N=8 Supergravity with Local Scaling Symmetry

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$N = 8$ supergravity with local scaling symmetry

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Abstract: We construct maximal supergravity in four dimensions with local scaling symmetry as deformation of the original Cremmer-Julia theory. The different theories which include the standard gaugings are parametrized by an embedding tensor carrying $56 + 912$ parameters. We determine the form of the possible gauge groups and work out the complete set of field equations. As a result we obtain the most general couplings compatible with $N = 8$ supersymmetry in four dimensions. A particular feature of these theories is the absence of an action and an additional positive contribution to the effective cosmological constant. Moreover, these gaugings are generically dyonic, i.e. involve simultaneously electric and magnetic vector fields.

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1 Introduction

$N = 8$ supergravity [1] is undoubtedly a highly distinguished field theory due to its high degree of symmetry and the remarkable structure of its amplitudes that has emerged in recent work, see e.g. [2, 3]. The continuous $E_{7(7)}$ symmetry underlying the classical field equations has important consequences for the structure of the counterterms [4–8]. The field content of maximal supergravity is the unique $N = 8$ supermultiplet

helicity	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+1$	$+\frac{3}{2}$	$+2$
d.o.f.	1	8	28	56	70	56	28	8	1

. (1.1)

On the other hand, the mutual couplings of the various fields are not uniquely determined, as supersymmetry allows for the introduction of particular (non-)abelian charges and the realization of different (non-)abelian gauge groups. After the original version of the theory with abelian gauge fields [1] the first maximal gauged supergravity was constructed in [9] with the 28 vector fields gauging a compact SO(8) subgroup of $E_{7(7)}$. Non-compact versions of this theory have been constructed and classified in [10–13] and later been extended to other non-semisimple gauge groups in [14, 15]. A general formalism for describing the gauging of subgroups in terms of an 'embedding tensor' has been established in [16, 17]. This constant tensor describes the embedding of the gauge group into the global $E_{7(7)}$ symmetry of the ungauged theory, and parametrizes all the couplings of the gauged theory.

The aim of this paper is the construction of all possible gaugings (and thus all possible couplings) of $N = 8$ supergravity, which in particular include a gauging of the global scaling symmetry of the theory. Their gauge groups are embedded in the product $E_{7(7)} \times \mathbb{R}$ of the Cremmer-Julia group $E_{7(7)}$ with the one-parameter scaling symmetry of the theory that generalizes the Weyl rescaling of general relativity and has been dubbed a 'trombone' symmetry of supergravity [18]. Supergravity theories that include a gauging of their scaling symmetry have first been constructed in ten dimensions by a generalized Scherk-Schwarz reduction from eleven dimensions [19, 20]. Lower dimensional examples of such theories include [21] and [22, 23]. As a generic feature, these theories are invariant under local rescaling of the fields (including the metric) with appropriate weights upon a compensating gauge transformation on the matter fields. They do not possess an action (since they result from the gauging of an on-shell symmetry) and typically support de Sitter geometries rather than Minkowski or AdS vacua. A systematic account to the construction of these theories has been put forward in [24]. Based on the algebraic structure of the duality groups of the ungauged theories, the representation content and the algebraic consistency constraints for the corresponding embedding tensor have been determined for the maximal supergravities. In [25, 26], these structures have been shown to be naturally embedded in the framework of the very-extended infinite-dimensional Kac-Moody algebra E_{11} [27, 28].

The general analysis reveals that four-dimensional $N = 8$ supergravity admits an embedding tensor transforming in the representation $\mathbf{56} + \mathbf{912}$ of $E_{7(7)}$, subject to a set of bilinear algebraic consistency constraints. Gaugings defined by an embedding tensor in the irreducible **912** representation describe gauge groups that entirely reside within $E_{7(7)}$ and have been constructed in [16, 17]. Additional non-vanishing components in the **56**

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representation on the other hand define gaugings that include the trombone generator, i.e. theories in which local scaling invariance is part of the gauge group. These are the theories to be constructed in this paper. While the analysis of [24] has been purely algebraic and based on the structure of non-abelian deformations of the underlying tensor gauge algebras, it is the aim of this paper to explicitly realize these theories by constructing the full set of supersymmetric field equations. Thereby we derive the most general couplings that are compatible with $N = 8$ supersymmetry in four dimensions. In particular, we confirm that the algebraic consistency constraints derived in [24] for the embedding tensor are indeed sufficient to ensure supersymmetry of the field equations.

