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Optimal siting and sizing of battery energy storage systems in unbalanced distribution systems: A multi objective problem under uncertainty

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ABSTRACT

In this paper the siting and sizing problem of battery energy storage systems in unbalanced active distribution systems is formulated as a mixed-integer, non-linear, constrained multi objective (MO) optimization problem under uncertainties. The problem is cumbersome from the computational point of view due to the presence of intertemporal constraints, a great number of state variables and the presence of uncertainties in the problem input data. A new approach based on the trade-off/risk analysis is proposed to obtain with acceptable computational efforts a solution that may not be the optimal solution but represents a reasonable and robust compromise. We use the trade-off/risk analysis, because it was specifically developed for power system planning problems in which we deal with a wide range of options, with possible conflicting objectives, and with uncertainty and risk. The proposed approach includes new procedures to select an adequate set of planning alternatives to be considered in the trade-off/risk analysis framework and to assist the planning engineer when difficulties arise in setting probabilities of the input data. Numerical applications to an IEEE unbalanced test system demonstrate the effectiveness of the proposed procedure and indicate the best alternatives of storage systems in the range from 450 kW to 600 kW globally installed in a reduced set of nodes.

1. Introduction

1.1. Background and motivation

In modern electrical distribution systems, a wide diffusion of storage systems is expected, and in particular of Battery Energy Storage Systems (BESSs). These systems compensate the unavoidable uncertainties of energy produced by solar and wind sources and are able to provide ancillary services such as frequency regulation, balancing, voltage support and black start capacity. Their use can also lead to transmission/distribution upgrade deferral and improve grid reliability, power quality and capability. Furthermore, BESSs can furnish a contribution in minimizing the costs of electricity and in balancing power in the islanded operating condition.

Unfortunately, BESSs are still characterized by significant costs and, therefore, one of the main problems recently addressed in the relevant literature has concerned with the need of maximizing the benefits deriving from their use. To maximize the benefits, it is mandatory to determine their best location in the network buses. An optimal siting, in fact, can help to optimize power quality and reliability levels, to mitigate peak demand and renewable energy resources integration, and to reduce costs [1].

The model to be formulated for solving the BESSs allocation problem (i.e., siting and sizing) is a mixed-integer, non-linear, constrained optimization model, with a great number of equality and inequality constraints; this model usually requires a high computational burden for its solution, mainly when the well-known uncertainties of renewable energy sources, such as photovoltaic and wind plants, and loads are considered. When the energy management problem of the resources [2-5] is included in the optimization problem to represent the short-term operation, the results of the planning problem will be more accurate but further complexities and computational efforts have to be accounted for.

1.2. Related literature

Analysing the contributions in the relevant literature on the BESSs

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Nomenclature

α	Discount rate
β	Annual rate of change of the electrical energy cost
$\gamma_{s,h,d,y}$	Efficiency of the battery energy storage system (BESS) at bus s, at hour h of the day d in the year y
$\vartheta^{p}_{b,h,d,y}$	Argument of voltage at phase p of bus b, at hour h of the day d in the year y
π_f	Probability of occurrence of scenario f
ω_i	Weight of the i^{th} objective function
Ω, Ω'	Global and significant global decision set
Ω_B, b	Set/index of the busses of the network
$\Omega_{ch,d,y}$	Set of BESS charging hours for the day d in the year y
$\Omega_{disch,d,y}$	Set of BESS discharging hours for the day d in the year y
Ω_D, d	Set/index of day
Ω_f, Ω'_f	Conditional decision set and significant conditional decision set for scenario f
Ω_H, h	Set/index of hour
Ω_L, l	Set/index of line
Ω_S, s	Set/index of bus with BESS
Ω_Y, y	Set/index of year
$c^{E}_{h,d,y}$	Cost of energy unit at hour h of the day d in the year y
f_i	i^{th} objective function
$kd_{b,h,d,y}$	Unbalance factor at bus b, at hour h of the day d in the year y
kd_{max}	Maximum allowable value of unbalance factor
p	Index of bus phase ($p = 1, 2, 3$)
r_a	Robustness of alternative a
r_i	Rank position of the objective function f_i
r_{s_y}	Parameter for battery replacement for the BESS installed at bus s at year y
C^E	Cost of the energy acquired from the upstream grid
C^{ES}	Cost of the BESS
E^{size}_s	Usable energy storage capacity of the BESS at bus s
EC_{B_y}	Unitary capacity cost of the batteries, at the year y
G^{pm}_{bk}, B^{pm}_{bk}	Terms of the three-phase admittance matrix
$F_{l,h,d,y}$, $F^{r}_{l,h,d,y}$	Current and ampacity of line l, at hour h of the day d in the year y
N_f	Number of scenarios
N_a, N_{af}	Number of alternatives and of feasible alternatives
$N_{ph,b}$	Number of phases at bus b
$P^{p}_{1_{h,d,y}}$	Active power at phase p of the slack bus #1, at hour h of the day d in the year y
$P^{p}_{b,h,d,y}$, $Q^{p}_{b,h,d,y}$	Active and reactive net powers at phase p of bus b, at hour h of the day d in the year y
P^{size}_s	Size (power) of the BESS installed at bus s
$P^{ES}_{s,h,d,y}$, $Q^{ES}_{s,h,d,y}$	Active and reactive power of the BESS installed at bus s, at hour h of the day d in the year y
SIC_{ES}	Initial cost of BESS for unit of power
S^{ES}_s	Size of the AC/DC interfacing converter of the BESS installed at bus s
$SM_{h,d,y}$	Security margin of the feeders at hour h of the day d in the year y
S_{trans}	Rating of the interfacing transformer
$V^{p}_{b,h,d,y}$	Magnitude of voltage at phase p of bus b, at hour h of the day d in the year y
V_{min}, V_{max}	Minimum and maximum allowable value of voltage magnitude
V^{slack}	Pre-fixed voltage at the slack bus

siting and sizing, mainly in recent years it arised that the optimization model complexities of such contributions are of a different nature and the involved solution algorithms are based on different mathematical techniques; furthermore, different storage services are considered and both deterministic and uncertain frameworks are considered.

A comprehensive review on single- and multi-objective optimization methods and outstanding issues of BESSs allocation strategies are reported in [1,6,7]. Optimization objectives (cost, capacity and lifetime, power quality and so on) and several constraints (charging/discharging, capacity, reliability, and so on) are analyzed in detail; the various approaches to solve the optimization models are also outlined and compared.

The analysis of the relevant literature reported in [1,6,7] and of other recent papers clearly reveals that the problem of the optimal allocation of BESSs is a topic of the greatest interest. It is also evident that nowadays several uncertain parameters (such as load powers and wind/photovoltaic powers) are recognized to be absolutely addressed when optimizing the BESSs allocation in active distribution systems; this trend is particularly clear in the most recent relevant literature and significant examples are reported in [8–17]. In [8] a two-step stochastic mixed-integer, linear programming problem is formulated to solve the allocation problem. In the first step the BESSs are optimized on a limited budget while, in the second step, a bilevel BESS arbitrage model is used to maximize the arbitrage revenue in the upper level and clears the distribution market in the lower level. Typical scenarios are obtained with statistic-based extraction algorithms. In [9] the problem of the optimal allocation of both BESSs and soft open points at minimum cost is approached in a probabilistic framework; demand response and conservation voltage reduction services are considered. The K-means clustering technique and a Monte Carlo simulation technique are applied to handle uncertainties. In [10] a hybrid optimization method based on metaheuristic evolutionary particle swarm optimization and linear programming is proposed. Uncertainties are considered by scenarios derived from historical data. In [11] uncertainties, such as the photovoltaic powers, are considered. The K-means data clustering algorithm is applied to reduce the computational efforts and the particle swarm optimization is employed to solve the optimization model. In [12] the DSO perspective in the BESS siting and sizing is considered and a multi-period convex AC-optimal power flow is applied. Wind, photovoltaic, and load powers uncertainties are modeled as symmetric and bounded variables. In [13] a distribution system with high penetration of PV units is considered and the uncertainty of load and PV generations is taken into consideration by means of probabilistic analysis and time-period clustering. In [14] unbalanced distribution systems with high penetration of PV units and electric vehicles charging stations are considered. The BESSs mitigate the negative effects of charging stations. A robust scenario-based method is applied to account for the uncertainties of charging powers, loads and PV generation. The optimization problem is solved using an algorithm that combines the alternating direction method of multipliers with a cutting plane procedure. In [15] the problem of the optimal BESSs location and sizing in a distribution system is approached considering their reactive power capability and the wind power uncertainties. A hybrid algorithm including the non-dominated sorting genetic algorithm, multi-objective particle swarm optimization and a decision-making algorithm is applied. In [16] the problem of the optimal allocation of both DERs and BESSs is faced considering the responsive load demand and the uncertainties of wind turbines and PV units. In [17] a multi-objective robust optimization allocation model of energy storage based on confidence gap decision is established for a power system with uncertain wind and PV units. A new adaptive harmonic aliasing multi-objective compound differential evolution algorithm is proposed to solve the model.

