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(Article begins on next page)

Evaluation of Near Singular Integrals for Computational Electromagnetics by Dimensionality Reduction

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***Abstract*—With the need for ever faster codes, a limiting factor that must be dealt with is the accurate yet efficient evaluation of interaction integrals between the more problematic near-field elements. Several recent works have together shown that all evaluations of source potential integrals and their derivatives for the most common bases and elements can be reduced to the evaluation of boundary line integrals; these can be evaluated by Gauss-Legendre quadrature, though integrand-smoothing transforms are often needed to accelerate their computation. In this paper, we modify the reported approach to eliminate cancellation errors in the line integral integrand, reinterpret the integral as a *vertex function*, and study the scalar potential integral form under the sinh transform and static subtraction acceleration methods.**

***Index Terms*—integral equations, moment methods, numerical analysis, singular integrals.**

I. INTRODUCTION

There is a strong need in Computational Electromagnetics (CEM) for accurately computing potential integrals and their derivatives for constant and linear source densities on triangular and tetrahedral domains, as well on other planar polygonal and polyhedral elements. In recent years, however, especially with the development of fast algorithms for moment methods, the problem has received renewed attention since interactions between elements in the near field cannot be easily aggregated, and are more difficult to accurately compute than for well-separated elements. In [1], a number of key results [2], [3] from the literature are gathered into a unified framework for dealing with the numerical evaluation of potential integrals and their derivatives, reducing them in each case to the evaluation of non-singular boundary *edge integrals*. The approach is applied to sources on triangular and tetrahedral elements for standard RWG and SWG bases, respectively, and the effectiveness of the so-called sinh [4] and double-sinh (sinh-sinh) [5] transforms in smoothing the integrals is demonstrated, permitting their accurate evaluation over a wide range of element shapes using only Gauss-Legendre (GL) quadrature. In this paper, we examine several important extensions to the approach of [1].

II. FORMULATION EXTENSIONS

A. Dimensionality Reduction (DR) details

We show that all the steps for the dimensionality reduction (DR) of surface or volumetric integrals to line integrals can be succinctly expressed as straightforward applications of appropriate Gauss theorems involving the grad, div, or curl operators in either surface or volumetric form. Similar approaches for static potentials have led to closed form results [6], but in the dynamic case, edge integrals remain that must be handled numerically. Several important caveats in deriving the appropriate forms for potential integrals and their derivatives for (projected) observation points both interior and exterior to source domains are identified.

B. Evaluation of Radial integrals

For the 3-D free space Green's function and linear bases, all intermediate integrals at each reduction step require evaluation of a radial or polar integral; these can always be performed in closed form (even for more general polynomial bases) and expressed in terms of elementary functions. The final 1-D line integrals themselves can be evaluated in closed form only for the static case.

C. Vertex potential interpretation of edge integrals

The simple partition of the resulting line integrals into two integrals,

$$\int_{\ell^-}^{\ell^+} d\ell = \int_0^{\ell^+} d\ell - \int_0^{\ell^-} d\ell,$$

expresses it as a difference between two functions of the form $I(u,\ell,d)$ where u, ℓ, and d are rectangular coordinates whose origin is at the projected observation point $\mathbf{r}_0$ in a (triangular) face or in a source triangle; u and ℓ locate a vertex position relative to $\mathbf{r}_0$ measured perpendicular and parallel to the associated edge, respectively. The distance from a triangular face or source to the observation point is d. As in the static case [7], the *vertex function* $I(u,\ell,d)$ is associated with a single vertex and the (extended) edge containing it; vertex functions are very useful for studying the (mis-)behavior of potential quantities near an isolated edge or vertex. The vertex function for scalar potentials and RWG bases on planar triangles, for example, is

$$I_{\Phi}^{\mathrm{RWG}}(u,\ell,d)=\int_0^{\ell}\frac{u}{P^2}\left[\frac{e^{-jkR}}{-jk}\right]_{R=|d|}^{R}d\ell$$
$$=\int_0^{\ell}\frac{u}{P^2}\left[Re^{-jkR/2}\mathrm{sinc}(kR/2)\right]_{R=|d|}^{R}d\ell \tag{1}$$

where $P^2=u^2+\ell^2$, $R^2=P^2+d^2$. GL quadrature may sometimes be applied directly to evaluate to (1) without acceleration, but as argued in [1], the sinh (S) [4] and sinh-sinh (S^2) [5] transforms are often effective in accelerating the convergence of the GL schemes used in evaluating these integrals.