This paper is organized as follows. In section 2, we analyze the general structure of the gauge groups induced by an embedding tensor in the $\mathbf{56} + \mathbf{912}$ representation. We explicitly construct the gauge group generators in terms of the embedding tensor and discuss the system of bilinear algebraic consistency constraints that the embedding tensor must satisfy. In case the gauge group includes the trombone generator, this system of constraints drastically reduces upon decomposing the embedding tensor into its $E_{6(6)}$ irreducible components, and we present a number of explicit solutions. We compute the Cartan-Killing metric of the gauge group and show that gaugings involving the local scaling symmetry are generically dyonic, i.e. genuinely involve electric and magnetic vector fields.

In section 3, we review the structure of the scalar target space $E_{7(7)}/\mathrm{SU}(8)$ and define the T tensor in terms of which the couplings of the gauged theory are expressed. Subsequently, in section 4, we determine the modified supersymmetry transformations by verifying the closure of their algebra on the bosonic fields of the theory. Based on these results, in section 5, we obtain the modified field equations of the gauged $N = 8$ supergravity by starting from an ansatz for the fermionic field equations and calculating their transformation under supersymmetry. This allows to uniquely determine the full set of field equations in lowest order of the fermions. As a particular feature of these theories, we find that gauging of the trombone generator leads to an additional positive contribution to the effective cosmological constant. In section 5.4, we determine the conditions for extremality, i.e. for solutions of the field equations with constant scalar and gauge fields and give explicit formulas for the mass matrices by linearizing the field equations around these solutions. Finally, in section 6, we present a simple example of a theory with local scaling symmetry that has its higher-dimensional origin as a generalized Scherk-Schwarz reduction from five dimensions upon twisting the field with a linear combination of an $E_{6(6)}$ generator and the five-dimensional trombone symmetry. We show that this theory admits a de Sitter solution with constant scalar fields and determine its mass spectrum which seems to indicate that the solution is not stable. We conclude with an outlook on the role and the applications of these theories.

2 Structure of gauge groups

Before explicitly constructing the full supersymmetric field equations, in this section we will present and analyze the structure of the possible gauge algebras that can be realized as local symmetry in maximally supersymmetric supergravity in four dimensions. Recall,

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that the global symmetry group of the ungauged maximally supersymmetric theory is given by [1]

$$G = E_{7(7)} \times \mathbb{R} \,, \tag{2.1}$$

where the second factor corresponds to the scaling (or trombone) symmetry of the equations of motion, under which the fields transform as

$$\begin{aligned} \delta g_{\mu\nu} &= 2\, g_{\mu\nu} \,, \qquad \delta \mathcal{A}^M_\mu \;=\; \mathcal{A}^M_\mu \,, \qquad \delta \phi^i \;=\; 0 \,, \\ \delta \psi_\mu &= \tfrac{1}{2}\, \psi_\mu \,, \qquad\quad \delta \chi \;=\; -\tfrac{1}{2}\, \chi \,. \end{aligned} \tag{2.2}$$

Here, the first line refers to the bosonic fields of spin 2, 1, and 0, while the second line gives the transformations of the spin 3/2 gravitinos and the spin 1/2 matter fermions, respectively. The $E_{7(7)}$ factor in (2.1) in contrast only acts on vector and scalar fields, with its generators t_α closing into the algebra

$$[t_\alpha, t_\beta] = f_{\alpha\beta}{}^\gamma\, t_\gamma \,. \tag{2.3}$$

The 28 electric vector fields A^Λ_μ combine with their magnetic duals $A_{\mu\,\Lambda}$ into the fundamental 56-dimensional representation A^M_μ of $E_{7(7)}$ while the 70 scalar fields transform in a non-linear representation parametrizing the coset space $E_{7(7)}/\mathrm{SU}(8)$. General gaugings will also require the introduction of two-form tensor fields $B_{\mu\nu\,\alpha}$ transforming in the adjoint 133-dimensional representation of $E_{7(7)}$.

2.1 Gauge group generators

In this paper, we will construct the most general supersymmetric theories in which a subgroup of (2.1) is gauged. Extending previous work [17], we will consider those theories in which the gauge groups include the scaling symmetry, i.e. the second factor in (2.1).