Table 1
Contributions of related papers.

Paper	Objective	Uncertainty sources	Proposed method
[8]	Maximization of profit	Hourly locational marginal prices and system loads	Karush-Kuhn Tucker optimality conditions used to convert the bilevel problem of the second stage to a single-level problem Two relaxation methods
[9]	Minimization of costs	Photovoltaic generation and load demand	Hybrid optimization solver for the large scale nonconvex mixed integer nonlinear programming problem (without linearization or relaxation)
[10]	Minimization of costs	Photovoltaic generation, load demand and energy prices	Evolutionary particle swarm optimization and linear programming
[11]	Minimization of costs	Photovoltaic generation and load demand	Particle swarm optimization
[12]	Minimization of costs	Wind and photovoltaic generation and load demand	Robust optimization
[13]	Minimization of costs	Photovoltaic generation and load demand	Mixed integer nonlinear programming (Gurobi solver)
[14]	Minimization of costs	Photovoltaic generation and load demand (including the demand of electric vehicles)	Alternating direction method of multipliers coupled with a cutting plane procedure
[15]	Minimization of costs, of the average voltage deviation and of the average power losses	Wind generation	Non-dominated sorting genetic algorithm, multi-objective particle swarm optimisation and TOPSIS as a decision-making algorithm
[16]	Minimization of active and reactive losses and of voltage deviation index	Wind and photovoltaic generation and responsive load demand	Weighted sum method
[17]	Maximization of voltage profile improvement index and minimization of investment costs	Wind and photovoltaic generation and load demand	Confidence gap decision method Adaptive harmonic aliasing multi-objective compound differential evolution algorithm
This paper	Minimization of costs, of voltage profile deviation and of security margin of feeders	Loading level and rating and siting of photovoltaic plants	Simultaneous perturbation stochastic approximation algorithm for the constrained single-objective optimization problem Trade-off/risk method

To better compare the related papers, Table 1 reports with more details the main features of the contributions.

Considering the uncertainties of active distribution systems usually requires solving a high-dimensional mixed-integer, optimization planning problem under uncertainties [5,18], which can involve enormous computational efforts. This implies that, to fill the research gaps, it is quite essential to develop approaches (models and algorithms) to carry out a useful and feasible study, even in realistic applications when the problem is further complicated by the great number of grid buses.

1.3. *Contributions*

To capture several impacts of BESSs in active, unbalanced distribution systems and consider the unavoidable loads and renewable energy system uncertainties, at first the problem of the BESS optimal siting and sizing is formulated as a mixed-integer, non-linear constrained multi-objective optimization problem under uncertainties. The objective functions to be minimized are technical and economic objectives (i.e., costs, grid voltage profile, security margin). The equality constraints refer to the three-phase power flow equations and to the BESSs balance equations whereas the inequality constraints refer to technical limitations of both network and BESSs.

To determine the planning solution under uncertainties, a risk-based approach is proposed. Firstly, a set of planning alternatives (i.e., BESSs siting and sizing) and a set of scenarios (i.e., loads and distributed generation power levels) with their probability are selected. In particular, the set of planning alternatives is determined by solving for each scenario a multi-objective optimization sub-problem with the weighted sum method and by means of the Simultaneous Perturbation Stochastic Approximation algorithm. Then, for each alternative and scenario, the values of the objective functions are calculated considering the equality constraints and verifying the inequality constraints of the formulated MO optimization problem. In this way, a matrix is obtained, i.e., the Decision matrix, whose columns are the scenarios and whose rows are the feasible alternatives; the matrix elements are the objective functions values. Finally, a "trade-off/risk analysis" [19,20], i.e. a decision analysis method, is performed on the decision matrix to find a sub-set of one or more feasible and robust BESSs planning alternatives that can be the most attractive to help the planning engineer (in the following, Decision Maker, DM) in selecting the final solution in term of BESS siting and sizing to be practically operated. The "trade-off/risk analysis" helps obtain a robust solution to the planning problem that may not be the optimal solution but certainly represents a reasonable compromise [20].

The trade-off/risk analysis was applied in several fields of power systems analysis [19 –22] and is recognized as a powerful tool for planning with conflicting objectives and risk under uncertainty. It has the following advantages:

- it allows to take into account uncertainties through a risk-based strategy involving both economic and technical risks;
- two or more conflicting objectives can be handled and the final solution represents a good compromise among the multiple objectives to be minimized;
- risk and alternatives robustness are handled in powerful and natural ways;
- the solving procedure of the planning problem takes into consideration the uncertainty in input data and the preferred planning alternatives are individuated by considering the robustness: a planning alternative is robust if it results preferred over all (or most of) the scenarios;
- the robust solution of the planning problems is provided with acceptable computational efforts.

Eventually, the main contributions of the new approach for BESS allocation under uncertainties proposed in this paper are:

(1) The formulated MO optimization problem under uncertainties is solved with the trade-off/risk analysis, which, in the authors' best knowledge, has never been proposed before in the relevant literature for BESS allocation. The proposed approach allows choosing the best siting and sizing of BESSs properly considering: (i) uncertainties of both loads and distributed generators, (ii) risk and alternatives robustness.
(2) New procedures are applied: (i) to select an adequate set of planning alternatives to be considered in the trade-off/risk analysis and (ii) to assist the DM when difficulties arise in the setting of scenario probabilities. This allows to avoid that either most of them or even all are discarded since not satisfying the problem constraints.

Finally, the proposed approach faces the BESS planning problem in a practical way (i.e., by considering only a discrete set of alternatives) and is a handy tool when facing with actual distribution systems in an uncertainty framework, with simplicity of implementation.

The rest of the paper is organized as follows. Section 2 illustrates the MO problem formulation. Section 3 shows the solving procedure under uncertain conditions. Section 4 contains the results of the case study application. Conclusions are drawn in Section 5.

2. Multi-objective formulation of the optimal siting and sizing of BESSs

Let us consider an unbalanced active distribution system where single-phase and three-phase BESSs must be allocated. The problem of the optimal BESSs planning can be formulated as a constrained, mixed-integer, non-linear MO optimization problem:

$$\min_{\boldsymbol{X},\boldsymbol{C}}\left\{f_1(\boldsymbol{X},\boldsymbol{C}),f_2(\boldsymbol{X},\boldsymbol{C}),\dots,f_{N_{obj}}(\boldsymbol{X},\boldsymbol{C})\right\} \tag{1}$$

subject to:

$$\boldsymbol{g}(\boldsymbol{X},\boldsymbol{C})=0 \tag{2}$$

$$\boldsymbol{h}(\boldsymbol{X},\boldsymbol{C})\leq 0 \tag{3}$$

where N_{obj} is the number of objective functions, $\boldsymbol{X}$ is the state vector, $\boldsymbol{C}$ is the decision variable vector, and $\boldsymbol{g}$ and $\boldsymbol{h}$ are the equality/inequality constraints vectors to be met.

