## Summary

The technological progress in terms of micro and nano-fabrication, together with the growing ability to enhance the performance of superconducting materials allowed in the last two decades to practically incorporate the rules of quantum mechanics into 2-dimensional electrical circuits. This combination produced from one side ordered and controllable quantum systems, that can be engineered and built by default to observe in laboratory exotic quantum phenomena, and from the other side pushed a whole new branch of theoretical physics that joins quantum mechanics and electrical engineering.

In this thesis we address the problem of noiseless microwave amplification for very low power signals, eventually reaching the single photon level. We tackle this problem adopting an approach that relies on superconducting transmission lines, since these objects put together two fundamental ingredients, low temperatures and a lossless nature. The key physical aspect that we exploit to achieve lossless amplification is parametric amplification, which allows a natural energy transfer from different modes, eventually transferring power from a strong pump tone to a weak signal tone, that we actually mean to amplify. To engineer such a transmission line we adopt circuit-Quantum ElectroDynamics techniques, hence we treat quantum mechanically microscopic systems modeling them as electrical circuits. The fundamental feature of these devices is the presence of Josephson junctions, a lossless nonlinear element which confers the needed nonlinearity to the system to trigger the parametric amplification process.

In this work we develop a quantum mechanical theory to describe a nonlinear transmission line amplifier, named Josephson Traveling Wave Parametric Amplifier, that works both in 3-wave mixing and 4-wave mixing regime. We perform analytical simulations based on the output of the quantum model and find a set of circuit parameters to realise a physical layout. Using electromagnetic simulations we design and test sections of the transmission line in order to ensure the best impedance matching possible, and after that we fabricate and characterise the device in a cryogenic environment. The cryogenic characterisation reveals a working mixing effect between the internal modes but low gain and narrow bandwidth, meaning that some important features of the device are not fully caught by the analytic quantum theory. For this reason a modified numerical approach is used to identify a working set of circuit parameters, together with a different structure of transmission line. The new circuit includes a periodic load of resonators that modifying the dispersion relation of the line avoids the creation of stray modes, which are a serious cause of gain reduction. This technique, which takes the name of Resonant Phase Matching, shows high gain in the numerical simulations, and we finally realise a physical layout of transmission line through an electromagnetic simulation approach.