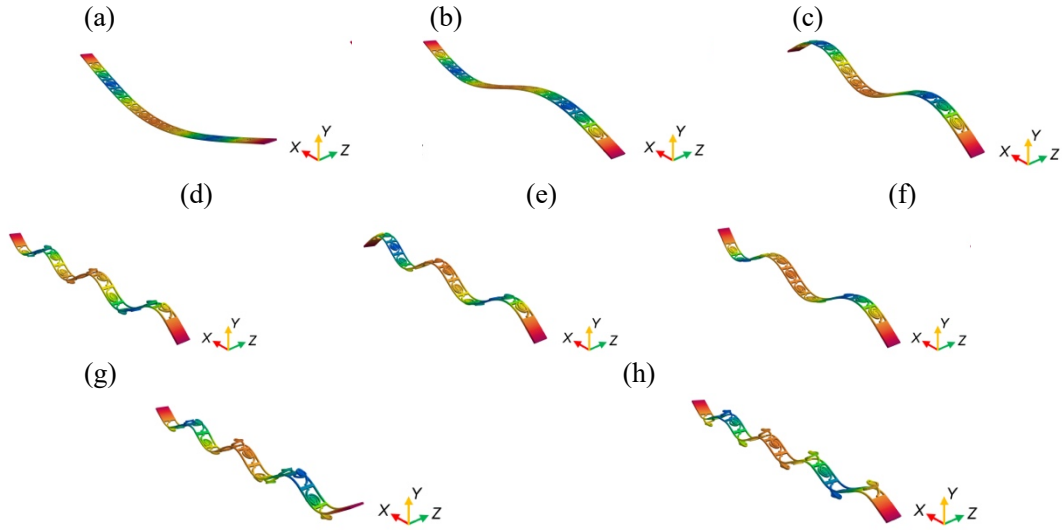


Figure 3.18 First eight flexural modes of HMB by LE4 and LE9: (a) $f_1 = 10.916$ Hz; (b) $f_2 = 31.294$ Hz; (c) $f_3 = 62.286$ Hz; (d) $f_4 = 101.70$ Hz; (e) $f_5 = 145.65$ Hz; (f) $f_6 = 198.74$ Hz; (g) $f_7 = 257.92$ Hz; (h) $f_8 = 307.9$ Hz.

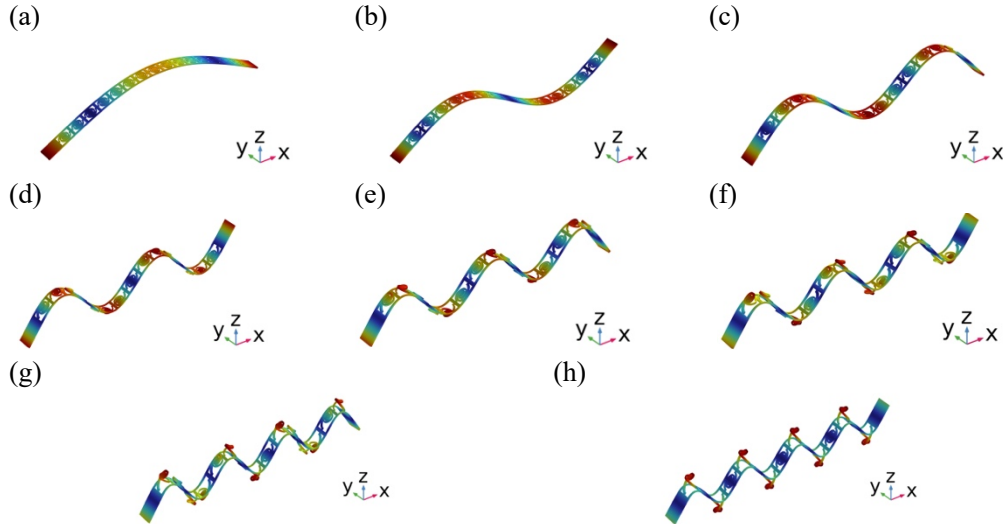


Figure 3.19 First eight flexural modes of HMB by COMSOL: (a) $f_1 = 10.780$ Hz; (b) $f_2 = 30.365$ Hz; (c) $f_3 = 60.039$ Hz; (d) $f_4 = 98.890$ Hz; (e) $f_5 = 145.35$ Hz; (f) $f_6 = 196.94$ Hz; (g) $f_7 = 250.07$ Hz; (h) $f_8 = 300.44$ Hz.

Comparing the frequencies obtained from the LE9LE4-B2 model and COMSOL-A, the LE9LE4-B3 model and COMSOL-B, and the LE9LE4-B4 model and COMSOL-C, the maximum discrepancy between them is less than 2.5%, indicating excellent agreement. Meanwhile, the degrees of freedom required for the CUF calculations are significantly lower than those for the COMSOL three-

dimensional finite element models. Figures 3.18 and 3.19 further validate the excellent agreement between the LE9LE4-B4 model and the COMSOL-C model regarding the first eight flexural vibration modes.

Table 3.5 presents the natural frequencies of the first eight flexural vibration modes for the HMBM. The mass of the concentrated mass sphere is assigned to LE9 element ①, as indicated in Figure 3.14(e). The equivalent density of element ① is calculated to be $\rho_e = 176954 \text{ kg/m}^3$. The results demonstrate that the natural frequencies computed with the LE9LE4 (B2, B3, B4) models are in good agreement with the COMSOL solutions.

Table 3.5 Natural frequencies of flexural modes of HMBM

Model	DOF	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
TE6-B4	336	18.487	53.116	108.56	461.59	862.20			
TE10-B2	396	19.585	54.736	110.04	202.73	402.89	1144.8	2669.9	
TE10-B3	594	18.489	51.755	103.58	210.59	443.97	1309.0	2735.2	
TE10-B4	792	18.477	51.744	104.12	207.95	412.18	1300.6	2611.4	
LE9LE4-B2	23304	10.541	29.579	57.876	93.297	130.66	173.28	217.65	250.38
LE9LE4-B3	34956	9.974	28.011	54.884	88.466	124.25	165.21	208.69	241.78
LE9LE4-B4	46608	9.944	27.925	54.687	88.036	123.40	163.71	206.08	237.33
COMSOL-A	54654	10.178	28.203	55.159	90.050	131.20	176.05	221.40	263.59
COMSOL-B	85209	10.010	27.701	53.974	87.712	126.93	168.87	209.63	245.25
COMSOL-C	169707	9.824	27.113	52.704	85.357	123.02	162.63	200.45	233.08

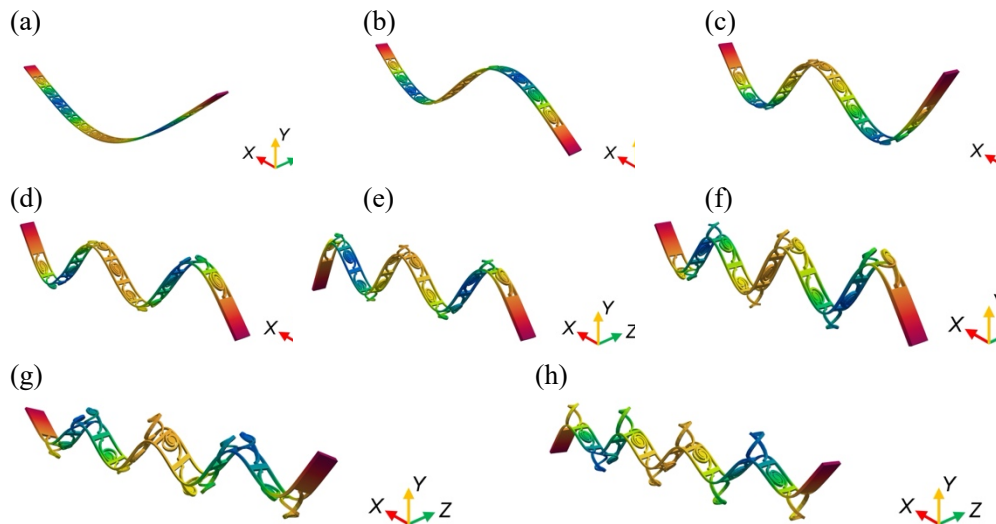


Figure 3.20 First eight flexural modes of HMBM by the LE4 and LE9: (a) $f_1 = 9.944 \text{ Hz}$; (b) $f_2 = 27.925 \text{ Hz}$; (c) $f_3 = 54.687 \text{ Hz}$; (d) $f_4 = 88.036 \text{ Hz}$; (e) $f_5 = 123.40 \text{ Hz}$; (f) $f_6 = 163.71 \text{ Hz}$; (g) $f_7 = 206.08 \text{ Hz}$; (h) $f_8 = 237.33 \text{ Hz}$.

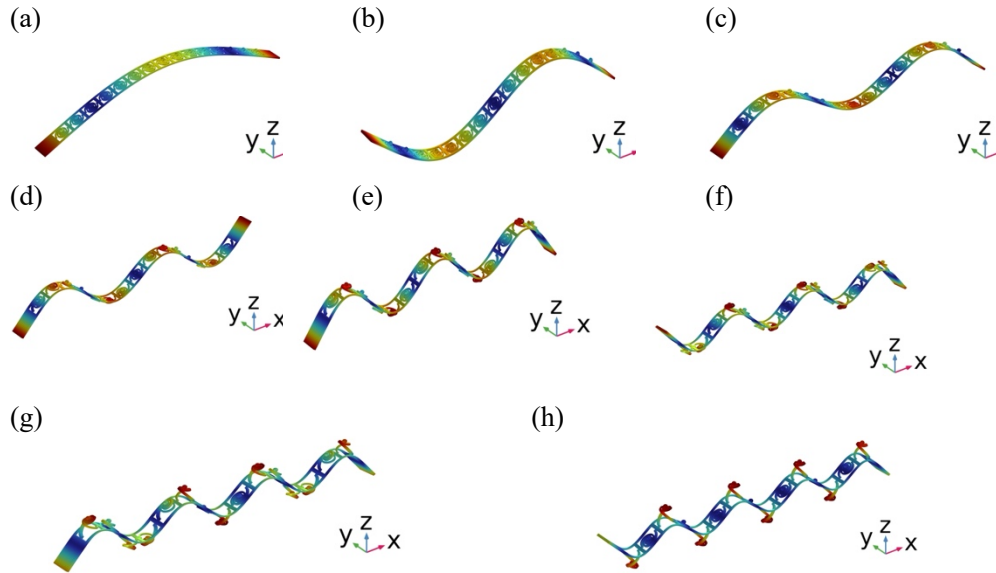


Figure 3.21 First eight flexural modes of HMBM by COMSOL: (a) $f_1 = 9.824$ Hz; (b) $f_2 = 27.113$ Hz; (c) $f_3 = 52.704$ Hz; (d) $f_4 = 85.357$ Hz; (e) $f_5 = 123.02$ Hz; (f) $f_6 = 162.63$ Hz; (g) $f_7 = 200.45$ Hz; (h) $f_8 = 233.08$ Hz.

For ease of comparison, Figures 3.20 and 3.21 show the first eight flexural vibration modes obtained from the LE9LE4-B4 model and the COMSOL-C model, respectively. The figures show that the mode shapes of flexural vibrations obtained by the two methods are highly consistent. Therefore, when calculating the displacement transmission curves for antisymmetric dual helix metamaterial beams, both with and without concentrated mass spheres, the LE9LE4-B4 theoretical model can be used.

(2) Calculation of displacement transmission curves

The displacement transmission curve for the rectangular cross-section beam is computed using the 1LE9-20B4 model, while the LE9LE4-B4 model is employed for the HMB and HMBM. The geometric parameters for the HMB and HMBM are listed in Table 3.1. The displacement transmission is calculated using Equation (2.33). It is emphasized that the CUF calculations adopt a point excitation scheme consistent with the experimental measurement's setup: the excitation point is located at the lower-middle point of the left end face, and the response measurement point at the upper-middle point of the right end face, as illustrated in Figure 3.15. For ease of comparison, the CUF and experimental results are analyzed together.

3.4.2 Experimental verification based on Laser Vibrometry

To verify the reliability of the experimental system, displacement transmission curve measurements were first conducted on a rectangular cross-section beam as a reference specimen, followed by experimental testing on the metamaterial beams. The experimental platform, based on the laser vibrometry system, is shown in Figure 3.22. Figure 3.22(a) displays the experimental setup for the rectangular

cross-section beam, where the red box highlights the custom fixture designed for the piezoelectric stack to ensure stable excitation transmission.

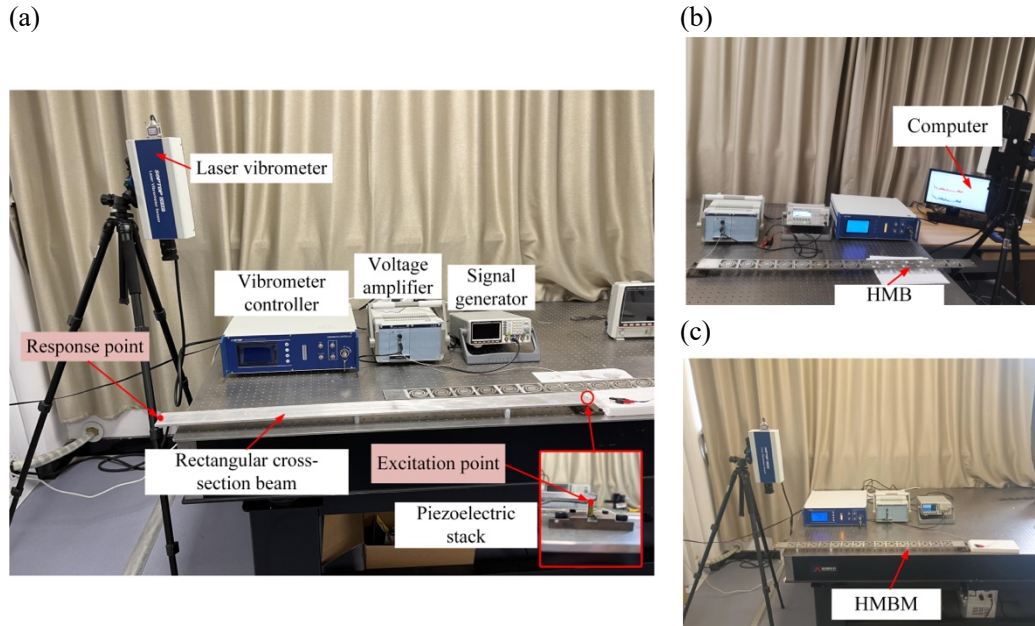


Figure 3.22 Experimental setup based on laser vibrometer system: (a) rectangular cross-section beam; (b) HMB; (c) HMBM.

Figures 3.22(b) and (c) present the experimental testing configurations for the HMB and the HMBM, respectively. During specimen preparation, the unit cells of the HMB and HMBM were bonded together using AB epoxy adhesive, with a curing time of at least 24 hours to ensure a stable connection. The mass spheres were subsequently fixed in place using the same adhesive.

During testing, one end of the beam was freely supported by the piezoelectric stack. Two or three rows (each consisting of three pads) of elastic rubber pads were uniformly arranged along the beam's axis at the bottom to effectively simulate free boundary conditions, ensuring consistency between the experimental and numerical boundary conditions.

In the experiment, a frequency sweep signal generated by a multi-channel signal generator was amplified by a voltage amplifier to drive the multi-layer piezoelectric stack actuator. The piezoelectric stack was fixed using the custom fixture at the midpoint of the beam's right side, serving as the excitation source. The vibration response at the midpoint of the left side of the beam was captured by the laser vibrometry system and transmitted to a computer. It should be noted that the experiment directly collected time-domain velocity signals. After the Fourier transformation, the signals were recorded and output using the specialized analysis software DSA 4.0. Finally, the frequency domain displacement transmission curve was calculated using Equation (2.35).

Figure 3.23 presents a comparison of the flexural wave displacement transmission curves for the rectangular cross-section beam. The geometric dimensions of the aluminum beam used in the experiment match those in the

computational model. In the figure, the blue solid line, green dashed line, and red dotted line represent the results from COMSOL, CUF, and experimental measurement, respectively.

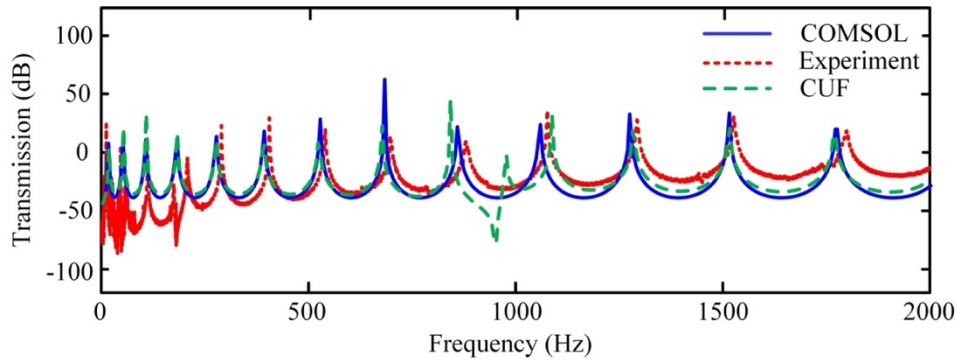


Figure 3.23 Comparison of flexural wave displacement transmission for a rectangular cross-section beam.

The results indicate that the three sets of curves are in good overall agreement, particularly in the frequency range below 900 Hz, where the COMSOL and CUF results overlap almost completely. The only notable difference is the appearance of an anti-resonance peak near 950 Hz in the CUF curve. This phenomenon is directly related to the excitation type used in the numerical calculation. When displacement excitation is employed, the CUF model captures an anti-resonance peak at 950 Hz, caused by the excitation point being located at a modal node. If the excitation method were changed to force excitation, this anti-resonance peak would disappear from the displacement transmission curve.

Figure 3.24(a) compares the displacement transmission curves for the HMB. The light blue shaded areas in the figure indicate the first (BG1: 430–670 Hz) and second (BG2: 1292–1643 Hz) flexural wave bandgaps calculated by COMSOL. The results show that the first bandgap ranges obtained from CUF and the experiment agree well with the COMSOL results. However, the starting frequency of the first bandgap obtained by COMSOL is slightly higher than that from CUF and the experiment, and its bandwidth is marginally narrower. The consistency among the three methods for the frequency range of the second bandgap is less than ideal, with some discrepancies. This is attributed to the fact that, in the finite element calculations, the material is assumed to be linearly elastic and the boundary conditions are set to perfectly free, whereas in the experiment, material nonlinearities and necessary elastic supports are unavoidable.

Figure 3.24(b) displays the full-field displacement distributions of the HMB at selected frequency points (A, 380 Hz; B, 460 Hz; C, 550 Hz; D, 670 Hz; E, 1300 Hz; F, 1400 Hz) obtained from CUF. The results indicate that at points B and C, located within the first bandgap, the vibration energy is highly concentrated in the central region of the first unit cell, and flexural wave propagation is significantly suppressed, suggesting that this bandgap is of the locally resonant type. At point A, situated in the passband, the flexural wave propagates smoothly along the structure.

At point D, located at the cutoff frequency of the first bandgap, vibrations are primarily distributed among the 4th to 8th unit cells of the HMB beam, indicating that flexural wave propagation remains hindered. At points E and F, within the second bandgap, flexural wave propagation is also suppressed, but the energy is concentrated in the spiral arm regions of the first HMB unit cell rather than its center, reflecting the distribution characteristics of higher-order locally resonant modes.

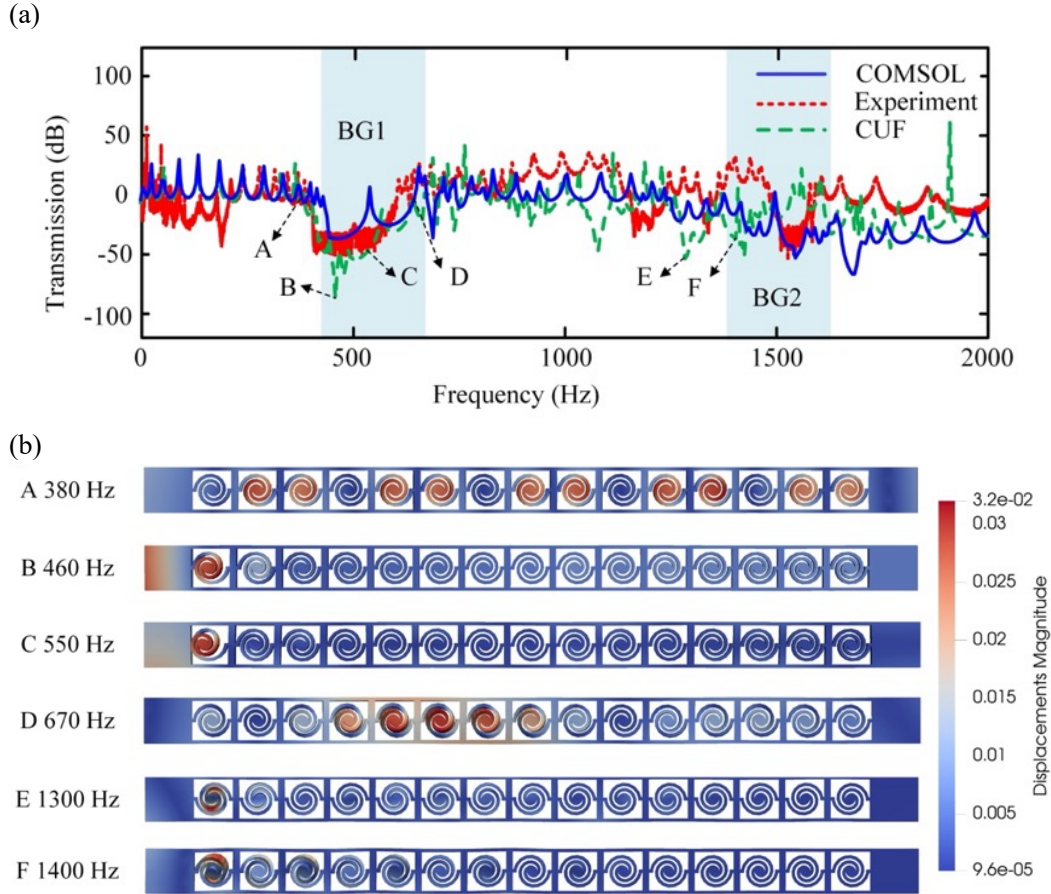


Figure 3.24 Flexural wave displacement transmission and full-field vibration responses of HMB: (a) displacement transmission curve; (b) full-field displacement at characteristic frequency points.

Figure 3.25(a) presents the flexural wave displacement transmission curves for the HMBM. Within the frequency range 0–2000 Hz, the structure exhibits two distinct bandgaps at 301–529 Hz and 1207–1484 Hz. A comparison of the results reveals that the attenuation level (bandgap depth) predicted by the CUF is superior to those obtained from COMSOL simulations and experimental measurements. Regarding the starting frequency of the first bandgap, although the CUF calculation result is slightly higher than the experimental value, it is closer to the experimental data than the COMSOL result, indicating higher predictive accuracy. It is noteworthy that the three methods show good consistency in the starting frequency and width of the second bandgap. This phenomenon can be attributed to the introduction of the concentrated mass sphere, which significantly strengthens and

amplifies the structure's local resonance. This effect not only substantially lowers the bandgap frequency of the HMBM compared to the HMB structure but also effectively increases the bandgap depth, thereby enhancing the wave suppression performance. The differences in peak-trough phases/amplitudes at some frequency points in the figure arise from distinct theoretical modeling frameworks and differing expressions of solution variables; moreover, discrepancies in boundary-condition treatments and degrees-of-freedom truncation methods may lead to slight inconsistencies in the positions of peaks and troughs at local frequencies. Nevertheless, the overall frequency ranges and bandwidths of the bandgaps remain consistent.

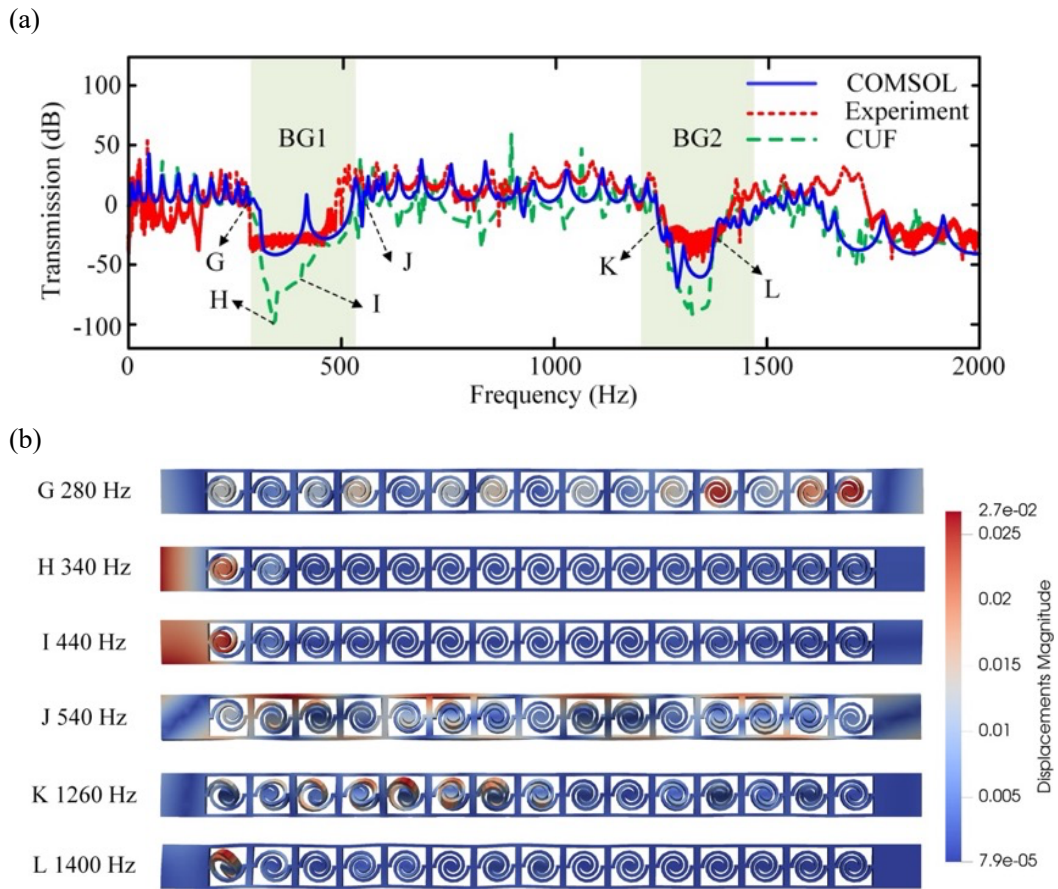


Figure 3.25 Flexural wave displacement transmission and full-field vibration responses of HMBM: (a) displacement transmission curve; (b) full-field displacement at characteristic frequency points.

To further investigate the propagation behavior of flexural waves in the HMBM, Figure 3.25(b) displays the full-field displacement distributions at six characteristic frequency points (G, 280 Hz; H, 340 Hz; I, 440 Hz; J, 540 Hz; K, 1260 Hz; L, 1400 Hz). The results indicate that at the frequency points G, J, and K within the passbands, flexural waves propagate effectively along the structure. In contrast, at points H, I, and L within the bandgaps, a significant wave-suppression effect is observed, with vibration energy highly concentrated in the first unit cell. Specifically, at points H and I, located in the first bandgap, the primary deformation

is concentrated in the central mass sphere and the helical region of the first unit cell. At point L, located in the second bandgap, the deformation energy is concentrated in the spiral arm structure of the first unit cell.

By comparing Figure 3.24(a) and Figure 3.25(a), the evolution of the bandgap can be further quantified: compared to the HMB, the starting frequency of the first bandgap for the HMBM is significantly reduced from 430 Hz to 301 Hz, demonstrating that the coupling effect between the concentrated mass sphere and the dual helix structure enhances the local resonance phenomenon. The width of its first bandgap narrows slightly from 240 Hz to 228 Hz. This phenomenon aligns with the common “frequency-bandwidth” trade-off observed in locally resonant systems: significantly lowering the central bandgap frequency by introducing a concentrated mass often sharpens the resonance peak, resulting in a corresponding narrowing of the absolute bandwidth.

3.5 Summary

Inspired by the nautilus shell, this chapter designed two types of antisymmetric dual helix metamaterial beams, one without and one with concentrated mass spheres, and investigated their low-frequency flexural wave bandgap characteristics using COMSOL, CUF, and laser vibrometry. The main conclusions are as follows:

(1) The band structures, displacement transmission curves, vibration modes, and full-field displacements of the two types of dual helix metamaterial beams were calculated using COMSOL. The results indicate good agreement between the bandgap positions of the infinite periodic band structures and those of the displacement transmission curves of the finite structures, particularly for the low-frequency bandgaps of interest in this study.

(2) The influence of the number of helical turns, the radius of the attached mass sphere, and the unit cell material properties on the bandgap was discussed. The results show that as the number of helical turns increases, the bandgap frequencies of both the HMB and HMBM shift towards lower frequencies, and the bandwidth narrows. As the mass sphere’s radius increases, the bandgap frequency decreases linearly, while the bandwidth remains constant. When the materials for the HMB and HMBM are aluminium, steel, and copper, respectively, the widest bandgap frequency is obtained with steel, and the lowest with copper. The most significant reduction in bandgap frequency occurs when the HMBM matrix material is aluminium, and the attached mass sphere is steel. Analysis of bandgap edge modes reveals that the bandgap formation mechanism in both structures is local resonance; this effect is enhanced by the combined action of the antisymmetric dual helix structure and the mass sphere, resulting in lower-frequency flexural wave bandgaps.

(3) CUF was extended to calculate the natural frequencies of flexural vibration, vibration modes, displacement transmission curves, and full-field displacements for the rectangular cross-section beam and the two types of metamaterial beams. The results demonstrate that CUF can accurately and effectively predict these dynamic characteristics for the helical metamaterial beams. For the rectangular cross-section beam, using either Taylor expansion (TE) or Lagrange expansion (LE) for the cross-

sectional function yields results similar to those from three-dimensional finite element analysis. However, for metamaterial beams with complex cross-sections, such as the HMB and HMBM, the LE model is recommended.

(4) The displacement transmission curves of the antisymmetric dual helix metamaterial beams were validated based on laser vibrometry technology. Experimental results show that the first bandgaps of HMB and HMBM obtained from CUF and COMSOL are in good agreement with the measured values. Good consistency was observed among the three methods (CUF, COMSOL, and experiment) for the second bandgap of the HMBM, although a slight discrepancy existed for the second bandgap of the HMB.

In summary, this chapter proposed two types of antisymmetric dual helix metamaterial beams and conducted an in-depth investigation of their bandgap characteristics. By varying geometric and material parameters, low-frequency and wide-range tunability of the flexural wave bandgap was achieved. The coupling between the concentrated mass sphere and the antisymmetric dual helix structure enhances local resonance, leading to lower-frequency flexural wave bandgaps. Furthermore, a comparison of the displacement transmission curves for the HMB and HMBM, obtained using three methods, ultimately confirms the reliability of the calculation results from both the COMSOL finite element software and CUF, laying the foundation for the research in subsequent chapters.

Chapter 4

Flexural wave bandgap characteristics of 2D dual helix metamaterial plates

4.1 Introduction

In engineering structures, a plate is a three-dimensional component characterized by a thickness significantly smaller than its other two dimensions and a flat geometry. Based on geometric dimensions and mechanical characteristics, plates can be categorized into thick plates, thin plates (including small-deflection rigid plates and large-deflection flexible plates), and membranes. Among these, small-deflection thin plates primarily undergo bending deformation under transverse loads, balancing external forces with internal moments and transverse shear, and exhibit mechanical behavior similar to that of beams. Membranes resist external loads mainly through uniform membrane forces. Large-deflection thin plates fall between these two categories, sustaining loads through a combination of internal moments, shear forces, and membrane forces. These differences in mechanical behavior directly influence the propagation characteristics of elastic waves and the vibration response within the plates.

From the perspective of wave theory, the propagation behavior of flexural waves in plates can be described by classical plate theories. The Kirchhoff thin plate theory, based on the assumption that the plate's cross-sections remain plane before and after deformation and that the stress distribution through the plate's thickness is uniform, establishes governing equations that neglect transverse shear effects, laying the theoretical foundation for wave analysis in thin plates. As plate thickness increases, shear deformation effects become significant. Reissner and Mindlin subsequently developed moderately thick plate theories by introducing independent rotational variables, thereby more accurately accounting for shear deformation and rotary inertia, thereby forming the Reissner-Mindlin theoretical framework applicable to moderately thick plates. These theories not only elucidate the mechanisms of flexural wave propagation, scattering, and mode conversion in plates but also provide a theoretical basis for the design of metamaterial plates for wave manipulation. However, the wave behavior of traditional homogeneous plate structures exhibits a continuous frequency distribution, making it challenging to achieve effective vibration suppression within specific frequency bands. The emergence of elastic metamaterials offers a novel approach to addressing this

challenge. By designing local resonances and periodic structures at subwavelength scales, specific bandgaps can be created to block elastic wave propagation.

The previous chapter investigated the bandgap characteristics of one-dimensional antisymmetric dual helix metamaterial beams, providing preliminary validation of the advantages of helical structures for manipulating flexural waves. Building on this foundation, the present chapter extends the investigation for two-dimensional plate structures, delving into the formation mechanism and tuning laws of flexural wave bandgaps in two-dimensional antisymmetric dual helix metamaterial plates. Specifically, this chapter constructs two types of dual helix metamaterial plates: those without mass spheres (Dual Helix Metamaterial Plate without mass sphere, NM-DHMP) and those with mass spheres (Dual Helix Metamaterial Plate with mass sphere, M-DHMP). Using COMSOL, their band structures, band edge modes, displacement transmission curves, and full-field displacements are calculated. Furthermore, the influence of parameters such as cross-sectional geometry, number of helical turns, helix width, helical orientation, plate thickness, and mass sphere radius on the flexural wave bandgap characteristics of two-dimensional dual helix metamaterial plates is discussed in detail.

4.2 Unit cell design and bandgap calculation

4.2.1 Unit cell modelling and FE solution

This subsection describes the unit cell structures of the two dual helix metamaterial plates. The modelling procedure and key geometric parameters follow those established in Section 3.2.1. However, two critical modifications have been implemented for the plate configurations, as illustrated in Figure 4.1. First, the number of helical turns is set to $N = 3.5$, where a turn is defined as a 180° rotation back to the starting axis, rather than the conventional 360° . Second, to elaborate, the equations governing each spiral have been uniformly adjusted to ensure that all connections between the core structure and the outer frame are consistently oriented horizontally.

After the adjustment, the equations for the four helical curves H_L^0, H_L^1, H_R^0 , and H_R^1 are:

$$x_L^0 = s \cos\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right), y_L^0 = s \sin\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right) \quad (4.1a)$$

$$x_L^1 = (s + w) \cos\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right), y_L^1 = (s + w) \sin\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right) \quad (4.1b)$$

$$x_R^0 = -s \cos\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right), y_R^0 = -s \sin\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right) \quad (4.1c)$$

$$x_R^1 = -(s + w) \cos\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right), y_R^1 = -(s + w) \sin\left(\frac{Ns\pi}{r_2} + \frac{\pi}{2}\right) \quad (4.1d)$$

Floquet-Bloch periodic boundary conditions are applied to the four sides of the unit cell along the x - and y -directions. As illustrated in Figure 4.1(c), periodic

constraints are imposed on the left/right boundaries (solid blue lines) and the upper/lower boundaries (red dashed lines), while all remaining boundaries are left free. The band structure is obtained by sweeping the wave vector along the ΓX , XM , and $M\Gamma$ paths in the first Brillouin zone (Figure 4.1(d)) and solving for the system's eigenfrequencies.

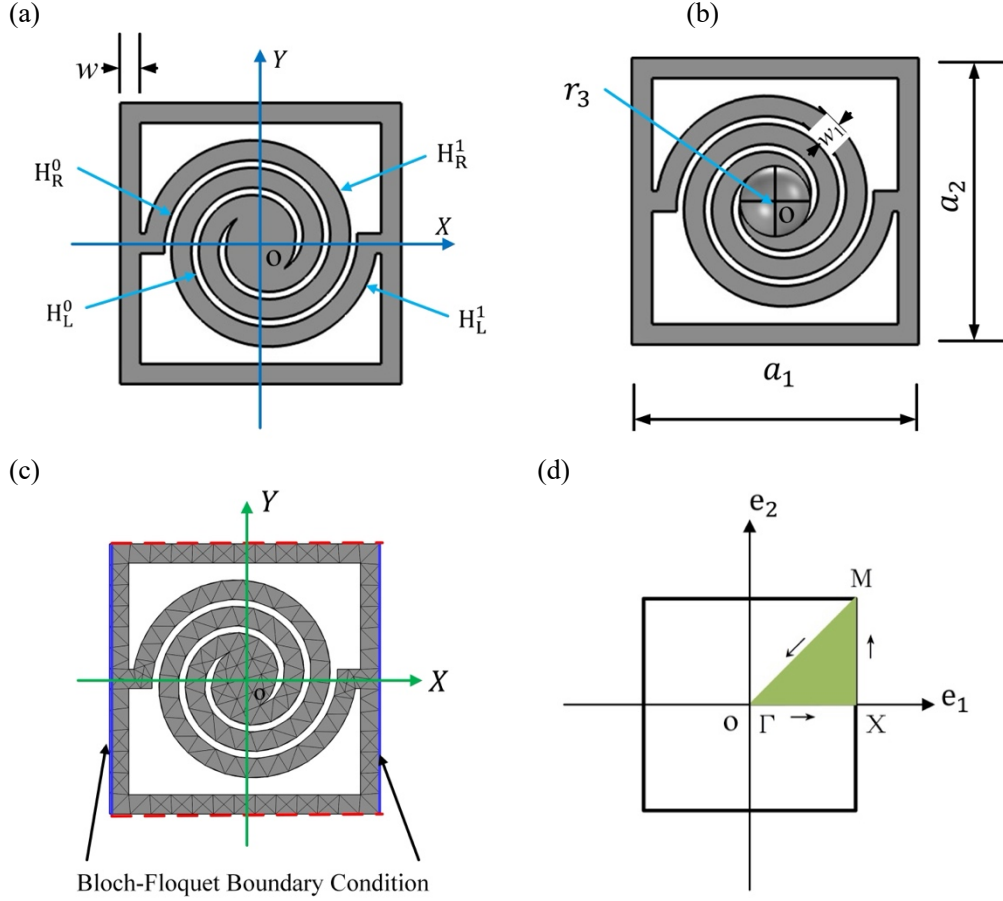


Figure 4.1 Configurations and analysis models of the dual helix metamaterial plate. Unit cell structures: (a) NM-DHMP (without mass sphere); (b) M-DHMP (with mass sphere). Computational models: (c) unit cell with Bloch-Floquet boundary conditions for the band structure calculation; (d) first Brillouin zone and its irreducible part.

To evaluate the wave response of finite periodic structures, both point and line excitations are applied in the frequency domain to the two-dimensional metamaterial plates. The displacement transmission coefficient T is then calculated using Equation (2.33). To accurately model wave propagation in an unbounded domain, low-reflecting boundary conditions are employed around the finite structure to eliminate non-physical reflections. All numerical simulations use physics-controlled tetrahedral meshes, with the mesh size refined via convergence analysis to ensure accuracy. Unless otherwise stated, the attached mass spheres in the structures investigated in this chapter are made of steel, while the base material

for all other components is aluminum. The material parameters are listed in Table 3.2.