The state vector $\boldsymbol{X}$ includes the magnitudes and arguments of phase voltages at all buses while the decision variables $\boldsymbol{C}$ are the sizing (power and energy) and siting of the single-phase and three-phase BESSs. The power sizing of the BESS is assumed to be discrete and multiple of an elementary size; the siting is clearly a discrete variable (the grid buses).

The input data (for instance, loads and distributed generation powers) are uncertain and then the optimization problem (1), (2), (3) is an MO optimization problem under uncertainties.

Sub-sections 2.A and 2.B detail the objective functions (1), the constraints (2) and (3). Section 3 shows the solving procedure of the MO problem.

A. Objective functions

Several objective functions have been considered in the relevant literature [1,6–17,23]. In this paper, without loss of generality, three objective functions are considered: i) the total costs of energy and installed BESSs f_1, ii) the grid voltage profile at all buses f_2, iii) the security margin f_3. Further objective functions can be very easily included.

The objective function f_1 refers to the cost of the energy acquired from the upstream grid over the planning period and to the BESSs costs:

$$f_1=PV\left(C^E\right)+PV\left(C^{ES}\right), \tag{4}$$

where the symbol PV means the present value.

Expanding the cost items in (4), it results in:

$$\begin{aligned} f_1 = &\sum_{y\in\Omega_Y}\left(\frac{1+\beta}{1+\alpha}\right)^{y-1}\sum_{d\in\Omega_D}\sum_{h\in\Omega_H}\sum_{p=1}^{3}c^E_{h,d,y}P^p_{1,h,d,y} \\ &+\sum_{s\in\Omega_S}\left[E^{size}_s\sum_{y\in\Omega_Y}\frac{r_{s_y}EC_{B_y}}{(1+\alpha)^{y-1}}+SIC_{ES}P^{size}_s\right]. \end{aligned} \tag{5}$$

The battery replacement costs are accounted through the parameter r_{s_y} in (5): when, during year y, there is a battery installation (only at $y=1$) or replacement, r_{s_y} is equal to 1, otherwise it is equal to zero. In particular, the number of cycles is assigned and, when the battery arrives to the end of its useful life, a replacement is required. A trend of the battery's installation costs is included in (5), thanks to the installation cost EC_{B_y} that varies versus the considered year y. We assume, finally, that the BESS static converters are never replaced during the planning period. The interconnection bus with the upstream grid is indicated as #1.

The objective function f_2 accounts for the profile of voltage at all busbars:

$$f_2=\frac{\sum_{y\in\Omega_Y}\sum_{d\in\Omega_D}\sum_{h\in\Omega_H}\sum_{b\in\Omega_B}\sum_{p=1}^{N_{ph,b}}\left(V^p_{b,h,d,y}-V_{nom}\right)^2}{\sum_{b=1}^{N_{bus}}N_{ph,b}} \tag{6}$$

where V_{nom} is the rated phase voltage.

The objective function f_3 is assumed to be:

$$f_3=\sum_{y\in\Omega_Y}\sum_{d\in\Omega_D}\sum_{h\in\Omega_H}SM_{h,d,y} \tag{7}$$

where the security margin of the feeders, $SM_{h,d,y}$, is defined as [24]:

$$SM_{h,d,y}=\left[1-\min_{l\in\Omega_L}\left|\frac{F^r_{l,h,d,y}-F_{l,h,d,y}}{F^r_{l,h,d,y}}\right|\right] \tag{8}$$

B. Constraints

The objective functions in (1) must meet a set of BESSs and network equality and inequality constraints (2)–(3).

a) BESSs constraints

In the following, only for the sake of simplicity, we refer to three phases BESSs. For each BESS and day, the energy charged must be equal to the energy discharged; then, the following intertemporal constraints apply:

$$\sum_{h\in\Omega_H}\gamma_{s,h,d,y}P^{ES}_{s,h,d,y}=0,\ s\in\Omega_S,d\in\Omega_D,y\in\Omega_Y \tag{9}$$

where the values of the efficiency $\gamma_{s,h,d,y}$ in charging and in discharging phases may be different. Moreover in (9): (i) the active power $P^{ES}_{s,h,d,y}$ is positive or negative, according to the discharge or charge steps; (ii) each BESS can be charged or discharged once per day, due to constraints on the expected life of the batteries; (iii) the hours of charging and discharging of the BESS depend on the structure of the electricity prices (tariffs).

The value of $P^{ES}_{s,h,d,y}$ for each battery cannot exceed admissible ranges. During the charging and discharging hours, the following constraints apply:

$$\begin{aligned} &-P^{size}_s\leq P^{ES}_{s,h,d,y}\leq 0,\ s\in\Omega_S,h\in\Omega_{ch,d,y},d\in\Omega_D,y\in\Omega_Y \\ &0\leq P^{ES}_{s,h,d,y}\leq P^{size}_s,\ s\in\Omega_S,h\in\Omega_{disch,d,y},d\in\Omega_D,y\in\Omega_Y \end{aligned} \tag{10}$$

The size of the BESS AC/DC interfacing converter imposes constraints on the active and reactive powers that the BESS can absorb/

Table 2
Decision matrix.

Alternative	Scenario 1 Probability π_1			… …	Scenario f Probability π_f			… …	Scenario N_f Probability π_{N_f}		
1	$f_{obj\,1(1,1)}$	$f_{obj\,2(1,1)}$	$f_{obj\,3(1,1)}$	…	$f_{obj\,1(1,f)}$	$f_{obj\,2(1,f)}$	$f_{obj\,3(1,f)}$	…	$f_{obj\,1(1,N_f)}$	$f_{obj\,2(1,N_f)}$	$f_{obj\,3(1,N_f)}$
2	$f_{obj\,1(2,1)}$	$f_{obj\,2(2,1)}$	$f_{obj\,3(2,1)}$	…	$f_{obj\,1(2,f)}$	$f_{obj\,2(2,f)}$	$f_{obj\,3(2,f)}$	…	$f_{obj\,1(2,N_f)}$	$f_{obj\,2(2,N_f)}$	$f_{obj\,3(2,N_f)}$
⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮	⋮
a	$f_{obj\,1(a,1)}$	$f_{obj\,2(a,1)}$	$f_{obj\,3(a,1)}$		$f_{obj\,1(a,f)}$	$f_{obj\,2(a,f)}$	$f_{obj\,3(a,f)}$		$f_{obj\,1(a,N_f)}$	$f_{obj\,2(a,N_f)}$	$f_{obj\,3(a,N_f)}$
	⋮	⋮	⋮		⋮	⋮	⋮		⋮	⋮	⋮
N_{af}	$f_{obj\,1(N_{af},1)}$	$f_{obj\,2(N_{af},1)}$	$f_{obj\,3(N_{af},1)}$	…	$f_{obj\,1(N_{af},f)}$	$f_{obj\,2(N_{af},f)}$	$f_{obj\,3(N_{af},f)}$	…	$f_{obj\,1(N_{af},N_f)}$	$f_{obj\,2(N_{af},N_f)}$	$f_{obj\,3(N_{af},N_f)}$

inject. For each BESS and hour of operation, it results:

$$\sqrt{\left(P^{ES}_{s,h,d,y}\right)^2 + \left(Q^{ES}_{s,h,d,y}\right)^2} \le S^{ES}_s, s \in \Omega_S, h \in \Omega_H, d \in \Omega_D, y \in \Omega_Y \tag{11}$$

Furthermore, for each BESS, the nominal discharging time (i.e., the ratio between the energy size E^{size}_s and the power size P^{size}_s) must be constrained into a range defined by the specific technology of the storage device.

b) Network constraints

The following three-phase load flow equations [25] apply at each three-phase bus:

$$P^p_{b,h,d,y} = V^p_{b,h,d,y} \sum_{k\in\Omega_B}\sum_{m=1}^{3} V^m_{k,h,d,y}\left[G^{pm}_{bk} cos\left(\vartheta^p_{b,h,d,y} - \vartheta^m_{k,h,d,y}\right) + B^{pm}_{bk} sin\left(\vartheta^p_{b,h,d,y} - \vartheta^m_{k,h,d,y}\right)\right]$$

$$Q^p_{b,h,d,y} = V^p_{b,h,d,y} \sum_{k\in\Omega_B}\sum_{m=1}^{3} V^m_{k,h,d,y}\left[G^{pm}_{bk} sin\left(\vartheta^p_{b,h,d,y} - \vartheta^m_{k,h,d,y}\right) - B^{pm}_{bk} cos\left(\vartheta^p_{b,h,d,y} - \vartheta^m_{k,h,d,y}\right)\right]$$

$$b \in \Omega_B, h \in \Omega_H,\ d \in \Omega_D,\ y \in \Omega_Y,\ p = 1,2,3 \tag{12}$$

Extending Equations from (12) to include single-phase and two-phase nodes is trivial.

