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Tear and interconnect DDM with efficient multibranch-multiresolution preconditioner for the simulation of highly complex realistic problems

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***Abstract*—In this work we present the combination of a high scalability implementation of the multibranch-multiresolution preconditioner with the domain decomposition method for the electromagnetic analysis of non-conformal complex problems with different levels of multi-scale features. A numerical experiment will be shown to illustrate the great flexibility of this approach for the solution of large multi-scale objects.**

***Index Terms*—Domain decomposition method (DDM), multiresolution (MR) preconditioner, multibranch RWG (MB-RWG) basis functions, method of moments (MoM), multilevel fast multipole algorithm (MLFMA).**

I. Introduction

Domain decomposition methods (DDMs) [1]–[6] based on surface integral equations (SIEs) have emerged as a powerful tool in computational electromagnetics (CEM) to improve or even enable the solution of realistic structures in a wide range of applications.

Essentially, the domain decomposition method splits the original problem into a collection of subdomains providing a block preconditioning matrix formed by the inverse of the impedance matrix of the subdomains. The key point of this efficient method is that the matrix vector product (MVP) corresponding to the preconditioner can be applied by solving the subdomain matrix systems independently. For small subdomains, direct factorization of MoM [7] matrices is the optimal option, both in terms of time and accuracy. Instead for large subdomains, fast iterative methods like highly scalable MLFMA-FFT algorithms [8], [9] become necessary to optimize computing resources.

However, the application of the DDM to realistic complex systems must be carefully approached. The optimization of the outer iterative method in the DDM scheme may result in a complexity shift towards local solvers, which must deal with inner strong interactions and multi-scale details like cavities or arrays of antennas, definitely, subdomains too large to apply the direct factorization of the MoM and too complex to obtain an accurate solution in reasonable times using fast solvers due to the ill-conditioning of the subdomain systems. In these cases, the inclusion of an efficient preconditioner is critical to improve the convergence of the fast solvers in the local stage, thus improving overall performance or even making it possible to get correct results in reasonable time for really challenging problems.

Among the different preconditioning alternatives, the multiresolution (MR) preconditioners [10]–[15] become a strong candidate to be integrated into a domain decomposition scheme in order to speed up the local solution of complex subdomains incorporating multi-scale features. The MR preconditioner introduces a set of multilevel basis functions to discretize the problem that improves the conditioning of the original system [13] while keeping the different scales of variation of the solution [15]. In part due to its multilevel nature, the MR preconditioner has proven to be a good choice to improve convergence in multi-scale problems [10], [16], especially in the case of medium-sized (not very large-scale) problems. An additional feature that makes this preconditioner appealing in this context is that it can be efficiently embedded into the MLFMA-FFT local solvers as a multiplicative preconditioner.

In this paper, an efficient implementation of the MR preconditioner is embedded into the MLFMA-FFT method, in the context of a SIE-DDM implementation. The objective is to provide the local solvers with capabilities to efficiently address medium-sized complex subdomains exhibiting deep-multiscale nature, avoiding bottlenecks in the application of DDM to real-life challenging problems. The multibranch RWG (MB-RWG) basis functions [17], which can be shown to be compatible with the MR preconditioner [18], are applied to facilitate problem description by allowing the use of non-conformal meshes. Numerical examples will demonstrate the versatility and efficiency of the proposed scheme.

II. Formulation

Let us consider the MoM solution of a problem that can be decomposed into different subdomains, according to its geometric and physical features. Then the original matrix system can be left-preconditioned by the block-diagonal matrix $\mathbf{M}$, posing the (isolated) local solutions of the individual subdomains. Each diagonal block of $\mathbf{M}$, namely $\mathbf{M}_i$, represents the inverse of the impedance matrix of the respective subdomain D_i, that is, $\mathbf{M}_i = \mathbf{Z}_i^{-1}$, where $\mathbf{Z}_i$ is the diagonal block of the original impedance matrix corresponding to the subdomain D_i.

The multiplication by matrix $\mathbf{M}$, denoting the application of the DDM preconditioner, can be written individually for each subdomain as $\hat{\mathbf{I}}_i = \mathbf{M}_i \cdot \hat{\mathbf{V}}_i$, i.e., $\hat{\mathbf{I}}_i = \mathbf{Z}_i^{-1} \cdot \hat{\mathbf{V}}_i$ where $\hat{\mathbf{V}}_i$ is the result of the global MVP, coupling the entire problem, restricted to the D_i subdomain. This last MVP can be written for each subdomain D_i as a new independent matrix system, $\mathbf{Z}_i \cdot \hat{\mathbf{I}}_i = \hat{\mathbf{V}}_i$, which can be solved by the method best tailored to the characteristics of each subdomain. In our case, we apply MoM for small subdomains and MLFMA-FFT for medium subdomains.

At this point, the multiresolution (MR) preconditioner [12] is applied to project the original matrix systems of certain selected subdomains into a new function subspace, with the aim of improving their eigenvalue distributions. The key point of this efficient preconditioner is the application of a quasi-Helmholtz decomposition deployed in a multilevel scheme. The complete set of original RWG bases is transformed into a set of multilevel bases, the so-called generalized RWG basis functions (gRWG) [12], and the unknown currents supported by the selected gRWG are then divided into their solenoidal and non-solenoidal parts. The MR preconditioner can be then applied for the solution of the local problems, posing the new equivalent systems for this problems, namely $\mathbf{Z}_i^{MR} \cdot \hat{\mathbf{I}}_i^{MR} = \hat{\mathbf{V}}_i^{MR}$. Additionally, the multibranch RWG basis functions can be applied in combination with the MR preconditioner to allow for the modeling and decomposition of currents defined on non-conformal discretizations.