4.2.2 Band structure and bandgap edge mode analysis

The geometric parameters used in this subsection are: $a_1 = a_2 = 70$ mm, $h = 6$ mm, $w = 5$ mm, $r_0 = 8$ mm. The helical parameters are $r_1 = 0$ mm, $r_2 = 24$ mm, and $N = 3.5$, with a steel sphere radius of $r_3 = 8$ mm. The dispersion relations, i.e., the band structures, are obtained by sweeping the wave vector along the ΓX , XM , and $M\Gamma$ directions. The band structures for the NM-DHMP and M-DHMP are shown in Figures 4.2(a) and 4.2(b), respectively. Figure 4.2(a) indicates that within the frequency range of 0–2000 Hz, the NM-DHMP exhibits two complete bandgaps, located at 408–596 Hz and 1199–1322 Hz, which are highlighted in red. Figure 4.2(b) shows that within the 0–2000 Hz frequency range, the M-DHMP possesses three complete bandgaps: 295–486 Hz, 1079–1207 Hz, and from 1950 Hz to 2000 Hz, with the first two complete bandgaps shaded in blue. The bandwidth of the first complete bandgap is significantly larger than that of the second bandgap for both the NM-DHMP and M-DHMP configurations. The introduction of mass spheres significantly lowers the frequency of the first bandgap, a trend consistent with the findings from the antisymmetric dual helix metamaterial beams, validating the universal tuning effect of concentrated mass spheres on locally resonant bandgaps. Further analysis reveals a unique tuning characteristic of the plate structure: while achieving a lower bandgap frequency, the mass spheres also cause a slight broadening of the bandwidth. This phenomenon, unlike that observed in the beam structure, unveils a new coupling mechanism between the two-dimensional in-plane constraints and the mass spheres, leading to a synergistic enhancement of bandgap performance through an optimized modal energy distribution. This indicates that the geometric dimension plays a crucial role in transitioning from one-dimensional to two-dimensional metamaterial systems, offering new insights for the design of low-frequency and broadband bandgap control.

According to the Bragg condition, the lowest Bragg bandgap frequency is calculated using the following formula [49]:

$$f_{B,n} = \frac{1}{2\pi} \left(\frac{n\pi}{a} \right)^2 \sqrt{\frac{D}{\rho h}} \quad (4.2)$$

where a is the lattice constant of the periodic system, h is the plate thickness, and D is the bending stiffness of the plate, given by $D = Eh^3 / [12(1 - \nu^2)]$. The calculated lowest Bragg bandgap frequency is $f_{B,1} = 3037.43$ Hz. A careful comparison of the first and second complete bandgap frequencies in the dispersion relations of both the dual helix metamaterial plate without mass spheres and with mass spheres reveals that these bandgap frequencies are significantly lower than the Bragg scattering frequency. Therefore, it can be concluded that the bandgaps observed here are not Bragg bandgaps.

The right side of Figure 4.2(a) displays the bandgap edge modes for the NM-DHMP, where I, II, III, and IV denote the mode shapes at the opening and closing frequency points of the first and second bandgaps, respectively. From the figure, it can be observed that for the first complete bandgap of the NM-DHMP, the mode shapes at both the opening and closing frequencies are dominated by vibrations of the central helix and the circular region. In contrast, the vibration modes at the opening and closing frequencies of the second complete bandgap are characterized by a rotational vibration around the central helical portion.

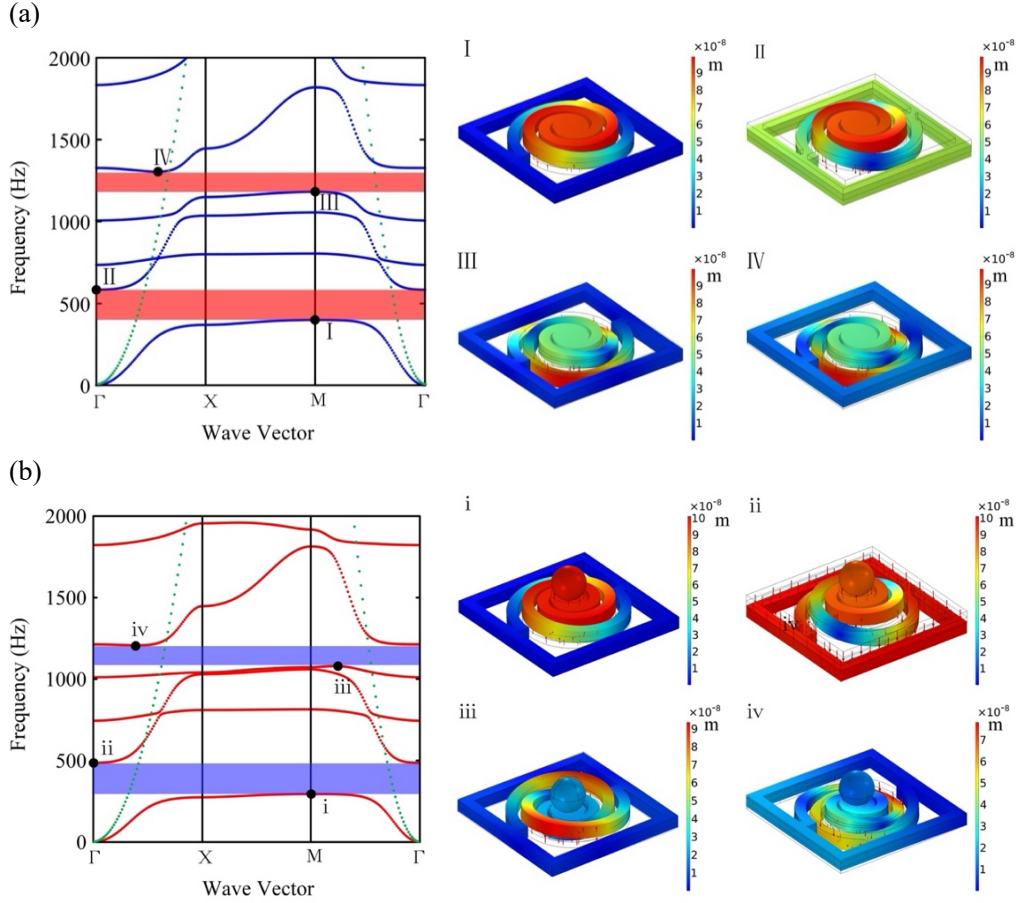


Figure 4.2 Band structure and bandgap edge vibration modes for the first two complete bandgaps of: (a) NM-DHMP (I $k_1 = 2.01$, $f_1 = 408.03$; II $k_2 = 0$, $f_2 = 595.60$; III $k_3 = 2.01$, $f_3 = 1199.5$; IV $k_4 = 0.57$, $f_4 = 1322.1$); (b) M-DHMP (i $k_1 = 2.01$, $f_1 = 294.83$; ii $k_2 = 0$, $f_2 = 485.67$; iii $k_3 = 2.22$, $f_3 = 1079.4$; iv $k_4 = 0.405$, $f_4 = 1207.1$).

The right side of Figure 4.2(b) presents the bandgap edge modes for the M-DHMP, where i, ii, iii, and iv denote the mode shapes at the opening and closing frequency points of the first and second bandgaps, respectively. The results indicate that for the first complete bandgap of the M-DHMP, the mode shape at the opening frequency is dominated by vibrations of the central helix and the concentrated mass sphere, while the vibration at the cutoff frequency is concentrated on the surrounding frame. The vibration modes at the opening and closing frequencies of

the second complete bandgap are similar to those of the NM-DHMP, characterized by a rotational motion around the central helical structure. The arrows in the mode shape figures indicate the direction of vibration, with their lengths representing the relative magnitudes of the vibrational displacements. From these band-edge modes, it can be concluded that both the first and second bandgaps for the two plate configurations are locally resonant bandgaps.

For comparison, the dispersion relations of a plain plate and a plain plate with equivalent attached mass spheres were calculated, with the results marked by green dots in Figure 4.2. The figure shows that neither case exhibits any bandgaps within the investigated frequency range.

4.2.3 Displacement transmission characteristics

To investigate the wave transmission characteristics of finite structures, displacement transmission curves are calculated for 10×10 unit cell configurations of the dual helix metamaterial plates, both with and without mass spheres, as shown in Figure 4.3. Low-Reflecting Boundary Conditions (LRB) are applied within the COMSOL Solid Mechanics module to simulate an infinite domain and prevent boundary reflections. Two excitation schemes are considered: point excitation at the lower-left corner (LL) with response measured at the upper-right corner (RU) (Figure 4.3a), and line excitation applied along the entire lower-left edge with the response characterized by the average displacement along the upper-right edge (Figure 4.3b).

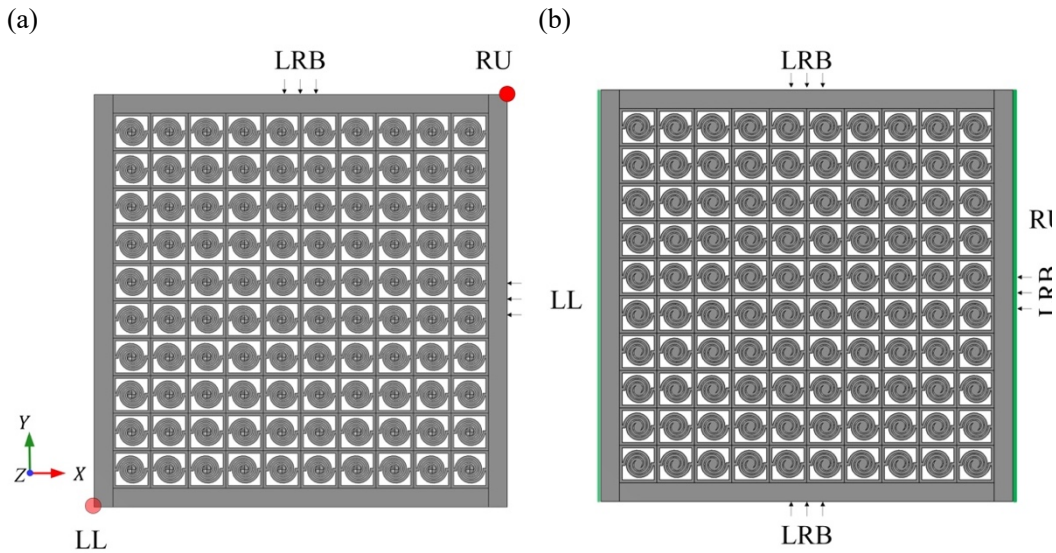


Figure 4.3 Structural model used for calculating displacement transmission spectra: (a) point excitation; (b) line excitation.

The transmission curve under point excitation is shown in Figure 4.4(b). To better observe the flexural wave propagation characteristics, four characteristic frequency points, A (241 Hz), B (511 Hz), C (1321 Hz), and D (1681 Hz), are selected to display their full-field displacements. The transmission curve under line

excitation is presented in Figure 4.4(c). Six characteristic frequency points, E (271 Hz), F (521 Hz), G (961 Hz), H (1321 Hz), I (1571 Hz), and J (1771 Hz), are chosen to illustrate their corresponding full-field displacements.

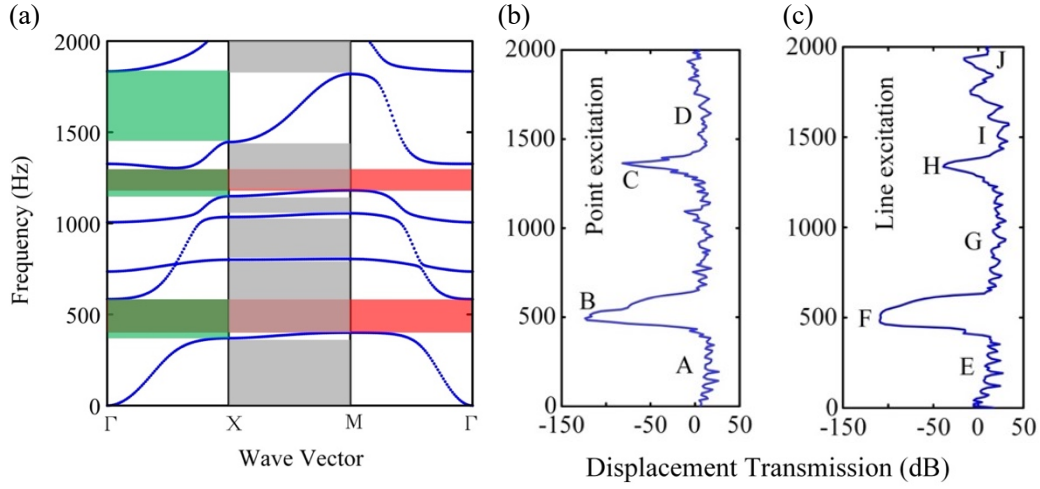


Figure 4.4 Band structure and displacement transmission spectra of NM-DHMP: (a) band structures; (b) displacement transmission along ΓM direction; (c) displacement transmission along ΓX direction.

Figure 4.5 presents the full-field displacement results. The figure shows that at the frequency points A, D, E, and G, flexural waves propagate effectively within the structure. In contrast, at frequency points B, C, F, H, I, and J, flexural waves are significantly suppressed, as these points lie within complete or directional bandgap frequency ranges.

At point A, with the excitation point located at the diagonal corner, the flexural wave propagates uniformly from the lower-left corner to the upper-right corner along the diagonal direction. At point D, flexural wave propagation is also observed, although the propagation path is relatively complex. Point B lies within the first locally resonant bandgap; here, flexural waves are markedly suppressed. Propagation is confined to the boundaries along the two edges connected to the excitation point, while no wave propagates through the interior of the finite metamaterial plate. Point C is situated within the second locally resonant bandgap. The excited flexural wave can only be observed within a few unit cells adjacent to the excitation point, with the remainder of the metamaterial plate remaining nearly motionless.

At points E and G, under line excitation, flexural waves propagate from left to right, exhibiting a relatively uniform overall propagation path. Points F and H, located within the first and second flexural wave bandgaps, respectively, show that elastic wave propagation is restricted to the first- and second-unit cells near the excitation line. The vibration is predominantly concentrated in the first unit cell, indicating that flexural waves are significantly inhibited. Points I and J lie within the ΓX directional bandgap. With the excitation line on the left side and the response

line on the right side, a clear bandgap phenomenon is observed: flexural waves fail to propagate from the left side to the right side.

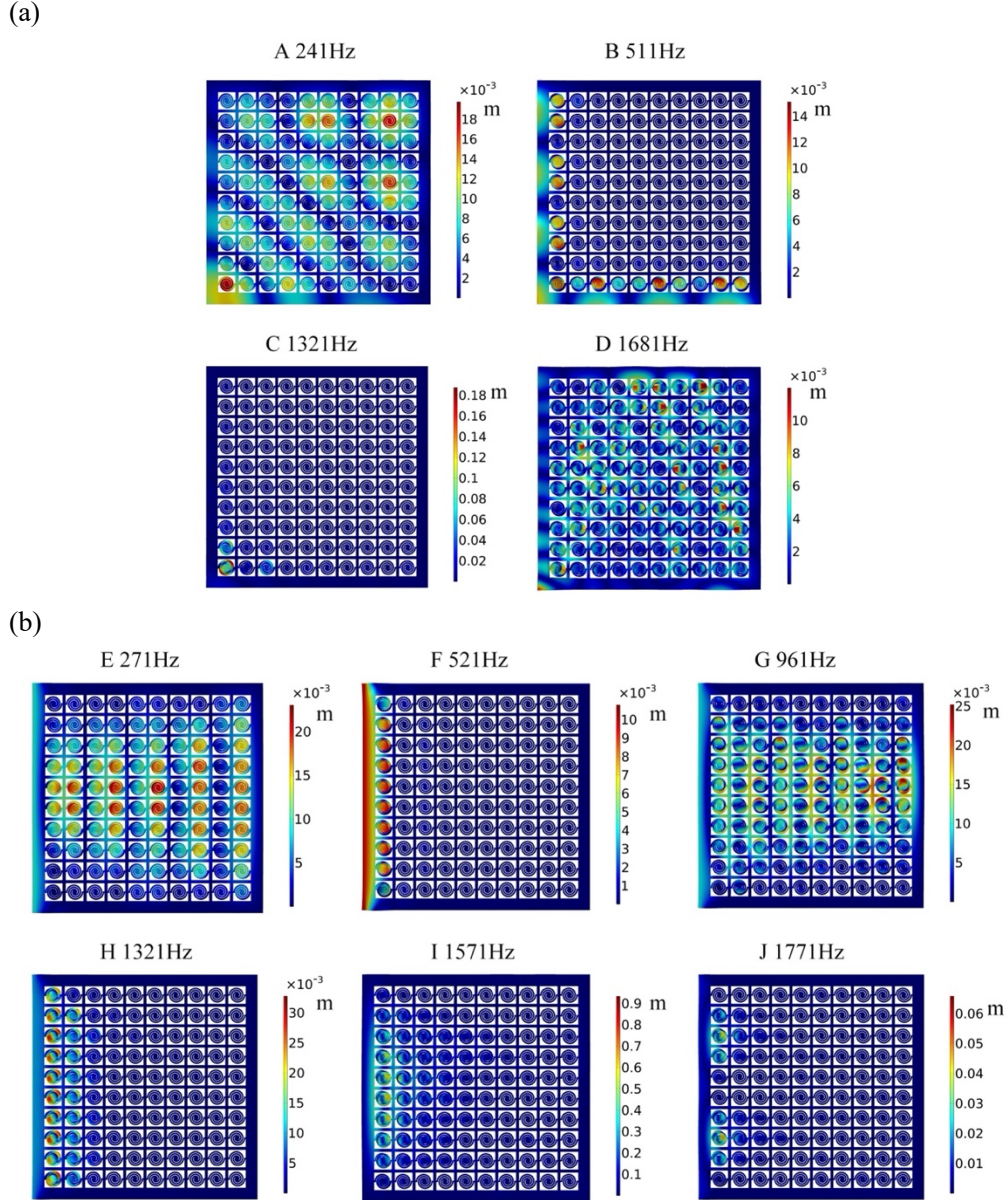


Figure 4.5 Full field displacement of NM-DHMP: (a) under point excitation at A, B, C, D; (b) under line excitation at E, F, G, H, I, J.

It is evident that the wave propagation phenomena revealed by the full-field displacements are in excellent agreement with the bandgap predictions obtained from the band structure analysis.

Figure 4.6 presents the band structure of the infinite metamaterial plate with mass spheres alongside the displacement transmission curves of its finite counterpart. Specifically, Figures 4.6(a), 4.6(b), and 4.6(c) display the band structure, the displacement transmission curve along the ΓM direction, and that along the ΓX direction, respectively. Consistent with the observations for the metamaterial plate without mass spheres, Figure 4.6(a) reveals significant

variations in the number, frequency distribution, and width of bandgaps across different wave vector directions. Along the ΓX direction, four bandgaps are observed, with frequency ranges of: 275–486 Hz, 1040–1208 Hz, 1447–1822 Hz, and 1917–2000 Hz. Along the XM direction, six bandgaps exist, covering: 0–370 Hz, 400–800 Hz, 804–1035 Hz, 1055–1149 Hz, 1181–1446 Hz, and 1819–2000 Hz. In the $M\Gamma$ direction, the first and second bandgaps coincide with the complete bandgaps, while the third bandgap is slightly wider than the third complete bandgap. Furthermore, the incorporation of mass spheres leads to a substantial reduction in the frequencies of the first and second complete bandgaps compared to the plate without mass spheres. Additionally, a third complete bandgap is observed near 2000 Hz; however, the present study focuses primarily on low-frequency bandgaps.

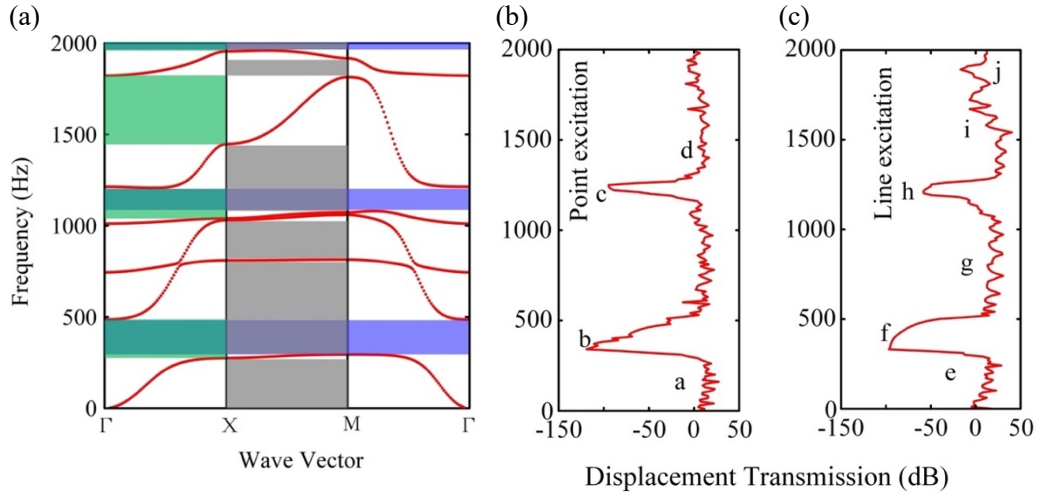


Figure 4.6 Band structure and displacement transmission spectra of M-DHMP: (a) band structures; (b) displacement transmission along ΓM direction; (c) the displacement transmission along ΓX direction.

Similarly, for the dual helix metamaterial plate with mass spheres, four characteristic frequency points, a (131 Hz), b (381 Hz), c (1171 Hz), and d (1401 Hz), are selected under point excitation, and six characteristic frequency points, e (151 Hz), f (391 Hz), g (761 Hz), h (1151 Hz), i (1641 Hz), and j (1961 Hz), are selected under line excitation to illustrate the corresponding full-field displacement distributions.

The full-field displacements are presented in Figure 4.7. The results indicate that at frequency points a, d, e, g, and j, flexural waves propagate effectively, whereas at points b, c, f, h, and i, flexural waves are significantly attenuated due to their location within complete or directional bandgaps. In Figure 4.7, at point a, the flexural wave propagates uniformly from the lower-left corner to the upper-right corner. At point d, the propagation path of the flexural wave is relatively scattered. Unlike the flexural wave propagation behavior observed in the dual helix metamaterial plate without mass spheres, at point b, the flexural wave propagates along only one boundary connected to the excitation point, with no wave propagation detected along the other boundary.

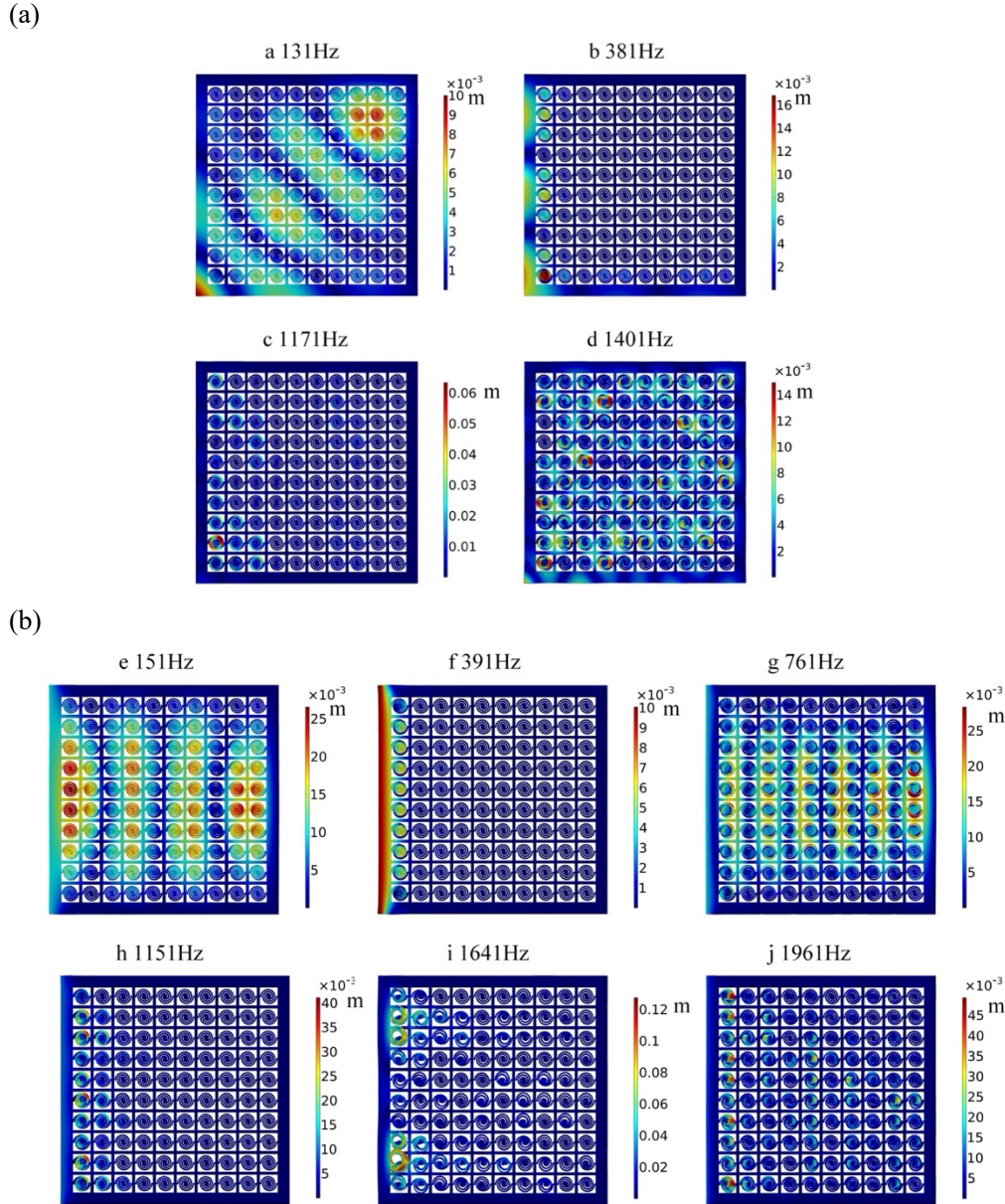


Figure 4.7 Full field displacement of M-DHMP: (a) under point excitation at a, b, c, and d; (b) under line excitation at e, f, g, h, i, and j.

At points f and h, which lie within the frequency range of the complete bandgap, noticeable vibration can only be observed in the first- and second-unit cells adjacent to the excitation line, while the remaining regions of the metamaterial plate remain nearly motionless. Point i, located within a directional bandgap, also exhibits suppressed elastic wave propagation. In contrast, at points e, g, and j, flexural waves propagate freely. The above results demonstrate that, for both the dual helix metamaterial plates with and without mass spheres, the bandgaps predicted by the flexural wave band structures obtained from COMSOL calculations are in good agreement with those reflected by the displacement transmission curves in terms of both location and range.

4.3 Parametric influence analysis

4.3.1 Effect of cross-section shapes

To gain deeper insight into the mechanism of flexural wave bandgaps in dual helix metamaterial plates, this section systematically investigates five unit-cell configurations. These configurations share the same cross-sectional area as the antisymmetric dual helix unit cell shown in Figure 4.1(a), but feature distinct cross-sectional shapes. Their bandgap characteristics are also analysed after incorporating the mass spheres illustrated in Figure 4.1(b). The five-unit cell configurations are presented in Figures 4.8(a)–4.8(e). In Figure 4.8(f), the outer frame is omitted to highlight the helical region with a width of w_1 .

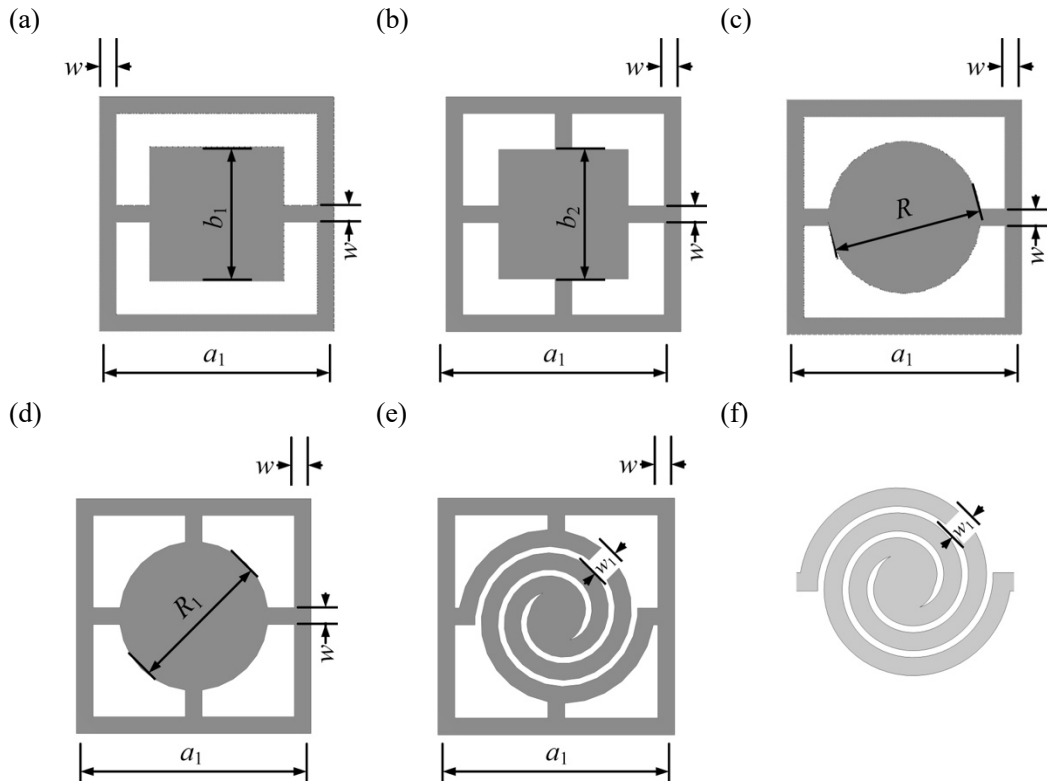


Figure 4.8 Configuration metamaterial plates with different cross-sectional shapes: (a) square; (b) four-point square; (c) circular; (d) four-point circular; (e) four-point NM-DHMP; (f) dual helix structure (outer frame omitted to show helix width w_1).

These five metamaterial plates share the same thickness as the antisymmetric dual helix metamaterial unit cell shown in Figure 4.1(a), i.e., $h = 6$ mm. As illustrated in Figures 4.8(a)–4.8(e), the geometric parameters for the unit cell configurations of the different metamaterial plates are as follows: $a_1 = 70$ mm, $w = 5$ mm, $D = 45.56$ mm, $D_1 = 44.36$ mm, $b_1 = 40.03$ mm, $b_2 = 38.576$ mm; the helical parameters are: $r_1 = 0$ mm, $r_2 = 24$ mm, $N = 3.5$, $w_1 = 0.9735w$. Here, the parameter w_1 represents the width of the helical structure, as defined in Figure 4.8(f). For

brevity, w_1 is referred to as the “helix width” hereafter. Unless otherwise specified, w_1 is taken to be equal to w by default, meaning the helix width is consistent with the width of the surrounding frame.

Figure 4.9 presents the infinite periodic band structures of metamaterial plates with different cross-sectional shapes, all with the same volume.

As shown in Figure 4.9(a), when the dual helix in the NM-DHMP unit cell is replaced with a square, the resulting square metamaterial plate exhibits a complete bandgap within the 0–5000 Hz frequency range. This bandgap, located between the 3rd and 4th bands, spans 3279–3444.3 Hz, with a bandwidth of 165.3 Hz. In contrast, when the central square is connected to the surrounding frame on all four sides, forming a four-point square metamaterial plate as shown in Figure 4.9(b), no complete bandgap exists in the structure. This absence is attributed to the four-way connections, which significantly enhance structural stiffness and thereby suppress the bandgap originally opened by the core’s rotational vibration.

Similarly, a circular metamaterial plate (Figure 4.9(c)), obtained by replacing the dual helix with a circle, generates a flexural wave bandgap within the 0–5000 Hz range. This bandgap lies between the 3rd and 4th bands, spanning 3334.9–3527.9 Hz and 193 Hz wide. However, when the central circle is connected to the outer frame via four-way linkages, forming a four-point circular metamaterial (Figure 4.9(d)), the sole complete bandgap present in the circular metamaterial plate disappears. This phenomenon can be attributed to the increased system stiffness introduced by the connecting structures.

Figure 4.9(e) shows the band structure of the NM-DHMP dual helix metamaterial plate within the 0–5000 Hz range, revealing four distinct bandgaps. The first complete bandgap, located between the 1st and 2nd bands, spans 408.03–595.6 Hz, with a bandwidth of 187.57 Hz. The second complete bandgap, located between the 4th and 5th bands, spans 1199.5–1322.1 Hz, with a bandwidth of 123 Hz. The third complete bandgap, between the 6th and 7th bands, covers 2124.74–2200.88 Hz, with a bandwidth of 76.14 Hz. The fourth complete bandgap, located between the 7th and 8th bands, spans 2933.47–3219.75 Hz, with a bandwidth of 286.28 Hz.

When the antisymmetric dual helix structure is connected to the outer frame on all four sides, forming the four-point NM-DHMP configuration, the resulting band structure is shown in Figure 4.9(f). Compared to the NM-DHMP, the original four bandgaps are reduced to two: one ranging from 623.99 to 823.79 Hz (bandwidth 199.8 Hz) and another from 2529.19 to 2559.14 Hz (bandwidth 29.95 Hz). This indicates that the four-point connection not only shifts the bandgap frequencies toward higher values but also suppresses the rotational motion of the central helix due to enhanced structural constraints, leading to the disappearance of bandgaps originally generated by this motion.

The above results demonstrate that the geometric configuration of the unit cell plays a decisive role in bandgap formation, while the number of connecting structures directly modulates the positions and number of bandgaps. Notably, the antisymmetric dual helix structure is conducive to the formation of complete bandgaps, particularly in the low-frequency region.

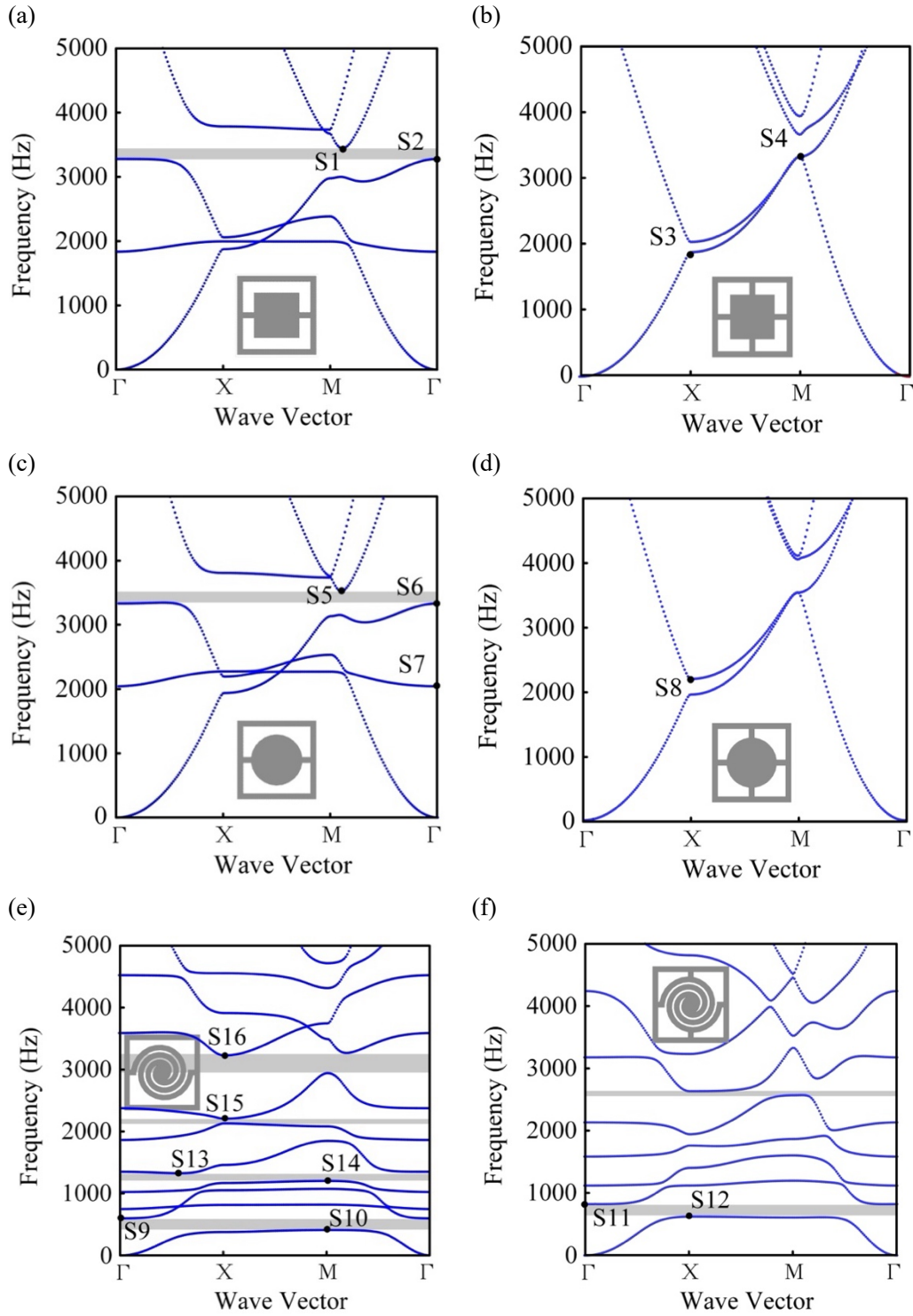


Figure 4.9 Band structures of metamaterial plates with different cross-sectional shapes: (a) square; (b) four-point square; (c) circular; (d) four-point circular; (e) NM-DHMP; (f) four-point NM-DHMP.

To better understand the mechanisms underlying bandgap formation, Figure 4.10 presents the vibration modes corresponding to the marked points in Figure 4.9. The modes at points S1, S2, S5, and S6 reveal that the lowest complete bandgaps

of the square and circular metamaterial plates are Bragg scattering bandgaps. Their frequency ranges are also situated precisely near the lowest Bragg bandgap frequency of 3037.43 Hz calculated using Equation (4.2). In contrast, the mode shapes at points S9, S10, S11, and S12 indicate that the lowest bandgaps of the dual helix metamaterial plate without attached mass spheres and the four-point NM-DHMP metamaterial plate are locally resonant bandgaps. Points S13, S14, S15, and S16 indicate that the second bandgap of the dual helix metamaterial plate without attached mass spheres is also a locally resonant bandgap, primarily generated by the rotational motion of the dual helix. The third bandgap is a hybrid bandgap that transitions from local resonance to Bragg scattering. The fourth bandgap is generated by Bragg scattering. Points S3 and S4 represent the vibration modes of the 1st and 2nd bands of the four-point square metamaterial plate, while points S7 and S8 correspond to the vibration modes of the 2nd band of the circular metamaterial plate and the four-point circular metamaterial plate; all of these modes are classified as flexural vibrations.