With reference to the slack bus, it is the bus of interconnection to the upstream network (i.e., bus #1) where the magnitude and the argument of phase voltages are specified as:

$$V^p_{1,h,d,y} = V^{slack};\ \vartheta^p_{1,h,d,y} = \frac{2}{3}\pi(1-p);\ h \in \Omega_H,\ d \in \Omega_D,\ y \in \Omega_Y,\ p = 1,2,3 \tag{13}$$

Moreover, at the interconnection bus #1, the apparent power flowing through the interfacing transformer is constrained by its rating S_{trans}:

$$\sqrt{\left(\sum_{p=1}^{3} P^p_{1,h,d,y}\right)^2 + \left(\sum_{p=1}^{3} Q^p_{1,h,d,y}\right)^2} \le S_{trans}, h \in \Omega_H, d \in \Omega_D, y \in \Omega_Y \tag{14}$$

Meeting PQ requirements at all buses leads to the following constraints:

$$V_{min} \le V^p_{b,h,d,y} \le V_{max}, b \in \Omega_B, h \in \Omega_H, d \in \Omega_D, y \in \Omega_Y, p = 1,2,3 \tag{15}$$

$$kd_{b,h,d,y} \le kd_{max}, b \in \Omega_B, h \in \Omega_H, d \in \Omega_D, y \in \Omega_Y \tag{16}$$

where the unbalance factors are expressed in function of the phase voltages.

Eventually, the lines phase currents of the system have a specified ampacity that cannot be exceeded:

$$I^p_{l,h,d,y} \le I^{max}_l, l \in \Omega_L, h \in \Omega_H, d \in \Omega_D, y \in \Omega_Y, p = 1,2,3 \tag{17}$$

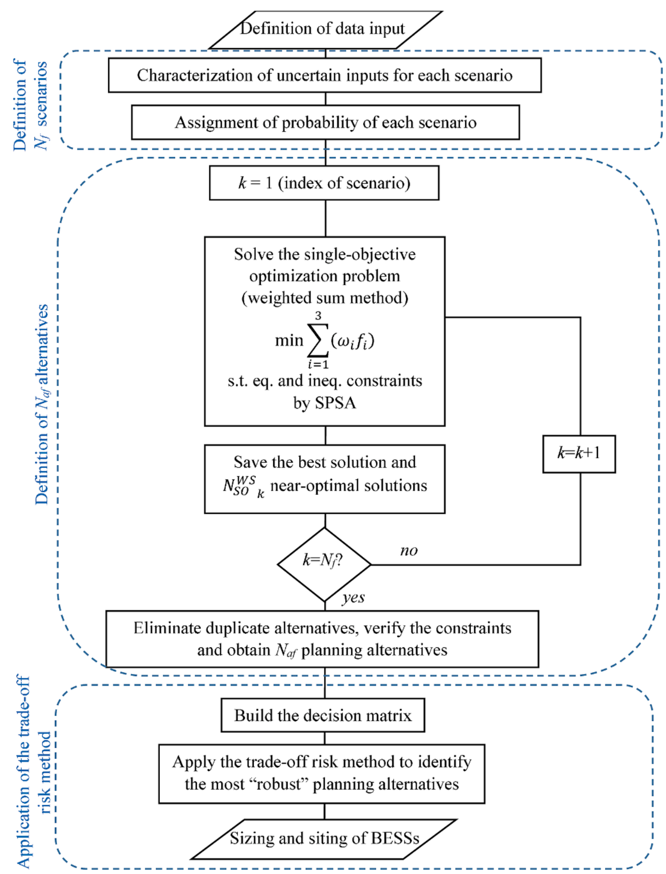


Fig. 1. Schematic representation of the procedure.

We note that considering all the days of all the years in the planning period may push the computational effort beyond reasonable time scales; therefore, it is rational to consider a reduced set of typical days for each year based on seasonal characteristics, holidays, weekdays (Monday to Friday), weekends (Saturday and Sunday), and so on. More details of the model are reported in [26].

3. Optimization problem solution under uncertain conditions

The optimization problem (1)-(3) is a MO optimization problem under uncertainties mainly due to the random nature of loads and renewable energies.

Several methods (fuzzy, probabilistic, robust optimization, etc.) are available in the relevant literature to solve such problems, each one with pros and cons; they differ on how the input random variables uncertainties are described. In the fuzzy-based approach, fuzzy variables and fuzzy memberships are introduced to solve the problem. In the probabilistic optimization methods, the probability density functions are used to represent the input randomness; Monte Carlo simulation approaches, Point estimate methods, scenario-based approaches are usually applied in this context. In the robust optimization the uncertainties are modeled with uncertainty set and the solution is based on the worst-case realization. In interval optimization, a range of values is assigned for each uncertain variable.

In this paper, we propose to solve the MO optimization problem under uncertainties formulated in Section 2 by applying a new approach based on the trade-off/risk analysis [19–22]. In the trade-off/risk analysis framework, at first the DM selects a number N_a of planning alternatives, each one characterized by a vector of decision variables (i.e., siting and sizing of BESSs), and a number N_f of scenarios, each one characterized by the values of the uncertain problem input data (for example, loads and distributed generation power levels) with their probability of occurrence. Once alternatives and scenarios are selected, for each alternative and for each scenario, the values of the objective functions (1) are calculated considering the equality and inequality constraints (2) and (3), and the unfeasible planning alternatives in at least one scenario are discarded, leading to N_{af} feasible planning alternatives. This procedure allows the DM to build the elements (objective functions values) of the so-called Decision Matrix, as the one reported in Table 2, in which the rows correspond to the N_{af} feasible planning alternatives and the columns to the scenarios with their probabilities. The elements of the matrix are the objective functions values. Finally, the trade-off/risk analysis is performed on the decision matrix elements to find a sub-set of one or more feasible and robust planning alternatives to be practically operated.

In Fig. 1 a schematic representation of the procedure applied to determine the best BESSs' sizing and siting under uncertainty is shown and, in the following subsections, the steps of the proposed approach are outlined in detail.

3.1. Definition of scenarios with their probabilities

Both the objective functions and the constraints of the MO optimization model formulated in Section 2 depend on several uncertain input data [27,28]. Uncertainties linked to load and renewable generation powers have the most significant influence and they are considered in the most relevant papers appeared in the relevant literature [1,6–17].

In the trade-off/risk analysis, the uncertain input data of the MO optimization problem are represented by a set of scenarios (i.e., selected set of values of considered uncertain input data, i.e. loads and renewable generation power levels), each one characterized by a probability of occurrence. Then, scenarios and probabilities have to be assigned.

The scenario probabilities can be assigned based on three different approaches: (i) the first approach is based completely on the observed information; (ii) the second approach is based completely on the subjective judgment of the DM; and (iii) the third approach is a mix of the above two approaches. More details on the three approaches can be found in [29].