Let us denote by $\mathbf{t}_0$ the generator of the scaling symmetry $\mathbb{R}$, and by $\mathcal{A}_\mu \equiv \vartheta_M \mathcal{A}^M_\mu$ the linear combination of vector fields that will be used to gauge this symmetry upon introduction of covariant derivatives. As this symmetry also acts in the gravitational sector by scaling the metric, its gauging necessitates a modification of the spin connection $\omega_\mu{}^{ab}$ and the Riemann tensor $R_{\mu\nu}{}^{ab}$ according to [24]

$$\begin{aligned} \widehat{\omega}_\mu{}^{ab} &= \omega_\mu{}^{ab} + 2\, e_\mu{}^{[a}\; e^{b]\,\nu} \mathcal{A}_\nu \,, \\ \widehat{\mathcal{R}}_{\mu\nu}{}^{ab} &\equiv 2\, \partial_{[\mu}\, \widehat{\omega}_{\nu]}{}^{ab} + 2\, \widehat{\omega}_{[\mu}{}^{ac}\, \widehat{\omega}_{\nu]c}{}^{b} \\ &= R_{\mu\nu}{}^{ab} - 4\, e_{[\mu}{}^{[a} \nabla(\omega)_{\nu]} \mathcal{A}^{b]} + 4\, e_{[\mu}{}^{[a} \mathcal{A}_{\nu]} \mathcal{A}^{b]} - 2\, e_{[\mu}{}^{a} e_{\nu]}{}^{b} \mathcal{A}_\lambda \mathcal{A}^\lambda \,, \end{aligned} \tag{2.4}$$

which are invariant under the joint transformation

$$\delta g_{\mu\nu} = 2\lambda(x)\, g_{\mu\nu} \,, \qquad \delta \mathcal{A}_\mu \;=\; \partial_\mu \lambda(x) \,. \tag{2.5}$$

Besides, they satisfy the generalized Bianchi identities

$$\widehat{\mathcal{R}}_{[\mu\nu\rho]}{}^{a} = \mathcal{F}_{[\mu\nu}\, e_{\rho]}{}^{a} \,. \tag{2.6}$$

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The most general gauging combines this symmetry with some subgroup of the $E_{7(7)}$ factor of (2.1). As shown in [16, 17] and [24] for the cases without and with the trombone factor, respectively, the parametrization of the general gauge group generators X_M allows for $56 + 912$ parameters spanning the 'embedding tensor' ϑ_M and $\Theta_M{}^\alpha$, according to

$$X_M \equiv \vartheta_M \, \mathbf{t}_0 + \left(\Theta_M{}^\alpha + 8\vartheta_N \, (t^\alpha)_M{}^N\right) \, t_\alpha \,, \tag{2.7}$$

with covariant derivatives given by $D_\mu \equiv \partial_\mu - \mathcal{A}^M_\mu X_M$.[1] Here, $(t_\alpha)_M{}^N$ are the $E_{7(7)}$ generators (2.3) in the fundamental representation,[2] and the matrix $\Theta_M{}^\alpha$ is constrained by the relations

$$\Theta_M{}^\alpha (t_\alpha)_N{}^M = 0 \,, \qquad \Theta_M{}^\alpha = -2(t_\beta \, t^\alpha)_M{}^N \, \Theta_N{}^\beta \,, \tag{2.8}$$

i.e. transforms in the **912** representation of $E_{7(7)}$. In absence of ϑ_M, it describes the gaugings whose gauge group entirely resides within the $E_{7(7)}$. The relative factors in (2.7) are chosen such that the tensor $Z^K{}_{MN} \equiv (X_{(M)})_N{}^K$ factors according to

$$Z^K{}_{MN} \equiv (X_{(M)})_N{}^K \;=\; -\tfrac{1}{2}(t_\alpha)_{MN} \left(\Theta^{K\alpha} - 16(t^\alpha)^{KL}\vartheta_L\right) \;\equiv\; (t_\alpha)_{MN} \, Z^{K\alpha} \,, \tag{2.9}$$

and thus projects onto the **133** representation in its indices (MN). This will be a central identity in the construction. For convenience, we also define the projection of the gauge group generators onto the $E_{7(7)}$ factor of (2.1) as

$$\check{X}_M \equiv \left(\Theta_M{}^\alpha + 8\vartheta_N \, (t^\alpha)_M{}^N\right) \, t_\alpha \,, \tag{2.10}$$

The gauged theory is invariant under the local symmetry

$$\begin{aligned} \delta_\Lambda \vec{\phi} &= \Lambda^M X_M \cdot \vec{\phi} \;\equiv\; \left(\Theta_M{}^\alpha + 8\vartheta_N \, (t^\alpha)_M{}^N\right) \vec{K}_\alpha(\phi) \,, \\ \delta_\Lambda \mathcal{A}^M_\mu &= D_\mu \Lambda^M \;\equiv\; \partial_\mu \Lambda^M + \mathcal{A}^K_\mu (X_K)_N{}^M \, \Lambda^N \,, \end{aligned} \tag{2.11}$$

where $\vec{K}_\alpha(\phi)$ represent the $E_{7(7)}$ Killing vector fields on the scalar target space, and the gauge group generators are given by evaluating (2.7) in the appropriate representation, i.e.