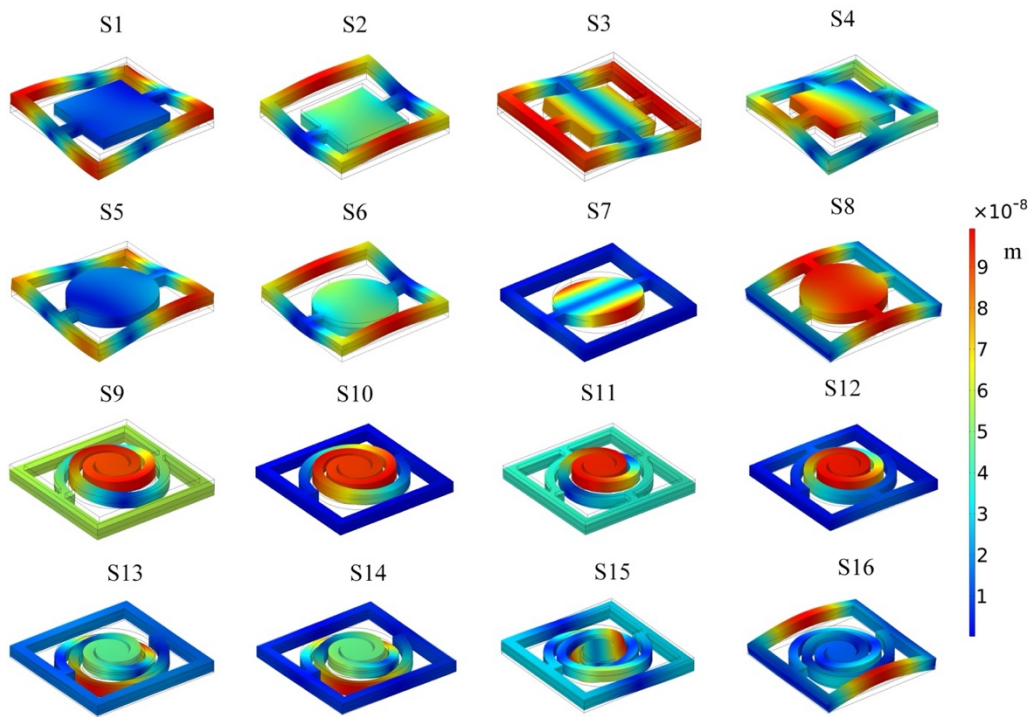


Figure 4.10 Vibration modes at selected frequency points for various unit cell shapes.

The wavenumber and frequencies corresponding to the marked points on the band structures of the metamaterial plates with different cross-sectional shapes are listed in Table 4.1.

Figure 4.11 presents the band structures of metamaterial plates with different cross-sectional shapes, all of which incorporate identical attached mass spheres. As shown in Figure 4.11(a), the square metamaterial plate with mass spheres exhibits a complete bandgap between the 3rd and 4th bands, ranging from 3038.4 Hz to 3477.4 Hz, with a bandwidth of 439 Hz. It is evident that adding mass spheres in

this configuration not only significantly broadens the bandgap but also substantially lowers its threshold frequency.

Table 4.1 Frequency and wavenumber of points selected in the band structure of NM-DHMP for different cross-sectional shapes

Marker	Frequency (Hz)	Wavenumber	Marker	Frequency (Hz)	Wavenumber
S1	3444.3	2.115	S9	595.6	0
S2	3279	3	S10	408.03	2.01
S3	1893.9	1.005	S11	823.79	0
S4	3341	2.01	S12	623.99	1.005
S5	3527.9	2.1	S13	1322.1	0.55
S6	3334.9	3	S14	1199.5	2.01
S7	2045.8	3	S15	2200.6	1.005
S8	2194.7	1.005	S16	3217.9	1.005

Figure 4.11(b) reveals that for the four-point square metamaterial with mass spheres, the 1st and 2nd bands are separated due to the presence of the mass spheres; however, this separation is insufficient to generate a complete bandgap. Figure 4.11(c) demonstrates that the circular metamaterial plate with mass spheres possesses a complete bandgap between the 3rd and 4th bands, spanning 3167.5 Hz to 3568.9 Hz, with a bandwidth of 401.4 Hz. The attached mass spheres broaden the first complete bandgap of the circular metamaterial and lower its starting frequency. However, this mass sphere effect is more pronounced in the square metamaterial plate, where the bandwidth increases by a factor of 1.66, compared to a factor of 1.08 for the circular metamaterial plate. Figure 4.11(d) indicates that while the mass spheres cause separation between the 1st and 2nd bands of the four-point circular metamaterial, a complete bandgap remains absent in this configuration.

As illustrated in Figure 4.11(e), the M-DHMP exhibits four complete bandgaps: 294.83–485.69 Hz (bandwidth 190.86 Hz); 1079.4–1207.1 Hz (bandwidth 127.7 Hz); 1966.02–2153.6 Hz (bandwidth 187.58 Hz); and 2915.34–3193.26 Hz (bandwidth 277.92 Hz). The results indicate that the mass spheres lower the starting frequencies of all four bandgaps, slightly broaden the first and second bandgaps, most significantly broaden the third bandgap, while the width of the fourth Bragg scattering bandgap narrows by less than 3%. In Figure 4.11(f), the bandgap frequencies for the four-point antisymmetric dual helix metamaterial with mass spheres are 412.3–613.01 Hz (bandwidth 200.71 Hz) and 1921.03–2019.87 Hz (bandwidth 98.84 Hz). These results demonstrate that the mass spheres substantially lower the starting frequencies of both bandgaps in the four-point antisymmetric dual helix configuration and broaden their bandwidths; notably, the second bandgap is expanded to 3.3 times that of the configuration without mass spheres.

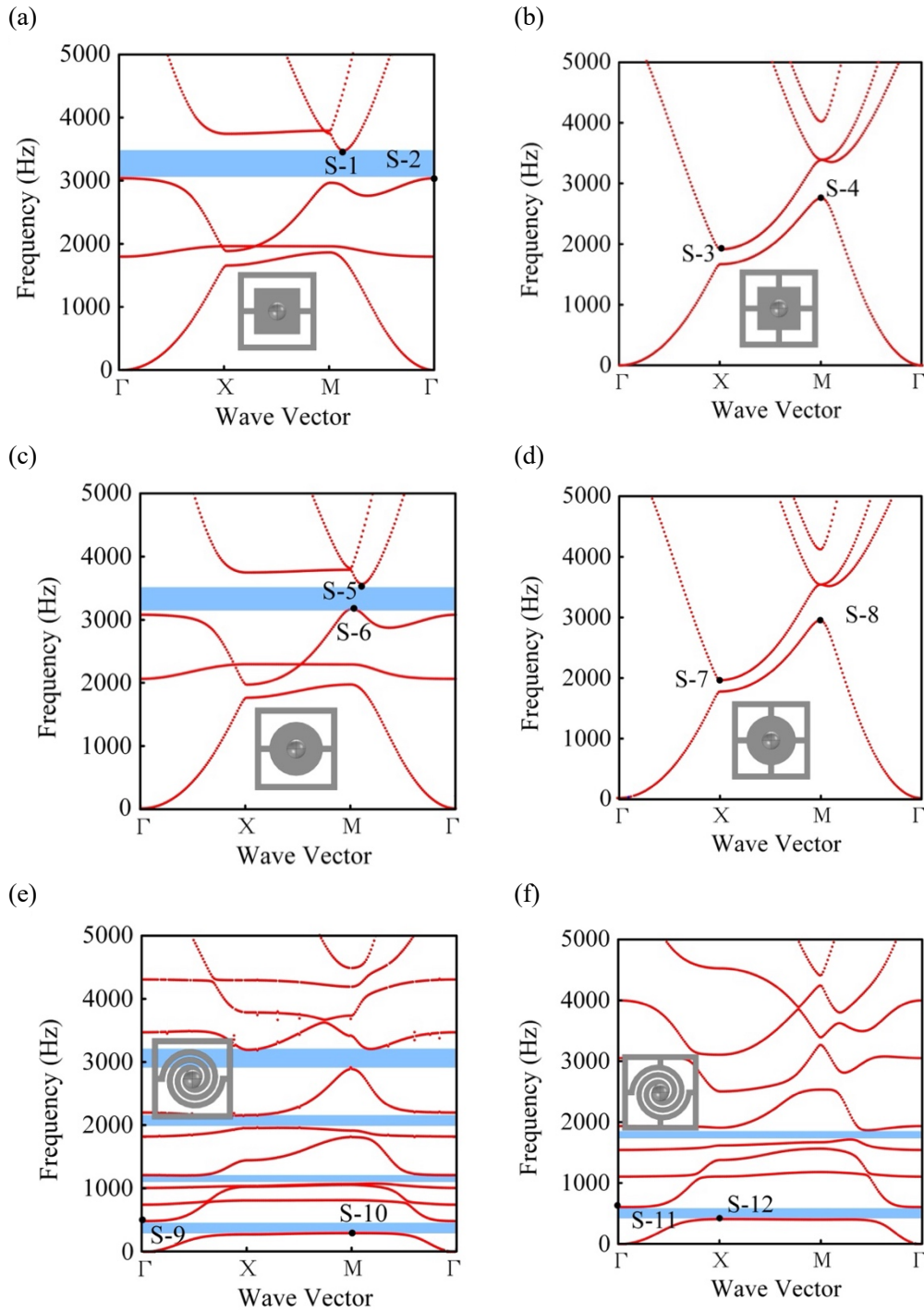


Figure 4.11 Band structure of metamaterial plates with attached spheres for different cross-section shapes: (a) square; (b) four-point square; (c) circular; (d) four-point circular; (e) M-DHMP; (f) four-point M-DHMP.

The above results indicate that, for both the square and circular metamaterial plates, the addition of mass spheres significantly broadens their first complete bandgap (Bragg bandgap) while also lowering its starting frequency. For the antisymmetric dual helix metamaterial plate, the mass spheres have a more pronounced effect on lowering the starting frequency of the first complete bandgap (locally resonant bandgap), achieving a reduction of approximately 28%, compared

to a modest broadening of the bandwidth by about 2%. Evidently, the coupling effect between the attached mass spheres and the dual helix curved beam significantly strengthens and amplifies the local resonance effect in the two-dimensional dual helix metamaterial plate, enabling the attainment of a lower-frequency first complete bandgap.

To better understand the mechanism underlying these bandgap variations, Figure 4.12 presents the vibration modes corresponding to the points marked in Figure 4.10. As observed from points S-1, S-2, S-5, and S-6, the band-edge modes of the square and circular metamaterial plates exhibit little change; the central mass spheres do not induce local resonance. The bandgaps in these configurations remain Bragg bandgaps, formed by multiple reflections and wave interference within the structure. From S-3 and S-7, a distinct blue band can be observed at the center of the mode shapes, attributed to additional connecting structures that alter wave reflection and interference, thereby disrupting the formation of the Bragg bandgap. S-3 and S-4, as well as S-7 and S-8, correspond to two distinct vibration modes, confirming the complete separation of the first and second bands in the band structures of the four-point square and four-point circular metamaterial plates with mass spheres. The vibration modes at S-10 and S-12 are very similar to those at S10 and S12, with the notable difference that the centrally located mass spheres evidently enhance the local resonance effect. The vibration modes at S-9 and S-11 indicate that local resonance is not confined to the central region but also extends to the frame portion. Consequently, the local resonance effect is amplified in both the dual helix metamaterial plate with attached mass spheres and the four-point M-DHMP metamaterial plate, resulting in lower and broader bandgap frequencies.

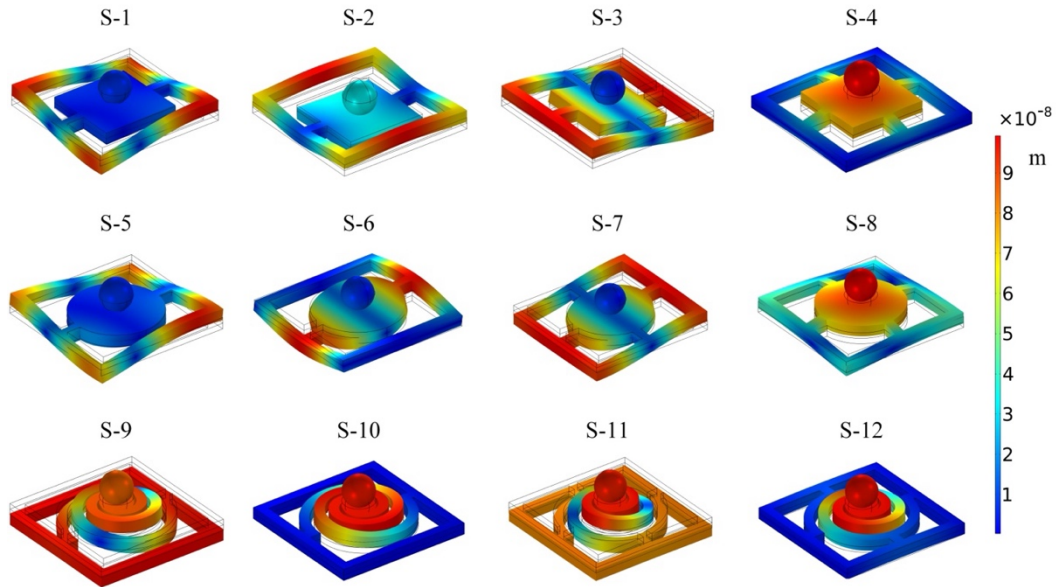


Figure 4.12 Vibration modes at selected frequency points for various unit cell shapes with mass spheres.

The wavenumbers and frequencies corresponding to the marked points on the band structures of metamaterial plates with different cross-sectional shapes incorporating mass spheres are listed in Table 4.2.

Table 4.2 Frequency and wavenumber of points selected in the band structure of M-DHMP for different cross-sectional shapes

Marker	Frequency (Hz)	Wavenumber	Marker	Frequency (Hz)	Wavenumber
S-1	3477.4	2.13	S-7	1915.7	1.005
S-2	3038.4	3	S-8	2707.4	2.1
S-3	1915.7	1.005	S-9	485.69	0
S-4	2756.1	2.01	S-10	294.83	2.01
S-5	3568.9	2.1	S-11	613.01	0
S-6	3167.5	3	S-12	412.3	1.005

The above analysis demonstrates that, compared to other configurations, the antisymmetric dual helix structure possesses unique advantages in bandgap formation, particularly for generating low-frequency bandgaps. Moreover, through synergistic interaction with attached concentrated mass spheres, the local resonance effect in the antisymmetric dual helix metamaterial plate is further strengthened and amplified, leading to the formation of even lower-frequency bandgaps. The following sections will discuss in detail the influence of key parameters on the bandgap characteristics of the antisymmetric dual helix metamaterial, including the number of helical turns N , helix width w , helical orientation, plate thickness h , and the radius of the attached mass sphere r_3 . Unless otherwise specified, the baseline geometric parameters for the unit cell used in the calculations are uniformly set as: $a_1 = a_2 = 70$ mm, $h = 6$ mm, $w = 5$ mm, $r_0 = 8$ mm; helical parameters: $r_1 = 0$ mm, $r_2 = 24$ mm, $N = 3.5$; and the steel sphere radius $r_3 = 8$ mm.

4.3.2 Effect of helical parameters

(1) Number of helical turns

The number of helical turns determines the length of the helix, which influences the equivalent stiffness of the structure and consequently affects the propagation characteristics of flexural waves in the NM-DHMP and M-DHMP. Figure 4.13 presents the band structures of the NM-DHMP for different numbers of helical turns.

Figure 4.13(a) shows that when the number of helical turns $N = 0.5$, no complete bandgap is generated in the NM-DHMP metamaterial plate. As N increases to 1 (Figure 4.13(b)), a complete bandgap forms between the 1st and 2nd bands, ranging from 1480 Hz to 1622 Hz. Figures 4.13(c)–4.13(e) illustrate that for $N = 1.5, 2$, and 2.5 , the NM-DHMP consistently exhibits a single complete bandgap: 981–1341 Hz, 717–1025 Hz, and 569–817 Hz, respectively. It is evident that the bandgap width progressively narrows as the number of turns increases. Notably, Figures 4.13(b)–4.13(d) reveal that the wave vector position of the bandgap cutoff

frequency varies with N . However, as shown in Figures 4.13(e)–4.13(h), once N exceeds 2, the wavenumbers of the starting and cutoff frequencies of the lowest bandgap become fixed at $k = 2.01$ and $k = 0$, respectively.

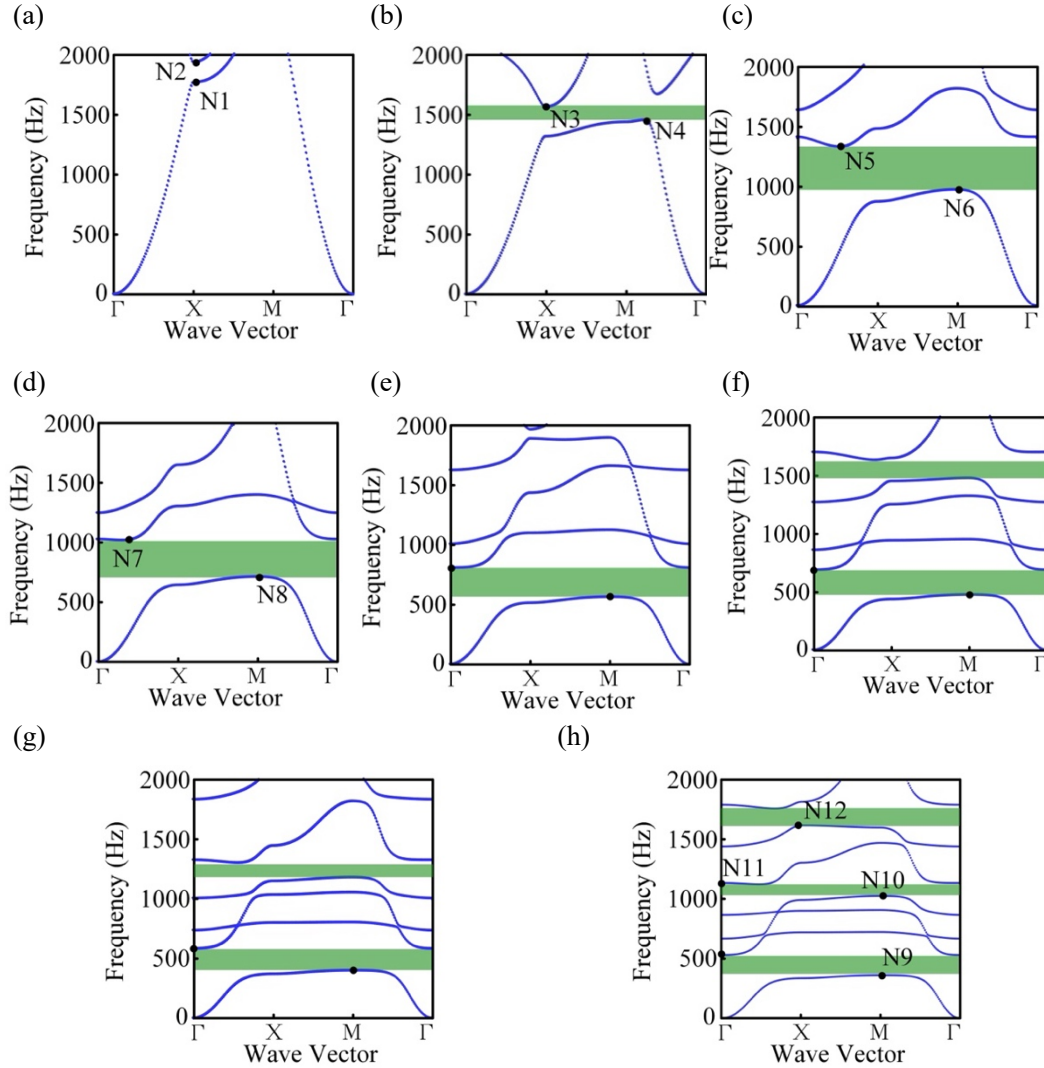


Figure 4.13 Band structure of NM-DHMP with various helical turns: (a) $N = 0.5$; (b) $N = 1.0$; (c) $N = 1.5$; (d) $N = 2.0$; (e) $N = 2.5$; (f) $N = 3.0$; (g) $N = 3.5$; (h) $N = 4.0$.

As depicted in Figure 4.13(f) ($N = 3$), the NM-DHMP exhibits two complete bandgaps: 475–686 Hz (bandwidth 211 Hz) and 1474–1628 Hz (bandwidth 154 Hz). For $N = 3.5$ (Figure 4.13(g)), two bandgaps are also present: 408–596 Hz (bandwidth 186 Hz) and 1199–1322 Hz (bandwidth 123 Hz). When N reaches 4.0 (Figure 4.13(h)), three complete bandgaps emerge: 361–531 Hz (bandwidth 170 Hz), 1030–1139 Hz (bandwidth 109 Hz), and 1625–1767 Hz (bandwidth 142 Hz).

To investigate the influence of the number of helical turns on the bandgap formation mechanism, Figure 4.14 shows the vibration modes at characteristic frequencies. In the figure, N1 and N2 both correspond to flexural modes. While N2 exhibits a tendency toward local resonance (with the most pronounced vibration

observed in the central circle and helix, as shown in the figure), this tendency is insufficient to open a bandgap. As the number of helical turns increases to 1, mode N4 displays typical characteristics of local resonance. This indicates that an increase in helix length favors the formation of locally resonant bandgaps. When the helix extends to 1.5 turns, both N5 and N6 exhibit pronounced locally resonant modes, resulting in the formation of a very wide locally resonant bandgap, as shown in Figure 4.13(c).

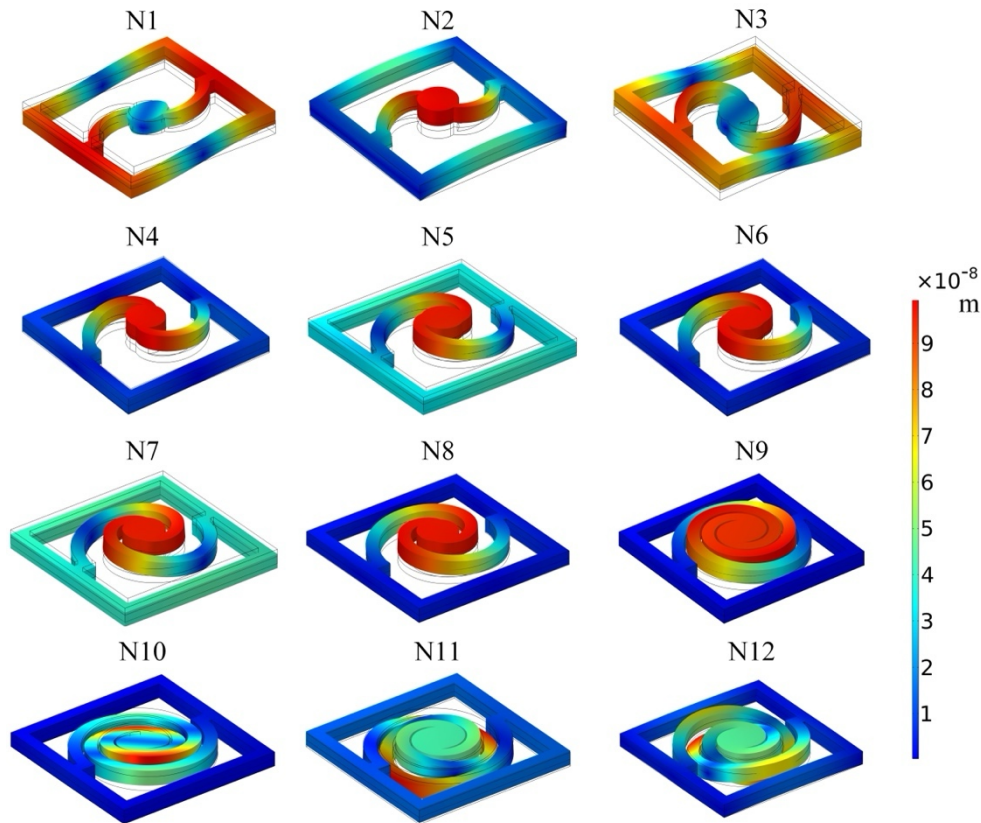


Figure 4.14 Vibration modes of NM-DHMP at selected frequency points for various helical turns.

Table 4.3 Frequency and wavenumber of points selected in the band structure of NM-DHMP for different helical turns

Marker	Frequency (Hz)	Wavenumber	Marker	Frequency (Hz)	Wavenumber
N1	1769	1.005	N7	1025	0.27
N2	1944	1.005	N8	2707.4	2.1
N3	1622	1.005	N9	717	2.01
N4	1480	1.995	N10	911	2.01
N5	1341	0.54	N11	613.01	2.01
N6	981	1.995	N12	1625	1.005

As the helix continues to lengthen, the local resonance effect is further strengthened, as evidenced by the modes at N7, N8, and N9. From these modes, it can be deduced that the lowest bandgaps in Figures 4.13(d)–4.13(h) are all formed by the vertical motion of the central circle and helix. The modes at N10, N11, and N12 indicate that the second and third bandgaps in the NM-DHMP metamaterial plate are locally resonant bandgaps generated by the first- and second-order rotational motions of the central helix. The wavenumbers and frequencies corresponding to the characteristic points on the NM-DHMP band structure are listed in Table 4.3.

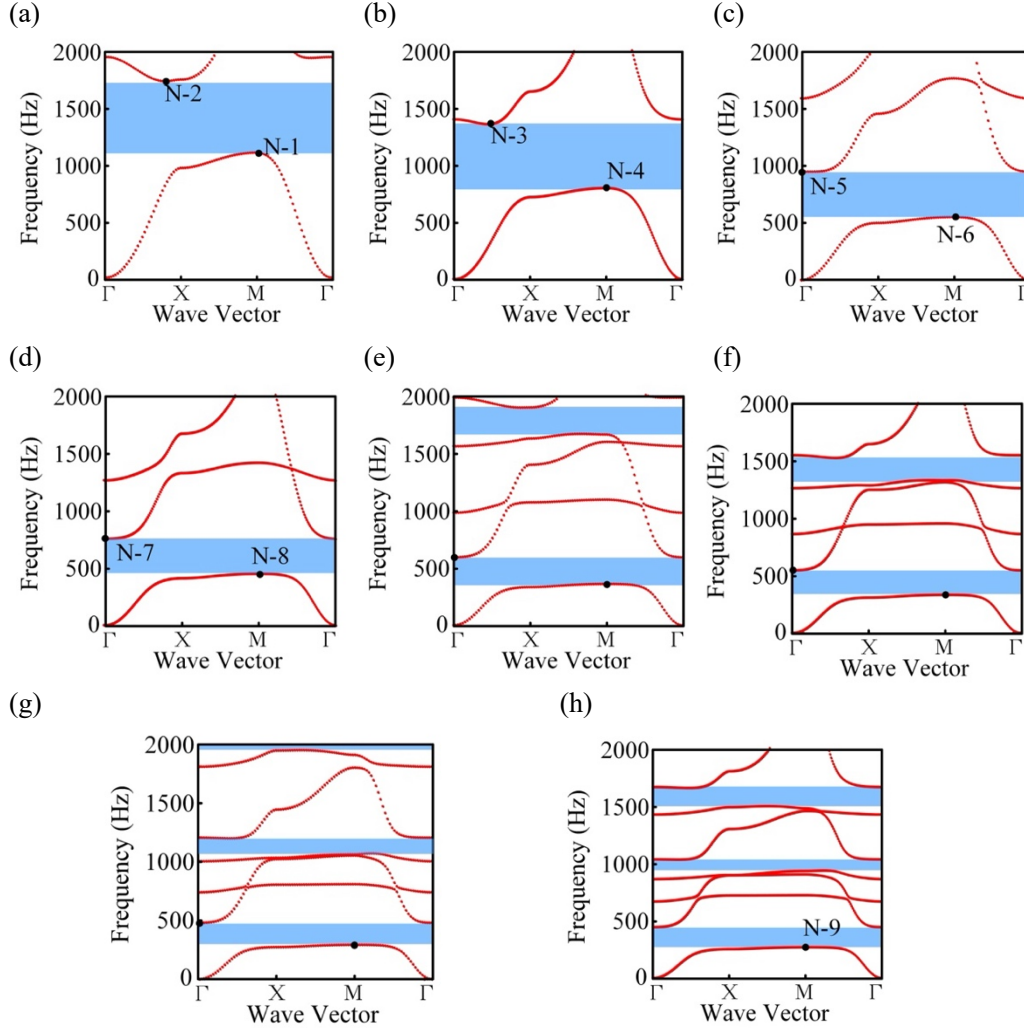


Figure 4.15 Band structure of M-DHMP with different helical turns: (a) $N = 0.5$; (b) $N = 1.0$; (c) $N = 1.5$; (d) $N = 2.0$; (e) $N = 2.5$; (f) $N = 3.0$; (g) $N = 3.5$; (h) $N = 4.0$.

Figure 4.15 presents the band structures of the M-DHMP for different numbers of helical turns. As shown in Figure 4.15(a), when $N = 0.5$, the M-DHMP exhibits an extremely wide complete bandgap spanning 1117–1763 Hz, with a bandwidth of 646 Hz. Figures 4.15(b) through 4.15(d) illustrate that for $N = 1, 1.5$, and 2 , the M-DHMP generates a first complete bandgap between the 1st and 2nd bands: 796–

1354 Hz (bandwidth 558 Hz), 565–970 Hz (bandwidth 405 Hz), and 448–751 Hz (bandwidth 303 Hz), respectively. As shown in Figures 4.15(e) through 4.15(h), for $N = 2.5, 3, 3.5$, and 4, the M-DHMP exhibits two or more complete bandgaps: 373–614 Hz and 1644–1940 Hz; 333–547 Hz and 1333–1529 Hz; 295–486 Hz, 1079–1207 Hz, and 1950–2000 Hz; 268–444 Hz, 937–1033 Hz, and 1499–1660 Hz, respectively. Unlike the NM-DHMP, Figures 4.15(c)–4.15(h) reveal that for the M-DHMP, the wave vector positions of both the starting and cutoff frequencies of the lowest bandgap are fixed.

To facilitate comparison of the bandgap formation mechanisms between the two configurations, Figure 4.16 shows the vibration modes at characteristic points on the band structures of the dual helix metamaterial plate with mass spheres (M-DHMP) for different numbers of helical turns. The results indicate that the modes at points N-1 and N1 are quite similar. However, the incorporation of mass spheres leads to a more pronounced local resonance at point N-2, with energy concentrated in the central mass sphere and helix. This explains the bandgap in M-DHMP, as shown in Figure 4.15(a). Points N-3, N-5, and N-7 correspond to the vibration modes at the cutoff frequencies of the first bandgap. Compared to their counterparts in the NM-DHMP band structure, N3, N5, and N7 exhibit not only concentrated vibration in the central mass sphere and helix but also noticeable vertical motion in the frame. The modes at points N-4, N-6, N-8, and N-9 are very similar to those at N4, N6, N8, and N9, with the key distinction being that the coupling between the dual helix structure and the mass spheres significantly enhances the local resonance, thereby lowering the starting frequency of the first complete bandgap in the metamaterial plate.

The wavenumbers and frequencies corresponding to the characteristic points of the band structure of the dual helix metamaterial plate with mass spheres are listed in Table 4.4.

Table 4.4 Frequency and wavenumber of points selected in the band structure of M-DHMP for different helical turns

Marker	Frequency (Hz)	Wavenumber	Marker	Frequency (Hz)	Wavenumber
N-1	1763	0.975	N-6	565	2.01
N-2	1117	2.01	N-7	751	0
N-3	1354	0.435	N-8	448	2.01
N-4	796	2.01	N-9	268	2.01
N-5	970	0			

To facilitate observation of the dependence of the lowest bandgap on the number of helical turns, Figure 4.17 illustrates the variation of the boundary frequencies of the first complete bandgap for different numbers of helical turns.

The results indicate that, unlike the behavior observed in the M-DHMP, the NM-DHMP exhibits no complete bandgap within the 0–2000 Hz frequency range when the number of helical turns is less than 1. As the number of turns increases, the starting frequency of the bandgap gradually decreases, while the bandgap width

initially broadens and then progressively narrows. For the M-DHMP, the starting frequency decreases continuously with increasing turns, and the bandwidth narrows steadily, with the rates of both frequency reduction and bandwidth narrowing gradually diminishing as the number of turns increases. This underscores the significant influence of the number of helical turns on low-frequency bandgap formation, particularly when the number of turns is less than 2.0. In summary, by varying the number of helical turns from 0.5 to 4, the first complete bandgap of the NM-DHMP can be tuned within the frequency range of 361 Hz to 1622 Hz, while that of the M-DHMP can be tuned within the range of 268 Hz to 1763 Hz.

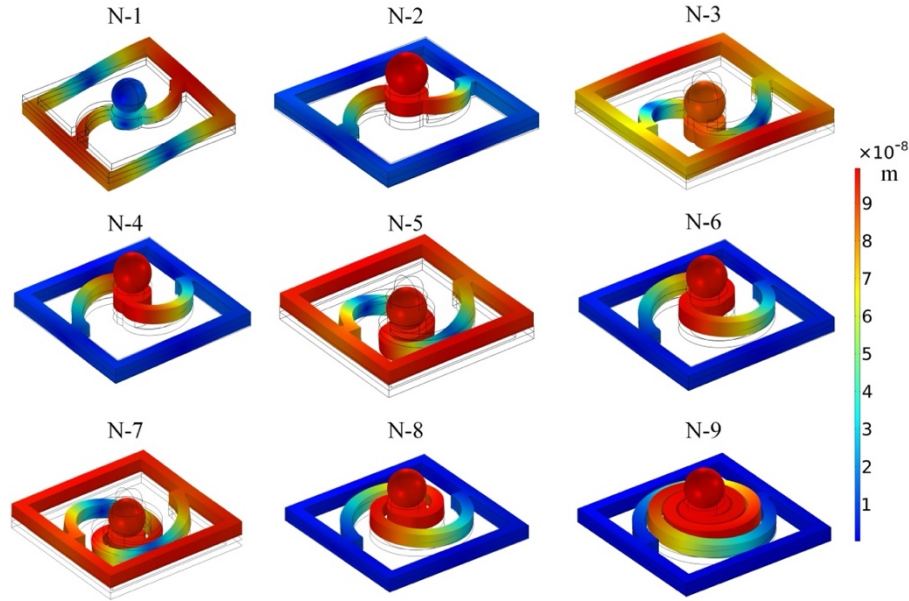


Figure 4.16 Vibration modes of M-DHMP at selected frequency points.

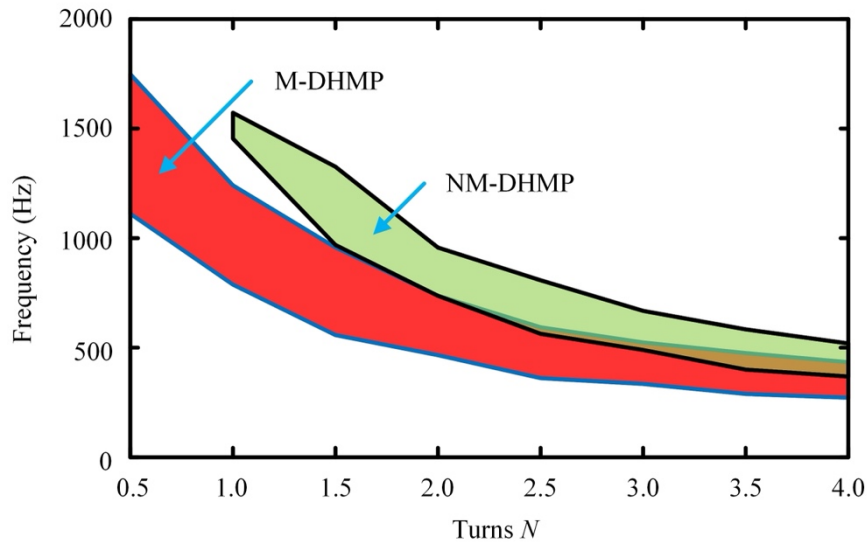


Figure 4.17 First complete bandgap frequency varies with the number of helical turns for the NM-DHMP and M-DHMP.

(2) Helix width

The following discussion examines the influence of the helix width w_1 on the bandgap characteristics. The geometric definition of w_1 is provided in Figure 4.8(f) in Section 4.3.1. The unit cell configuration and geometric parameters remain consistent with those shown in Figures 4.1(a) and 4.1(b), with only the helix width varying from 0.2 to 1.0 times the reference width. Figure 4.18 presents the band structures of the dual helix metamaterial plate without mass spheres (NM-DHMP) within the 0–1000 Hz frequency range for $w_1 = 0.2w$, $0.4w$, $0.6w$, $0.8w$, and $1.0w$, respectively.

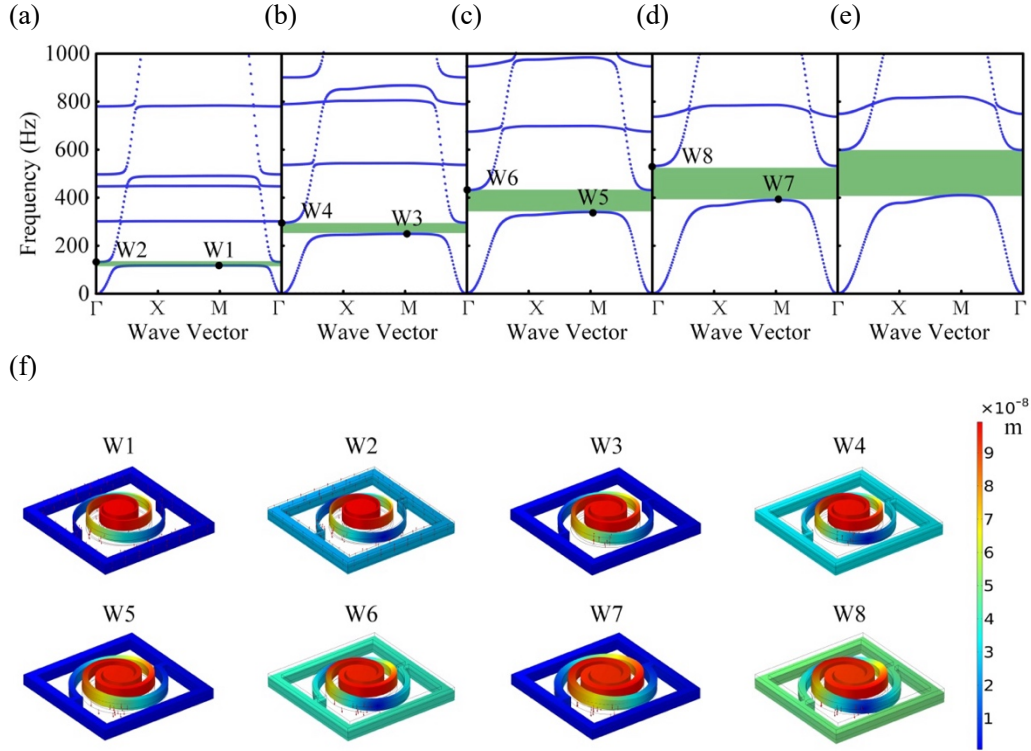


Figure 4.18 Band structures of NM-DHMP for various helix widths: (a) $w_1 = 0.2w$; (b) $w_1 = 0.4w$; (c) $w_1 = 0.6w$; (d) $w_1 = 0.8w$; (e) $w_1 = 1.0w$; (f) corresponding vibration modes at the marked boundary points: W1(116 Hz), W2(131 Hz), W3(247 Hz), W4(294 Hz), W5(336 Hz), W6(428 Hz), W7(388 Hz), W8(529 Hz).

The results indicate that, in all considered cases, the NM-DHMP exhibits a complete bandgap between the 1st and 2nd bands. The frequency ranges and corresponding bandwidths are as follows: 116–131 Hz (bandwidth 15 Hz), 247–294 Hz (bandwidth 47 Hz), 336–428 Hz (bandwidth 92 Hz), 388–529 Hz (bandwidth 141 Hz), and 408–596 Hz (bandwidth 188 Hz), for $w_1 = 0.2w$, $0.4w$, $0.6w$, $0.8w$, and $1.0w$, respectively.

To elucidate the underlying mechanism of bandgap formation, Figure 4.18(f) presents the vibration modes at the bandgap boundaries for different helix widths, with arrows indicating the direction of motion. For all configurations, the bandgap is induced by the localized vertical motion and resonance of the core structure,

comprising the central circle and the dual helix. In Figure 4.18(f), W1, W3, W5, and W7 correspond to the vibration modes at the lower bandgap boundaries. These modes are nearly identical: the frame portion remains stationary, with all vibration concentrated in the central region. W2, W4, W6, and W8 represent the vibration modes at the upper bandgap boundaries. The results reveal that as the helix width increases, an increasing amount of the vibration energy, initially confined to the central region, is transmitted to the frame structure.