It is worth noting that, even if to assign probabilities with little or no observed information might appear risky, it is well known that, surprisingly, good problem solutions can be obtained when the DM uses only his own understanding of the nature of the uncertainties to assign probabilities [29–33].

In this paper, we used the second approach (subjective judgment of the DM) for assigning both scenarios and probabilities. Anyway, in this paper also a new procedure will be proposed in Section 3.C to assist the DM when difficulties arise in the setting of scenario probabilities, as it can happen in practical situations.

We outline also that more sophisticated techniques to select scenarios and probability values exist; a survey is given in [34,35]. These scenario-based techniques could be easily applied to the proposed probabilistic approach without any difficulty of implementation.

3.2. Definition of alternatives

The DM will make the final decision choosing among a set of starting planning alternatives, that must be accurately pre-selected. When the problem of the BESS siting and sizing is dealt with, this is a crucial and critical step of the trade-off/risk analysis-based procedure, as the planning alternatives can be difficult and cumbersome to be pre-assigned by the DM; this is mainly due to due to the lack of experience that has been acquired to date in the installation of storage systems. In addition, it would be desirable that the selected planning alternatives have some characters of optimality to avoid that either most of them or even all of them are discarded since they do not satisfy the optimization problem constraints in all scenarios. In this paper, to individuate an adequate set of alternatives to be pre-assigned, the following procedure is applied:

i) in each scenario, the MO constrained optimization problem (1)-(3) is solved transforming it into a single-objective (SO) constrained optimization problem by applying the "weighted sum method" for an assigned set of weights [36];
ii) the optimal solution (BESS's siting and sizing) provided by the "weighted sum method" plus $N_{SO}^{WS}k$ (for each scenario k) near-optimal solutions are saved;
iii) steps (i) and (ii) are executed for all N_f scenarios and, after removing the duplicates, the obtained alternatives constitute the searched pre-selected set of planning alternatives to be used as rows of the Decision matrix.

This step-procedure has proved to be very useful in identifying a set of planning alternatives whose number and optimality characteristics are particularly effective in the subsequent application of the trade-off/risk analysis, as it is shown also in the numerical applications. On the other hand, each selected planning alternative either belongs to the Pareto surface constituted by the MO solution points for each scenario or is a point quite close to it.

It is worth noting that the planning alternatives could have been also chosen as a number of solutions of the MO optimization problem belonging to the Pareto front determined by solving the problem with the weighted sum approach. The choice of the approach of step ii) was imposed by the need of a reduced computational burden. Indeed, each optimization problem (even if with a single-objective) is computationally demanding due to many factors (e.g., the three-phase representation of the network, the intertemporal constraints of BESS and so on).

Some details of the abovementioned procedure are provided below for the steps (i)-(iii).

We recall that among the solution methods of MO problems [36], the weighted sum method has limited CPU time requirements; in addition, this method is also characterized by the least programming complexity.

The weighted sum method consists in solving the following SO optimization problem[1]:

$$min \sum_{i=1}^{3} \omega_i f_i(\boldsymbol{X}, \boldsymbol{C}) \tag{18}$$

subject to constraints (2), (3).

In (18) $f_i(\boldsymbol{X}, \boldsymbol{C})$ is the i-th objective function and ω_i is the corresponding weight. The weights are assumed to be positive values; this assumption is a sufficient condition that the solution of the SO optimization problem satisfies the Pareto optimality conditions [36].

The SO optimization problem with the objective function (18) and the constraints (2), (3) is solved by means of the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm.

The SPSA method, firstly proposed by J. C. Spall [37], is based on the

[1] For the sake of clarity, (18) does not contain the indication of the scenario; of course, for each scenario, a SO optimization problem has to be solved.

simultaneous stochastic perturbation of all unknown variables and on the differentiation approximation. Although SPSA is a stochastic optimization solution method, simultaneous population-free perturbations make the computation very effective. This method makes available, in addition to the optimal solution, also a certain number of near-optimal solutions.

Here, for sake of brevity, the algorithm implemented for solving the constrained optimization problem is not explained but the reader is referred to [26,37] to deepen the procedure. In [26], it is also shown how the complexity effort introduced by the constraints of an inter-temporal nature is overcome by applying an inner simplified but effective algorithm that allows to set the BESSs optimal daily charging/discharging powers.

Once solved the SO optimization problems for all the scenarios and removed the duplicates among the scenarios, a set of planning alternatives N_{sol} is available. Since the MO optimization problem is constrained by (2) and (3), only the alternatives that in all the scenarios satisfy the inequality constraints are considered discarding the unsatisfying alternatives. Therefore, at the end of this step, $N_{af} \le N_{sol}$ alternatives are available.

3.3. Application of the trade-off risk method

Once the N_f scenarios are defined (as in sub-section 3.A) and the N_{af} planning alternatives are individuated (as in sub-section 3.B), the trade-off risk analysis can be applied. Let us assume initially that, for each scenario f ($f = 1,\cdots,N_f$), a probability of occurrence π_f is assigned by the DM.

For the planning alternative a ($a = 1,\cdots,N_{af}$), and assuming as input data the ones corresponding to the scenario f ($f = 1,\cdots,N_f$), the values of the objective functions (5), (6) and (7) are calculated considering the equality constraints (9), (12), (13) and verifying the inequality constraints (10), (11), (14), (15), (16) and (17). Once all the alternatives and all the scenarios are considered, the Decision matrix elements are obtained. We outline that no optimization problem is solved in setting the elements of the decision matrix which instead involves very low computational efforts.

When dealing with MO problems, the DM is interested in selecting the alternatives that are good compromises (i.e., trade-offs) among the objectives to be minimized [38]. These alternatives are also referred to as non-dominated alternatives and are Pareto optimal according to the well-known definition [36]. This applies to problems under certainty; instead, when the MO optimization problem is under uncertainties, the individuation of the compromise solutions has to consider more aspects.

For each scenario $f = 1,\cdots,N_f$, we first find a set of alternatives that realize the best compromise among the objective functions, that is the set of non-dominated alternatives. This set is denominated conditional decision set Ω_f (conditional on the specified scenario f), which can be derived by selecting, among the N_{af} alternatives, the non-dominated alternatives. The union of the N_f conditional decision sets will provide the global decision set:

$$\Omega = \bigcup_{f=1}^{N_f} \Omega_f \tag{19}$$

The concept of the conditional significant dominance can also be applied to select the trade-off solutions [20]. The concept of significant dominance is determined by the "much worse" index (Δ_{mw}) and the "significantly better" index (Δ_{sb}) and can be stated as follows: "The alternative i significantly dominates the alternative j if at least one objective function of j is much worse than the corresponding one of i and if no objective function of j is significantly better than the corresponding objective functions of i" [20].

For each scenario $f = 1,\cdots,N_f$, for assigned values of Δ_{mw} and Δ_{sb}, the set Ω_f' of the alternatives that are non-dominated in a significant way is determined and the union of the N_f conditional decision sets will provide the global significant decision set:

$$\Omega' = \bigcup_{f=1}^{N_f} \Omega_f' \tag{20}$$

The decision sets Ω and Ω', thus, contain all the alternatives that, in at least one scenario, are non-dominated or non-significantly dominated, respectively.

It is worth noting that the proposed procedure to individuate the alternatives (see Section 3.B), for each scenario, is certainly able to provide some non-dominated solutions because the weighted sum method can determine points belonging to the Pareto fronts.

The alternatives of the sets Ω and Ω' can be ordered based on their robustness [19–22]. In particular, the robustness of the alternative a is the sum of the probabilities of the scenarios that exhibit this alternative in the corresponding conditional decision set, that is:

$$r_a = \sum_f \pi_f, \ \forall f : a \in \Omega_f \tag{21}$$

The DM, handling the values of robustness, could choose the preferred planning alternative as the one that has the highest value of the robustness index:

$$A_{opt} = \underset{a \in \Omega}{argmax} r_a \tag{22}$$

Relationships (21) and (22) can be applied to the case of significant dominance, by considering the sets Ω_f' $(f = 1,\cdots,N_f)$ and Ω' instead of Ω_f and Ω.