D. Vertex functions in sinc function format, static limits

In [1], only the exponential term involving the upper limit R would be retained in the vertex function integrand from the bracketed expression in (1) above, since at the lower limit the term is constant, and the factor u/P^2 can be integrated in closed form. But that form does not properly handle the (quasi-)static limit, $k\to 0$, whereas the sinc function form of the second equality of (1) circumvents all such difficulties. As $k\to 0$, integral (1) approaches [6], [7]

$$I_{\Phi}^{\mathrm{RWG}}(u,\ell,d)\xrightarrow{k\to 0}u\int_0^{\ell}\frac{1}{R+|d|}\,d\ell$$
$$=u\sinh\left(\frac{\ell}{\sqrt{u^2+d^2}}\right)-|d|\tan^{-1}\left(\frac{u\ell}{u^2+d^2+|d|R}\right) \tag{2}$$

a result suggesting a simple acceleration scheme similar to singularity subtraction: *static subtraction* (SS).

III. Numerical Results

In Fig. 1, and for a wide range of the parameter u, we plot the number of significant digits achieved in computing I_{Φ}^{RWG}, i.e. $-10\log_{10}|(I_{\Phi}^{\mathrm{RWG}}-I^{\mathrm{REF}})/I^{\mathrm{REF}}|$, where I^{REF} is a reference value. I^{REF} is obtained using the GL scheme with a very high number of points and independently verified using an iterative subdivision method. A challenging case from [1], $d=0.01$[m], $\ell=1.0$[m], and $k=0.3$[m^{-1}], is chosen and GL sampling schemes of 1, 3, 5, and 7 points are used. The results for two accelerating methods, sinh (S) applied alone, and sinh applied together with the static subtraction scheme (SSS), are shown. Very usable results are achieved with but a few sample points; convergence to full machine precision is exponential.

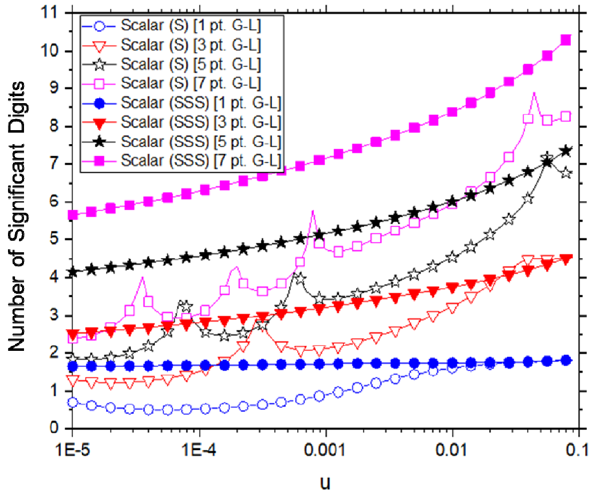

Fig. 1. Number of significant digits of the numerically evaluated scalar potential vertex function using the sinh transform acceleration (S) alone, and both sinh and static subtraction accelerations (SSS) for 1, 3, 5 and 7 point GL quadrature.

IV. Conclusions

We present a number of extensions to the Dimensionality Reduction (DR) framework presented in [1]. We also show numerical results using the sinc function representation with the sinh transform and static subtraction approaches to accelerate the integral computations. It is found that convergence is exponential, implying the combination yields high accuracy with but a few sample points.

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