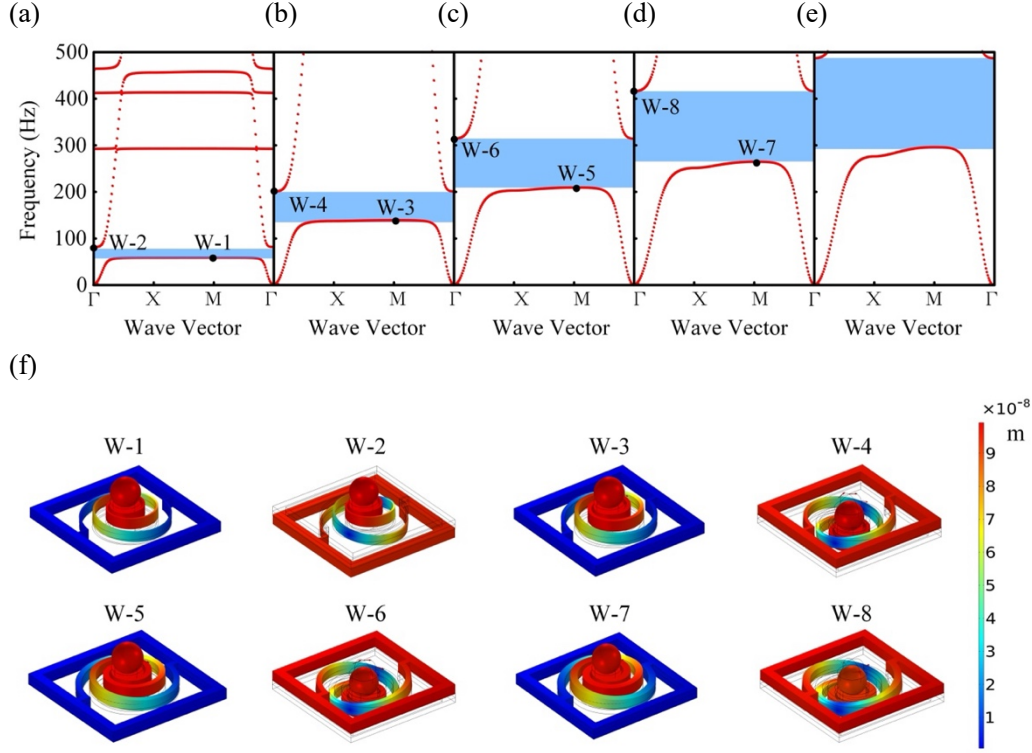


Figure 4.19 Band structures of M-DHMP for various helix widths: (a) $w_1 = 0.2w$; (b) $w_1 = 0.4w$; (c) $w_1 = 0.6w$; (d) $w_1 = 0.8w$; (e) $w_1 = 1.0w$; (f) corresponding vibration modes at the marked boundary points: W-1, 56 Hz; W-2, 79 Hz; W-3, 137 Hz; W-4, 198 Hz; W-5, 208 Hz; W-6, 313 Hz; W-7, 262 Hz; W-8, 412 Hz.

For comparison, Figure 4.19 shows the band structures of the dual helix metamaterial plate with mass spheres (M-DHMP) in the 0–500 Hz frequency range for helix widths $w_1 = 0.2w$, $0.4w$, $0.6w$, $0.8w$, and $1.0w$, respectively. The results indicate that a bandgap exists for all considered M-DHMP configurations: 56–79 Hz (bandwidth 23 Hz), 137–198 Hz (bandwidth 61 Hz), 208–313 Hz (bandwidth 105 Hz), 262–412 Hz (bandwidth 150 Hz), and 295–486 Hz (bandwidth 191 Hz). A comparative analysis reveals that the bandgap broadening induced by identical-mass spheres varies with helix width. The corresponding bandgap broadening ratios are 53%, 30%, 14%, 6%, and 2%, respectively.

Figure 4.19(f) displays the vibration modes at the bandgap boundaries of the M-DHMP. Points W-1, W-3, W-5, and W-7 correspond to the modes at the lower bandgap boundaries, while W-2, W-4, W-6, and W-8 represent the modes at the

upper bandgap boundaries. Compared to the NM-DHMP, the vibration modes at the upper bandgap boundaries of the M-DHMP exhibit comparable vibration intensity between the frame and the central sphere-helix assembly, both regions depicted in red, indicating equal participation in the mode shape.

Figure 4.20 illustrates the variation of the bandgap boundary frequencies for different helix widths. The results indicate that for the NM-DHMP plate, the upper bandgap boundary shifts upward at a relatively uniform rate, while the lower boundary initially rises more rapidly, with its upward movement becoming gradual once the helix width exceeds $0.8w$. For the M-DHMP plate, both the upper and lower bandgap boundaries shift toward higher frequencies at relatively uniform rates, with the lower boundary exhibiting a comparatively gentler slope. The bandgap width is highly sensitive to variations in the helix width. By adjusting the helix width, the first complete bandgap of the NM-DHMP can be tuned within the frequency range of 116 Hz to 596 Hz, while that of the M-DHMP can be tuned within the range of 79 Hz to 486 Hz.

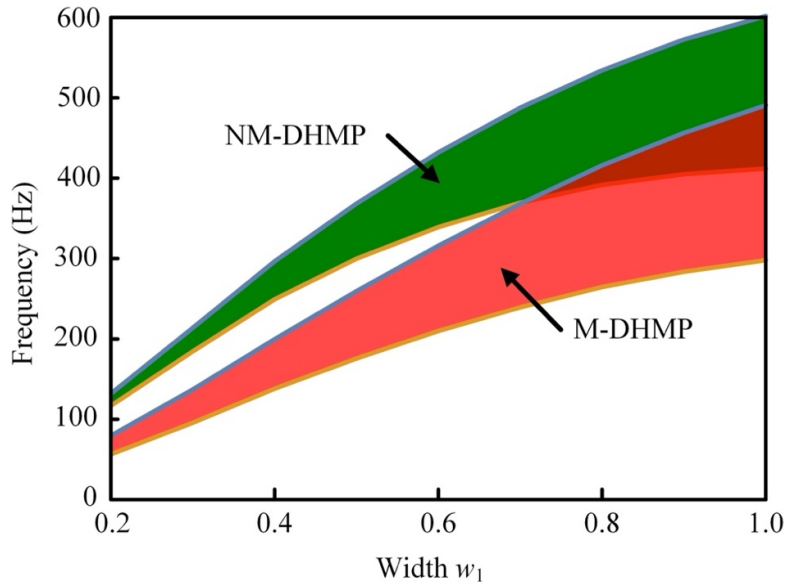


Figure 4.20 First complete bandgap frequency varies with the width of the helix for NM-DHMP and M-DHMP.

(3) Helix orientation

Four distinct unit-cell configurations of the dual helix metamaterial plate, denoted DHMP0, DHMP, DHMP1, and DHMP2, are defined based on different helix orientations. Their geometric configurations are illustrated in Figures 4.21(i)–4.21(m), with the number of helical turns uniformly set to $N = 3.5$. Specific symmetry relationships exist among these structures: DHMP can be obtained by rotating DHMP0 by 90° , and similarly, DHMP1 rotated by 90° yields DHMP2. Consequently, DHMP0 and DHMP2, as well as DHMP and DHMP1, form two pairs of chiral structures. Notably, the connecting structures in DHMP and DHMP1

are oriented horizontally, while those in DHMP0 and DHMP2 are oriented vertically.

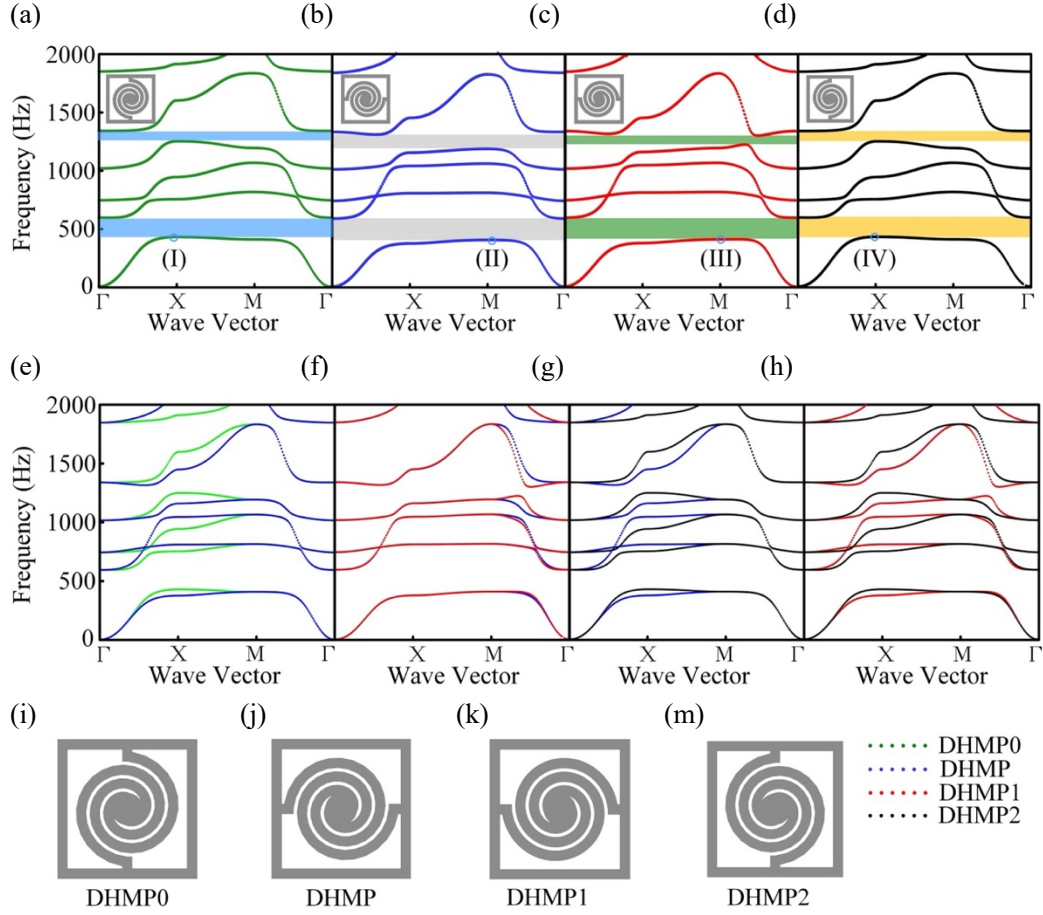


Figure 4.21 Band structure of metamaterial plates with different helical orientations and the definitions of various DHMP configurations. Band structures: (a) DHMP0; (b) DHMP; (c) DHMP1; (d) DHMP2; (e) DHMP0 vs. DHMP; (f) DHMP vs. DHMP1; (g) DHMP vs. DHMP2; (h) DHMP0, DHMP, DHMP1, and DHMP2 superimposed; (i)–(m) definitions of the four DHMP configurations.

The corresponding band structures of these four metamaterial plates are presented in Figures 4.21(a)–4.21(d). The results indicate that two bandgaps exist in each DHMP configuration. For DHMP0, the bandgaps are 429–596 Hz and 1256–1346 Hz; for DHMP, 408–596 Hz and 1199–1322 Hz; for DHMP1, 410–596 Hz and 1229–1322 Hz; and for DHMP2, 429–596 Hz and 1255–1345 Hz. To elucidate the relationships among these band structures, Figures 4.21(e)–4.21(h) sequentially present comparisons of the following: (e) DHMP0 versus DHMP, (f) DHMP versus DHMP1, (g) DHMP versus DHMP2, and (h) all four structures superimposed.

From Figure 4.21(e), it can be observed that the band structures of DHMP0 and DHMP do not coincide in the left portion (low-frequency region). The bands of DHMP are shifted downward relative to those of DHMP0, resulting in a lower

starting frequency for the lowest bandgap in DHMP. However, the upper bandgap boundaries for the two configurations are completely aligned.

Figure 4.21(f) shows that the 1st, 2nd, and 3rd bands of DHMP and DHMP1 essentially overlap, while differences in the 4th, 5th, and 6th bands are confined to the right portion of the band structure. Figure 4.21(g) indicates that the band structure differences between DHMP and DHMP2 are also concentrated in the left portion. From the superimposed band structures of all four configurations presented in Figure 4.21(h), it is evident that the band structures of DHMP0 (green) and DHMP2 (black) almost completely coincide. In contrast, the left portion of the band structure of DHMP (blue) overlaps with that of DHMP1 (red), while its right portion aligns with that of DHMP2 (black).

The above results demonstrate that altering the rotational direction of the helix leads to slight variations in the lower boundary frequency of the lowest bandgap. When the connecting structures are oriented vertically, the band structures of the corresponding chiral pair (DHMP0 and DHMP2) are almost identical. When the connecting structures are oriented horizontally, the band structures of the other chiral pair (DHMP and DHMP1) exhibit slight differences. Overall, a horizontal orientation of the connecting structures favors a lower starting frequency for the first complete bandgap, while the upper boundary of the first complete bandgap remains identical across all four configurations.

4.3.3 Effect of plate thickness

To investigate the influence of plate thickness on the bandgap formation mechanism, Figure 4.22 shows the evolution of the band structures of dual helix metamaterial plates with and without mass spheres over the 0–2000 Hz frequency range for different plate thicknesses.

Figure 4.22(a) shows that when the plate thickness $h = 3$ mm, the NM-DHMP exhibits five bandgaps of varying widths, located between the 1st and 2nd, 4th and 5th, 5th and 6th, 6th and 7th, and 7th and 8th bands. As the thickness increases to $h = 5$ mm, Figure 4.22(b) shows four bandgaps, located between the 1st and 2nd, 4th and 5th, 5th and 6th, and 6th and 7th bands, with the bandgap boundary frequencies shifting toward higher values. Figures 4.22(c) through 4.22(e) demonstrate that for $h = 7$ mm, 9 mm, and 11 mm, the plate exhibits two bandgaps located between the 1st and 2nd and between the 4th and 5th bands. As the thickness increases, the boundary frequencies of these bandgaps gradually rise, accompanied by a slight broadening of the bandwidth. This trend is attributed to the system's increasing stiffness with greater plate thickness, which progressively shifts the bands toward higher frequencies.

As illustrated in Figure 4.22(f), the M-DHMP exhibits four bandgaps, located between the 1st and 2nd, 4th and 5th, 6th and 7th, and 7th and 8th bands. Notably, the introduction of mass spheres induces a degeneracy between the 5th and 6th bands at point A, causing the bandgap between them to disappear. This degeneracy between bands 5 and 6 significantly broadens the third bandgap, located between bands 6 and 7. Furthermore, the incorporation of mass spheres broadens the

frequency ranges of the first and second bandgaps and lowers their boundary frequencies. In Figure 4.22(g), the degeneracy between bands 5 and 6 is lifted, resulting in three relatively wide bandgaps located between the 1st and 2nd, 4th and 5th, and 6th and 7th bands. Figures 4.22(h) through 4.22(j) reveal that the M-DHMP exhibits two bandgaps. Their boundary frequencies gradually shift toward higher values with increasing plate thickness, while the bandwidths show no significant variation.

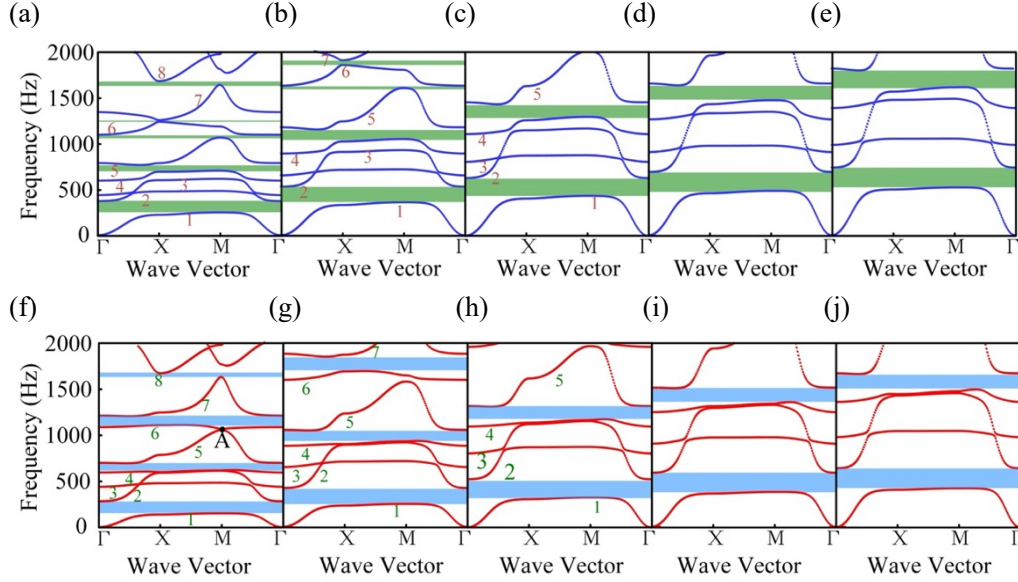


Figure 4.22 Band structures of DHMP for different plate thicknesses: (a)–(e) NM-DHMP with $h = 3, 5, 7, 9$, and 11 mm, respectively; (f)–(j) M-DHMP with $h = 3, 5, 7, 9$, and 11 mm, respectively.

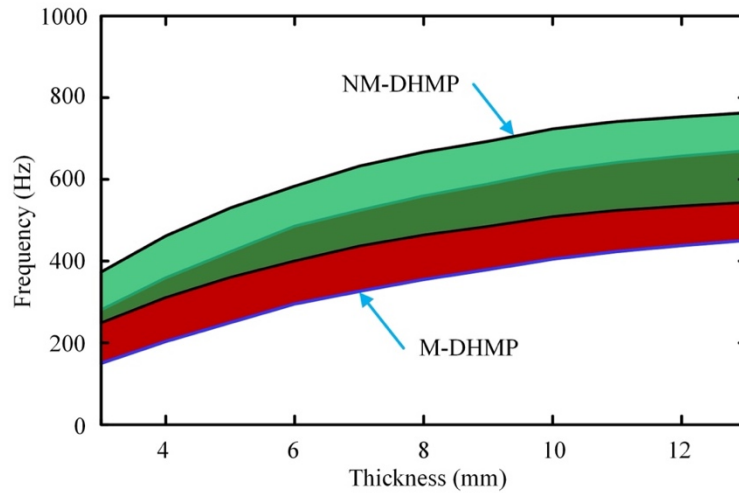


Figure 4.23 First bandgap frequency changes with the thickness of NM-DHMP and M-DHMP.

To reveal the influence of plate thickness on the first complete bandgap, Figure 4.23 presents the variation of the upper and lower boundary frequencies of the first

bandgap for different plate thicknesses for both the NM-DHMP and M-DHMP configurations. The results indicate that for both types of dual helix metamaterial plates, the boundary frequencies of the first complete bandgap progressively increase with increasing plate thickness. The rate of increase in frequency is initially rapid, then gradually diminishes. Furthermore, the width of the first complete bandgap gradually broadens as the plate thickness increases, although it remains essentially unchanged once the thickness exceeds 8 mm. Ultimately, by varying the plate thickness from 3 mm to 13 mm, the first complete bandgap of the NM-DHMP can be tuned within the frequency range of 249 Hz to 763 Hz, while that of the M-DHMP can be tuned within the range of 150 Hz to 670 Hz.

4.3.4 Effect of mass sphere radius

To analyze the influence of the mass sphere radius on the bandgap characteristics, the band structures of the dual helix metamaterial plate with mass spheres (M-DHMP) were calculated for various sphere radii, considering two representative plate thicknesses: $h = 4$ mm and $h = 6$ mm.

Figures 4.24(a)–4.24(e) show that for a fixed plate thickness of $h = 6$ mm, the M-DHMP plates with different mass sphere radii exhibit six bands and two flexural wave bandgaps within the 0–2000 Hz frequency range. These bandgaps consistently appear between the 1st and 2nd bands and between the 4th and 5th bands. Figures 4.24(f) through 4.24(j) demonstrate that for $h = 4$ mm, the M-DHMP plates possess seven bands and three bandgaps, located between the 1st and 2nd, 4th and 5th, and 6th and 7th bands, respectively.

The results indicate that for a fixed plate thickness, the radius of the mass sphere primarily modulates the bandgap frequencies rather than altering the topological structure of the bands. This is because varying the sphere radius does not change the geometric symmetry of the unit cell; it merely acts as a perturbation parameter that tunes the frequencies of existing vibrational modes without exciting new modes. Notably, when the mass sphere radius increases to 9 mm, degeneracy is observed in both plate thickness configurations (points B and C in Figure 4.24). Beyond this critical radius, an approximately flat band emerges as the 4th band in the band structure (highlighted in red in Figure 4.24(j)). This flat band implies strong elastic-wave localization within the structure, preventing propagation. Due to its constant frequency across all wave vectors, the entire Brillouin zone constitutes its isofrequency surface, forming an ideal platform for vibration localization and energy harvesting.

To provide a detailed examination of the influence of the mass sphere radius on the bandgap characteristics, Figure 4.25 presents the variation of the boundary frequencies of the first and second complete bandgaps for different sphere radii. As shown in Figure 4.25(a), for a fixed plate thickness, both the starting and cutoff frequencies of the first complete bandgap decrease in an approximately linear manner with increasing mass sphere radius, accompanied by a slight broadening of the bandgap width. Figure 4.25(b) demonstrates that the upper and lower boundary frequencies of the second complete bandgap also decrease approximately linearly

with increasing sphere radius, while the variation in bandgap width remains negligible.

The results ultimately demonstrate that for the M-DHMP with a plate thickness of $h = 6$ mm, varying the mass sphere radius from 3 mm to 14 mm enables tuning of the first complete bandgap within 166–586 Hz and the second complete bandgap within 1021–1310 Hz. For $h = 4$ mm, the first complete bandgap can be tuned within 108–452 Hz, and the second complete bandgap within 755–967 Hz.

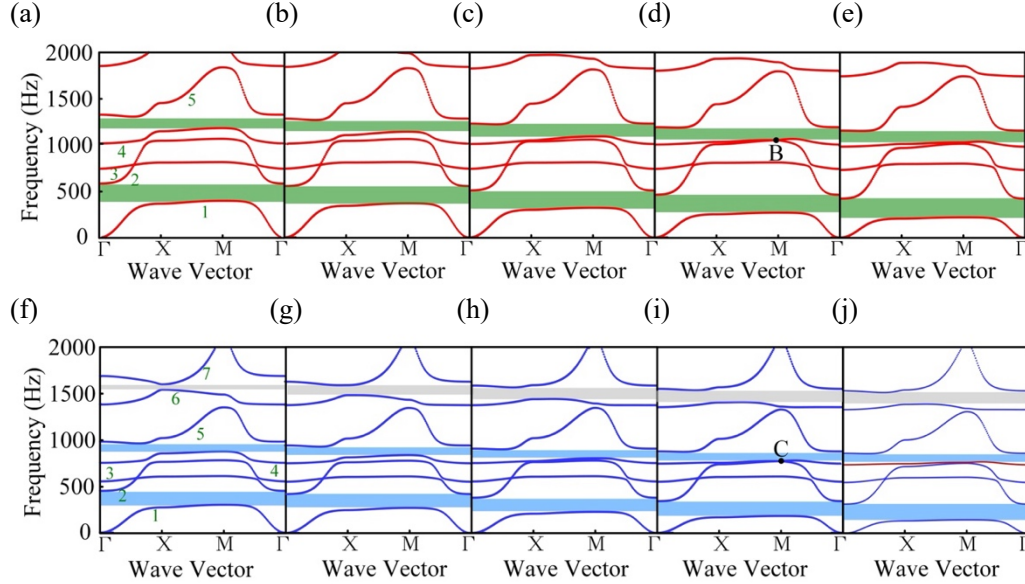


Figure 4.24 Band structures of M-DHMP for different mass sphere radii at two plate thicknesses. When $h = 6$ mm: (a) $r_3 = 3$ mm; (b) $r_3 = 5$ mm; (c) $r_3 = 7$ mm; (d) $r_3 = 9$ mm; (e) $r_3 = 11$ mm. When $h = 4$ mm: (f) $r_3 = 3$ mm; (g) $r_3 = 5$ mm; (h) $r_3 = 7$ mm; (i) $r_3 = 9$ mm; (j) $r_3 = 11$ mm.

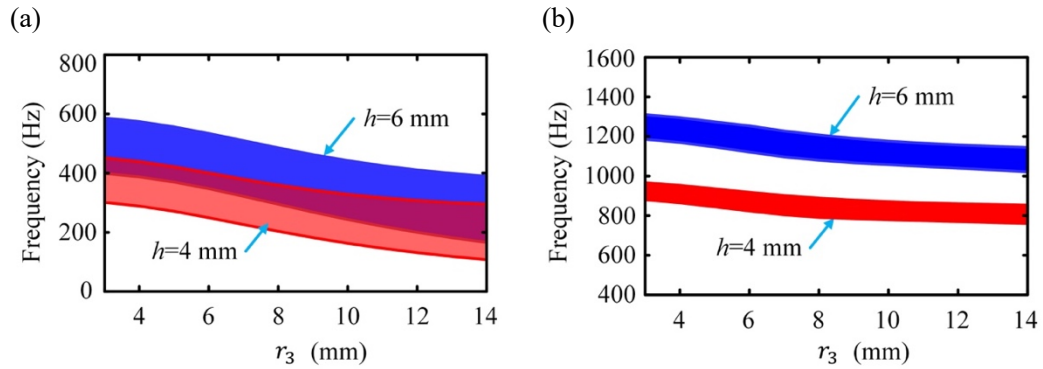


Figure 4.25 First and second complete bandgap frequencies vary with mass sphere radius r_3 for M-DHMP at different plate thicknesses: (a) first complete bandgap; (b) second complete bandgap.

4.4 Summary

The flexural wave bandgap characteristics of two-dimensional dual helix metamaterial plates were investigated using the finite element method. The band structures, band-edge modes, displacement transmission curves of finite structures, and full-field displacements were systematically analysed for both configurations with and without mass spheres. A comprehensive comparative study was conducted on the band structures and band-edge modes of square, circular, four-point square, four-point circular, and four-point dual helix metamaterial plates, with and without mass spheres. The effects of key parameters, including the number of helical turns, helix width, helix orientation, plate thickness, and mass-sphere radius, on the bandgap characteristics of dual helix metamaterial plates were discussed in detail. The main findings are as follows:

(1) Within the frequency range of 2000 Hz, the dual helix metamaterial plates with and without mass spheres exhibit three and two complete locally resonant bandgaps, respectively. The flexural wave bandgap characteristics derived from band structure analysis and displacement transmission curves are in good agreement. Notably, the number and frequency ranges of directional bandgaps vary significantly along different wave vector directions.

(2) The geometric configuration of the unit cell plays a crucial role in the bandgap formation mechanism, with the dual helix structure being particularly advantageous for generating low-frequency bandgaps. The lowest bandgaps of the square and circular metamaterial plates are Bragg scattering bandgaps, characterized by relatively high frequencies. In contrast, the lowest bandgap of the dual helix metamaterial plate is a locally resonant bandgap, exhibiting significantly lower frequencies. The introduction of connecting elements in the four-point square and four-point circular configurations eliminate bandgaps within the investigated frequency range, demonstrating that the number of connecting structures influences bandgap formation. The attachment of concentrated mass spheres lowers the starting frequency of the first bandgap in both the dual helix and four-point dual helix metamaterial plates, although its effect on bandwidth broadening is less pronounced.

(3) The number of helical turns significantly affects the upper and lower boundary frequencies of the lowest bandgap. When the number of turns is less than one, the NM-DHMP exhibits no complete bandgap within the investigated frequency range. As the number of turns increases, the boundary frequencies of the bandgap in the NM-DHMP gradually decrease, while the bandwidth first broadens and then narrows. For the M-DHMP, increasing the number of turns also leads to a gradual decrease in bandgap frequency, accompanied by a monotonic narrowing of the bandwidth. The helix width has a pronounced effect on the width of the lowest bandgap. Increasing the helix width raises the bandgap frequencies and broadens the bandwidth for both NM-DHMP and M-DHMP. Varying the helix orientation and the positions of connecting structures results in only minor changes in the lowest bandgap frequency. The upper and lower boundary frequencies of the lowest bandgap gradually increase with increasing plate thickness, while the bandwidth

remains relatively stable. Both the first and second bandgap frequencies decrease linearly with increasing mass sphere radius, with negligible changes in bandwidth.

The results demonstrate that by adjusting parameters such as helix width, plate thickness, and the number of helical turns, the bandgap frequency of the NM-DHMP can be tuned over 116–1622 Hz. With the addition of mass spheres, the bandgap frequency of the M-DHMP can be tuned to a lower, broader range, specifically 79–1763 Hz. Furthermore, by adjusting the mass-sphere radius and plate thickness, phenomena such as the degenerate state and flat-band behavior are observed in metamaterial plates with mass spheres. In summary, the formation mechanisms and tuning laws of the bandgaps in antisymmetric dual helix metamaterial plates, with and without mass spheres, as discussed in this chapter, provide theoretical guidance for the design of plate structures targeting low-frequency bandgaps required for applications such as vibration and noise reduction.

Chapter 5

Longitudinal wave bandgap characteristics of DNA double helix metamaterials

5.1 Introduction

The preceding sections have systematically investigated the bandgap characteristics of flexural waves in beam and plate structures. These waves, characterized by transverse structural vibrations and cross-sectional rotations, have wavelengths closely related to the structural dimensions and are a primary contributor to low-frequency noise and macroscopic vibrations in thin-walled structures. However, in engineering practice, longitudinal waves, compression waves with a vibration direction parallel to their propagation, exert an equally profound influence and pose even greater control challenges. Longitudinal waves induce periodic compressive and tensile deformations in the medium, characterized by high propagation speeds and strong penetration. They serve not only as the primary carriers of energy transfer in large load-bearing structures such as aircraft trusses and ship keels but also as the root cause of “structure-borne sound”, which propagates efficiently through structural frameworks to distant locations. Specifically, in high-speed trains, longitudinal waves excited by wheel-rail impact can travel long distances through the carriage framework, exciting panel vibrations and radiating low-frequency noise. In the aerospace domain, engine vibrations are also primarily transmitted through truss systems via longitudinal waves, posing a serious threat to the proper functioning of precision instruments. Therefore, achieving effective suppression of longitudinal waves has become a critical aspect of advancing vibration control and fundamentally addressing structure-borne sound. Its research significance is on par with that of flexural wave control and is particularly urgent in specific engineering scenarios.

Elastic metamaterials have demonstrated immense potential for wave manipulation. However, the design of existing tunable metamaterials often relies on complex auxiliary structures or continuous external energy input, which, to some extent, limits their engineering applications. Against this backdrop, bio-inspired metamaterials have garnered widespread attention for their unique structural-functional integration. Drawing inspiration from the multiscale structural features of biological systems, researchers have developed various bio-inspired designs that exhibit exceptional mechanical properties. These include impact-resistant materials

inspired by the microstructure of sea cucumber dermis, QZS vibration isolation systems that mimic the anatomical features of avian necks, and wave-manipulating metamaterials based on the cochlear spiral structure.

Notably, the discovery of the double helix structure of DNA (deoxyribonucleic acid), the carrier of genetic information, stands as one of the most significant scientific breakthroughs of the twentieth century. A typical B-form DNA consists of two antiparallel polynucleotide chains coiled around the same central axis, forming a right-handed helix: phosphate and deoxyribose sugars alternately connect to form the molecular backbone, while the four bases, adenine (A), thymine (T), guanine (G), and cytosine (C), achieve cross-linking between the two strands through specific base pairing. This exquisite helical stacking and complementary pairing mechanism not only ensures the efficient storage and transmission of genetic information but also provides an ideal paradigm for achieving dynamically tunable wave control.

To overcome the limitations of traditional metamaterials, namely, their fixed performance and difficulty in achieving dynamic tunability, this chapter proposes a novel bio-inspired metamaterial with tunable longitudinal wave bandgap characteristics, inspired by the B-form DNA double helix structure, designated the B-form DNA Dual Helix Metamaterial (BDHM). Drawing inspiration from the information encoding function of the four bases (A, T, C, G) in DNA, four materials with different densities, steel, aluminium, nylon, and foam, were heuristically selected to serve as mass blocks. These were embedded within a pre-3D-printed double-helix framework to construct metamaterial units with encoding capabilities. The research encompasses the following: designing the BDHM unit cell; solving the band structure characteristics of the infinite periodic structure and the displacement transmission response of a finite structure consisting of six unit cells; systematically analyzing the longitudinal wave bandgap characteristics under three configurations, uniform mass blocks of a single material, mixed arrangements of mass blocks, and the absence of mass blocks, to elucidate the tunable encoding mechanism of the bandgap; investigating in depth the influence of key parameters such as the helix's minor diameter, the end plate's thickness, and the mass block's length on the bandgap characteristics; and finally, constructing a laser vibrometry experimental platform to experimentally validate the finite element results.

5.2 Structural design and FE modelling

5.2.1 Unit cell design of the DNA double helix metamaterials

Figure 5.1 illustrates the unit cell structure of the novel BDHM, inspired by the B-form DNA double helix [217]. The unit cell structure of this metamaterial consists of three parts. The first part is the framework, composed of a double helix and two hollow cylindrical end plates, as indicated by the blue region in Figure 5.1(a). The second part comprises eleven connecting beams that link the two strands of the DNA double helix, highlighted in blue in Figure 5.1(b). The third part consists of

22 mass blocks placed on these connecting beams, depicted in the blue regions of Figure 5.1(c).

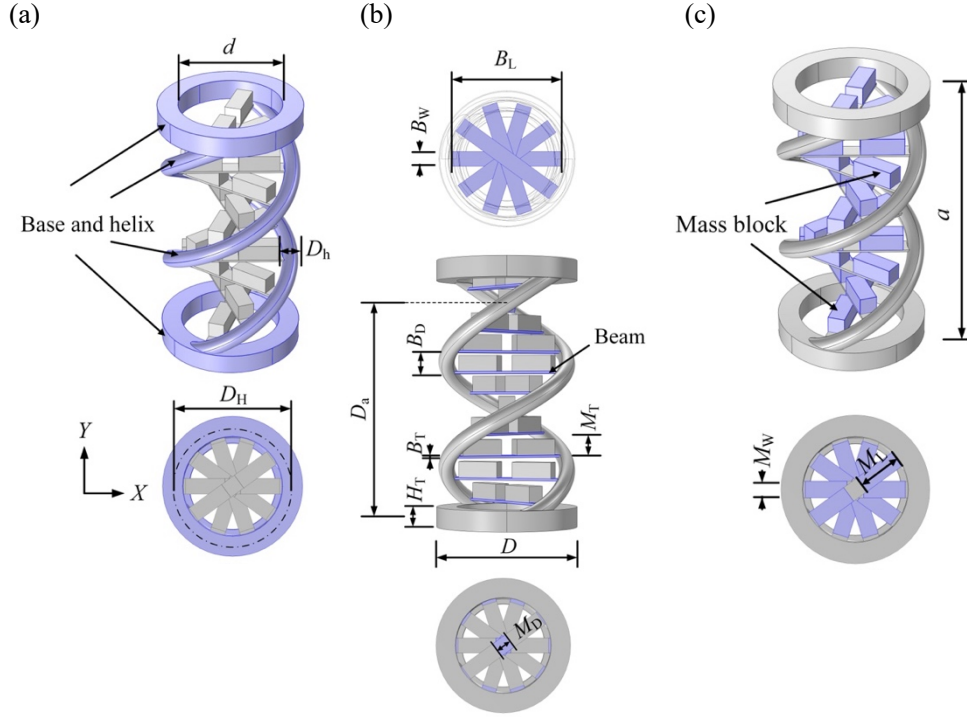


Figure 5.1 Schematic of the B-form DNA-inspired double helix metamaterial unit cell: (a) end plate and helix structure (isometric and top views); (b) connecting beams (top, main, and bottom views); (c) mass blocks (isometric and top views).

Additionally, Figure 5.1 indicates the dimensional parameters of each structural component. It is particularly important to note that the spacing between adjacent connecting beams (denoted B_D) is not uniform. The spacing from the 1st to the 4th beam and from the 9th to the 11th beam is $B_D = 0.1D_a + B_T$, whereas the spacing from the 4th to the 9th beam is $B_D = 0.1D_a$. This design is implemented to achieve a more perfect connection between the beams and the helix. Here, D_a represents the axial distance between the two DNA strands, and B_T is the thickness of the beams. The connecting beams are created by a combination of rotation and translation operations: each successive beam is rotated by 36 degrees relative to the previous one and translated upward by B_D . The number of helical turns within the unit cell is $N = 1.2$. Excluding the portions occupied by the hollow cylindrical end plates, the central section comprises exactly 1.1 turns of the helix, resulting in eleven connecting beams per unit cell. In the experiments, the substrate plates and the B-form DNA double helix metamaterial were fabricated from Nylon PA 2200 via 3D printing. The eleven connecting beams were made from aluminium sheets 1 mm in thickness, while the mass blocks were composed of four materials with different densities: steel, aluminium, nylon, and foam. To achieve longitudinal wave bandgaps below 200 Hz, multiple simulations and parametric optimizations were performed on the BDHM unit cell using COMSOL. The final optimized structural parameters are listed in Table 5.1 [218].

Table 5.1 Geometrical dimensions of B-form DNA double helix metamaterial

Symbol	Definition	Value
D_a	Pitch of the axial of the helix	102 mm
D_H	Major diameter of the helix	60 mm
D_h	Minor diameter of the helix	8 mm
D	Primary diameter of the hollow end plate	70 mm
d	Minor diameter of the hollow end plate	50 mm
H_T	Thickness of the hollow end plate	10 mm
B_L	Length of the beam	60 mm
B_W	Width of the beam	8 mm
B_T	Thickness of the beam	1 mm
M_L	Length of the mass block	20 mm
M_W	Width of the mass block	8 mm
M_T	Thickness of the mass block	8 mm
M_D	Distance between two mass blocks	8 mm
a	Height of BDHM	132.4 mm

5.2.2 FE model development

To characterize the dispersion properties of the proposed novel BDHM and determine its bandgap positions, this subsection employs COMSOL for finite element numerical simulations. Periodic boundary conditions are applied to the two sides of the unit cell along the z -axis, while all other boundaries are set as free. The corresponding finite element model is shown in Figure 5.2(a).

Furthermore, to investigate the dynamic response of elastic waves in the BDHM, a frequency-domain analysis is conducted to determine its displacement transmission curve. A finite chain model comprising six unit cells is established, with the unit cells reliably connected via an array instruction. This model is depicted in Figure 5.2(b). An axial harmonic displacement excitation with an amplitude of 0.01 m is applied to the left end face of the model, while all other boundaries are set free. By performing a frequency sweep within the range of 0–1000 Hz, the average displacement response on the right end face is obtained, from which the displacement transmission curve is plotted. The transmissibility is calculated according to Equation (2.33), where U_{out} and U_{in} represent the average axial displacement at the excitation end and the response end, respectively.

In the simulation calculations, physics-controlled tetrahedral mesh elements are employed. All materials are treated as linearly elastic, with the specific material parameters listed in Table 5.2. For the band-structure calculations, a standard mesh size is used, resulting in 35,612 elements with an average element quality of 0.6272. For the finite structure analysis, an extra-fine mesh is used, comprising 155,217 elements with an average element quality of 0.6212.