It is worth noting that the output of the proposed procedure can be a single or a (limited) set of preferred planning alternatives; the latter occurs when there is more than one alternative with the same value of robustness or when the DM prefers to handle a reduced set of preferred alternatives instead of only one alternative.

It is important to also note that, based on the definition (21), the probabilities of occurrence of the scenarios affect the value of the robustness index. However, it could be interesting to assess the solutions suggested by the proposed method also when the scenario probabilities are unknown and, to this aim, a new procedure is proposed to individuate the problem solutions.

In this new framework, the application of the trade-off/risk method is repeated for a number of sets N_{set} of scenario probabilities that are randomly derived and, for each set, the value of robustness of each alternative belonging to Ω (or Ω') is evaluated. A statistical analysis of the values of robustness is carried out. Then, based on the statistical measures, that can be percentiles, the DM can select the best alternatives, as it is shown in the numerical applications.

Each set of random value of scenario probabilities has to meet the condition that their sum is unitary, and the sets have to be uniformly distributed. These characteristics comply with the multivariate Dirichlet distribution and, therefore, this distribution with unitary coefficients was chosen to generate the random values of scenario probabilities.

4. Numerical applications

The MO approach for optimal allocation of BESSs is solved for the IEEE unbalanced 34-bus test system (without the series voltage regulators). In the test system, both single- and three-phase lines are present as well as single- and three-phase loads. In particular, the system lines 808–810, 816–818, 818–820, 820–822, 824–826, 854–856, 858–864, and 862–838 are single-phase while the remaining ones are three-phase lines. [39,40] report all the network data. Single-phase and three-phase buses are considered as BESSs' candidate buses.

The following assumptions are taken into consideration:

Table 3
Scenarios.

Scenario	Loading level (%)	Photovoltaic plants(rating and siting)
1 (reference case)	70	–
2	60	–
3	50	–
4	70	150 kWp @ # 844
5	60	150 kWp @ # 844
6	50	150 kWp @ # 844
7	70	150 kWp @ # 844
		150 kWp @ # 860
8	60	150 kWp @ # 844
		150 kWp @ # 860
9	50	150 kWp @ # 844
		150 kWp @ # 860
10	70	150 kWp @ # 832
		150 kWp @ # 844
		150 kWp @ # 860
11	60	150 kWp @ # 832
		150 kWp @ # 844
		150 kWp @ # 860
12	50	150 kWp @ # 832
		150 kWp @ # 844
		150 kWp @ # 860

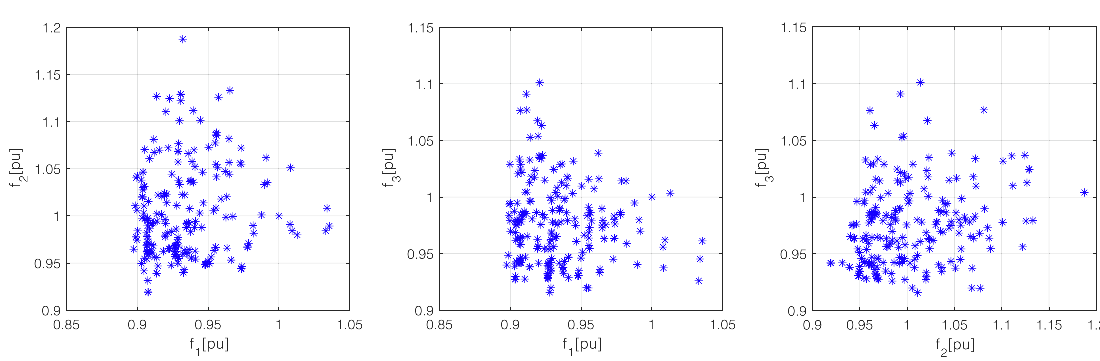
- for the calculation of costs of electrical energy brought from the utility (see eq. (5)), the Time-of-Use pricing is assumed; the considered tariffs are reported in [26];
- three kind of days (i.e., working day, Saturday, Holiday) and four seasons are considered [26,41]. Experimental daily variations of loads in each of the 12 typical days are applied. As a result, for each year, 12 active power daily load curves and 12 reactive power daily load curves, with hourly values, have been assumed; also for PV generation, typical daily production curves have been assumed mainly based on the season. In each typical day, the unbalances of the data of the IEEE test system are used [40];
- the load is assumed to increase over the planning period with an annual rate of 2 % and both the effective rate of change and the discount rate are assumed equal to 3 %;
- the maximum line currents are fixed at the ratings as reported in [39,40] and the maximum value of the unbalance factor at each bus is set at 3 %. The minimum and the maximum values of the voltage at each phase of each bus are set at 90 % and 110 % of the rated value;
- Na-NiCl$_2$ batteries are taken into consideration due to their modularity and to economic considerations [42]: the unit storage system available at any phase of each bus is assumed to come in discrete sizes of 50 kVA; the standard value of the power/energy ratio is assumed 1/3 [43]; the charging/discharging efficiencies are set at 0.9 [43]; the installation cost at year 1 is assumed to be 400 $/kWh [42]; the expected evolution of battery costs in the next years is also provided in [42];
- the BESSs are used only in the summer months: during the off-peak hours they can charge, and during the rest of the day the battery can discharge. In fact, due to the higher electrical energy tariffs in the summer months, the strategy that resulted the most convenient one is to charge/discharge the battery considering the active power only for the days falling in the summer months [26]. For the days of the other seasons, the BESS is operated only to obtain the optimal value of reactive power according to constraints (11).

The first step for the application of the proposed procedure for an optimal BESS's allocation is to define the scenarios. The uncertainties considered refer to the levels of total load and of the installed distributed generation. Three levels of peak powers are considered and set, respectively, to 70 %, 60 % and 50 % of the nominal power of the total load, i.e. 1769 kW and 1051 kVar, respectively, as reported in [39,40]. With respect to the presence of distributed generators, photovoltaic (PV) plants are considered, and several configurations are assumed; specifically, the case without any PV is considered along with three different allocations of PVs (respectively, with one, two and three PV plants). The combination of all the load and PV levels leads to 12 scenarios that are listed in Table 3. A limited number of scenarios was selected in order to make it easier to understand the application of the various procedure steps.

For each scenario, the MO constrained optimization problem (1)-(3) is solved by applying the weighted sum method by means of the SPSA procedure (as explained in Section 3). The weights are assigned applying a surrogate weight method (in particular, the equal weights and the rank-order centroid method) [44].

For each scenario, the optimal solution in terms of BESSs allocation and the successive suboptimal solutions provided by SPSA are saved. Then, the eventual duplicates are removed, and the constraints are verified for all scenarios; the remaining feasible configurations constitute the set of N_{af} planning alternatives. In this case study, we obtained $N_{af} = 241$ feasible planning alternatives.

Eventually, the decision matrix is determined by evaluating the objective functions f_1, f_2, and f_3 for each planning alternative in each scenario; it results a matrix with 241 rows and 12 columns. Since it is impossible to report such a matrix, as an example, a graphical representation of the couples of the objective functions is provided in Fig. 2 with respect to scenario 1. The values of the objective functions in Fig. 2 are in p.u. of the corresponding values calculated when any BESS is installed. When the normalized objective function is less than one, it means that the installation of BESSs' improves the considered objective; on the contrary, a normalized value greater than 1 implies a deterioration of the considered objective. All alternatives show a good behaviour with values of the objective functions quite similar. This confirms the



(a) (b) (c)

Fig. 2. Objective functions values for scenario 1 – Objective function f_1 versus f_2 (a); - Objective function f_1 versus f_3 (b); Objective function f_2 versus f_3 (c).