$$(X_K)_N{}^M \equiv -\vartheta_K \delta^M_N + \left(\Theta_K{}^\alpha + 8\vartheta_L \, (t^\alpha)_K{}^L\right) \, (t_\alpha)_N{}^M \,. \tag{2.12}$$

Finally, covariant field strengths are defined by

$$\mathcal{H}^M_{\mu\nu} \equiv 2\partial_{[\mu}\mathcal{A}^M_{\nu]} + (X_N)_P{}^M \, \mathcal{A}^N_{[\mu}\mathcal{A}^P_{\nu]} + Z^{M\alpha} \, B_{\mu\nu\,\alpha} \,, \tag{2.13}$$

with a Stückelberg-type coupling to the two-forms $B_{\mu\nu\,\alpha}$ and the (constant) intertwining tensor $Z^{M\alpha}$ defined in (2.9). They transform covariantly under the gauge transformations (2.11) provided the two-forms transform as

$$\delta_\Lambda B_{\mu\nu\,\alpha} = -2(t_\alpha)_{MN} \left(\Lambda^M \mathcal{H}^N_{\mu\nu} - \mathcal{A}^M_{[\mu} \, \delta\mathcal{A}^N_{\nu]}\right) \,. \tag{2.14}$$

[1] For transparency we have suppressed explicit coupling constants, which can at any stage be reintroduced by rescaling $\vartheta_M \to g\vartheta_M$, $\Theta_M{}^\alpha \to g\Theta_M{}^\alpha$.

[2] We raise and lower adjoint indices with the invariant metric $\kappa_{\alpha\beta} \equiv \mathrm{Tr}\,[t_\alpha t_\beta]$. Fundamental indices are raised and lowered with the symplectic matrix Ω^{MN} using north-west south-east conventions: $X^M = \Omega^{MN} X_N$, etc.

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Moreover, the covariant field strengths (2.13) are invariant under the tensor gauge transformations[3]

$$\begin{aligned}\delta_\Xi B_{\mu\nu\,\alpha} &= 2\mathcal{D}_{[\mu}\Xi_{\nu]\,\alpha} + 2(t_\alpha)_{MN}\,\mathcal{A}^M_{[\mu}\,\delta\mathcal{A}^N_{\nu]}\,,\\ \delta_\Xi \mathcal{A}^M_\mu &= -Z^{M\alpha}\,\Xi_{\mu\,\alpha}\,.\end{aligned} \tag{2.15}$$

The covariant field strengths (2.13) satisfy the generalized Bianchi identities

$$\mathcal{D}_{[\mu}\mathcal{H}^M_{\nu\rho]} = \tfrac{1}{3} Z^{M\alpha}\,\mathcal{H}_{\mu\nu\rho\,\alpha}\,, \tag{2.16}$$

with the covariant non-abelian field strength $\mathcal{H}_{\mu\nu\rho\,\alpha}$ of the two-form tensor fields, given by:

$$\mathcal{H}_{\mu\nu\rho\,\alpha} = 3D_{[\mu}B_{\nu\rho]\alpha} + 6\,t_{\alpha PQ}\,A^P_{[\mu}\left(\partial_\nu A^Q_{\rho]} + \frac{1}{3}\,X_{RS}{}^Q\,A^R_\nu A^S_{\rho]}\right)\,, \tag{2.17}$$

where

$$D_{[\mu}B_{\nu\rho]\alpha} = \partial_{[\mu}B_{\nu\rho]\alpha} + 2\,t_{\alpha PQ}Z^{Q\beta}A^P_{[\mu}B_{\nu\rho]\beta}\,. \tag{2.18}$$

2.2 Consistency constraints

The previous construction leads to a consistent (closed) gauge algebra, if the irreducible components ϑ_M, $\Theta_M{}^\alpha$ satisfy the following system of quadratic constraints [24]

$$\vartheta_M\,\Theta^{M\alpha} \overset{!}{=} 16\,(t^\alpha)^{MN}\,\vartheta_M\,\vartheta_N\,, \tag{2.19}$$

$$(t_\gamma)_{[M}{}^P\,\Theta_{N]}{}^\gamma\,\vartheta_P \overset{!}{=} 0\,, \tag{2.20}$$

$$\Theta_M{}^\alpha\,\Theta^{M\beta} \overset{!}{=} 8\,\vartheta_M\,\Theta_N{}^{[\alpha}\,t^{\beta]MN} - 4\,f^{\alpha\beta}{}_\gamma\,\vartheta_M\,\Theta^{M\gamma}\,, \tag{2.21}$$

transforming in the **133**, the **1539** and the **133** + **8645**, of $E_{7(7)}$, respectively. For $\vartheta_M = 0$ they consistently reduce to the quadratic conditions of [17]. As we will show in the following, any solution to the constraints (2.19)–(2.21), will define a viable maximally supersymmetric gauged supergravity.

