Appendix A. *Low-Fidelity* (LF) database mesh independence study

The computational meshes adopted in the LF database runs were generated using `snappyHexMesh` utility in OpenFOAM. To perform the mesh independence study, a series of progressively refined meshes were generated by varying the base cell count of the initial background mesh generated by `blockMesh`. Specifically, refinement was achieved by increasing the number of cells along each axis of the `blockMesh` domain, which directly controls the resolution at which the STL geometry is intersected and captured by `snappyHexMesh`. For the construction of the simulation database, a background mesh resolution of (40, 40, 40) in a cubic block of $1 \times 1 \times 1$ cm was employed, representing a good compromise between solution accuracy and computational cost. To evaluate mesh independence, additional simulations were carried out using coarser (15, 15, 15) and finer (92, 92, 92) resolutions, allowing assessment of the sensitivity of the numerical results to mesh refinement. All the meshes consist of mixed elements (tetrahedral and hexahedral), and 7 prism layers, with a relative first layer thickness of 0.1, and expansion ratio of 1.2. The mesh independence study has been carried out on the gyroid wall thicknesses 0.25, 2, and 3.75 mm, at the pressure drops 100, 250, and 500 Pa. The comparison between the three grids for the gyroid wall thickness 2 mm is reported in Figure Appendix A.1. The results of the different runs (with the setup described in Section 4 were compared in terms of the Darcy velocity $\langle u \rangle$ and the heat transfer coefficient $h_{(sf)}$. The comparison is reported in Table Appendix A.1.

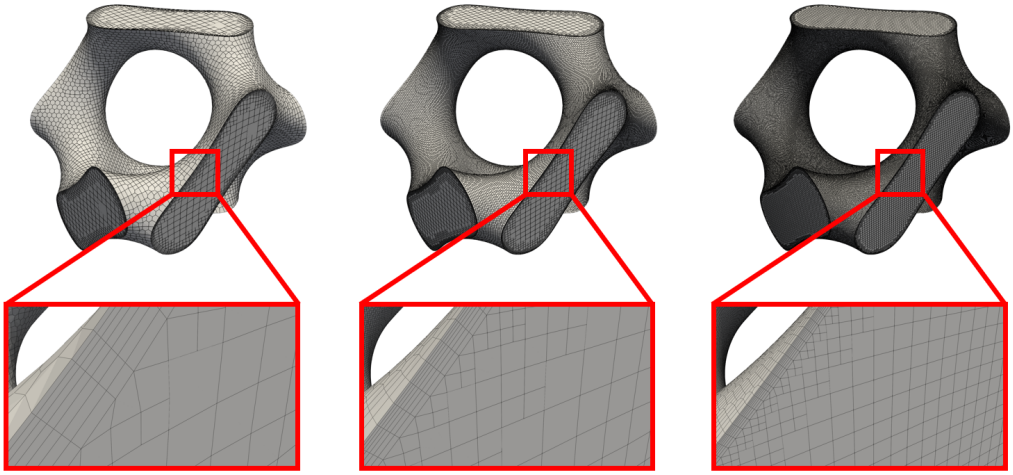

Figure Appendix A.1: Computational mesh at three different resolutions for a gyroid with wall thickness of 2 mm, with detailed views on the prismatic boundary layer. From left to right: coarse grid ((15, 15, 15) background resolution, 50,000 elements), medium grid ((40, 40, 40) background resolution, 400,000 elements), and fine grid ((92, 92, 92) background resolution, 3,000,000 elements).

th [mm]	Δp [Pa]	# elements [–]	$\langle u \rangle$ [m/s]	$e_{(\langle u \rangle)}$ [–]	$h_{(sf)}$ [W/m^2·K]	$e_{(h_{(sf)})}$ [–]
0.25	100	100,000	0.222913	5.51 %	4,584.51	9.25 %
0.25	100	500,000	0.212442	0.56 %	4,680.20	7.36 %
0.25	100	3,500,000	0.211265	–	5,052.06	–
0.25	250	100,000	0.343342	5.24 %	5,978.37	6.01 %
0.25	250	500,000	0.360067	0.64 %	6,082.76	4.36 %
0.25	250	3,500,000	0.362319	–	6,360.37	–
0.25	500	100,000	0.498208	5.95 %	7,441.80	3.83 %
0.25	500	500,000	0.517856	2.24 %	7,308.51	1.82 %
0.25	500	3,500,000	0.529734	–	7,737.94	–
2	100	50,000	0.072608	4.05 %	4,407.47	12.83 %
2	100	400,000	0.070055	0.40 %	4,674.28	7.55 %
2	100	3,000,000	0.069779	–	5,056.10	–
2	250	50,000	0.120680	3.88 %	5,695.25	8.90 %
2	250	400,000	0.113973	1.89 %	5,723.02	8.46 %
2	250	3,000,000	0.116171	–	6,251.64	–
2	500	50,000	0.163975	5.96 %	6,888.65	6.66 %
2	500	400,000	0.182824	4.85 %	6,940.56	5.06 %
2	500	3,000,000	0.174371	–	7,380.06	–
3.75	100	15,000	0.003879	2.32 %	4,927.23	10.89 %
3.75	100	250,000	0.004024	1.33 %	4,365.67	1.74 %
3.75	100	1,200,000	0.003971	–	4,443.19	–
3.75	250	15,000	0.007772	4.50 %	5,929.94	8.09 %
3.75	250	250,000	0.008287	1.62 %	5,441.53	0.81 %
3.75	250	1,200,000	0.008155	–	5,486.53	–
3.75	500	15,000	0.012774	6.60 %	7,081.06	4.87 %
3.75	500	250,000	0.013831	1.13 %	6,735.64	0.25 %
3.75	500	1,200,000	0.013677	–	6,752.20	–

Table Appendix A.1: Mesh independence study for LF runs at different resolution levels. Table rows highlighted in yellow correspond to the finest grid.