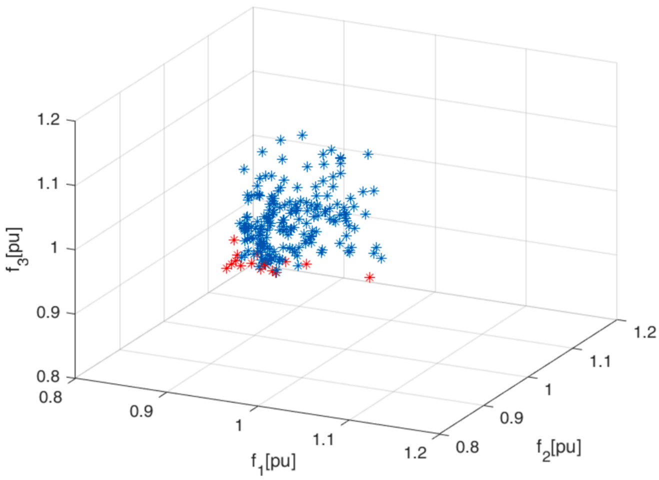


Fig. 3. Values of the objective functions for scenario 1 (* all the 241 planning alternatives; * non-dominated planning alternatives)

strength of the selection process of the alternatives, which produces quite good solutions in each scenario considered.

The analysis of all the objective function values also pointed out that, in scenario 12, all the alternatives exhibit values of the objective functions f_1 and f_2 greater than 1 p.u., indicating that the objectives surpass their respective values in the reference case. This implies that the economic viability and technical advantages derived from BESS installations are diminished when there is a substantial penetration of PV generators and low load levels (characteristic of scenario 12). It is important to note that these outcomes are dependent on the chosen BESS charging strategy, economic assumptions, and the considered tariffs.

Once the decision matrix is built, the trade-off/risk method can be applied. First, the conditional decision sets Ω_f ($f = 1,\ldots,12$) are individuated by applying the concept of the strict dominance. As an example, Fig. 3 shows the non-dominated alternatives (i.e., alternatives belonging to the conditional decision set) and all the alternatives in scenario 1 in a three-dimensional graph. Table 4 reports the cardinality of the conditional decision sets determined for all the scenarios and the global decision set contains 67 alternatives. From Fig. 3 and Table 4, it is evident that, for each scenario, many alternatives are non-dominated. For the alternatives belonging to the global decision set, the robustness (eq. (21)) is calculated, and the alternatives associated with a robustness greater than 0.75 are listed in Table 5 with the value of the robustness and the corresponding configuration of BESSs. Two alternatives (i.e., #98 and #239) are non-dominated in all the considered scenarios and then, are 100 % robust irrespective of the probabilities of scenarios occurrence. The mean values of the objective functions over the 12 scenarios for the alternatives associated with a robustness greater than 0.75 have been calculated and reported in Table 5.

As expected, due to the conflicting nature of the objectives, situations with a contemporaneous improvement and worsening of objectives are frequent.

To provide more insights about the alternatives of Table 5, the values of the objective functions (costs, voltage profile, and security margin) have been reported in Table 6 to compare the objectives functions values before (case no BESS; alternative #121) and after the placement of BESS (case with BESS; alternatives # 98, 239, 95, 23 and 81). For some scenarios (typically the ones characterized by the presence of PVs), the objectives cannot experience an improvement when the BESSs are installed. This is expected due to the reduction of benefits, mainly in terms of voltage profile, that the BESSs can obtain in a system with a significant penetration of PV generators and/or low load levels. The results are also dependent on the adopted strategy to determine the charging/discharging cycle of BESSs [26]. Of course, if further objectives are taken into account, the economic and technical considerations will be different.

The trade-off/risk method was applied also considering the significant dominance. Table 7 reports the conditional significant decision sets for all the scenarios, when equal scenario probabilities are imposed and the “much worse” and the “significantly better” indices are both equal to 10 %. After determining the global decision set, the values of robustness are calculated, and they are reported in Table 8. The planning alternatives exhibiting the highest robustness (i.e., the alternatives #91 and #98) are characterized by the presence of three-phase BESSs’ with a total installed power equal to 600 kW even if with different configurations and different allocation. Also in this case, the mean values of the objective functions over the 12 scenarios are reported in Table 8.

Table 4
Cardinality of the conditional decision sets.

Scenario f	1	2	3	4	5	6	7	8	9	10	11	12
Cardinality of Ω_f	25	22	27	26	23	21	22	20	16	22	18	16

Table 5
Alternatives with a robustness greater than 0.75 (with the concept of dominance) – Equal scenario probability.

Alternative	Robustness	BESS		Mean values of the objectives		
		Type	Sizing and siting	f_1 (M$)	f_2 (pu)	f_3 (pu)
98	1	3-phase	300 kW @ #806	15.17	0.0291	0.0664
		3-phase	300 kW @ #842			
239	1	3-phase	150 kW @ #814	15.32	0.0314	0.0655
		3-phase	150 kW @ #836			
		3-phase	150 kW @ #860			
95	0.917	3-phase	300 kW @ #806	15.37	0.0282	0.0681
		3-phase	150 kW @ #844			
23	0.833	1-phase	50 kW @ #810	15.62	0.0309	0.0656
		3-phase	150 kW @ #836			
		3-phase	150 kW @ #860			
81	0.833	3-phase	150 kW @ #806	15.16	0.0301	0.0664
		1-phase	50 kW @ #810			
		1-phase	50 kW @ #818			
		3-phase	150 kW @ #844			
		3-phase	150 kW @ #860			
121	0.75	–	Without BESS	15.64	0.0274	0.0717

Table 6
Values of the objective functions for alternatives with a robustness greater than 0.75 (with the concept of dominance) − Equal scenario probability.

	Scenario	Alternative #98	Alternative #239	Alternative #95	Alternative #23	Alternative #81	Alternative #121
f_1 (M$)	1	19.63	20.05	20.17	20.36	19.76	21.63
	2	16.54	16.87	16.94	17.17	16.58	18.36
	3	13.99	13.75	13.78	14.04	13.81	15.16
	4	18.50	18.92	19.03	19.23	18.64	19.78
	5	15.50	15.79	15.85	16.09	15.50	16.54
	6	13.01	12.71	12.72	13.00	12.82	13.36
	7	17.50	17.93	18.01	18.24	17.64	17.96
	8	14.51	14.81	14.85	15.11	14.52	14.75
	9	9.11	8.88	8.89	9.10	8.97	8.77
	10	16.52	16.95	17.02	17.26	16.66	16.18
	11	13.55	13.84	13.88	14.14	13.55	12.99
	12	13.68	13.30	13.31	13.66	13.45	12.16
f_2 (pu)	1	0.0251	0.0261	0.0256	0.0260	0.0256	0.0273
	2	0.0260	0.0278	0.0254	0.0274	0.0267	0.0255
	3	0.0290	0.0310	0.0276	0.0306	0.0300	0.0265
	4	0.0261	0.0278	0.0260	0.0274	0.0269	0.0267
	5	0.0277	0.0301	0.0266	0.0294	0.0288	0.0257
	6	0.0306	0.0336	0.0294	0.0331	0.0319	0.0275
	7	0.0272	0.0293	0.0268	0.0287	0.0282	0.0267
	8	0.0292	0.0319	0.0280	0.0312	0.0304	0.0266
	9	0.0348	0.0380	0.0334	0.0376	0.0360	0.0311
	10	0.0285	0.0307	0.0278	0.0300	0.0295	0.0273
	11	0.0309	0.0336	0.0296	0.0330	0.0320	0.0279
	12	0.0338	0.0367	0.0326	0.0363	0.0349	0.0303
f_3 (pu)	1	0.0823	0.0811	0.0843	0.0811	0.0820	0.0874
	2	0.0701	0.0689	0.0717	0.0690	0.0703	0.0739
	3	0.0576	0.0571	0.0595	0.0574	0.0581	0.0605
	4	0.0770	0.0760	0.0794	0.0760	0.0764	0.0834
	5	0.0650	0.0638	0.0667	0.0639	0.0649	0.0699
	6	0.0531	0.0525	0.0545	0.0527	0.0537	0.0566
	7	0.0725	0.0715	0.0748	0.0716	0.0717	0.0796
	8	0.0608	0.0597	0.0622	0.0597	0.0607	0.0661
	9	0.0673	0.0673	0.0684	0.0672	0.0681	0.0723
	10	0.0684	0.0673	0.0705	0.0674	0.0675	0.0760
	11	0.0571	0.0558	0.0580	0.0559	0.0569	0.0627
	12	0.0662	0.0654	0.0670	0.0658	0.0667	0.0721

Table 7
Conditional significant decision sets and global significant decision set (Δ_{mw} = 10 %; Δ_{sb} = 10 %).