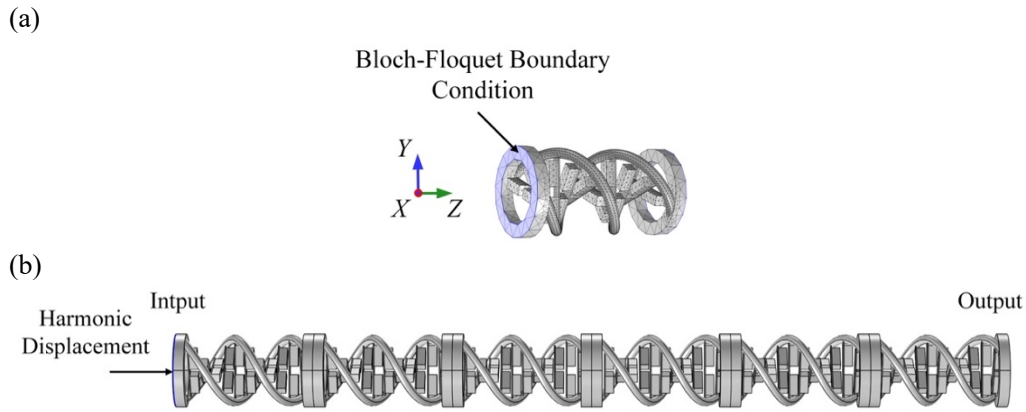


Figure 5.2 FEM Models of BDHM: (a) unit cell with periodic Bloch-Floquet boundary conditions for infinite periodic analysis; (b) finite chain consisting of 6-unit cells for transmission analysis.

Table 5.2 Material parameters of B-form DNA double helix metamaterial

Material Parameter	Steel	Aluminum	Nylon PA 2200	Foam
$E(\text{GPa})$	210	70	1.2	0.8
$\rho(\text{kg/m}^3)$	7800	2700	950	18
ν	0.33	0.33	0.35	0.33

5.3 Numerical computation and result analysis

5.3.1 Band structure and displacement transmission

In this subsection, four materials, steel, aluminium, nylon, and foam, are employed to analogously represent the four bases (A, T, C, and G) in DNA, respectively, leading to the construction of four types of BDHM: BDHM-A, BDHM-T, BDHM-C, and BDHM-G. The pitch-to-diameter ratio of these structures strictly follows the geometric characteristics of natural DNA. The infinite periodic band structures, displacement transmission curves of finite structures, and full-field displacement distributions of the aforementioned metamaterials are systematically calculated. It should be noted that, due to the dense distribution of modes in the band structures, the longitudinal wave modes are highlighted to clearly illustrate their propagation characteristics, while the remaining modes are dimmed for enhanced visualization and focused analysis.

Figure 5.3 presents the calculation results for BDHM-A, where all attached mass blocks are made of steel. Figure 5.3(a) shows the band structure of BDHM-A within the frequency range of 0–1000 Hz. The results reveal the presence of six relatively wide longitudinal wave bandgaps, with their frequency ranges and corresponding bandwidths as follows: 88.58–123.30 Hz (bandwidth 34.72 Hz), 185.72–241.72 Hz (bandwidth 56.00 Hz), 277.42–319.98 Hz (bandwidth 42.56 Hz), 344.55–552.01 Hz (bandwidth 207.46 Hz), 604.56–674.47 Hz (bandwidth 69.91 Hz), and 846.16–1000 Hz (bandwidth 153.84 Hz).

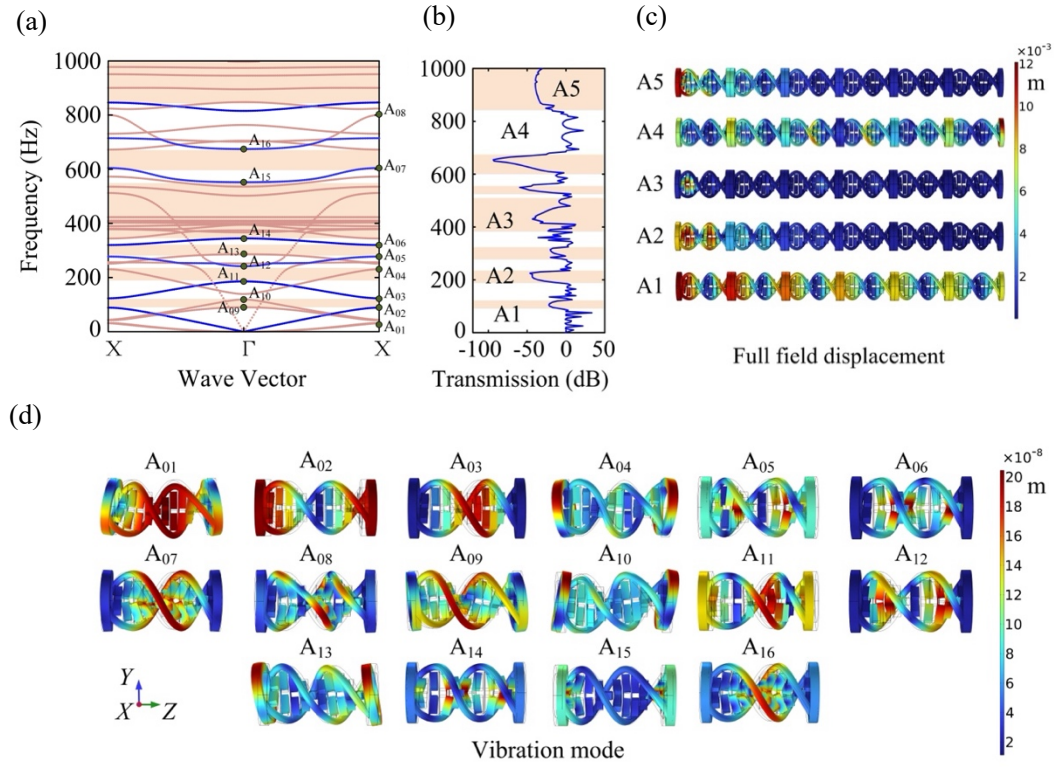


Figure 5.3 FEM results for the BDHM-A: (a) band structure of the infinite periodic structure; (b) displacement transmission of the finite structure; (c) full field displacement at characteristic frequency points; (d) vibration modes at band-edge frequencies.

Figure 5.3(b) displays the corresponding displacement transmission curve. Notably, the fourth bandgap manifests as a narrower attenuation region in the transmission curve, containing a narrow passband. This discrepancy arises because the displacement transmission curve reflects the forced-vibration response of a finite structure, whereas the band structure describes the intrinsic properties of an infinite periodic system.

To further analyse the wave propagation behavior, five characteristic frequency points (A1: 91 Hz, A2: 216 Hz, A3: 416 Hz, A4: 771 Hz, A5: 911 Hz) were selected to illustrate their full-field displacement distributions (Figure 5.3(c)). The results indicate that at points A1 and A4, located within passbands, longitudinal waves propagate effectively. In contrast, at points A2, A3, and A5, within bandgaps, longitudinal wave propagation is strongly suppressed, further validating the role of bandgaps in tuning elastic wave propagation.

Figure 5.3(d) illustrates the characteristic vibration modes of the unit cell at the boundary frequencies of each bandgap, with the specific frequency values of the characteristic points listed in Table 5.3.

In mode A₀₂, vibrations are primarily concentrated in the end plates and in the 1st, 2nd, 10th, and 11th beams, along with their attached mass blocks, with the overall structure in a tensile state. Mode A₀₃ shows vibrations concentrated in the central helix, the 5th, 6th, and 7th beams, and their attached mass blocks. In mode

A₀₅, vibrations are concentrated in the middle sections of the 4th, 5th, 7th, and 8th beams, where bending deformation occurs, resulting in an overall compressive state of the structure. Mode A₀₆ displays bending deformation in the 2nd through 10th beams. In mode A₀₇, significant deformation occurs in the 3rd through 9th beams, leading to contact between them. Mode A₁₁ exhibits the most pronounced deformation in the 5th through 7th beams, with the overall structure shifting to the right. Mode A₁₂ is symmetric about the central 6th beam, with maximum deformation occurring in the 3rd, 4th, 7th, and 8th beams. In mode A₁₄, energy is concentrated in the central regions of the 3rd, 4th, 5th, 6th, and 9th beams and their attached mass blocks. Mode A₁₅ shows the most significant deformation in the 1st, 2nd, 10th, and 11th beams and their mass blocks. Mode A₁₆ has energy primarily distributed in the 3rd through 9th beams and the middle section of the double helix.

Mode A₀₁ shows compression in the upper part and tension in the lower part, with deformation concentrated in the 4th through 8th beams and the double helix, resulting in a clear bending deformation of the overall structure. Mode A₀₄, conversely, exhibits tension in the upper part and compression in the lower part, with energy concentrated in the end plates. Mode A₀₈ shows energy concentrated in the 2nd through 10th beams and the connected double helix, presenting an overall torsional state. Mode A₀₉ displays a coupled torsional-bending state, with energy most concentrated in the central segment of the double helix. Mode A₁₀ also exhibits a coupled torsional-bending state, but the energy is most concentrated in the end plates. Mode A₁₁ exhibits a mixed state of bending, torsion, and tension, with energy concentrated in the central 5th and 6th beams and the connected double helix segments.

The results indicate that modes A₀₂, A₀₃, A₀₅, A₀₆, A₀₇, A₁₁, A₁₂, A₁₄, A₁₅, and A₁₆ are longitudinal wave vibration modes. Modes A₀₁ and A₀₄ are flexural vibration modes. Mode A₀₈ is a torsional vibration mode. Modes A₀₉, A₁₀, and A₁₃ are coupled vibration modes. Upon close examination of the full-field displacements of the longitudinal modes, it is observed that the beams arranged periodically around the DNA double helix undergo bending or rotation, causing the mass blocks to move accordingly. The helical portion primarily deforms through tension, compression, and torsion. The various parts of the structure absorb energy from propagating elastic waves through different deformation mechanisms from different directions, collectively leading to the formation of bandgaps.

Figure 5.4 presents the calculation results for BDHM-T, where all attached mass blocks are made of aluminium. As shown in Figure 5.4(a), within the frequency range of 0–1000 Hz, BDHM-T exhibits five relatively wide longitudinal elastic wave bandgaps: 114.07–182.06 Hz (bandwidth 67.99 Hz), 255.76–366.62 Hz (bandwidth 110.86 Hz), 403.90–498.55 Hz (bandwidth 94.65 Hz), 531.40–763.46 Hz (bandwidth 232.06 Hz), and 863.90–983.52 Hz (bandwidth 119.62 Hz). Similar to the BDHM-A case, the fourth longitudinal wave bandgap of BDHM-T contains passbands at 591–601 Hz, 636–641 Hz, and 656 Hz. Figure 5.4(b) displays the corresponding displacement transmission curve, where only the relatively wide narrow passband (591–601 Hz) within the bandgap is indicated. Following the same approach, five characteristic frequency points, T1 (143 Hz), T2 (225 Hz), T3 (456

Hz), T4 (835 Hz), and T5 (931 Hz), are selected to illustrate the full-field displacement response of the finite BDHM-T metamaterial structure. As shown in Figure 5.4(c), at the frequency points T2 and T4, which lie within the passbands, elastic waves propagate normally. In contrast, at frequency points T1, T3, and T5, which are situated within bandgaps, elastic waves are clearly suppressed.

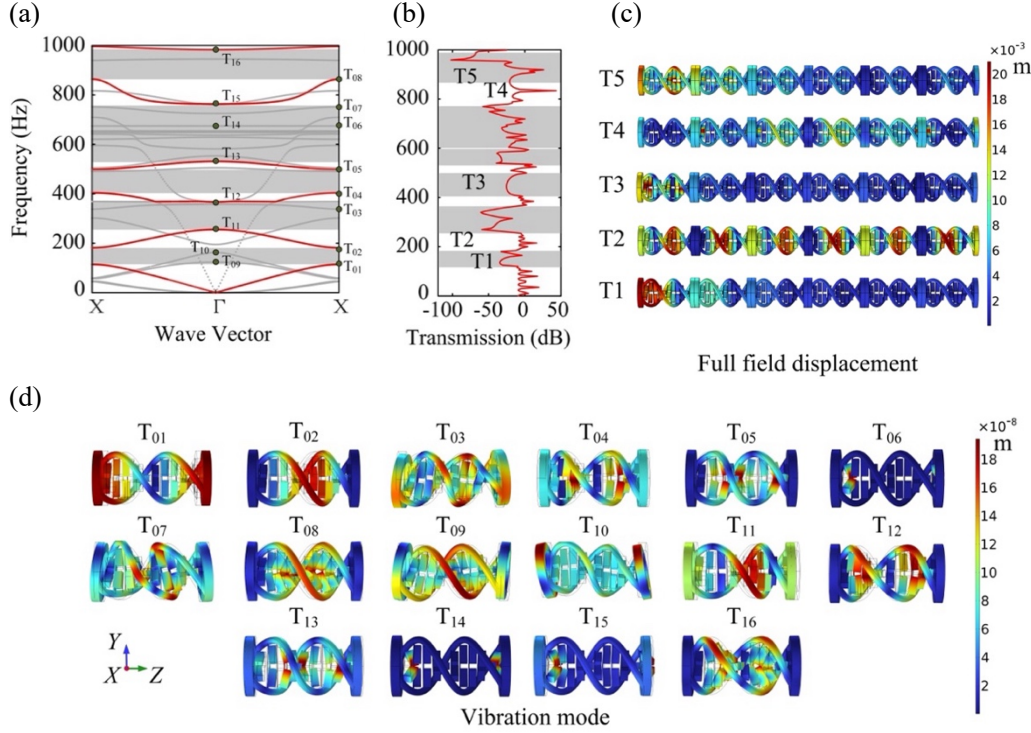


Figure 5.4 FEM results for the BDHM-T: (a) band structure of the infinite periodic structure; (b) displacement transmission of the finite structure; (c) full field displacement at characteristic frequency points; (d) vibration modes at band-edge frequencies.

Figure 5.4(d) presents the vibration modes of the unit cell at the characteristic band-edge frequencies, which are listed in Table 5.3.

At mode T₀₁, the full-field displacement reveals an overall compressive state of the structure, with deformation gradually decreasing from the ends toward the center. Mode T₀₂ also exhibits an overall compressive state, but deformation progressively increases from the ends toward the center. Mode T₀₄ displays an overall compressive state, with the most significant deformation occurring in the 4th and 8th beams, which undergo bending. Mode T₀₅ shows an overall compressive state; the 2nd through 10th beams rotate with the double helix, experiencing varying degrees of bending, with the 3rd, 6th, and 9th beams exhibiting the most pronounced bending. Mode T₀₈ reveals slight torsion of the double helix, with bending deformation occurring in the 2nd through 10th beams, most prominently in the 3rd through 9th beams. Mode T₁₁ shows the overall structure in a compressive state, shifting to the right, with the 5th, 6th, and 7th beams rotating together with the double helix and exhibiting the most significant

deformation. Mode T_{12} displays maximum deformation in the 3rd, 4th, 8th, and 9th beams, along with their connected helices and mass blocks. Mode T_{13} shows the most pronounced deformation in the 3rd, 4th, 5th, 6th, 9th, 10th, and 11th beams. Mode T_{15} exhibits deformation concentrated in the 1st and 11th beams, which undergo bending. Mode T_{16} shows deformation concentrated in the 2nd through 5th beams and the 7th through 10th beams, along with their connected helices. Modes T_{03} and T_{07} display a bending state where one half of the structure is compressed, and the other half is tensioned. Modes T_{06} and T_{14} show energy concentrated in the 1st and 11th beams, respectively, and in their attached mass blocks. Mode T_{09} exhibits energy concentrated in the central segment of the double helix, with the structure in a coupled torsional-tensile state. Mode T_{10} shows energy concentrated in the end plates, with the double helix also in a coupled torsional-tensile state.

The results indicate that modes T_{01} , T_{02} , T_{04} , T_{05} , T_{08} , T_{11} , T_{12} , T_{13} , T_{15} , and T_{16} are longitudinal wave modes. Modes T_{03} and T_{07} are bending modes. Modes T_{06} , T_{09} , T_{10} , and T_{14} are coupled vibration modes. Notably, modes T_{06} and T_{14} lie on the same flat band, where pronounced local resonance can be observed in the full-field displacement of the unit cell.

Figure 5.5 presents the calculation results for BDHM-C, where all attached mass blocks are made of nylon. As shown in Figures 5.5(a) and 5.5(b), within the frequency range of 0–1000 Hz, BDHM-C exhibits three relatively wide bandgaps and four narrower bandgaps. The three wide bandgaps are: 104.57–200.28 Hz, 262.10–430.53 Hz, and 814.68–1000 Hz. The four narrow bandgaps, identified from the frequency transmission curve, are: 456–461 Hz, 601–621 Hz, 661–691 Hz, and 721–761 Hz. As shown in Figure 5.5(b), the displacement transmission curve of the finite structure reveals a passband (341–351 Hz) within the second wide bandgap. Following the same approach, five characteristic frequency points at different locations, C1 (116 Hz), C2 (251 Hz), C3 (386 Hz), C4 (536 Hz), and C5 (881 Hz), are selected to illustrate the full-field displacement of BDHM-C, enabling further observation of the dynamic response of the finite structure across different frequency bands. As shown in Figure 5.5(c), at points C1, C3, and C5, which lie within the bandgap frequency ranges, elastic wave propagation is significantly attenuated. At points C2 and C4, located in passband regions, longitudinal waves propagate effectively.

Figure 5.5(d) presents the vibration modes of the unit cell at the band-edge frequencies, with the specific frequency values listed in Table 5.3. As illustrated, mode C_{02} exhibits an overall tensile state, with energy concentrated in the end plates and the 1st, 2nd, 10th, and 11th beams. Mode C_{03} displays an overall compressive state, with energy concentrated in the central helix, beams, and mass blocks. In mode C_{07} , deformation is concentrated in beams 2–5 and 7–10. Mode C_{08} shows deformation concentrated in beams 2, 4, 6, 8, and 10. Mode C_{11} exhibits energy concentrated in the central region of the structure, which is overall compressive, with a rightward shift. Mode C_{16} shows energy concentrated in the middle region of the 11th beam.

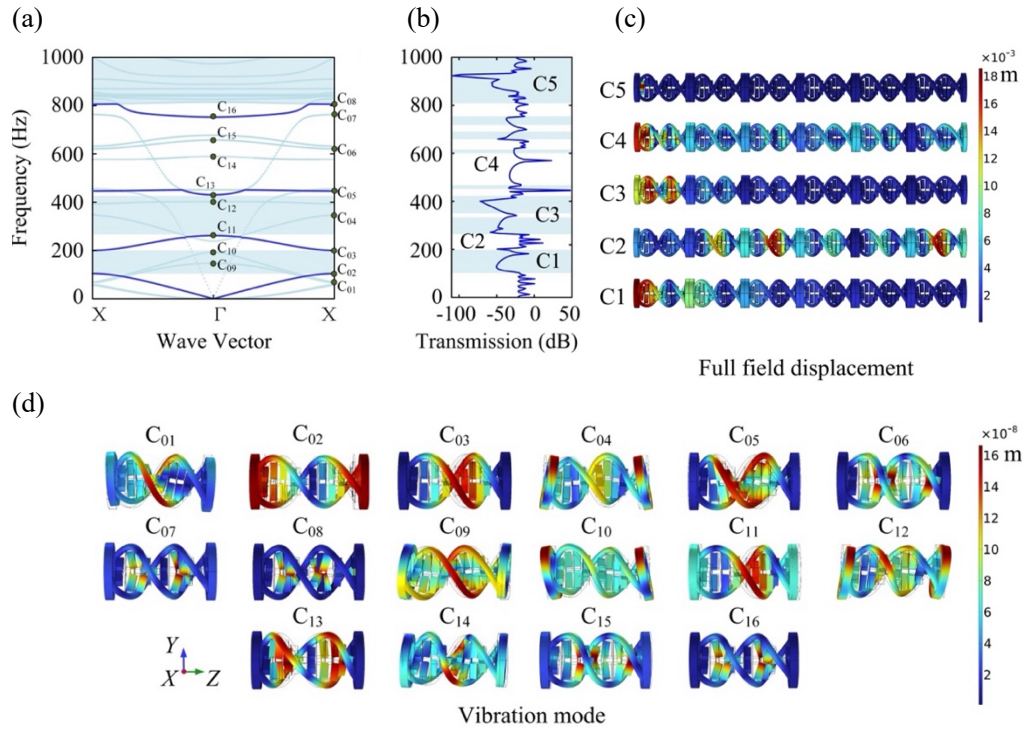


Figure 5.5 FEM results for the BDHM-C: (a) band structure of the infinite periodic structure; (b) displacement transmission of the finite structure; (c) full field displacement at characteristic frequency points; (d) vibration modes at band-edge frequencies.

Modes C₀₅, C₀₆, C₁₃, and C₁₅ display an overall torsional state, with the beams and mass blocks rotating together with the double helix. Modes C₀₉, C₁₀, C₁₂, and C₁₄ exhibit combined tensile-torsional deformation. Energy is concentrated in the central double helix for mode C₀₉, and in the hollow end plates for mode C₁₀, while modes C₁₂ and C₁₄ show no obvious energy concentration.

The results indicate that modes C₀₂, C₀₃, C₀₇, C₀₈, C₁₁, and C₁₆ are longitudinal wave modes. Modes C₀₁ and C₀₄ are flexural modes. Modes C₀₅, C₀₆, C₁₃, and C₁₅ are torsional modes. Modes C₀₉, C₁₀, C₁₂, and C₁₄ are coupled vibration modes. The consistent vibration characteristics observed at different frequency points within the same band, for example, C₀₃ and C₁₁, C₀₆ and C₁₅, and C₀₈ and C₁₆, demonstrate that the individual bands in the band structure can be identified by the consistency of their vibration modes.

Figure 5.6 presents the band structure, displacement transmission curve, and full-field displacement for BDHM-G, where all attached mass blocks are made of foam. As shown in Figure 5.6(a), within the frequency range of 0–1000 Hz, BDHM-G exhibits three relatively wide bandgaps: 98.48–227.82 Hz, 279.86–518.28 Hz, and 938.21–1000 Hz. Additionally, several narrow bandgaps, such as 526–531 Hz, 826–831 Hz, and 856–861 Hz, can be observed in the displacement transmission curve, though they are not individually marked in the figure. A closer examination of the displacement transmission curve in Figure 5.6(b) reveals two narrow

passbands within the second wide bandgap of the finite structure, specifically at 361–396 Hz and 466–471 Hz, where elastic waves can propagate.

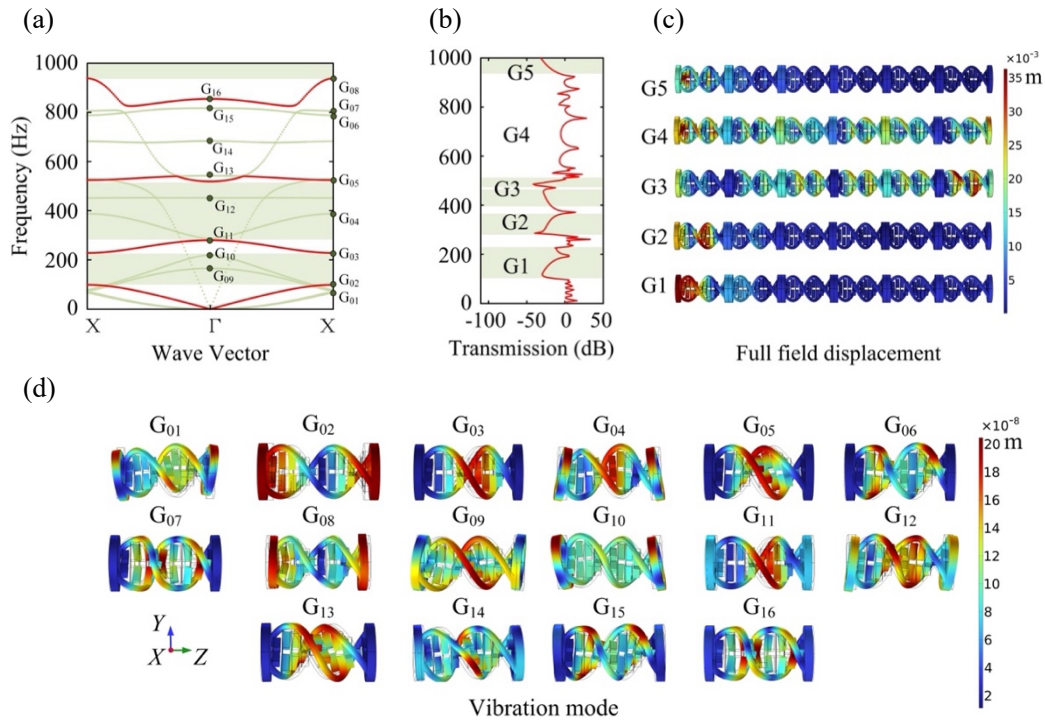


Figure 5.6 FEM results for the BDHM-G: (a) band structure of the infinite periodic structure; (b) displacement transmission of the finite structure; (c) full field displacement at characteristic frequency points; (d) vibration modes at band-edge frequencies.

To further illustrate the propagation characteristics of longitudinal elastic waves, the full-field displacements at five frequency points, G1 (141 Hz), G2 (306 Hz), G3 (471 Hz), G4 (676 Hz), and G5 (981 Hz), are presented. Points G1, G2, and G5 lie within bandgap frequency ranges, where elastic wave propagation is significantly suppressed. Point G4, located in a passband region, allows normal longitudinal wave propagation. Notably, although G3 falls within the second wide bandgap, it corresponds to a narrow passband that still permits elastic wave transmission.

Figure 5.6(d) illustrates the vibration modes of the unit cell at the band-edge frequencies, with the corresponding frequency values detailed in Table 5.3. Modes G02, G03, G08, G11, and G16 exhibit an overall tensile, compressive, or laterally displaced state. Modes G01 and G04 exhibit a state in which one half of the structure is under tension and the other half under compression. Modes G05, G06, G07, G13, G14, and G15 show an overall torsional state, with the beams and mass blocks rotating together with the double helix. Modes G09, G10, and G12 exhibit a coupled tensile-torsional state. The results indicate that modes G02, G03, G08, G11, and G16 are longitudinal wave modes. Modes G01 and G04 are flexural modes. Modes G05,

G_{06} , G_{07} , G_{13} , G_{14} , and G_{15} are torsional modes. Modes G_{09} , G_{10} , and G_{12} are coupled vibration modes.

The specific values of the characteristic frequency points at the band edges for the novel B-form DNA double helix metamaterials under different configurations presented in Figures 5.3–5.6 are listed in Table 5.3.

Table 5.3 Band-edge frequency points selected in the band structure of BDHM

Marker	Frequency (Hz)	Marker	Frequency (Hz)	Marker	Frequency (Hz)	Marker	Frequency (Hz)
A ₀₁	30.98	T ₀₁	114.07	C ₀₁	68.75	G ₀₁	66.20
A ₀₂	88.57	T ₀₂	182.06	C ₀₂	104.57	G ₀₂	98.48
A ₀₃	123.3	T ₀₃	338.11	C ₀₃	200.28	G ₀₃	227.82
A ₀₄	230.63	T ₀₄	403.9	C ₀₄	345.53	G ₀₄	387.01
A ₀₅	277.42	T ₀₅	498.55	C ₀₅	446.32	G ₀₅	523.7
A ₀₆	319.98	T ₀₆	674.69	C ₀₆	620.84	G ₀₆	788.36
A ₀₇	604.56	T ₀₇	751.29	C ₀₇	762.8	G ₀₇	806.46
A ₀₈	800.34	T ₀₈	863.9	C ₀₈	805.95	G ₀₈	938.21
A ₀₉	90.042	T ₀₉	124.1	C ₀₉	147.27	G ₀₉	165.49
A ₁₀	119.02	T ₁₀	162.63	C ₁₀	192.86	G ₁₀	218.57
A ₁₁	185.72	T ₁₁	255.76	C ₁₁	262.1	G ₁₁	279.86
A ₁₂	241.72	T ₁₂	366.62	C ₁₂	401.75	G ₁₂	451.37
A ₁₃	286.17	T ₁₃	531.4	C ₁₃	430.53	G ₁₃	541.63
A ₁₄	344.55	T ₁₄	674.77	C ₁₄	589.6	G ₁₄	683.43
A ₁₅	552.01	T ₁₅	763.46	C ₁₅	657.28	G ₁₅	817.74
A ₁₆	674.47	T ₁₆	983.52	C ₁₆	752.32	G ₁₆	855.26

5.3.2 Characteristics of low-frequency longitudinal wave bandgaps

To elucidate the formation mechanism and tuning laws of low-frequency bandgaps, this section systematically investigates the band structures, displacement transmission curves, and vibration modes below 250 Hz in four BDHM helix metamaterials with uniform mass blocks.

As shown in Figure 5.7(a), BDHM-A exhibits two distinct bandgaps below 250 Hz: 89–123 Hz and 186–250 Hz. The displacement transmission curve demonstrates excellent agreement with the band structure regarding bandgap positions. Figure 5.7(b) further shows the vibration modes at selected characteristic frequencies. Among these, a_1 and a_3 correspond to the modes at the opening and closing frequencies of the first longitudinal bandgap, respectively. Point a_2 represents the mode at 90 Hz, located on the first band within the first bandgap, while a_4 corresponds to the mode at 186 Hz, the lower boundary of the second bandgap. The results indicate that at point a_1 (lower boundary of the first bandgap), vibrations are primarily concentrated in the hollow cylindrical end plates, gradually diminishing from the ends toward the centre. At point a_3 (upper boundary of the

first bandgap), vibrations are predominantly concentrated in the central helix, crossbeams, and mass blocks, gradually weakening from the centre outward. This confirms that the first bandgap is dominated by local resonance. At point a_2 , the most pronounced vibrations occur in the central helix. At point a_4 , vibrations are mainly concentrated in the central helix and its connected crossbeams and mass blocks, with noticeable vibrations also present in the end plates.

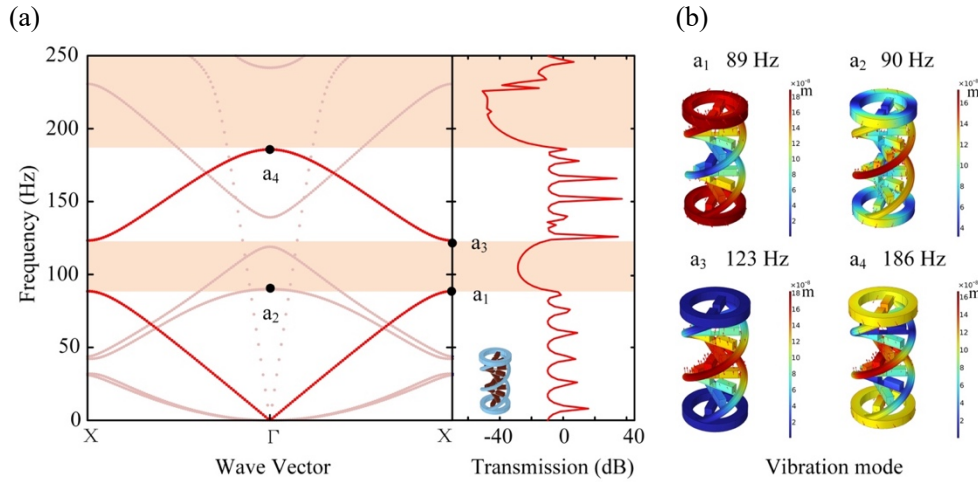


Figure 5.7 Longitudinal wave bandgap characteristics and corresponding vibration modes for BDHM-A below 250 Hz: (a) band structure and displacement transmission curve; (b) vibration modes at selected frequency points.

From Figure 5.8(a), it can be observed that BDHM-T exhibits only one longitudinal bandgap below 250 Hz, spanning 114–182 Hz. The band structure and transmission curve agree well on the bandgap. Figure 5.8(b) presents the vibration modes at selected frequency points, where t_1 and t_3 correspond to the modes at the lower and upper boundaries of the first longitudinal bandgap, respectively. t_2 represents the vibration mode at 124 Hz, located on the first band within the first longitudinal bandgap, while t_4 corresponds to the mode at 195 Hz, situated on the third band within the same bandgap. The results indicate that at point t_3 (the upper boundary of the lowest bandgap), vibrations are predominantly concentrated in the central helix, crossbeams, and mass blocks. At point t_1 (the lower boundary), vibrations are primarily concentrated in the hollow cylindrical end plates. This confirms that the lowest bandgap is also predominantly formed by local resonance. The vibration mode at t_3 is similar to that at a_2 , while the vibrations at t_4 are concentrated in the centre of the unit cell.

Figure 5.9(a) shows that BDHM-C exhibits a single longitudinal bandgap spanning 105–200 Hz, with excellent agreement between the band structure and the transmission curve regarding the bandgap position. As shown in Figure 5.9(b), c_1 and c_4 represent the vibration modes at the lower and upper boundaries of the first longitudinal bandgap, respectively. Their vibration characteristics are similar to those observed in the previous two configurations. c_2 and c_3 correspond to the vibration modes at characteristic points on two different bands within the first

bandgap, exhibiting relatively dispersed vibrations. Specifically, vibrations at c_1 are primarily concentrated in the hollow cylindrical end plates, while vibrations at c_4 are predominantly concentrated in the central helix, attached mass blocks, and crossbeams of the unit cell.

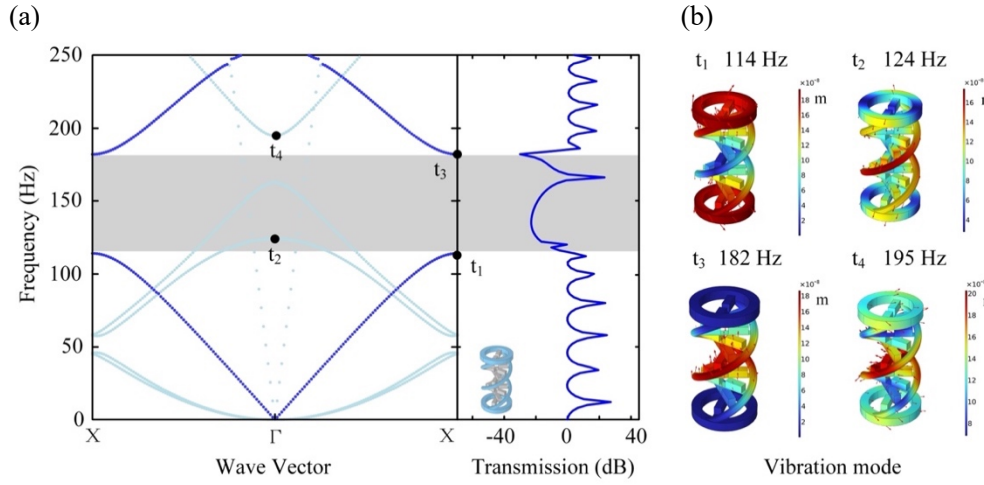


Figure 5.8 Longitudinal wave bandgap characteristics and corresponding vibration modes for BDHM-T below 250 Hz: (a) band structure and displacement transmission curve; (b) vibration modes at selected frequency points.

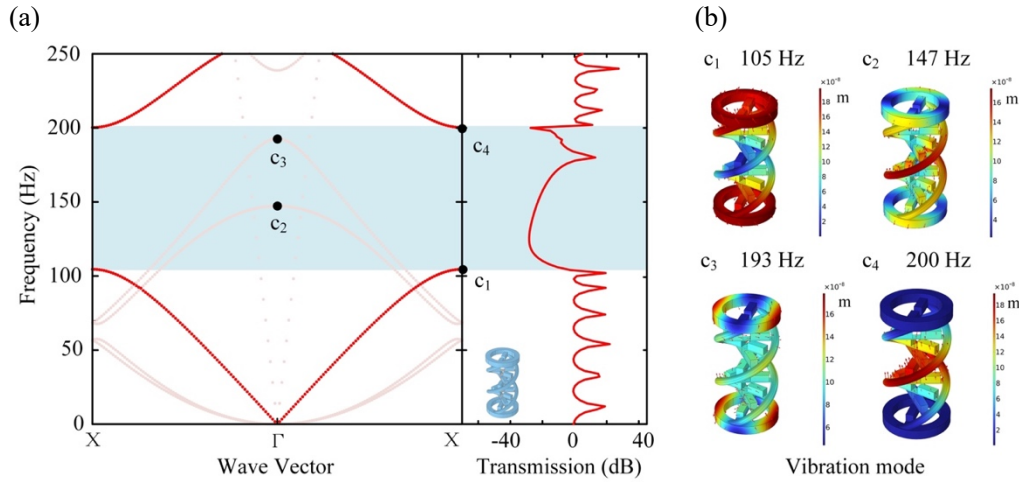


Figure 5.9 Longitudinal wave bandgap characteristics and corresponding vibration modes for BDHM-C below 250 Hz: (a) band structure and displacement transmission curve; (b) vibration modes at selected frequency points.

Figure 5.10(a) presents the band structure and displacement transmission curve of BDHM-G within 250 Hz. The results indicate that the lowest longitudinal bandgap of BDHM-G ranges from 98 Hz to 228 Hz, with good agreement between the band structure and the transmission curve regarding the bandgap position. Figure 5.10(b) shows the vibration modes at the lower and upper boundary points of the lowest bandgap (g_1 and g_4), as well as at two selected points on two different

bands within the bandgap (g_2 and g_3). The modes at g_1 and g_4 exhibit pronounced local resonance, while no local resonance is observed at g_2 and g_3 .

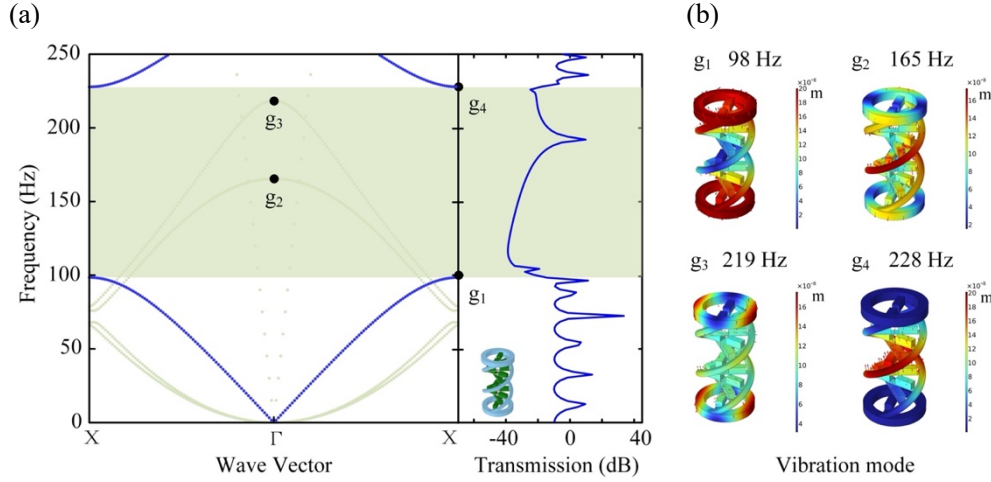


Figure 5.10 Longitudinal wave bandgap characteristics and corresponding vibration modes for BDHM-G below 250 Hz: (a) band structure and displacement transmission curve; (b) vibration modes at selected frequency points.

The above calculations demonstrate that for the four B-form DNA double helix metamaterial configurations with uniform mass blocks, namely BDHM-A, BDHM-T, BDHM-C, and BDHM-G, the vibration modes at the lower and upper boundary frequencies of the first longitudinal bandgap are remarkably similar, all exhibiting bandgaps dominated by local resonance. At the lower boundary frequency, significant vibrations occur in the hollow cylindrical end plates, with the overall structure exhibiting a tensile state. At the upper boundary frequency, energy is concentrated in the helix-beam-mass block region at the center of the unit cell, with the overall structure exhibiting a compressive state.