III. Numerical results

A numerical result is presented to demonstrate the efficiency and versatility of the proposed approach to solve large problems with local deep-multi-scale features. Let us consider an array antenna mounted on a finite plane. The mesh is adapted to the fine detail features of the strips allowing a non-conforming discretization in the connections with the plane (mesh details in Fig1). The dimensions of the plane are 10x10 m length ($33\lambda \times 33\lambda$ at the working wavelength, λ). The detailed model of this structure is shown in Fig 1.

The DDM was applied in conjunction with the MR preconditioner to get an accurate prediction of the array antenna radiation at 1 GHz. A total of 468 792 unknowns were required to mesh the entire geometry, which exhibits multi-scale features, with a mesh density varying from $\lambda/10$ to $\lambda/40$ in the array antenna.

For the analysis of this structure, the problem was partitioned into 4 subdomains, as shown in Fig 1. The subdomains are solved using 4 independent MLFMA-FFT solvers. The complexity of this structure is localized in the array with 400 strip antennas. Although the electrical size of the antennas that make up the array is small, due to the extremely coupling between each antenna, the entire array is considered as a single large domain instead of different MoM domains to improve the overall convergence of the DDM. This strong interaction also makes necessary the inclusion of the MR preconditioner to the local solver MLFMA-FFT to achieve a good convergence for this complex domain.

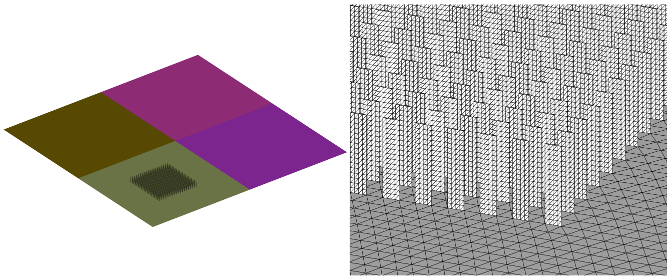

Fig. 1. Partition into subdomains and non-conformal mesh details.

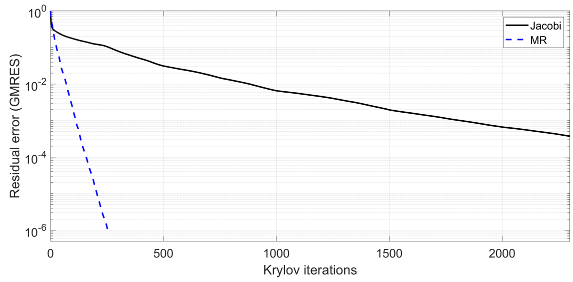

Fig. 2. Convergence performance of the inner GMRES when solving one outer GMRES iteration of the array domain using the proposed MR preconditioner method and the Jacobi preconditioner.

Fig 2 shows the residual error of the local iterative solution (at a given DDM iteration) of the subdomain corresponding to the array antenna using the MR preconditioned, compared with the Jacobi preconditioner. It can be observed that the MR preconditioner greatly reduces the iteration count for this multi-scale problem, thus enabling the solution of this domain in the context of the DDM iterative resolution.

Fig 3 shows the residual error for whole problem using DDM for the two cases of local preconditioners mentioned in Fig 2. Although the outer Krylov convergence should be similar for both local preconditioner cases, the poor residual error posed by the Jacobi preconditioner impairs the outer GMRES convergence, thus ruining the accuracy of the solution. The final equivalent currents calculated applying DDM and the MR preconditioner for the local array antenna subdomain is shown in Fig 4.

IV. Conclusion

In this work, we present the efficient combination of the multiresolution preconditioner with the domain decomposition

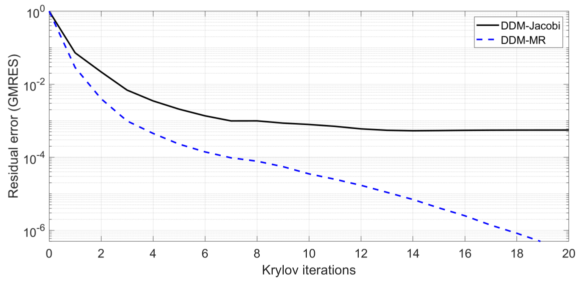

Fig. 3. Convergence performance of the outer GMRES when solving the EMC problem using the proposed MR and the Jacobi preconditioner in the local solver.

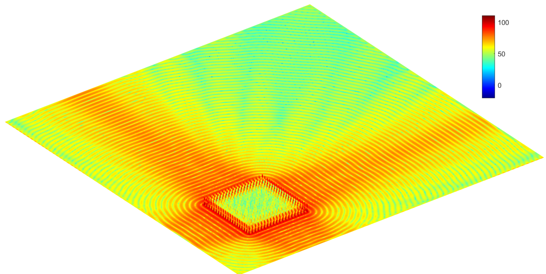

Fig. 4. Real part of the equivalent electric surface current distribution ($dB\mu A/m$) provided by the SIE-DDM approach.

method to obtain the accelerated solution of complex multi-scale domains in the local stage. This strategy equip the local solvers with an efficient tool to address medium-sized complex subdomains exhibiting multi-scale features, ensuring the optimal convergence of the DDM in real-life challenging problems. The efficiency and accuracy of the proposed methods was demonstrated through the solution of a numerical example.

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