	Alternatives of the conditional decision sets
Ω'_1	{43, 91, 93, 98}
Ω'_2	{43, 91, 93, 98}
Ω'_3	{90, 91, 92, 93, 95, 98, 100}
Ω'_4	{43, 91, 93, 98}
Ω'_5	{90, 91, 92, 93, 95, 98, 100,121}
Ω'_6	{89,90, 91, 92, 95, 98, 121}
Ω'_7	{91, 93, 98}
Ω'_8	{90, 92, 95, 100, 121}
Ω'_9	{90, 92, 95, 121}
Ω'_{10}	{43, 91, 93, 94, 98, 121}
Ω'_{11}	{90, 92, 95, 121}
Ω'_{12}	{71, 76, 78, 90, 92, 95, 121}
Ω'	{43, 71, 76, 78, 89, 90, 91, 92, 93, 94, 95, 98, 100, 121}

In conclusion, it is worth highlighting i) the alternative #121 in certain conditional decision sets presented in Table 7 particularly for scenarios characterized by significant PV plant penetration and/or lower load levels, and ii) the values of the variations of the objectives with respect to the case without BESS reported in Tables 5, 6 and 8. As previously mentioned, the extensive presence of PV installations reduces the feasibility of BESS installations when considering the costs and the benefits in terms of voltage profiles and security margins. This finding reinforces the complexity of optimal BESS planning in an active unbalanced distribution system, considering uncertainties and encompassing both economic and technical considerations. Such a planning problem requires a robust solution approach to ensure effective outcomes.

Finally, to consider the case of not known scenario probabilities, the trade-off/risk method is applied (see Section 3.C) for a number of sets (10000) of scenario probabilities according to a Dirichlet distribution with unitary factors. For each set, the value of robustness of each alternative belonging to Ω' reported in Table 7 is evaluated and a statistical analysis of these values is carried out. Table 9 reports the statistical parameters (mean values and some percentiles) of the robustness index for each alternative belonging to the global significant decision set Ω'. Finally, Fig. 4 reports, for each alternative, the frequency of values of robustness index greater than 0.90. It is evident, from the statistical analysis, that the alternatives #91 and #98 are the ones the DM should consider for the choice of the planning alternative to be operated.

5. Conclusions

A non-linear, constrained MO optimization model has been formulated to select the allocation of single- and three-phase BESSs in an unbalanced distribution system, under uncertainties. It has been solved by a trade-off/risk analysis-based approach.

New procedures have been proposed to help the planning engineer in some critical steps of the trade-off/risk analysis application, i.e., the selection of planning alternatives and the setting of probabilities of the input data.

The main outcomes of the paper are that:

Table 8
Alternatives with a robustness greater than 0.5 (with the concept of significant dominance) – Equal scenario probability.

Alternative	Robustness	BESS		Mean values of the objectives		
		Type	Sizing and siting	f_1 (M$)	f_2 (pu)	f_3 (pu)
91	0.67	3-phase	300 kW @ #806	15.17	0.0291	0.0665
		3-phase	150 kW @ #836			
		3-phase	150 kW @ #860			
98	0.67	3-phase	300 kW @ #806	15.17	0.0291	0.0664
		3-phase	300 kW @ #842			
90	0.58	3-phase	300 kW @ #806	15.37	0.0282	0.0681
		3-phase	150 kW @ #860			
92	0.58	3-phase	300 kW @ #806	15.37	0.0282	0.0681
		3-phase	150 kW @ #848			
93	0.58	3-phase	300 kW @ #806	15.17	0.0291	0.0665
		3-phase	300 kW @ #848			
95	0.58	3-phase	300 kW @ #806	15.37	0.0282	0.0681
		3-phase	150 kW @ #844			
121	0.58	–	Without BESS	15.64	0.0274	0.0717

Table 9
Statistical parameters of the robustness index of alternatives of the global significant decision set.

Alternative	Statistical measures of robustness index			
	Mean value	25th percentile	95th percentile	99th percentile
43	0.334	0.238	0.561	0.654
71	0.084	0.027	0.242	0.342
76	0.084	0.027	0.242	0.342
78	0.084	0.027	0.242	0.342
89	0.082	0.025	0.234	0.328
90	0.583	0.489	0.800	0.862
91	0.666	0.580	0.863	0.915
92	0.583	0.489	0.800	0.862
93	0.584	0.491	0.798	0.862
94	0.084	0.027	0.246	0.345
95	0.583	0.489	0.800	0.862
98	0.666	0.580	0.863	0.915
100	0.251	0.161	0.470	0.576
121	0.583	0.490	0.800	0.865

- The trade-off/risk analysis appears to be a powerful tool to support the DM in the choice of a robust BESSs location in an unbalanced distribution system, as it has been for a lot of other important planning problems under uncertainties in power systems. It allows a good compromise among multiple objectives, considering both economic and technical risks.
- The proposed procedure of selection of the planning alternatives allows the planning engineer to deal with a wide range of sizing and siting options with obvious advantages in identifying the final solution.
- The proposed procedure of setting the scenario probabilities allows the DM to be relieved from attributing probabilities, when he/she has a not good understanding of the nature of the uncertainties.
- The robustness indices values provide the DM of extensive information in identifying the most robust BESSs' location, particularly useful in presence of high penetration levels of renewable generation powers.
- In the case study, the best alternatives of sizing and siting of BESS are individuated; they range from 450 kW to 600 kW globally installed and they are characterized by a reduced set of nodes that are suggested siting of BESS.

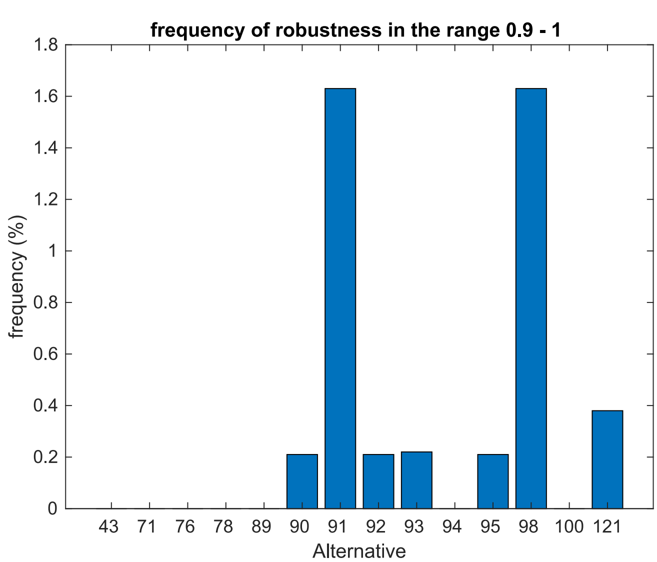


Fig. 4. Frequency of the robustness index falling in the range 0.9–1, for each alternative of the global significant decision set.

Future research will be devoted to the analysis of new scenario-based probabilistic approaches and the case of many objective functions will be considered. Future work will also consider the problem of the best allocation of electric vehicle charging station with a techno-economic analysis.

CRediT authorship contribution statement

Guido Carpinelli: Writing – review & editing, Writing – original draft, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Christian Noce:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Angela Russo:** Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Pietro Varilone:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Paola Verde:** Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data used in the research are either indicated in the paper or available in the included references.

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