5.3.3 Study on tunable longitudinal wave bandgaps

The BDHM, inspired by the base-pair arrangement of B form-DNA, employs four materials with different densities, steel, aluminium, nylon, and foam, to construct the mass blocks. By spatially encoding the distribution of these mass blocks while keeping the geometric structure fixed, the bandgap characteristics can be actively tuned. By varying the permutations and combinations of the four mass block types (A, T, C, G) within the structure, the bandgap position and width can be systematically adjusted. Since each unit cell contains 22 mass blocks, in theory, up to 4^{22} distinct band structure configurations can be realized. In this subsection, ten typical permutations and combinations are selected, and the band structures of the corresponding B-form DNA double helix metamaterials are calculated. Taking BDHM-M1 (with the mass block arrangement ATCCGGGCCTA) as an example, the formation mechanism of its low-frequency bandgap is analysed in detail.

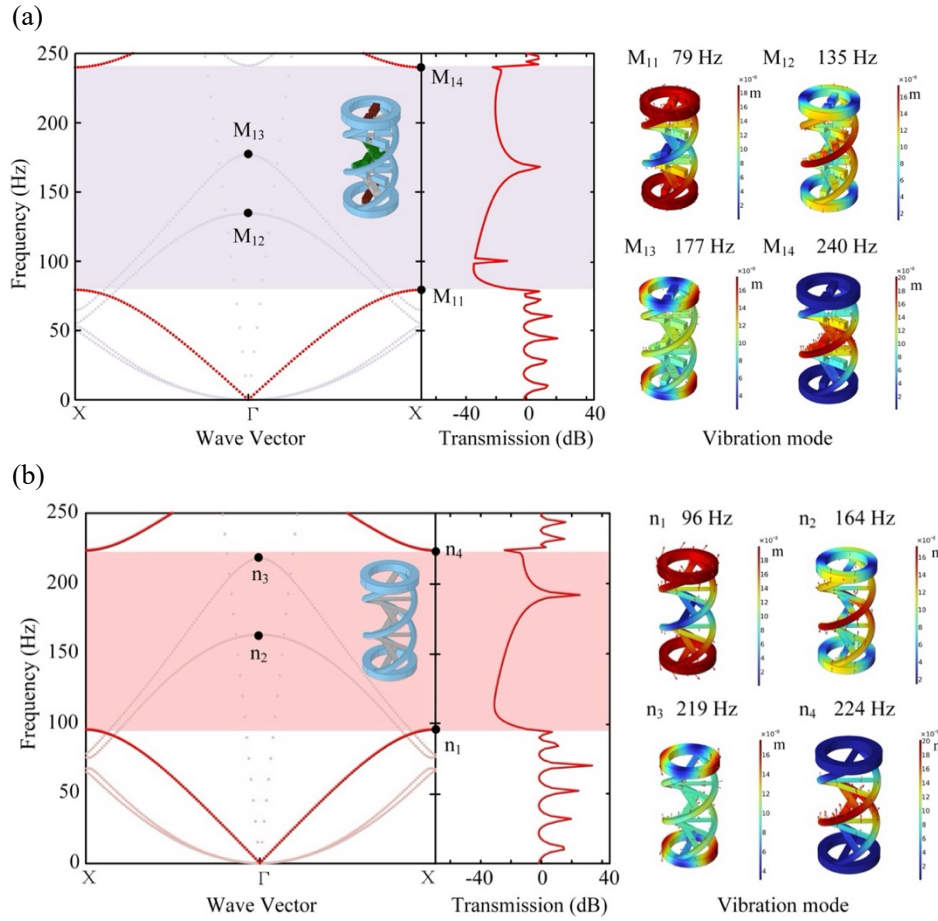


Figure 5.11 Longitudinal wave bandgap and corresponding displacement field response for two BDHM configurations: (a) BDHM-M1 (mass block arrangement from bottom to top as ATCCGGGCCTA); (b) BDHM-N (without mass blocks).

Figure 5.11(a) illustrates the low-frequency bandgap characteristics of BDHM-M1. In the figure, the dark brown, grey, light blue, and green regions within the unit cell represent the mass blocks made of the four materials: steel (A), aluminium (T), nylon (C), and foam (G), respectively. The results demonstrate that this hybrid arrangement achieves a wider low-frequency bandgap than configurations with uniform mass blocks (BDHM-A, BDHM-T, BDHM-C, BDHM-G), with the first bandgap ranging from 79 Hz to 240 Hz. This broadening phenomenon is attributed to the synergistic effect of multiple local resonance mechanisms excited by the mass blocks of different densities within BDHM-M1, which collectively dissipate longitudinal wave energy over a broader frequency range. The right side of Figure 5.11(a) further presents the vibration modes at characteristic frequency points: M_{11} (79 Hz, lower boundary of the first bandgap) exhibits vibration energy concentrated in the hollow cylindrical end plates; M_{14} (240 Hz, upper boundary) shows energy localized in the central helix-beam-mass block region; while M_{12} (135 Hz) and M_{13} (177 Hz), located within the bandgap, do not exhibit pronounced energy concentration.

For comparative analysis, Figure 5.11(b) presents the low-frequency bandgap characteristics of the B-form DNA double helix structure without attached mass

blocks (BDHM-N). The first bandgap of this structure ranges from 96 Hz to 224 Hz, which is relatively close to the bandgap characteristics of the configuration with foam mass blocks (BDHM-G). The right side of Figure 5.11(b) displays the vibration modes at characteristic frequency points: n_1 (96 Hz) corresponds to the mode at the lower boundary of the first bandgap, n_4 (224 Hz) to the mode at the upper boundary, and n_2 (164 Hz) and n_3 (219 Hz) to characteristic modes on two bands within the bandgap. The distribution patterns of the bandgap boundary modes are consistent with those observed in the aforementioned uniform mass block configurations: at the lower boundary (n_1), vibration energy is concentrated in the end plates, while at the upper boundary (n_4), energy is localized in the central helix-beam region. The numerical results demonstrate that by tuning the permutations and combinations of the mass units, effective encoding and tuning of the longitudinal wave bandgaps in the BDHM can be achieved while maintaining the geometric structure unchanged.

5.3.4 Influence of geometric parameters on bandgaps

Using BDHM-C as an example, this subsection systematically analyzes the influence of the hollow cylindrical end plate thickness H_T and the DNA minor helix diameter D_h on the first longitudinal bandgap.

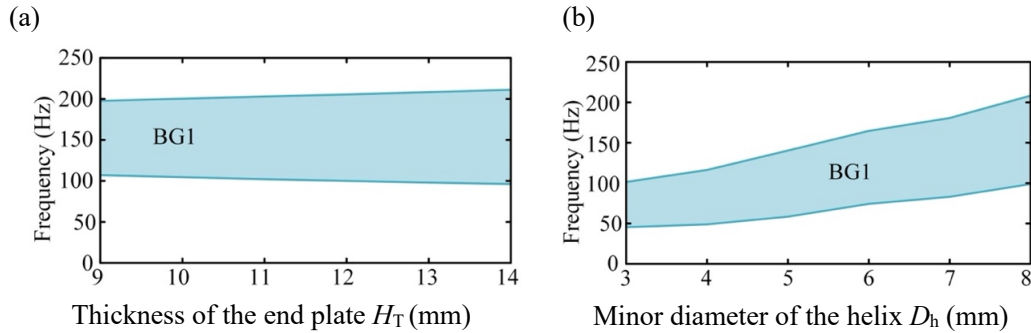


Figure 5.12 First bandgap frequency of BDHM-C varies with key geometric parameters: (a) thickness of the end plate H_T ; (b) minor diameters of the helix D_h (with mass block length fixed at $M_L = 10$ mm)

Based on the structural parameters listed in Table 5.1, the tuning effect of the end plate thickness on the bandgap characteristics is first investigated by varying H_T . Figure 5.12(a) illustrates the variation of the upper and lower boundary frequencies of the first longitudinal bandgap for different end plate thicknesses. As the end plate thickness increases from 9 mm to 14 mm, the starting frequency of the bandgap decreases from 107.11 Hz to 96.18 Hz, while the bandwidth correspondingly expands from 90.48 Hz to 114.89 Hz.

Then, with the mass block length fixed at $M_L = 10$ mm and other parameters held constant (as specified in Table 5.1), the influence of the minor helix diameter D_h on the bandgap characteristics is systematically investigated. As shown in Figure 5.12(b), as D_h increases from 3 mm to 8 mm, the frequency range of the first

longitudinal bandgap expands and shifts from 45.39–101.45 Hz (bandwidth 56.06 Hz) to 98.81–208.2 Hz (bandwidth 109.39 Hz).

Taking BDHM-A as an example, the influence of the mass block length M_L on the first longitudinal bandgap is systematically analyzed. With the center-to-center distance between adjacent mass blocks fixed at $M_C = D_h/2$ and other structural parameters held constant (as specified in Table 5.1), the variation of the first longitudinal bandgap is investigated by altering M_L . Figure 5.13 illustrates the variation of the upper and lower boundary frequencies of the lowest bandgap for different M_L . As M_L increases from 6 mm to 20 mm, the bandgap's starting frequency rises from 87 Hz to 93 Hz, while the cutoff frequency decreases from 137 Hz to 111 Hz, resulting in a significant narrowing of the bandwidth.

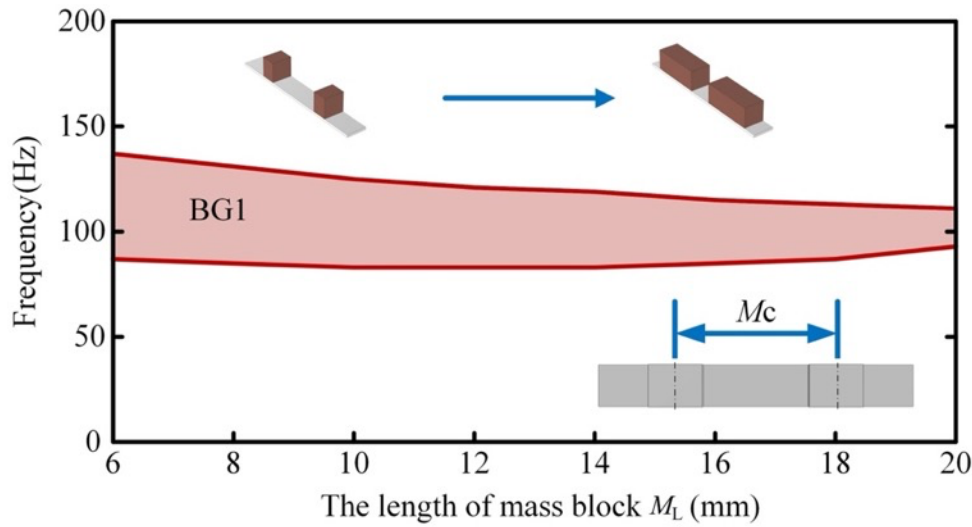


Figure 5.13 First bandgap frequency of BDHM-A varies with mass block length M_L , with center distance fixed at $M_C = 30$ mm.

The parametric analysis results indicate that, in addition to tuning the bandgap characteristics by varying permutations and combinations of mass blocks, the longitudinal bandgap can also be customized by adjusting key geometric parameters, such as the end plate thickness, minor helix diameter, and mass block length. This provides flexible design strategies for meeting wave control requirements in engineering applications.

5.4 Experimental verification

In this subsection, the bandgap characteristics predicted by the infinite periodic band structure are evaluated by measuring the displacement transmission curves of finite BDHM structures. The physical prototype of the finite structure and the experimental platform based on laser vibrometry are shown in Figure 5.14. The experimental specimen consists of six-unit cells. The helix and end plates of each unit cell were fabricated from imported nylon using 3D printing and laser sintering. The four types of mass blocks and the aluminum crossbeams were adhered to the

helical structure using epoxy adhesive, and the mass blocks of different densities were likewise attached to the crossbeams with AB adhesive. As shown in Figure 5.14(a), four supports made of acrylic plates and rubber pads were placed beneath the experimental sample to maintain structural stability and facilitate excitation.

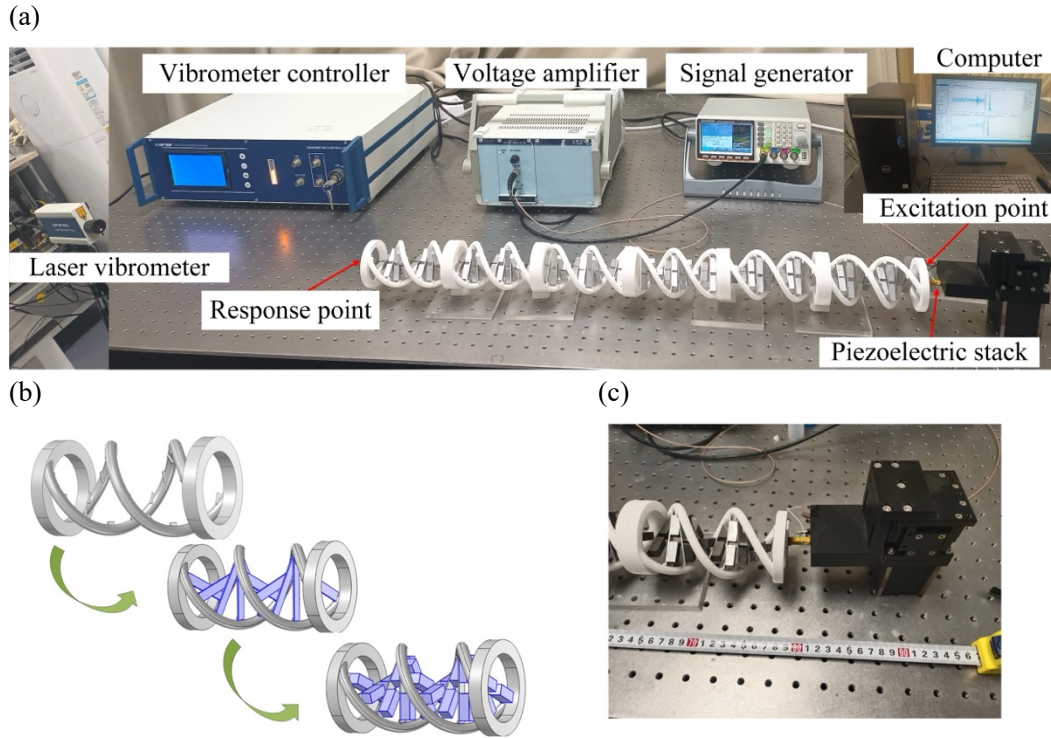


Figure 5.14 Experimental setup for BDHMs based on a laser vibrometer system: (a) photograph of the experimental setup; (b) assembly process of a BDHM unit cell; (c) assembled DNA double helix metamaterial structure.

During the experiment, to obtain the displacement transmission curve of the 3D-printed encoded helical metamaterial, a longitudinal wave sweep signal ranging from 1 to 2000 Hz was first generated by an arbitrary waveform generator. This signal was then amplified and transmitted to a piezoelectric stack actuator. The piezoelectric stack applied a longitudinal excitation parallel to the axial direction to the right end of the metamaterial structure. The displacement response at the left end of the structure was captured using a laser vibrometry system, and the time-domain vibration response signal was recorded for 90 seconds. The data were processed according to Equation (2.35) to obtain the corresponding displacement transmission.

Figure 5.15 presents the displacement transmission curves of six types of B-form DNA double helix metamaterial finite structures: BDHM-A, BDHM-T, BDHM-C, BDHM-G, BDHM-M1, and BDHM-N. As shown in Figure 5.15(a), the first (90–121 Hz), second (178–245 Hz), fifth (552–679 Hz), and sixth (839–1000 Hz) longitudinal bandgaps of the BDHM-A metamaterial exhibit good agreement with the finite element simulation results. However, the experimentally measured

third bandgap (270–345 Hz) is wider than the simulated value, while the fourth bandgap (399–510 Hz) is narrower than predicted.

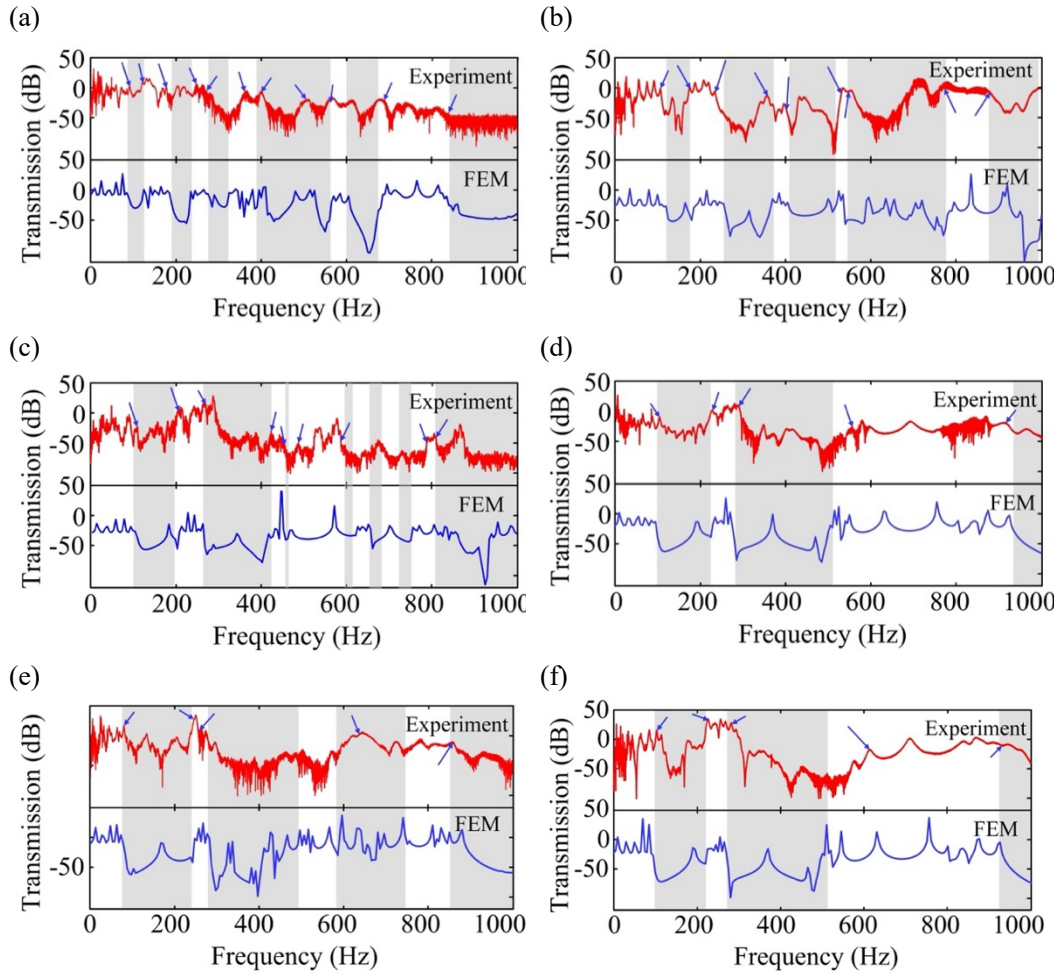


Figure 5.15 Experimental and FEM results for the longitudinal displacement transmission of six BDHM configurations in the 0–1000 Hz range: (a) BDHM-A; (b) BDHM-T; (c) BDHM-C; (d) BDHM-G; (e) BDHM-M1; (f) BDHM-N.

As illustrated in Figure 5.15(b), the four longitudinal bandgaps of the BDHM-T metamaterial show excellent agreement with the finite element results in terms of both position and width. Specifically, the measured frequencies of the first (107–176 Hz) and second (236–352 Hz) bandgaps are slightly lower than the simulated values, while the bandwidths remain consistent. The experimental bandwidth of the third bandgap (405–524 Hz) exceeds the predicted value, whereas the fourth (554–766 Hz) and fifth (880–980 Hz) bandgaps are narrower than the finite element results.

Figure 5.15(c) demonstrates that the three wide bandgaps of BDHM-C (110–206 Hz, 268–426 Hz, and 810–1000 Hz) are consistent with the finite element results. Notably, the experimentally measured first narrow bandgap (458–485 Hz) exhibits a wider bandwidth than the simulated value, while the second through

fourth narrow bandgaps merge experimentally, forming a continuous wide bandgap spanning 587–788 Hz.

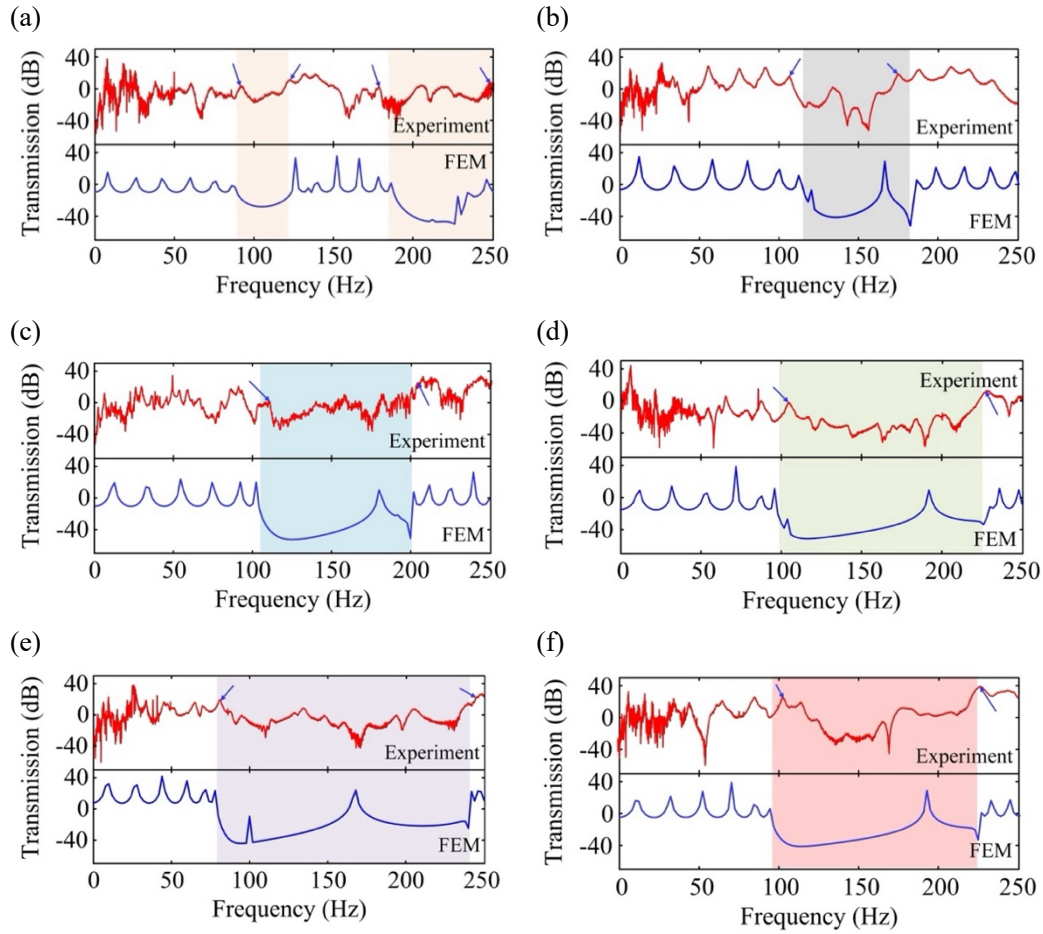


Figure 5.16 Experimental and FEM results for the longitudinal displacement transmission of six BDHM configurations in the 0–250 Hz range: (a) BDHM-A; (b) BDHM-T; (c) BDHM-C; (d) BDHM-G; (e) BDHM-M1; (f) BDHM-N.

Figure 5.15(d) shows that for the BDHM-G metamaterial, the experimental measurements are in good agreement with the finite element predictions for the first bandgap (103–228 Hz) and the third bandgap (920–1000 Hz). However, the experimentally obtained second bandgap (288–579 Hz) is significantly wider than the numerical simulation results.

Figure 5.15(e) presents a comparison between the experimentally measured and finite element predicted displacement transmission curves for BDHM-M1. It can be observed that the first bandgap (80–245 Hz) and the fourth bandgap (848–1000 Hz) exhibit excellent agreement with the finite element predictions, while the second and third bandgaps merge into a wider continuous bandgap (265–645 Hz) in the experimental results. The experimental findings also confirm that hybrid arrangements of mass blocks can yield broader, lower-frequency longitudinal wave bandgaps.

Figure 5.15(f) displays the experimentally measured and numerically calculated displacement transmission curves for BDHM-N. The results indicate that the experimentally measured first bandgap (100–226 Hz) and third bandgap (932–1000 Hz) are in good agreement with the finite element predictions, while the second bandgap (280–611 Hz) is considerably wider than the finite element calculation results.

Figure 5.16 presents the low-frequency displacement transmission curves for all six BDHM configurations. The results show that while the experimental first bandgap depth for BDHM-A is less pronounced, good agreement between measurement and simulation is achieved for the lowest bandgaps of the other five configurations (BDHM-T, -C, -G, -M1, and -N). Multiple instances of bandgap merging, narrowing, or broadening are observed experimentally. These discrepancies are attributed to the idealized linear elasticity assumptions in the numerical model, which contrast with the inevitable viscosity, nonlinearity, manufacturing defects, and non-ideal free boundary conditions present in the actual experimental setup.

5.5 Summary

Inspired by the arrangement of the four base pairs in B-form DNA, this chapter employed four materials with different densities to construct mass blocks. By spatially encoding the mass distribution while keeping the geometric structure fixed, active tuning of the bandgap was achieved, leading to the proposal of a double helix metamaterial structure (BDHM) with encodable longitudinal wave propagation characteristics. Using the finite element method, the band structures of different BDHM configurations (namely BDHM-A, BDHM-T, BDHM-C, BDHM-G, BDHM-M1, and BDHM-N) and the displacement transmission curves of finite structures comprising six unit cells were obtained. Furthermore, taking BDHM-C as an example, the influence of the hollow cylindrical end plate thickness H_T and the minor helix diameter D_h on the first longitudinal bandgap characteristics was investigated. Using BDHM-A as an example, the effect of mass block length on the first longitudinal bandgap was studied. Additionally, a laser vibrometry platform was established to validate the displacement transmission characteristics of the various BDHM finite structures obtained from finite element simulations. The main conclusions are as follows:

(1) Within the frequency range of 0–1000 Hz, both the number and range of bandgaps vary among different BDHM configurations. The bandgap characteristics observed in the band structures and in the displacement transmission curves are consistent. BDHM-A, BDHM-T, BDHM-C, and BDHM-G exhibit 6, 5, 3, and 3 bandgaps, respectively, with first bandgap frequency ranges of 89–123 Hz, 114–182 Hz, 105–200 Hz, and 98–228 Hz. Notably, BDHM-M1, featuring a hybrid arrangement (ATCCGGGCCTA), achieves the lowest and widest first bandgap (79–240 Hz). It should be noted that theoretically, there exist 4^{22} permutations and combinations of mass blocks within the BDHM unit cell, which are not exhaustively covered in this chapter. Therefore, by optimizing the sequence of mass blocks, even lower-frequency bandgaps can be achieved.

(2) Parametric analysis results indicate that as the end plate's thickness increases, the lower boundary frequency of the lowest bandgap gradually decreases, the upper boundary frequency slowly increases, and the bandwidth broadens. As the helix's minor diameter increases, both the upper and lower boundary frequencies of the lowest bandgap gradually increase, accompanied by a slight broadening of the bandwidth. As the mass block's length increases, both the upper and lower boundary frequencies of the lowest bandgap decrease, while the bandwidth narrows.

(3) The experimentally measured displacement transmission curves exhibit good agreement with the numerical simulation results regarding the number and width of bandgaps, particularly within the frequency range of the first bandgap. The B-form DNA-inspired structure constructed in this chapter provides a novel approach for the design of tunable, low-frequency, broadband elastic metamaterials. The calculated results provide valuable references for the application of DNA double helix elastic metamaterials.

Chapter 6

Double helix metamaterials with ultra-broadband and ultra-low frequency longitudinal wave attenuation

6.1 Introduction

Low-frequency vibration and noise (typically below 100 Hz) persistently pose a significant challenge in vibration and noise control due to their inherent characteristics: long wavelengths, strong penetration, and weak attenuation. For example, a 50 Hz vibration can have a wavelength of approximately 6.8 meters in a medium at room temperature, far exceeding the typical thickness of traditional vibration isolation materials such as rubber and foam, rendering them nearly ineffective at low frequencies. More critically, low-frequency vibrations can penetrate dense media such as concrete and soil, posing a continuous threat to building safety and the operation of precision instruments. Low-frequency vibrations induced by structures such as engines and large bridges can not only precipitate metal fatigue but may even lead to catastrophic failure. Elastic metamaterials, with their artificially engineered special structures and the ability to generate elastic wave bandgaps, offer a novel pathway to address this challenge. In recent years, significant progress has been made in this field: in structural vibration damping, metamaterial beams with low-frequency bandgaps have been successfully constructed by periodically integrating damping absorbers or inerter elements into beams; for seismic protection, periodic steel resonators have been proven effective in attenuating surface waves, offering protection for infrastructure; in acoustic control, negative Poisson's ratio structures and acoustic black hole designs have significantly enhanced low-frequency sound insulation and vibration absorption performance; furthermore, combined with 3D printing and smart actuation technologies, researchers have developed intelligent metamaterials capable of real-time tuning of bandgap characteristics. These achievements collectively propel metamaterials towards ultra-broadband, ultra-low-frequency, and dynamically tunable functionalities.

The previous chapter proposed a B-form DNA double helix metamaterial that reduced the longitudinal wave bandgap frequency to 79 Hz by manipulating the distribution of mass blocks. Theoretical analysis revealed that its bandgap arises

from a bending-torsional coupling resonance mechanism inherent to the DNA double helix. However, further reducing the bandgap frequency below 50 Hz presents numerous challenges. To achieve ultra-broadband and ultra-low-frequency longitudinal-wave attenuation, this chapter proposes a novel dual helix metamaterial incorporating nodal mass disks (Dual Helix Metamaterial with nodal Mass, DHMM). The research content of this chapter primarily includes: first, designing two types of metamaterial unit cells, one without and one with nodal mass disks; second, systematically analyzing their band structures, displacement transmission curves, vibration modes, and full-field displacements using the finite element method, with a focus on investigating the influence of key parameters such as the minor helix diameter, mass disk thickness, and axial pitch on the first complete bandgap (BG1) and the first longitudinal bandgap (LBG1); subsequently, experimentally validating the displacement transmission characteristics of finite structures using a laser vibrometry platform; and finally, providing a comprehensive summary of the work presented in this chapter.

6.2 Structural design and FE modelling

6.2.1 Unit cell design

Figure 6.1(a) illustrates the unit cell structure of the double helix metamaterial without nodal mass disks (DHMN). The DHMN unit cell is defined by the following geometric parameters: the major diameter of the DNA double helix, D_h ; the minor diameter of the DNA double helix, d_h ; the axial pitch of the double helix, D_a ; the number of helical turns, N ; the thickness of the hollow side plates connecting the double helix at both ends, T_h ; the major diameter D ; and the minor diameter d .

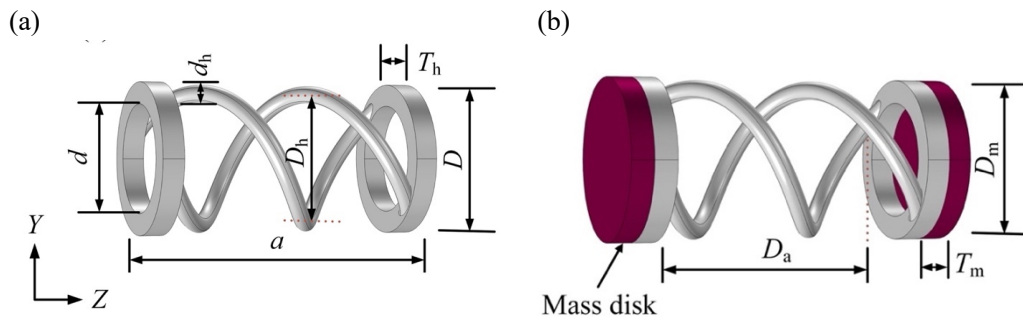


Figure 6.1 Schematics of the DNA double helix metamaterial unit cell: (a) DHMN (without nodal mass disks); (b) DHMM (with nodal mass disks).

Figure 6.1(b) depicts the unit cell structure of the double helix metamaterial with nodal mass disks (DHMM). DHMM is based on the DHMN unit cell, with mass disks added on both sides of the helical structure. The geometric dimensions of the mass disks are defined by their diameter, D_m , and thickness, T_m . Table 6.1 lists the specific geometric parameters for the unit cells of the two types of DNA double helix metamaterials, DHMs (DHMN and DHMM).

Table 6.1 Geometrical parameters of the unit cell of the DHMs

Symbol	Definition	Value
D_a	Pitch of the axial of the helix	102 mm
D_h	Major diameter of the helix	60 mm
d_h	Minor diameter of the helix	8 mm
D	Primary diameter of the hollow side plate	70 mm
d	Minor diameter of the hollow side plate	50 mm
T_h	Thickness of the hollow side plate	10 mm
T_m	Thickness of the mass disk	10 mm
D_m	Diameter of the mass disk	70 mm
N	The number of helical turns	1.2 mm
a	Total length of the DHMN cell	132.4mm

6.2.2 FE model development

This chapter continues to employ COMSOL for numerical modelling and finite element calculations. To obtain the band structures of the metamaterials designed herein, Bloch-Floquet periodic boundary conditions are applied to the two sides of the unit cell, as illustrated in Figure 6.2(a), while all remaining boundaries are set as free surfaces. By sweeping the wave vector $\mathbf{k}$ along the ΓX direction of the first irreducible Brillouin zone boundary from $-\pi/a$ to π/a and solving for the eigenfrequencies, the band structure is obtained.

To analyse the wave propagation characteristics of finite structures, an axial harmonic displacement excitation with an amplitude of 0.01 m is applied to one end face of the DHMs. A frequency sweep is performed from 0–2000 Hz, and the elastic wave displacement transmission curve is obtained by calculating the frequency domain response. The computational model is depicted in Figure 6.2(b). To simulate an open space, obtain reliable simulation results, and reduce the computational domain, low-reflecting boundary conditions are applied at both ends of the finite structure during the calculation of the displacement transmission curve. In the numerical simulations, a finite-length chain of the double helix metamaterial, consisting of six unit cells, is constructed using the array command in the COMSOL geometry module to ensure robust, reliable connections between units. To further facilitate the application of low-reflecting boundary conditions, two mass disks, each with a thickness equal to that of the hollow side plates, are added to both ends of the finite chain. As shown in Figure 6.2(b), the low-reflecting boundary conditions are applied to the end faces of the finite structure. Meanwhile, within the Solid Mechanics module, all other external boundaries are set as free boundary conditions. Meshing is performed using a physics-controlled mesh with the “Finer” element size selected.

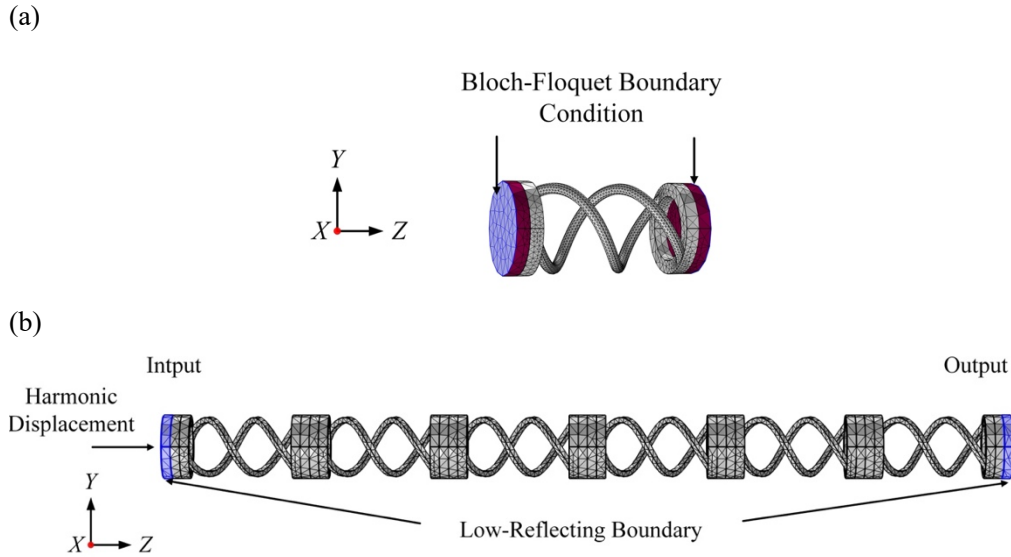


Figure 6.2 Finite element models for DHMs simulations: (a) unit cell with periodic Bloch-Floquet boundary condition; (b) finite DHMs chain with 6-unit cells.

The displacement transmissibility is defined by Equation (2.33), where U_{out} and U_{in} represent the average axial displacement at the excitation plane and the response plane, respectively. All materials are modelled as linearly elastic, with the specific material parameters listed in Table 6.2.

Table 6.2 Material parameters of the DHMs

Parameter	Nylon PA 2200	Aluminum	Steel	Copper (T2)
$E(\text{GPa})$	1.2	70	210	110
$\rho(\text{kg/m}^3)$	950	2700	7800	8900
ν	0.35	0.33	0.33	0.33

6.3 Numerical analysis

To analyse the formation mechanism of ultra-broadband and ultra-low-frequency longitudinal wave bandgaps in the DNA double helix metamaterial, this chapter systematically calculates the infinite periodic band structures, displacement transmission curves of finite structures, vibration modes, and full-field displacements for two configurations: the dual helix metamaterial without nodal mass disks and those with nodal mass disks (where the disks are made of aluminium, steel, and copper, respectively). Building on this, to explore the tuning mechanisms of broad low-frequency bandgaps, the influence of key parameters, namely, the minor helix diameter d_h , the mass disk thickness T_m , and the axial pitch D_a , on the first complete bandgap (BG1) and the first longitudinal bandgap (LBG1) is investigated.

6.3.1 Ultra-broadband and ultra-low-frequency bandgap

Figure 6.3 presents the band structure and vibration modes at the bandgap boundary frequencies for the double helix metamaterial without nodal mass disks (DHMN), along with the displacement transmission curve and corresponding full-field displacement distributions for a finite structure composed of six-unit cells.

The DHMN is fabricated using Nylon PA 2200. To clearly characterize the distribution of longitudinal wave bandgaps, all bands corresponding to longitudinal modes are highlighted in Figure 6.3(a) and in the band structures for various configurations, while bands associated with other modes are dimmed. The calculation results reveal that the structure exhibits three longitudinal bandgaps within the 0–400 Hz range: first bandgap: 77.40–131.96 Hz (bandwidth 54.56 Hz); second bandgap: 141.54–240.53 Hz (bandwidth 99.02 Hz); third bandgap: 278.11–400 Hz (bandwidth 121.89 Hz). Collectively, these bandgaps cover 68.9% of the frequency range. It should be noted that the DHMN does not exhibit a complete bandgap below 100 Hz. The positions of the longitudinal bandgaps are consistent between the band structure and the displacement transmission curve, with the only discrepancy being that the starting frequency of the third bandgap observed in the transmission curve is slightly higher than the value predicted by the band structure, although the bandwidths are in perfect agreement. The band structure, based on an infinite periodic model, is suitable for obtaining a macroscopic understanding of the bandgap distribution, while the displacement transmission curve, derived from the response of a finite structure, is more representative of practical applications.

Figure 6.3(b) displays the vibration modes at different frequency points, n_1 , n_2 , n_3 , and n_4 , located on the 4th, 7th, 8th, and 10th bands of Figure 6.3(a), respectively (band numbers are counted from the leftmost side of the wave vector axis, the same applies hereinafter). The results indicate that n_1 and n_3 are bending-torsional coupled vibration modes. The vibrations in mode n_1 are concentrated in the middle section of the double helix, while mode n_3 exhibits less concentrated vibrations, with the most pronounced deformation occurring in the hollow cylindrical side plates at both ends. Mode n_2 is a compressive longitudinal wave mode, and mode n_4 is a dilatational longitudinal wave mode; the vibrations in both n_2 and n_4 are concentrated in the central part of the double helix.