Appendix B. *High-Fidelity* (HF) database mesh independence study

The computational meshes used in the HF computations were generated in BOXER. The refinement was achieved by decreasing the size of cubic cells relative to an $8.4 \times 1.4 \times 4.4\,\text{cm}$ box, which represents the domain enclosing the HX geometry. To evaluate the mesh independence, simulations were performed using a coarse mesh (relative cell size equal to 0.002), an intermediate mesh (relative cell size of 0.0011), and a fine mesh (relative cell size equal to 0.0008). All the meshes consist of mixed elements (tetrahedral and hexahedral), and 8 prism layers, with a relative first layer thickness of 0.1, and expansion ratio of 1.1. The mesh independence study was performed on a simplified HX model with homogeneous gyroid matrix with wall thicknesses of 0.25, 1.5, and 3.75 mm, and an imposed total pressure drops of 100, 500, and $1,000\,\text{Pa}$. A close-up view of the three grids is shown in Figure Appendix B.2 for a gyroid matrix with a wall thickness of 1.5 mm. Numerical results in terms of the computed inlet-averaged velocity $u_{(in)}$ are reported in Table Appendix B.2.

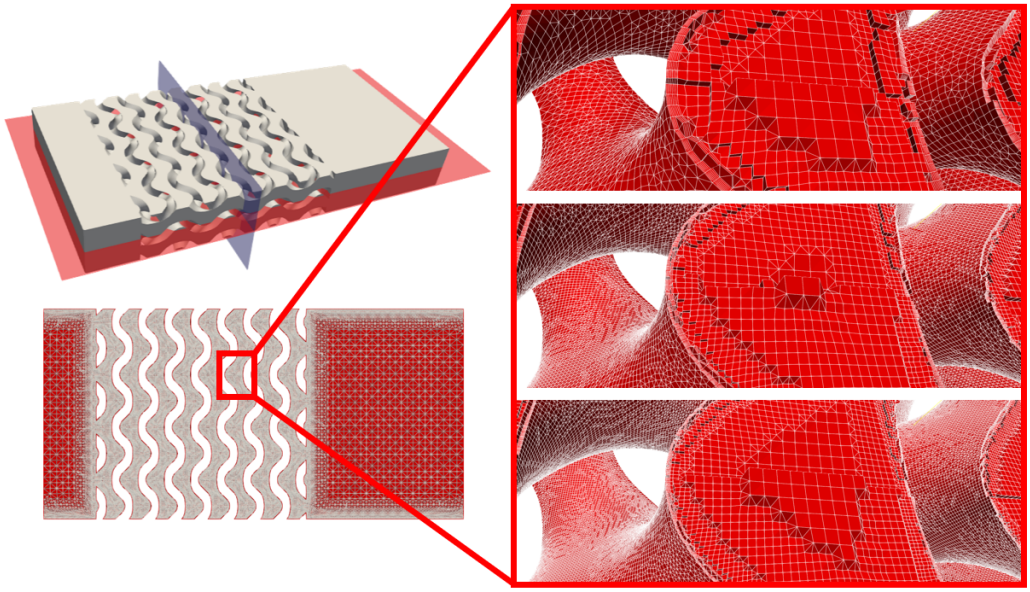

Figure Appendix B.2: Computational mesh at three different resolutions for the mesh independence study performed on a simplified HX. From top to bottom: coarse grid (relative cell size 0.002, number of elements equal to $17,000,000$), medium grid (relative cell size 0.0011, number of elements equal to $63,000,000$), and fine grid (relative cell size 0.0008, number of elements equal to $114,000,000$).

th [mm]	Δp° [Pa]	# elements [–]	$u_{(in)}$ [m/s]	$e_{(u_{(in)})}$ [–]
0.25	100	19,000,000	0.088825	0.57 %
0.25	100	73,000,000	0.088277	0.05 %
0.25	100	131,000,000	0.088323	–
0.25	500	19,000,000	0.214392	0.53 %
0.25	500	73,000,000	0.213337	0.04 %
0.25	500	131,000,000	0.213258	–
0.25	1,000	19,000,000	0.305062	2.20 %
0.25	1,000	73,000,000	0.308972	0.95 %
0.25	1,000	131,000,000	0.311931	–
1.5	100	17,000,000	0.040791	0.46 %
1.5	100	63,000,000	0.040561	0.10 %
1.5	100	114,000,000	0.040603	–
1.5	500	17,000,000	0.097673	3.30 %
1.5	500	63,000,000	0.099880	1.12 %
1.5	500	114,000,000	0.101008	–
1.5	1,000	17,000,000	0.147344	1.94 %
1.5	1,000	63,000,000	0.146742	1.52 %
1.5	1,000	114,000,000	0.144539	–
3.75	100	15,000,000	0.001158	1.03 %
3.75	100	48,000,000	0.001166	0.34 %
3.75	100	110,000,000	0.001170	–
3.75	500	15,000,000	0.004272	1.11 %
3.75	500	48,000,000	0.004250	0.59 %
3.75	500	110,000,000	0.004225	–
3.75	1,000	15,000,000	0.006806	1.79 %
3.75	1,000	48,000,000	0.006758	1.08 %
3.75	1,000	110,000,000	0.006686	–

Table Appendix B.2: Results of the mesh independence study for different wall thicknesses and pressure drops imposed across the HX. Table rows highlighted in yellow correspond to the finer grid.