According to Figure 6.3(c), the bandgap characteristics of the structure are further validated by selecting five characteristic frequency points (N1, N2, N3, N4, N5) on the displacement transmission curve and analyzing their full-field displacements. The results show that N2, N3, and N5, located within the first, second, and third longitudinal bandgaps respectively, exhibit vibration energy concentrated in the first unit cell, leading to significant attenuation of longitudinal wave propagation. In contrast, N1 and N4, situated outside the bandgap frequency ranges, allow effective longitudinal wave propagation. The analysis results based on full-field displacements, the displacement transmission curve, and the band structure are highly consistent, mutually validating one another and collectively revealing the structure's longitudinal wave attenuation characteristics.

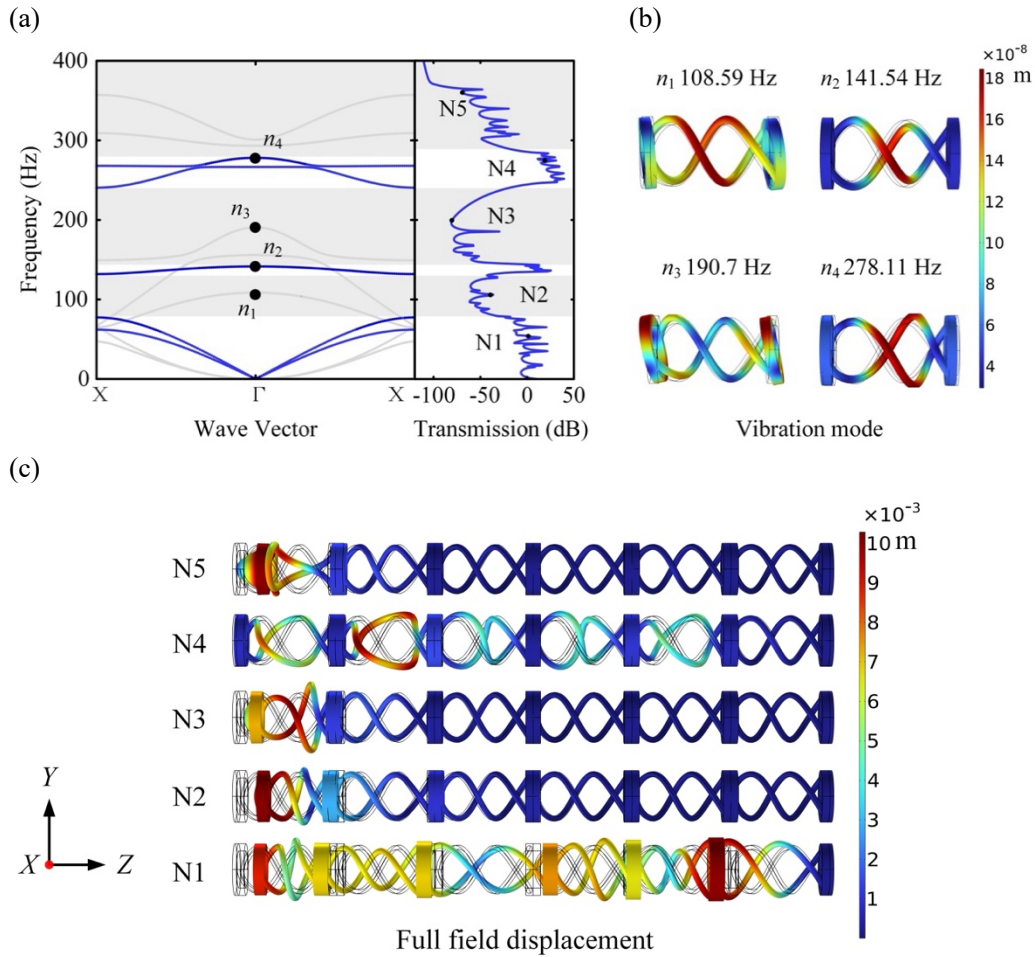


Figure 6.3 FEM results for the DHMN: (a) band structure and the displacement transmission curve; (b) vibration modes at band-edge frequencies (n_1 – n_4 marked in (a)); (c) full field displacement at selected frequency points (N1, 58 Hz; N2, 103 Hz; N3, 199 Hz; N4, 276 Hz; N5, 355 Hz).

Figure 6.4 presents the finite element calculation results for the configuration with aluminum mass disks on both sides, denoted as DHMA, including its band structure, displacement transmission curve, vibration modes, and full-field displacements. The results shown in Figure 6.4(a) indicate that the number and width of bandgaps observed in the band structure are in good agreement with those in the displacement transmission curve. However, the bandgap onset frequencies in the transmission curve are slightly shifted toward higher frequencies, with the most pronounced upward shift observed for the third longitudinal bandgap and the least pronounced for the first longitudinal bandgap. DHMA also exhibits three longitudinal bandgaps below 400 Hz, with frequency ranges of 35.77–133.66 Hz (bandwidth 97.89 Hz), 142.74–248.48 Hz (bandwidth 95.74 Hz), and 292.42–400 Hz (bandwidth 107.58 Hz). Collectively, these three bandgaps cover 77.8% of the frequency range from 0 to 400 Hz.

Figure 6.4(b) displays the vibration modes at four frequency points selected on the 6th, 8th, and 12th bands of the band structure: a_1 (104.17 Hz), a_2 (142.74 Hz), a_3 (277.35 Hz), and a_4 (292.42 Hz). As illustrated, except for a_1 , which corresponds

to a shear wave mode with dispersed vibrations and the most pronounced regions located in the upper parts of the hollow cylindrical end plates and mass disks, a_2 , a_3 , and a_4 are all longitudinal wave modes. Among these, vibrations in a_2 are concentrated in the middle section of the double helix, while a_3 and a_4 exhibit vibrations concentrated in the middle section of the double helix with displacement fields symmetric about the helical axis. Notably, the frequency points a_3 and a_4 lie on the same band, and their vibration modes are identical.

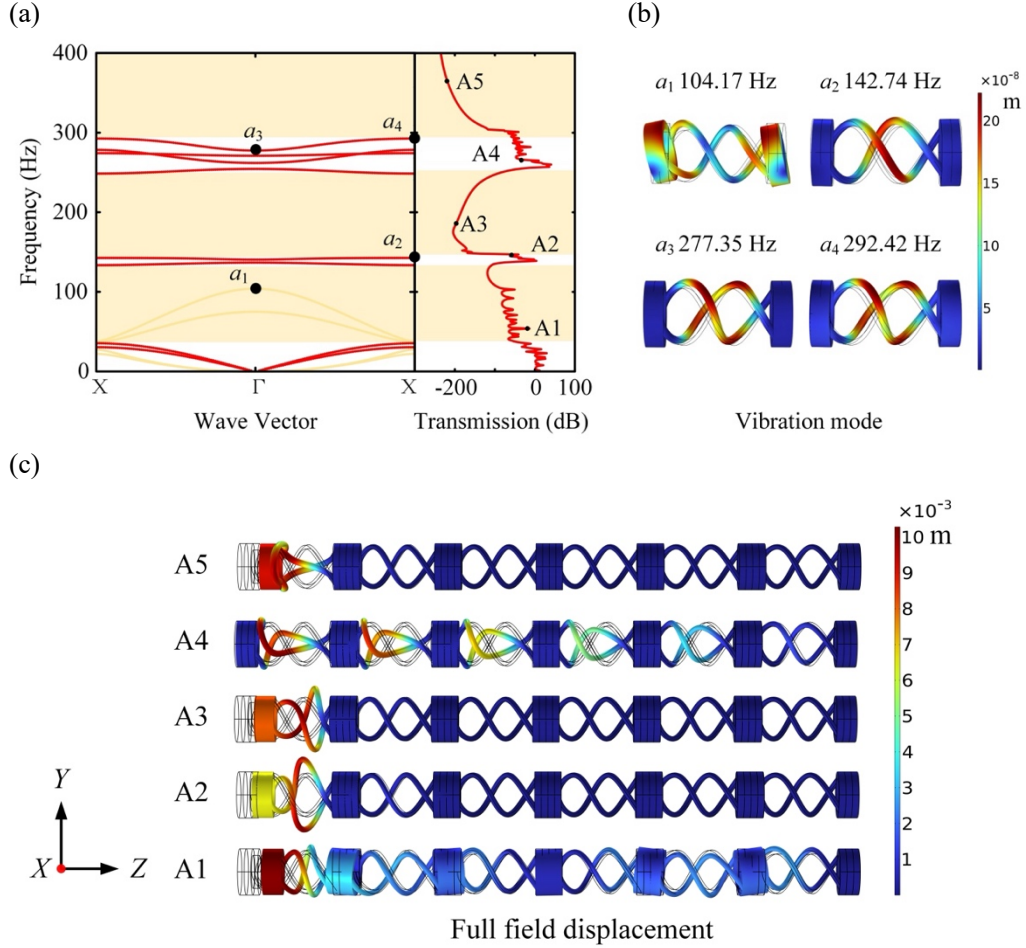


Figure 6.4 FEM results for the DHMA: (a) band structure and the displacement transmission curve; (b) vibration modes at band-edge frequencies (a_1 – a_4 marked in (a)); (c) full field displacement at selected frequency points (A1, 54 Hz; A2, 148 Hz; A3, 184 Hz; A4, 263 Hz; A5, 327 Hz).

Figure 6.4(c) presents the full-field displacements at five distinct frequency points: A1 (54 Hz), A2 (148 Hz), A3 (184 Hz), A4 (263 Hz), and A5 (327 Hz).

The results indicate that although A1 lies within the first bandgap, it corresponds to a frequency at which longitudinal waves can still propagate. A4 lies outside the bandgap, allowing longitudinal waves to propagate freely. For A2, A3, and A5, which are frequency points within the bandgaps, the full-field displacements reveal that vibrations are almost entirely confined to the first unit cell, indicating that longitudinal wave propagation is significantly suppressed.

Figure 6.5 presents the finite element results for the configuration with steel mass disks on both sides of the DNA double helix metamaterial (DHMF), including its band structure, displacement transmission curve, vibration modes, and full-field displacement distributions.

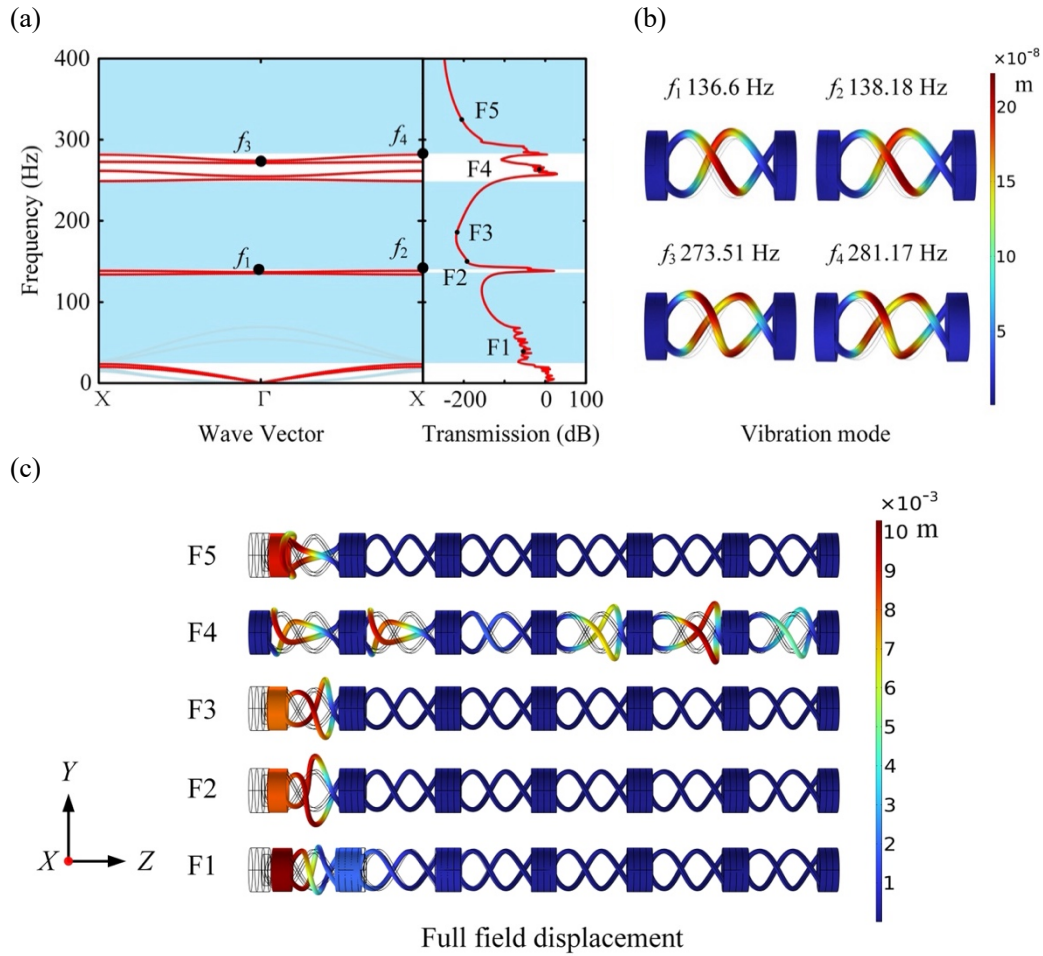


Figure 6.5 FEM results for the DHMF: (a) band structure and the displacement transmission curve; (b) vibration modes at band-edge frequencies (f_1 – f_4 marked in (a)); (c) full field displacement at selected frequency points (F1, 30 Hz; F2, 151 Hz; F3, 186 Hz; F4, 259 Hz; F5, 325 Hz).

As shown in Figure 6.5(a), both the infinite periodic band structure and the displacement transmission curve of the finite structure indicate that DHMF exhibits three wide longitudinal bandgaps below 400 Hz. The frequency ranges of these bandgaps are: 22.97–133.67 Hz (bandwidth 110.7 Hz), 138.18–261.21 Hz (bandwidth 123.03 Hz), and 281.17–400 Hz (bandwidth 118.83 Hz). Collectively, these three bandgaps cover 88.0% of the frequency range from 0 to 400 Hz.

Figure 6.5(b) displays the vibration modes at frequency points marked on the 8th and 12th bands: f_1 (136.6 Hz) and f_2 (138.18 Hz) on the 8th band, and f_3 (273.51 Hz) and f_4 (281.17 Hz) on the 12th band. The results show that the modes at f_1 and f_2 exhibit energy concentrated in the middle section of the double helix, with identical displacement fields. Similarly, the modes at f_3 and f_4 show energy

concentrated in the double helix, with displacement fields symmetric about the helical axis, and are identical. This is because each pair of frequency points lies on the same band, resulting in identical vibration modes.

Figure 6.5(c) presents the full-field displacements at five selected frequency points: F1 (30 Hz), F2 (151 Hz), F3 (186 Hz), F4 (259 Hz), and F5 (325 Hz). The results indicate that F1, F2, F3, and F5 lie within the first, second, and third longitudinal bandgaps, respectively, where longitudinal wave propagation is significantly suppressed. Notably, the energy of these suppressed longitudinal waves is almost entirely absorbed by the first unit cell, with vibrations concentrated in the left-side disk of the first unit cell and the adjacent helix. F4, located within the passband, allows the effective propagation of elastic longitudinal waves.

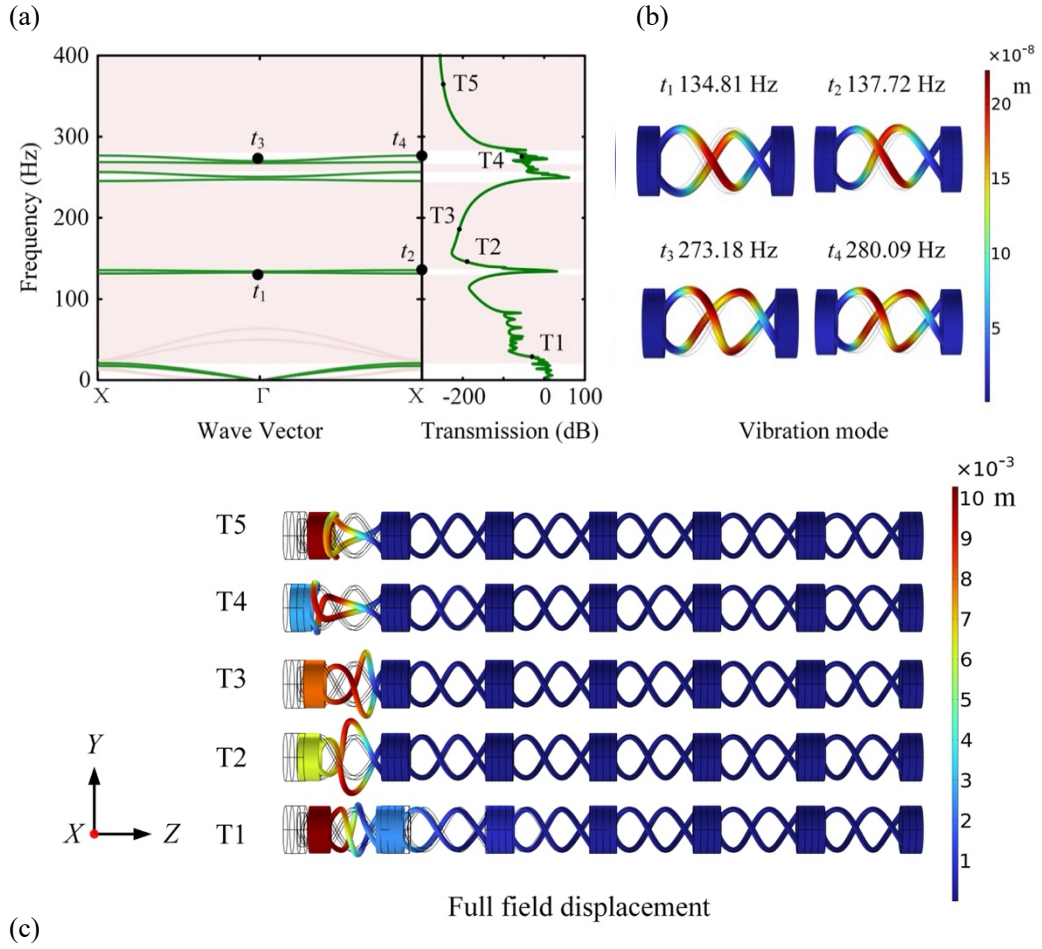


Figure 6.6 FEM results for the DHMT: (a) band structure and the displacement transmission curve; (b) vibration modes at band-edge frequencies (t_1 – t_4 marked in (a)); (c) full field displacement at selected frequency points (T1, 26 Hz; T2, 147 Hz; T3, 185 Hz; T4, 276 Hz; T5, 364 Hz).

Figure 6.6 presents the band structure, displacement transmission curve, vibration modes, and full-field displacement results for the configuration with copper mass disks (T20), denoted as DHMT. Figure 6.6(a) shows that DHMT exhibits three even wider longitudinal bandgaps below 400 Hz: 21.64–133.65 Hz

(bandwidth 112.01 Hz), 137.72–248.44 Hz (bandwidth 110.72 Hz), and 280.09–400 Hz (bandwidth 119.91 Hz). Collectively, these three bandgaps cover 93.6% of the frequency range from 0 to 400 Hz.

Figure 6.6(b) displays the vibration modes at selected frequency points on the 8th and 12th bands: t_1 (134.81 Hz) and t_2 (137.72 Hz) on the 8th band, and t_3 (273.18 Hz) and t_4 (280.09 Hz) on the 12th band. The results indicate that all four modes are longitudinal wave modes, with vibrations concentrated in the middle section of the double helix.

Figure 6.6(c) presents the full-field displacements at the marked frequency points: T1 (26 Hz), T2 (147 Hz), T3 (185 Hz), T4 (276 Hz), and T5 (364 Hz). Among these, T1, T3, and T5 lie within the first, second, and third longitudinal bandgaps, respectively, where longitudinal waves are completely suppressed. Observation suggests that these three bandgaps are locally resonant bandgaps. T2, located outside the bandgaps, allows elastic wave propagation. T4 falls within a narrow bandgap, leading to the suppression of elastic waves.

To better understand the formation mechanism of the ultra-low-frequency bandgaps in the double helix metamaterial without mass disks (DHMN) and those with mass disks (DHMM, including DHMA, DHMF, and DHMT), Figures 6.7(a)–6.7(d) present the low-frequency band structures below 100 Hz for DHMN, DHMA, DHMF, and DHMT, respectively. The results indicate that as the density of the additional mass disks increases, the starting frequency of the first longitudinal bandgap progressively decreases, while the bandgap width gradually broadens. Figure 6.7(e) displays the vibration modes at the characteristic frequency points on the band edges (the corresponding values are listed in Table 6.3).

Table 6.3 Band-edge frequency points selected in the band structure of DHMs

Marker	frequency (Hz)	Marker	Frequency (Hz)	Marker	Frequency (Hz)
m_1	47.129	m_6	75.176	m_{11}	15.241
m_2	62.059	m_7	37.819	m_{12}	12.958
m_3	63.995	m_8	69.121	m_{13}	13.399
m_4	66.805	m_9	27.119	m_{14}	51.14
m_5	66.817	m_{10}	21.638	m_{15}	65.272

The results indicate that at the boundary frequency point m_2 of the DHMN without mass disks, the mode corresponds to a dilatational longitudinal wave, with vibration energy concentrated in the hollow cylindrical end plates on both sides. For the DHMT with mass disks, the modes at m_{10} , m_{11} , and m_{12} are compressive longitudinal wave modes, with their vibration energy concentrated in the hollow cylindrical end plates and the mass disks. Meanwhile, at frequency points not located on the longitudinal bandgap edges, other vibration modes are observable, such as shear vibration modes (m_1 and m_{13}) and coupled shear-torsional/bending-torsional vibration modes (m_3 , m_4 , m_5 , m_6 , m_7 , m_8 , m_9 , m_{14} , and m_{15}). Notably, m_6 , m_7 , and m_{14} belong to the same vibration mode, while m_8 , m_9 , and m_{15} share an identical vibration mode. Furthermore, due to the crossing of the 6th band with the 4th and

5th bands below 100 Hz, a complete bandgap fails to form in the DHMN. DHMA exhibits a very narrow complete bandgap below 40 Hz, while DHMF and DHMT possess both a narrow complete bandgap below 40 Hz and a wider complete bandgap above 60 Hz.

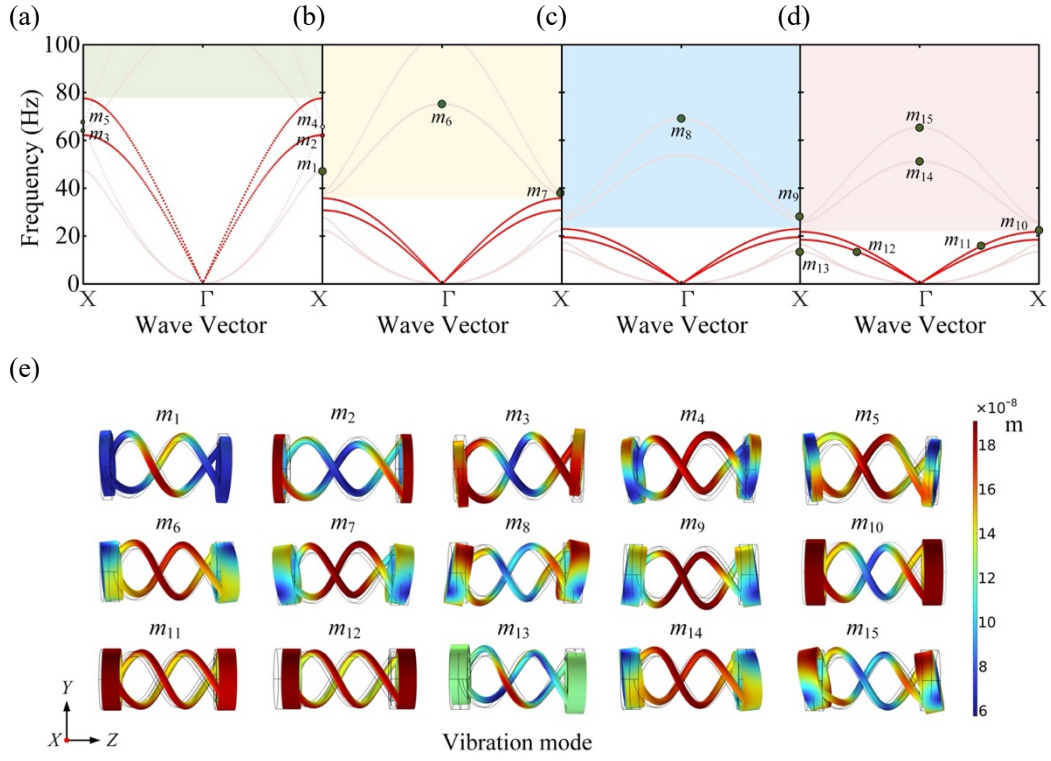


Figure 6.7 Band structures of DHMs below 100 Hz and corresponding bandgap edge modes: (a) DHMN; (b) DHMA; (c) DHMF; (d) DHMT; (e) vibration modes at the band-edge frequencies (points m_1 – m_{15} marked in (a)–(d)).

Figure 6.8 presents the calculated displacement transmission curves for finite structures of the four types of double helix metamaterials over the 0–2000 Hz range, facilitating further observation and understanding of their ultra-broad longitudinal wave attenuation characteristics. In the figure, the red dotted line represents DHMN, the blue long-dashed line DHMA, the green short-dashed line DHMF, and the purple solid line DHMT. The results indicate that DHMA, DHMF, and DHMT exhibit extremely wide bandgaps across the 2000 Hz frequency range. They possess four relatively wide bandgaps (I, II, III, and IV) and four relatively narrow bandgaps (i, ii, iii, and iv), whose combined coverage approaches nearly the entire frequency spectrum. In contrast, the DHMN structure without mass disks exhibits only three relatively narrow bandgaps. Therefore, attaching a mass disk to each side of the DHMN unit cell significantly expands the bandgap frequency range.

Further quantitative analysis reveals that for the structure without mass disks, the attenuation depth in frequency bands below 200 Hz is less than 100 dB, with a maximum attenuation throughout the entire frequency domain not exceeding 150 dB. After the introduction of nodal mass disks, the attenuation depth in the low-frequency range (<200 Hz) is enhanced to 150–200 dB, reaching 200–300 dB in

other frequency bands. The study also reveals that the bandgap attenuation depth increases monotonically with the density of the mass disk material. In summary, the addition of mass disks not only significantly broadens the effective attenuation frequency range but also greatly enhances the vibration suppression capability of the bandgaps. Based on these findings, subsequent investigations will focus on the influence of unit cell geometric parameters on the ultra-broadband and ultra-low-frequency bandgap characteristics of the DHMM structure.

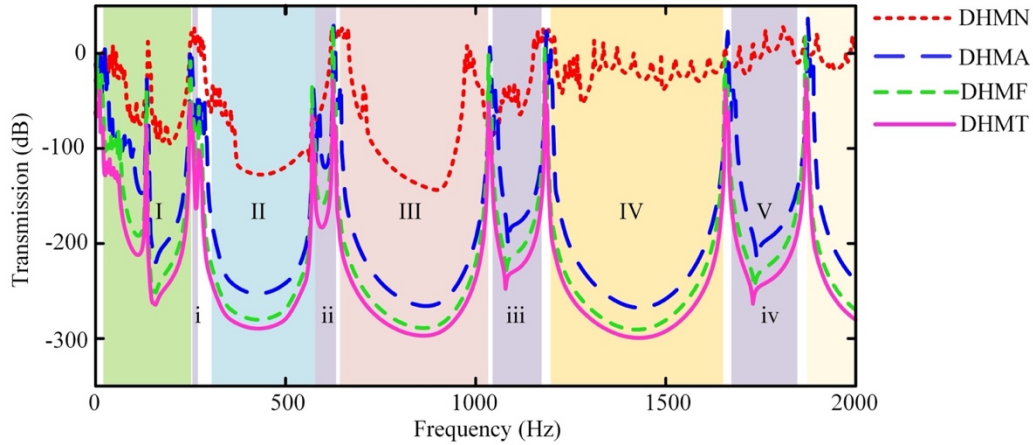


Figure 6.8 Displacement transmission curves for various DHMs within 2000 Hz.

6.3.2 Influence of key geometric parameters

This subsection primarily discusses the influence of the minor diameter d_h , the thickness of the mass disks T_m , and the axial pitch of the helix D_a on the first complete bandgap (BG1) and the first longitudinal bandgap (LBG1) of the DNA double helix metamaterial with nodal mass disks.

Figure 6.9 illustrates the variation of the upper and lower boundary frequencies of the first complete bandgap for each nodal mass double helix metamaterial configuration as the minor helix diameter d_h increases from 4 mm to 9 mm. In the figure, the green-shaded region represents the range bounded by the upper and lower boundary frequencies for the configuration with copper mass disks (DHMT). The blue-shaded region with a blue-dashed border indicates the corresponding range for the configuration with steel-mass disks (DHMF). The purple shaded region represents the range for the configuration with aluminum mass disks (DHMA).

The results indicate that for DHMA, both the upper and lower boundary frequencies of the first complete bandgap gradually increase with the minor diameter, while the bandwidth initially broadens slowly and then progressively narrows. At $d_h = 4$ mm, the BG1 frequency is the lowest, with a bandgap range of 9.16–10.82 Hz and a bandwidth of 1.66 Hz. The widest BG1 occurs at $d_h = 6$ mm, with a range of 20.39–22.92 Hz and a bandwidth of 2.53 Hz.

For DHMF, the upper and lower boundary frequencies of the first complete bandgap increase gradually with the minor diameter, accompanied by a progressive broadening of the bandgap. The lowest BG1 frequency occurs at $d_h = 4$ mm, with a bandgap of 5.8–7.2 Hz and a bandwidth of 1.4 Hz. The widest BG1 is observed at $d_h = 9$ mm, with a range of 28.96–33.58 Hz and a bandwidth of 4.62 Hz.

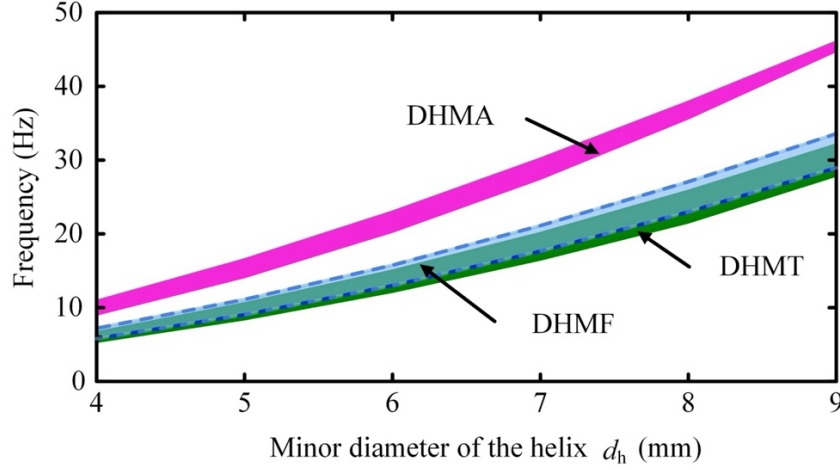


Figure 6.9 First complete bandgap frequency vary with the helix minor diameter.

For DHMT, the upper and lower boundary frequencies of the first complete bandgap shift progressively toward higher frequencies as the minor helix diameter increases, while the bandwidth initially broadens and then narrows slightly. The lowest BG1 frequency occurs at $d_h = 4$ mm, with a bandgap of 5.46–6.8 Hz and a bandwidth of 1.34 Hz. The widest BG1 is observed at $d_h = 8$ mm, with a range of 21.64–25.75 Hz and a bandwidth of 4.11 Hz.

For all three configurations, the lowest bandgap frequency occurs at the smallest minor helix diameter. Among them, DHMT achieves the lowest minimum complete bandgap frequency with the narrowest bandwidth, while DHMF attains the widest first complete bandgap bandwidth. This is because increasing the minor helix diameter increases structural stiffness; consequently, the lowest frequency occurs at the minimum stiffness, i.e., the smallest minor helix diameter. Notably, the first complete bandgaps of the three nodal mass double helix metamaterials have onset frequencies as low as 9.16 Hz, 5.8 Hz, and 5.46 Hz. For context, an ultra-low-frequency bandgap with a starting frequency of 77.3 Hz has been reported in the literature [99].

Figure 6.10 illustrates the variation of the upper and lower boundary frequencies of the first longitudinal bandgap (LBG1) for different minor helix diameter d_h . Figures 6.10(a), 6.10(b), and 6.10(c) present the results for configurations with aluminum (DHMA), steel (DHMF), and copper (DHMT) mass disks, respectively.

The results indicate that, for DHMA, DHMF, and DHMT, the frequency of the first longitudinal bandgap shift increases progressively as d_h increases, accompanied by a gradual broadening of the bandwidth. Notably, the cutoff

frequencies of the first longitudinal bandgap remain nearly identical across the different mass disk materials; the primary distinction lies in their starting frequencies.

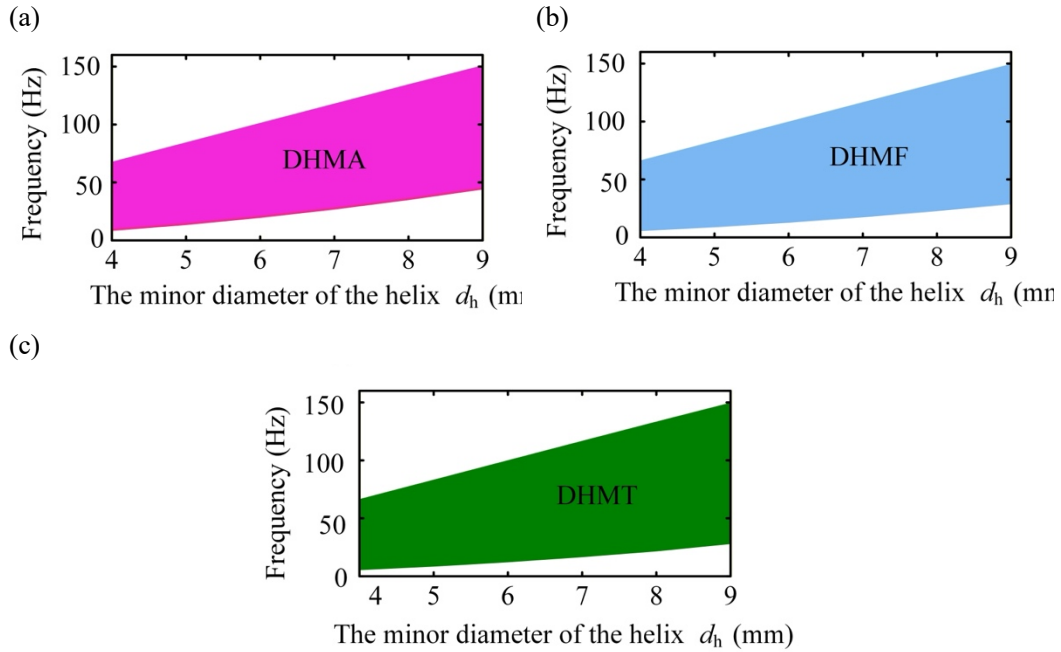


Figure 6.10 First longitudinal wave bandgap frequency varies with helix minor diameter: (a) DHMA; (b) DHMF; (c) DHMT.

At the smallest diameter, $d_h = 4$ mm, all three configurations exhibit their lowest and narrowest LBG1. The bandgaps are: 9.16–66.79 Hz (bandwidth 57.63 Hz) for DHMA; 5.8–66.78 Hz (bandwidth 60.98 Hz) for DHMF; and 5.46–66.79 Hz (bandwidth 61.33 Hz) for DHMT. At the largest diameter, $d_h = 9$ mm, the widest LBG1 are observed: 44.90–150.02 Hz (bandwidth 105.12 Hz) for DHMA; 28.96–150.05 Hz (bandwidth 121.09 Hz) for DHMF; and 27.99–150.02 Hz (bandwidth 122.02 Hz) for DHMT. Within the 0–150 Hz frequency range, as d_h increases from 4 mm to 9 mm, the coverage of the first longitudinal bandgap expands significantly: for DHMA, from 38.42% to 70.08%; for DHMF, from 40.65% to 80.73%; and for DHMT, from 40.89% to 81.35%. Therefore, increasing the minor helix diameter shifts the longitudinal bandgap frequency range toward higher frequencies, effectively broadening its bandwidth.

Figure 6.11 illustrates the variation of the first complete bandgap (BG1) for the three configurations, DHMA, DHMF, and DHMT, as the thickness of the attached mass disks T_m increases from 2 mm to 10 mm. The results reveal the following:

For DHMA, there is no complete bandgap when the aluminum disk thickness is less than 2 mm. At $T_m = 6$ mm, the first complete bandgap is very narrow, ranging from 44.92–45.38 Hz with a bandwidth of 0.46 Hz. As T_m increases to 10 mm, the first complete bandgap shifts to 35.77–37.82 Hz, with a bandwidth of 2.05 Hz.

For DHMF, as the steel disk thickness increases, both the upper and lower boundaries of the first complete bandgap gradually shift toward lower frequencies,

accompanied by a progressive narrowing of the bandwidth. At $T_m = 2$ mm, the first complete bandgap spans 39.26–50.67 Hz, with a bandwidth of 11.41 Hz. At $T_m = 10$ mm, the bandgap shifts to 22.97–27.08 Hz, with a bandwidth of 4.11 Hz.

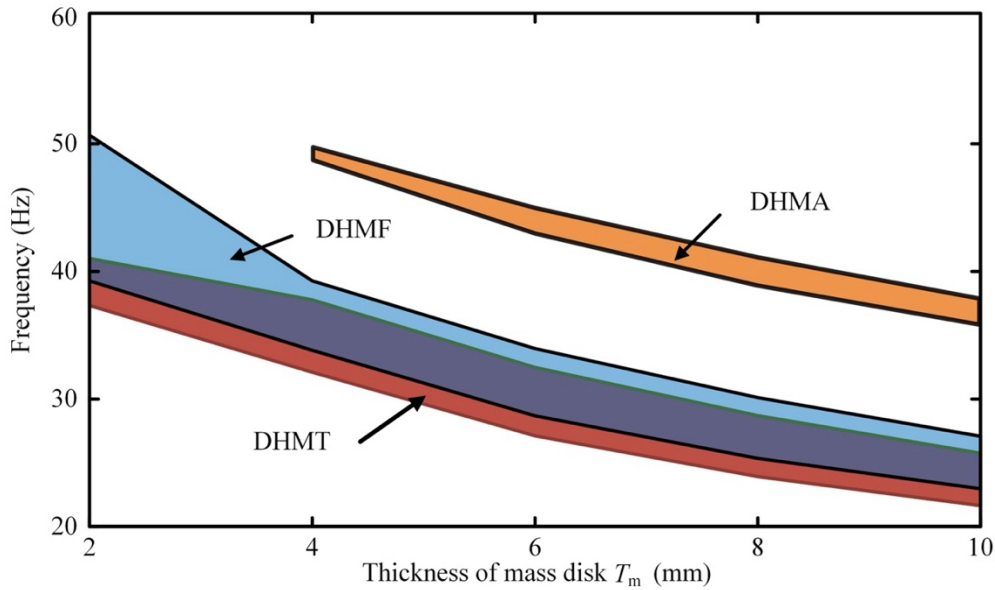


Figure 6.11 First complete bandgap frequency change with mass disk thickness.

For DHMT, as the copper disk thickness increases, the upper and lower boundaries of the first complete bandgap also shift toward lower frequencies. The bandwidth initially broadens rapidly and then narrows slowly. At $T_m = 2$ mm, the first complete bandgap spans 37.33–41.02 Hz, with a bandwidth of 3.67 Hz. At $T_m = 10$ mm, the bandgap shifts to 21.64–25.75 Hz, with a bandwidth of 4.11 Hz. The widest bandgap occurs at $T_m = 6$ mm, spanning 27.12 Hz to 32.48 Hz and yielding a bandwidth of 5.36 Hz.

These observations demonstrate that the mass disk thickness is a critical parameter for tuning the first complete bandgap. For DHMA, it directly determines whether a complete bandgap can be formed below 50 Hz. For DHMF and DHMT, it significantly influences the bandgap's frequency range.

To better understand the influence of mass disk thickness on the formation mechanism of the first complete bandgap, Figure 6.12 presents the band structures and band-edge modes of DHMA for different mass disk thicknesses. Figures 6.12(a)–6.12(e) sequentially show the band structures at thicknesses of 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm, while Figure 6.12(f) displays the vibration modes at selected frequency points.

The results indicate that at $T_m = 2$ mm, DHMA does not exhibit a complete bandgap. At $T_m = 4$ mm, the first complete bandgap of DHMA (48.69–49.70 Hz, bandwidth 1.01 Hz) appears between the 4th and 5th bands. For aluminum disk thicknesses greater than 4 mm, the first complete bandgap consistently lies between the 4th and 5th bands. The band-edge modes A1 (49.70 Hz), A3 (45.38 Hz), and

A4 (44.92 Hz) are bending vibration modes, while A2 (48.69 Hz) and A5 (38.85 Hz) are dilatational longitudinal wave modes.

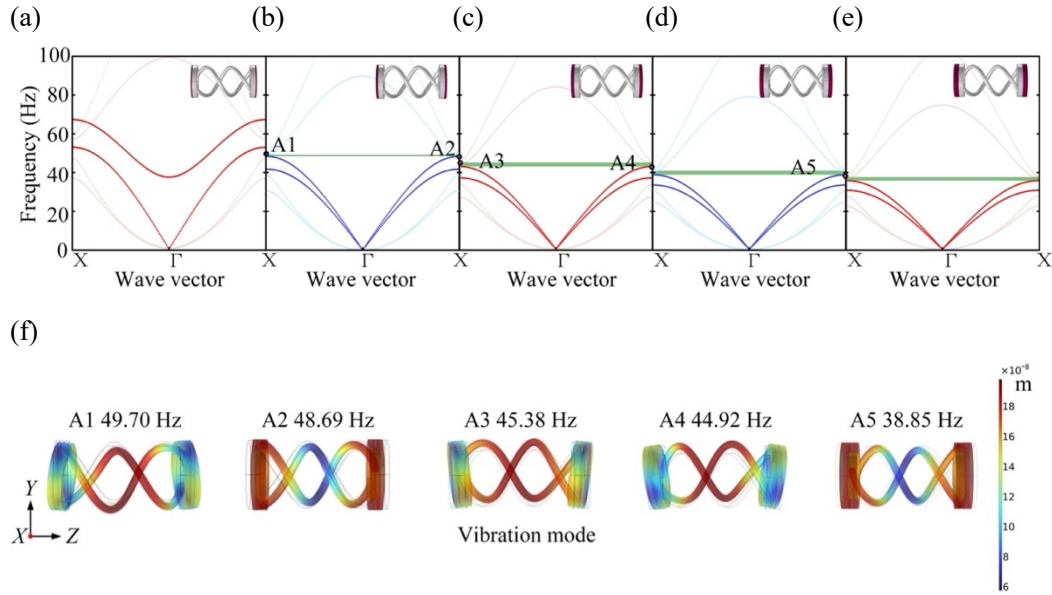


Figure 6.12 Band structure of DHMA for different mass disk thickness and vibration modes: (a) $T_m = 2$ mm; (b) $T_m = 4$ mm; (c) $T_m = 6$ mm; (d) $T_m = 8$ mm; (e) $T_m = 10$ mm; (f) the vibration modes at marked points.

Figure 6.13 presents the band structures and band-edge modes for the DHMF at various thicknesses. Figures 6.13(a)–6.13(e) sequentially show the band structures at thicknesses of 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm, while Figure 6.13(f) displays the vibration modes at selected frequency points.

The results indicate that at $T_m = 2$ mm, the first complete bandgap of the DHMF lies between the 3rd and 6th bands. At $T_m = 4$ mm, the first complete bandgap appears between the 4th and 5th bands. For thicknesses greater than 4 mm, the first complete bandgap remains consistently positioned between the 4th and 5th bands.

Figure 6.13(f) reveals that F1 (50.67 Hz) and F3 (39.26 Hz) are bending vibration modes; F2 (39.26 Hz) is a dilatational longitudinal wave mode; and F4 (33.82 Hz) and F5 (28.68 Hz) are compressive longitudinal wave modes. For the DHMT, the bandgap formation characteristics are very similar to those observed for DHMF; therefore, a detailed discussion is omitted here.

Figures 6.14(a)–6.14(c) illustrate the variation of the first longitudinal bandgap (LBG1) frequency with mass disk thickness for DHMA, DHMF, and DHMT, respectively. The results show that as the disk thickness increases, the bandgap onset frequency decreases for all three configurations, while the cutoff frequency remains stable.

For DHMA, at $T_m = 2$ mm, the first longitudinal bandgap ranges from 70.13 Hz to 133.47 Hz, with a bandwidth of 63.34 Hz, covering 42.23% of the 0–150 Hz frequency range. At $T_m = 10$ mm, the bandgap shifts to 35.77–133.66 Hz, with a bandwidth of 97.89 Hz, increasing the coverage to 65.26%.

For DHMF, at $T_m = 2$ mm, the first longitudinal bandgap ranges from 39.26 Hz to 133.41 Hz, with a bandwidth of 94.15 Hz, covering 62.77% of the 0–150 Hz range. At $T_m = 10$ mm, the bandgap shifts to 22.97–133.67 Hz, with a bandwidth of 110.7 Hz, expanding the coverage to 73.8%.

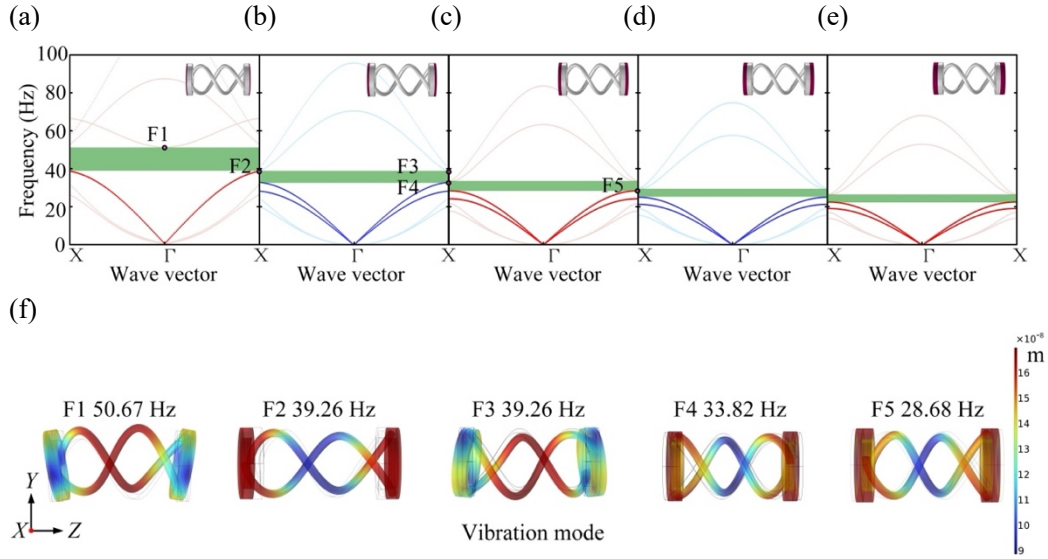


Figure 6.13 Band structure of DHMF for different mass disk thickness and vibration modes: (a) $T_m = 2$ mm; (b) $T_m = 4$ mm; (c) $T_m = 6$ mm; (d) $T_m = 8$ mm; (e) $T_m = 10$ mm; (f) the vibration modes at marked points.

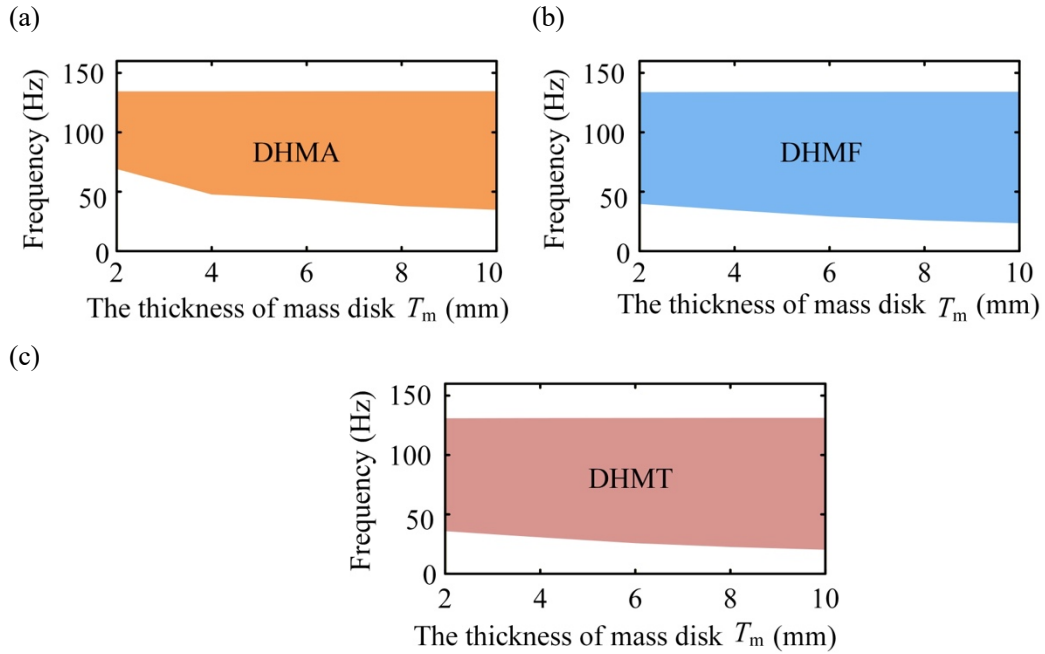


Figure 6.14 First longitudinal wave bandgap frequency change with the mass disk thickness: (a) DHMA; (b) DHMF; (c) DHMT.

For DHMT, at $T_m = 2$ mm, the first longitudinal bandgap ranges from 37.33 Hz to 133.29 Hz, with a bandwidth of 95.96 Hz, covering 63.97% of the 0–150 Hz range.

range. At $T_m = 10$ mm, the bandgap shifts to 21.64–133.65 Hz, with a bandwidth of 112.01 Hz, achieving a coverage of 74.67%.

Figure 6.15(a) shows the influence of the axial pitch on the first longitudinal bandgap of DHMT. The results indicate that as the axial pitch increases, the cutoff frequency of the first longitudinal bandgap gradually decreases, while the starting frequency remains relatively unchanged. At $DA = 0.5$, the first longitudinal bandgap spans 25.24–205.13 Hz, with a bandwidth of 178.89 Hz, covering 89.95% of the 0–200 Hz frequency range. The decrease in cutoff frequency with increasing axial pitch is attributed to the reduction in the equivalent stiffness of the helix. Conversely, a smaller axial pitch results in a more densely wound helix, enabling longitudinal wave attenuation over a broader frequency range; hence, a smaller axial pitch leads to a wider first longitudinal bandgap.

Figure 6.15 illustrates the variation of the first longitudinal bandgap (LBG1) and the first complete bandgap (BG1) for the DHMT structure for different dimensionless axial pitch DA (where $DA = D_a/102$).

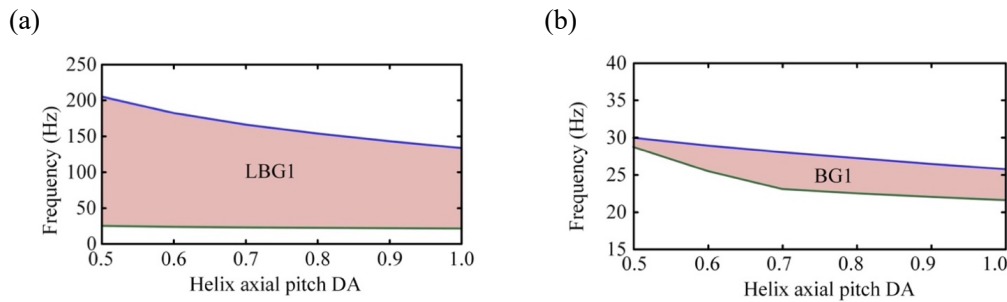


Figure 6.15 First longitudinal wave and complete bandgap frequencies change with the axial pitch of the helix for the DHMT: (a) LBG1; (b) BG1.

Figure 6.15(a) shows the influence of the axial pitch on the first longitudinal bandgap of DHMT. The results indicate that as the axial pitch increases, the cutoff frequency of the first longitudinal bandgap gradually decreases, while the starting frequency remains relatively unchanged. At $DA = 0.5$, the first longitudinal bandgap spans 25.24–205.13 Hz, with a bandwidth of 178.89 Hz, covering 89.95% of the 0–200 Hz frequency range. The decrease in cutoff frequency with increasing axial pitch is attributed to the reduction in the equivalent stiffness of the helix. Conversely, a smaller axial pitch results in a more densely wound helix, enabling longitudinal wave attenuation over a broader frequency range; hence, a smaller axial pitch leads to a wider first longitudinal bandgap.

Figure 6.15(b) presents the variation of the first complete bandgap with axial pitch. The results show that as the axial pitch increases, the frequency range of the first complete bandgap gradually shifts toward lower frequencies. The bandwidth initially broadens and then narrows very slowly. The widest bandwidth occurs at $DA = 0.7$ (corresponding to $D_a = 71.4$ mm), where the first complete bandgap spans 23.11–28.01 Hz, with a bandwidth of 4.8 Hz.

Figure 6.16 presents the band structures and vibration modes at characteristic frequency points for DHMT within 100 Hz. Figures 6.16(a)–6.16(e) show the band structures for $DA = 0.5, 0.6, 0.7, 0.8$, and 0.9 , respectively.

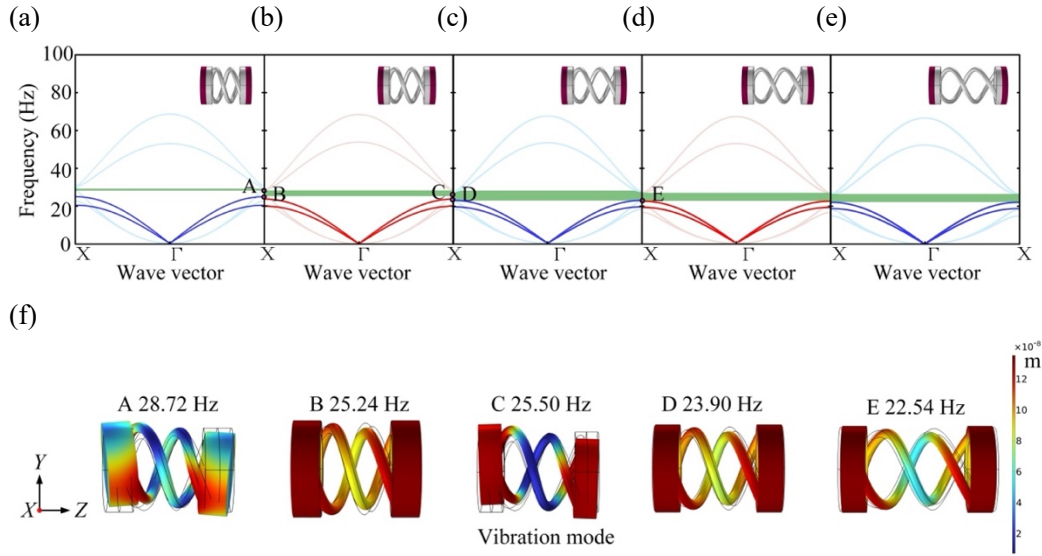


Figure 6.16 Band structure of DHMT for different axial pitches and corresponding vibration modes: (a) $DA = 0.5$; (b) $DA = 0.6$; (c) $DA = 0.7$; (d) $DA = 0.8$; (e) $DA = 0.9$; (f) the vibration modes of marked points.

Figure 6.16(f) displays the vibration modes at four selected points (A, B, C, D) on the band structures. The results indicate that at $DA = 0.5$, the first complete bandgap (BG1) lies between the 4th and 5th bands, spanning 28.72–29.94 Hz. The starting point of BG1 is point A (28.72 Hz) on the 4th band in Figure 6.16(a). The starting point of the first longitudinal bandgap (LBG1) is pointing B (25.24 Hz) on the 3rd band. At $DA = 0.6$, BG1 also lies between the 4th and 5th bands, ranging from 25.50 Hz to 28.89 Hz, with its starting frequency at point C (25.50 Hz) on the 4th band in Figure 6.16(b). The starting frequency of LBG1 is at point D (23.90 Hz) on the 3rd band. For $DA \geq 0.7$, the lower boundaries of BG1 and LBG1 coincide and lie on the 4th band. Mode A exhibits upward motion on the left side and downward motion on the right side of the structure, accompanied by some torsion of the helix, characterizing it as a torsional-shear mode. Mode B shows slight overall tension in the structure, identifying it as a longitudinal wave mode. Mode C displays upward motion on one side and downward motion on the other, with energy concentrated in the hollow cylindrical end plates and mass disks, and some torsion in the helix, indicating a shear mode. Mode D shows energy concentrated in the mass disks on both sides, with the overall structure in a compressive state, representing a longitudinal wave mode.

6.4 Experimental results and discussions

Figure 6.17 shows the laser vibrometry platform used in the experiments of this chapter. During the experiment, a multi-channel function generator was used to

generate a random spectral excitation signal. This signal was then amplified by a voltage amplifier and transmitted to a multilayer piezoelectric stack actuator, which applied longitudinal wave excitation to the right end of the DHMs metamaterial chain. At the left end of the metamaterial chain, the displacement response was captured using a laser vibrometer, with the time-domain vibration signal collected over 100 seconds. The displacement transmission curve of the finite metamaterial structure was obtained using the formula $T_d(\omega) = 20 \log_{10} |S_{out}(i\omega)/S_{in}(i\omega)|$, where $S_{out}(i\omega)$ is the spectral density function of the output displacement and $S_{in}(i\omega)$ is the auto-spectral density function of the input sweep signal.

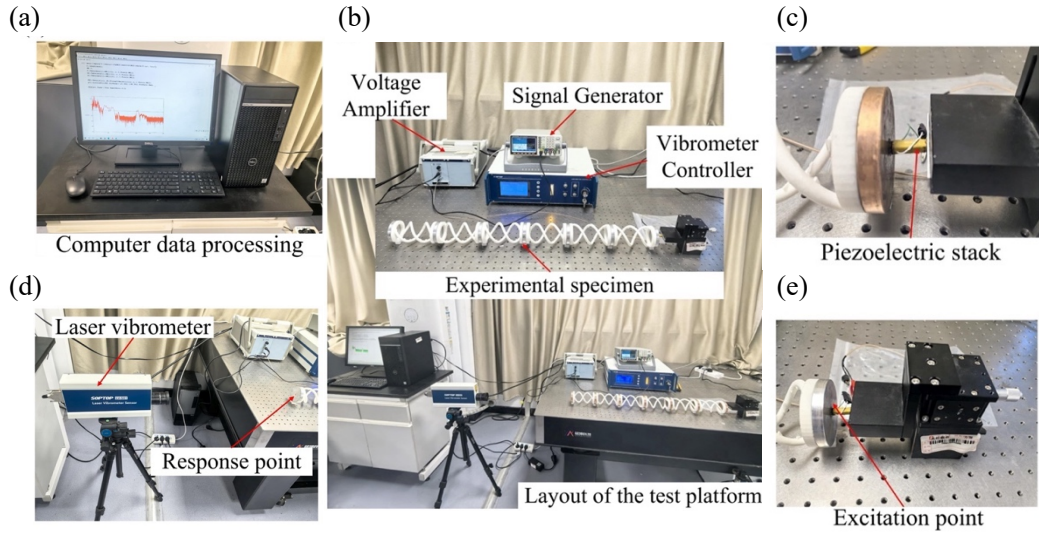


Figure 6.17 Experimental setup for DHMs based on the laser vibrometer system: (a) data processing computer; (b) test specimen and overview of the experimental setup; (c) piezoelectric stack actuator; (d) Laser vibrometer head; (e) location of vibration excitation point.

Figures 6.18(a)–6.18(d) present the measured displacement transmission curves for the four DNA double helix metamaterial configurations: DHMN, DHMA, DHMF, and DHMT, respectively. A consistent structural model was employed for both the COMSOL simulations and experimental tests. The double helix framework was fabricated using 3D laser sintering technology. Mass disks 10 mm thick were attached to one end of the 3D-printed helical framework. Consequently, the effective thickness of the mass disks on each side of a unit cell was 5 mm. However, to equivalently achieve the low-reflecting boundary conditions and maintain consistency with the simulation setup, the mass disks at both ends of the finite metamaterial structure had a thickness of 10 mm. The specific structure is illustrated in Figure 6.17(b).

Figure 6.18(a) compares the finite element simulation and experimental measurement results for the DHMN, where the blue solid line represents the COMSOL calculation curve and the red dashed line represents the experimental data. The comparison indicates that within the 0–400 Hz range, both simulation and experiment exhibit three distinct bandgaps, showing good agreement in bandgap

width. However, the experimentally measured bandgap attenuation depth is significantly lower than the simulation predictions. This discrepancy primarily arises from system damping and boundary condition dissipation in the experimental environment, which reduces the actual attenuation of vibrational energy.

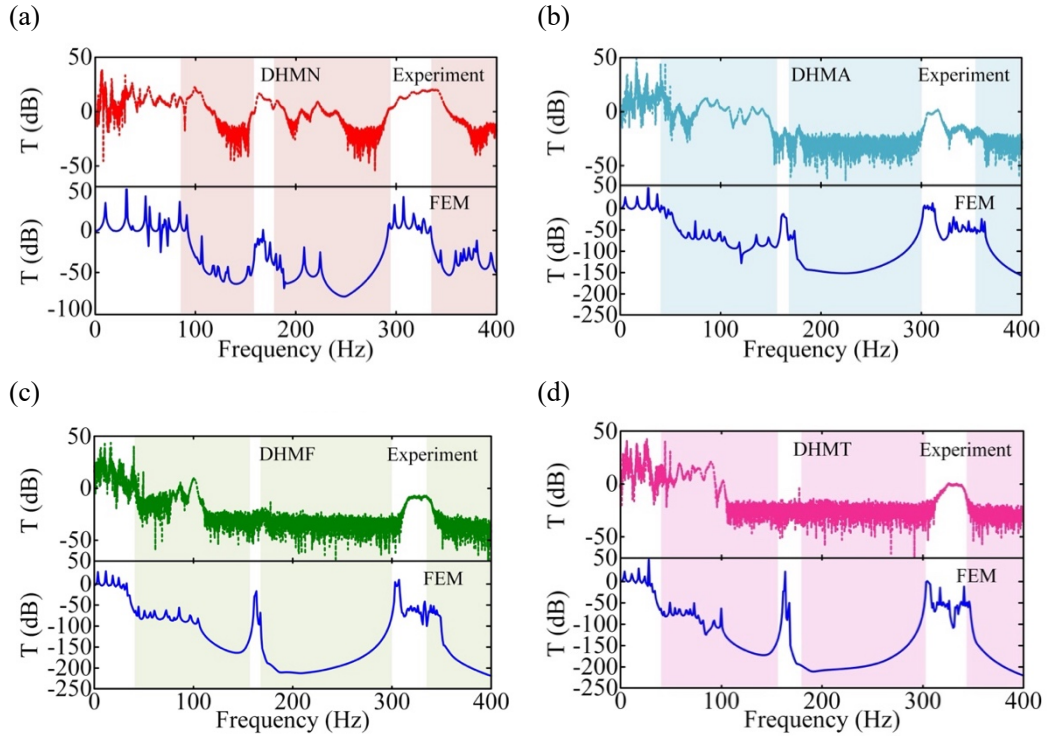


Figure 6.18 FEM and experimental results of the displacement transmission for DHMs: (a) DHMN; (b) DHMA; (c) DHMF; (d) DHMT.

Figure 6.18(b) compares the finite element and experimental transmission curves for the DHMA, with the light blue dashed line representing the experimental results and the blue solid line representing the finite element results. The results show that while the finite element predictions reveal three distinct bandgaps within the 400 Hz range for DHMA, only two distinct bandgaps are observed experimentally. This is because the first and second bandgaps merge into a broader first bandgap in the experimental results, and the experimentally measured third bandgap is slightly wider than the finite element calculation.

Figure 6.18(c) displays the results for the DHMF, where the green dashed line represents the experimental results and the blue solid line represents the simulation results. A bandgap merging phenomenon similar to that observed in DHMA is evident in the experimental results for DHMF. In the experiment, the second bandgap and part of the first bandgap merge, resulting in an experimentally measured first bandgap narrower than the theoretical calculation. However, excellent agreement is observed between experiment and simulation for the third bandgap.

Figure 6.18(d) shows the results for the DHMT, where the purple dashed line represents the experimental results and the blue solid line represents the COMSOL

results. In the experimental results for DHMT, the first bandgap almost completely merges with the second bandgap, although the width of the merged bandgap is narrower than the sum of the calculated first and second bandgaps. Good consistency between experiment and simulation is again observed for the third bandgap.

The discrepancies between the finite element calculations and the experimental measurements can be attributed to several factors. First, the experimental setup required the use of rubber pads to support the test specimen, making it impossible to ensure perfectly free boundary conditions for the finite structure. Second, the materials used in the measurements inevitably exhibit viscosity and nonlinear behavior, whereas the simulations treat the materials as ideally linearly elastic. Third, in the finite element simulations, the unit cells are perfectly connected via array instructions, whereas in the experiments, they are bonded together using AB adhesive. Finally, the experimental specimens were fabricated using 3D printing, which inevitably introduces internal defects such as voids and imperfections.

6.5 Summary

This chapter proposes a novel DNA double helix metamaterial with ultra-low-frequency and ultra-broadband bandgap characteristics. The formation mechanisms are systematically investigated for configurations without and with nodal mass disks (DHMN, DHMA, DHMF, DHMT). For each configuration, the band structures, displacement transmission curves, vibration modes, and full-field displacements are obtained. A detailed parametric study is conducted to examine the influence of the minor helix diameter, mass disk thickness, and axial pitch on the first complete bandgap (BG1) and first longitudinal bandgap (LBG1). Finally, the displacement transmission curves are experimentally validated using a laser vibrometry platform. The main findings are as follows:

(1) After attaching nodal mass disks, the proposed DNA double helix metamaterial exhibits a broader frequency range of vibration attenuation and more pronounced attenuation effects. Within the 0–400 Hz frequency range, DHMN possesses three longitudinal bandgaps: 77.40–131.96 Hz, 141.54–240.53 Hz, and 278.11–400 Hz, achieving a bandgap coverage of 68.9%. DHMA also exhibits three longitudinal bandgaps: 35.77–133.66 Hz, 142.74–248.48 Hz, and 292.42–400 Hz, with a coverage of 77.8%. DHMF has three longitudinal bandgaps: 22.97–133.67 Hz, 138.18–261.21 Hz, and 281.17–400 Hz, covering 88.0% of the range. DHMT achieves three longitudinal bandgaps: 21.64–133.65 Hz, 137.72–248.44 Hz, and 280.09–400 Hz, with a coverage reaching 93.6%.

(2) For DHMA, DHMF, and DHMT, the lowest complete bandgap occurs at the smallest minor helix diameter. Among them, DHMT achieves the lowest minimum complete bandgap frequency, while DHMF attains the widest first complete bandgap bandwidth. The first longitudinal bandgap frequency for all three configurations shifts toward higher frequencies as the minor helix diameter increases, accompanied by a gradual broadening of the bandwidth. The mass disk thickness significantly influences the first complete bandgap, directly determining

whether a low-frequency complete bandgap exists for DHMA, while also affecting the bandgap range for DHMF and DHMT. Additionally, the mass disk thickness influences the starting frequency of the first longitudinal bandgap for DHMA, DHMF, and DHMT, but has little effect on its cutoff frequency. As the axial pitch of the double helix increases, the frequency of the first complete bandgap for DHMT gradually shifts toward lower frequencies, with the bandwidth initially broadening and then slowly narrowing. The cutoff frequency of the first longitudinal bandgap gradually decreases, while its starting frequency remains relatively unchanged.

(3) Good agreement is observed between the displacement transmission curves obtained from finite element calculations and experimental measurements. The number and width of bandgaps for DHMN show the best agreement. Experimental results for DHMA, DHMF, and DHMT reveal merging phenomena between the first and second bandgaps, while good consistency is maintained for the third bandgap.

The DNA double helix metamaterial with nodal mass disks proposed in this chapter exhibits extremely wide longitudinal bandgaps spanning 0–2000 Hz, along with excellent vibration attenuation performance. By adjusting various structural parameters, longitudinal bandgaps below 50 Hz can be achieved, positioning this structure as a promising candidate for ultra-low-frequency and ultra-wideband vibration attenuation and noise control applications.

Chapter 7

Conclusions and future work

7.1 Conclusions

This dissertation, centred on the core theme of “elastic wave manipulation in helical metamaterials,” systematically investigates the formation mechanisms and tuning laws of elastic wave bandgaps in a range of helical metamaterials. The research employs a comprehensive methodology that integrates the finite element method, the Carrera Unified Formulation (CUF), and laser vibrometry. The work encompasses the entire research pipeline, from structural design and theoretical modelling to numerical simulation and experimental validation, thereby establishing a comprehensive framework for analysing and verifying bandgaps in helical metamaterials. This provides a systematic theoretical foundation and engineering guidance for achieving low-frequency, broadband, and tunable elastic wave bandgaps. The main research achievements and conclusions are as follows:

(1) Inspired by the nautilus shell morphology, antisymmetric dual helix metamaterial beams were proposed, along with an improved design incorporating mass spheres. Through finite element analysis and CUF theoretical calculations, the mechanism underlying the formation of low-frequency flexural wave bandgaps in these beams was revealed. The influence of key parameters, including the number of helical turns, the radius of the mass spheres, and material properties, on bandgap characteristics was elucidated. The research demonstrates that the low-frequency bandgaps in the antisymmetric dual helix structure primarily originate from local resonance effects, and that coupling between the mass spheres and the helical structure can significantly enhance this effect, leading to lower-frequency, broader bandgaps. Displacement transmission curves calculated using CUF showed excellent agreement with COMSOL simulations and laser vibrometry experiments, validating the high accuracy and applicability of CUF for vibration analysis of structures with complex geometries.

(2) Two-dimensional dual helix metamaterial plate models were constructed. The influence of geometric parameters, including the number of helical turns, cross-sectional shape, helix width, helix orientation, plate thickness, and mass sphere radius, on the flexural wave bandgap characteristics was systematically analysed. The findings reveal that the unit cell geometry significantly affects the bandgap properties. The number of helical turns determines the frequency range where bandgaps appear, with no complete bandgap forming when the number of turns is less than one. The helix width has the most pronounced effect on both the bandgap position and bandwidth; increasing the helix width shifts the bandgap upward and

broadens its width. An approximately linear relationship was observed between the mass sphere radius and the bandgap frequency. These results indicate that appropriate tuning of geometric parameters can effectively extend the low-frequency bandgaps of dual helix plates, offering a viable approach for bandgap design in multi-dimensional structures.

(3) To address the challenge of achieving tunable bandgaps in traditional structures, a DNA double helix metamaterial with tunable longitudinal wave bandgaps was proposed. The longitudinal wave bandgap characteristics were systematically investigated using finite element calculations for configurations with uniform mass blocks and various permutations and combinations of mass blocks. The band structures, displacement transmission curves, vibration modes, and full-field displacement distributions for different mass distributions were obtained. The research revealed that by adjusting the sequence of mass block arrangements, encodable bandgap tuning could be achieved without altering the unit cell's geometric parameters. Further adjustments to the geometric parameters enabled precise design and optimization of the bandgap frequency. Experimental tests on specimens with uniform mass blocks yielded bandgap ranges in good agreement with simulation results, demonstrating particularly high consistency within the first longitudinal bandgap region.

(4) To achieve even lower-frequency and broader longitudinal wave bandgaps, a DNA double helix metamaterial incorporating nodal masses was further proposed. The formation mechanisms and tuning laws of ultra-low-frequency and ultra-wideband longitudinal-wave bandgaps were systematically analysed for configurations with and without nodal mass disks, and for nodal mass disks made of three different materials (aluminium, steel, and copper). The results demonstrate that the nodal-mass disk structure generates wide and deep longitudinal wave bandgaps over the 0–2000 Hz range. By adjusting the minor helix diameter, mass disk thickness, and axial pitch, longitudinal wave bandgaps below 50 Hz can be achieved. Experimental results exhibited excellent agreement with simulation predictions, further validating the superior performance of this structure for low-frequency vibration control.

In summary, through the systematic design and investigation of various helical metamaterial configurations, this dissertation demonstrates the feasibility of achieving low-frequency broadband elastic wave bandgaps using helical structures. The research demonstrates that the coupling effect between the double helix structure and attached masses can significantly enhance local resonance, thereby yielding lower-frequency and broader bandgap characteristics. By appropriately tuning geometric parameters, material properties, and mass distributions, bandgap performance with ultra-low frequency, ultra-wide bandwidth, strong vibration attenuation, and tunable, encodable performance can be achieved. Furthermore, this work extends the application range of CUF to the elastic wave analysis of complex structures. The results obtained from CUF calculations, doubly validated by COMSOL simulations and laser vibrometry experiments, prove its efficiency and reliability for engineering bandgap prediction. The research presented herein

provides a systematic theoretical foundation and experimental support for the low-frequency broadband tunable design of elastic metamaterials.

7.2 Future work

Although this dissertation has yielded systematic research results on elastic wave manipulation in helical metamaterials, several directions warrant further exploration. Future research can be pursued from the following aspects:

(1) Investigation of multi-field coupling and nonlinear effects

Future work can introduce multi-physical fields, such as temperature, electric, magnetic, and strain fields, to investigate the evolution laws of bandgaps and the dynamic response characteristics of helical metamaterials under coupled-field conditions. Incorporating geometric and material nonlinear analyses can reveal the mechanisms by which multi-field and nonlinear coupling influence local resonance and energy propagation. Establishing multi-field coupling models and experimental validation systems can provide a theoretical foundation and technical support for achieving multifunctional bandgap tuning with environmental adaptability.

(2) Multi-scale modelling and optimization design

The complex geometry and local resonance mechanisms of helical metamaterials exhibit multi-scale dependence on bandgap characteristics. Future research can leverage topology optimization, response surface methods, and machine learning algorithms to establish multi-scale parametric models, enabling rapid mapping and optimization between structural parameters and performance. By integrating data-driven approaches with physical modelling methods, modelling accuracy can be enhanced while significantly shortening design cycles, providing new theoretical and numerical tools for the efficient construction of low-frequency broadband metamaterials.

(3) Expansion of engineering applications

Leveraging their wide low-frequency bandgaps and strong vibration attenuation, helical metamaterials have broad application potential across aerospace, marine engineering, energy equipment, and acoustic protection. Future work can extend the design concepts proposed in this dissertation to complex loading and dynamic operating environments, developing engineered devices featuring lightweight construction, high damping, and multifunctionality. By combining structural optimization with composite material design, functions such as efficient vibration damping, low-frequency noise suppression, and energy harvesting can be realized, advancing metamaterials from theoretical research toward engineering practice.

(4) Extension and deepening of CUF theory

This dissertation systematically applied CUF to elastic wave modelling and bandgap prediction in helical metamaterials for the first time, validating its high accuracy and adaptability for complex geometric structures. Future research can further extend CUF in four directions: first, extending it to three-dimensional helical structures to achieve wave analysis under thermo-acoustic-mechano-electric multi-field coupling; second, introducing nonlinear and dissipative mechanisms to

develop higher-order CUF models suitable for strongly nonlinear systems; third, integrating multi-scale theory with higher-order displacement expansions to construct a unified analysis framework spanning from macroscopic structures to microscopic unit cells; fourth, fusing CUF with machine learning and topology optimization methods to establish data-driven bandgap prediction and structural optimization systems. Further extensions of CUF will provide new numerical frameworks and theoretical support for refined analysis of helical metamaterials under complex multi-field conditions.

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Appendix A

List of publications

A.1 Journal Articles

1. **Yumei Chen**, Shabnam Kisasat, Matteo Filippi, Xiang Fang, Ji Wang, Erasmo Carrera. Numerical and experimental investigation of inertia amplified dual helix metamaterial beam with low frequency flexural wave band gaps. *Engineering Structures*, 2025, 343: 121112.
2. **Yumei Chen**, Lei Yang, Jia Lou, Ji Wang, Matteo Filippi, Erasmo Carrera, Xiang Fang. Bio-inspired elastic metamaterial by B-form DNA: Programmable dual helix structures for low-frequency longitudinal wave prohibition. *Composite Structures*, 2025, 360: 118986.
3. **Yumei Chen**, Xiang Fang, Ji Wang, Matteo Filippi, Erasmo Carrera. An analysis of bandgap characteristics of metamaterial plates with dual helix cells. *Mechanics of Advanced Materials and Structures*, 2024, 31(1): 92-102.
4. **Yumei Chen**, Lei Yang, Xiang Fang, Ji Wang. Shear-horizontal surface waves in half-space of piezoelectric semiconductor with initial stresses. *Acta Physica Polonica A*, 2022, 142(2): 273-279.
5. **Yumei Chen**, Lei Yang. SH wave propagation in piezoelectric semiconductor plate with initial stress. *Chinese Journal of Solid Mechanics*, 2022, 43(4): 467-476 (in Chinese).
6. Xiang Fang, **Yumei Chen**, Tingfeng Ma, Jia Lou, Ji Wang, Erasmo Carrera, Huimin Wu, Zhilong Huang, Kuo-Chin Chuang. Optomechanical-coupled tristable oscillations in a nonlinear light-driven system. *International Journal of Mechanical Sciences*, 2025, 299: 110424.
7. Xiang Fang, Jia Lou, **Yumei Chen**, Ji Wang, Ming Xu, Kuo-Chih Chuang. Broadband Rayleigh wave attenuation utilizing an inertant seismic metamaterial. *International Journal of Mechanical Sciences*, 2023, 247: 108182.
8. **Yumei Chen**, Bing Xu, Lei Yang, Huimin Wu, Ji Wang, Matteo Filippi, Erasmo Carrera, Xiang Fang. Manipulating longitudinal waves in a hybrid bio-inspired double helix metamaterial. *International Journal of Mechanical Sciences*, Major revisions.

A.2 Conference Proceeding

1. **Yumei Chen**, Ji Wang. Analysis of temperature-frequency relationship in quartz crystal resonators considering equivalent viscous dissipation, In: *2nd National Academic Forum on PhD Students in Mechanics*, Hangzhou, China, 15-17 October 2021.
2. **Yumei Chen**, Xiang Fang, Ji Wang, Erasmo Carrera. An analysis of band gap characteristics of metamaterial plates with dual helix cells, In: *International Conference on Advanced Topics in Mechanics of Materials, Structures and Construction (AToMech-I)*, Al Khobar, Saudi Arabia, 12-14 March 2023.
3. **Yumei Chen**, Shabnam Kiasat, Xiang Fang, Matteo Filippi, Erasmo Carrera, Ji Wang. Investigation of flexural wave propagation in dual helix metamaterial beam, In: *The 18th Symposium on Piezoelectricity, Acoustic Waves, and Device Applications (SPAWDA)*, Dongguan, China, 8-11 November 2024.