It is straightforward to show that (2.19)–(2.21) imply several direct consequences for the gauge group generators, such as the closure of the gauge algebra according to

$$[X_M, X_N] = -X_{MN}{}^K\,X_K\,, \tag{2.22}$$

and orthogonality between gauge group generators and the intertwining tensor Z

$$X_{MN}{}^K\,Z^{M\alpha} \;=\; 0 \;=\; \vartheta_M\,Z^{M\alpha}\,. \tag{2.23}$$

The reason for the fact that the gauge transformations consistently close into an algebra when properly extended to the two-form tensor fields even in presence of the gauging of the scaling symmetry is the underlying structure of a hierarchy of non-abelian tensor gauge transformations [29, 30] which is not based on the existence of an action. The relative factors in (2.7) and the identity (2.9) are central in this construction. What we will show explicitly in this paper is that the non-abelian deformations defined in the previous section are precisely the ones that are moreover compatible with maximal supersymmetry of the field equations.

[3] W.r.t. reference [17] we have rescaled the two-form fields and associated tensors as $B_{\mu\nu\alpha} \to -B_{\mu\nu\alpha}$, $\Xi_{\mu\alpha} \to -\Xi_{\mu\alpha}$, $Z^{M\alpha} \to -Z^{M\alpha}$.

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2.3 Solution to the quadratic constraints

In general, it is a hard task to construct solutions to the quadratic constraints of gauged supergravity. However, it turns out that in presence of the trombone (i.e. non-vanishing ϑ_M), the system (2.19)–(2.21) can be reduced to a much simpler one in terms of a reduced number of components. The strategy for solving the quadratic constraints follows the case of pure trombone gaugings [24] by decomposing all objects with respect to the $E_{6(6)} \times \mathrm{SO}(1,1)$ subgroup of $E_{7(7)}$. Explicitly, this means that the adjoint representation branches as

$$\begin{aligned} t_\alpha &\to (t_\circ,\ t_a,\ t_m,\ t^m)\ , \\ \text{according to}\ \ \mathbf{133} &\to \mathbf{1}^0 + \mathbf{78}^0 + \mathbf{27}^{-2} + \bar{\mathbf{27}}^{+2}\ , \end{aligned} \tag{2.24}$$

while the fundamental representation breaks into

$$\begin{aligned} \vartheta_M &\to (\vartheta_\bullet,\ \vartheta_m,\ \vartheta^m,\ \vartheta^\bullet)\ , \\ \text{according to}\ \ \mathbf{56} &\to \mathbf{1}^{+3} + \mathbf{27}^{+1} + \bar{\mathbf{27}}^{-1} + \mathbf{1}^{-3}\ , \end{aligned} \tag{2.25}$$

and the embedding tensor $\Theta_M{}^\alpha$ decomposes into

$$\begin{aligned} \Theta_M{}^\alpha &\to \left(\xi^a_+,\ \xi_m,\ \xi^{mn},\ \xi_{mn},\ \xi^m,\ \xi^a_-\right)\ , \\ \text{according to}\ \ \mathbf{912} &\to \mathbf{78}^{+3} + \mathbf{27}^{+1} + \overline{\mathbf{351}}^{+1} + \mathbf{351}^{-1} + \bar{\mathbf{27}}^{-1} + \mathbf{78}^{-3}\ , \end{aligned} \tag{2.26}$$

with its explicit (56×133) matrix form given in (B.2) in the appendix. We use indices $a, b, \dots = 1, \dots, 78$ and $m, n, \dots = 1, \dots, 27$ to label the adjoint and the fundamental representation of $E_{6(6)}$, respectively.

In appendix B we derive an important result: for non-vanishing ϑ_M and up to $E_{7(7)}$ rotations, the general solution to the system (2.19)–(2.21) is parametrized by a real constant κ, an $E_{6(6)}$ matrix $\Xi_m{}^n \equiv \Xi^a (t_a)_m{}^n$ and two real tensors ζ^m, $\zeta^{[mn]}$, as follows

$$\begin{aligned} (\vartheta_\bullet, \vartheta_n, \vartheta^n, \vartheta^\bullet) &= (\kappa, 0, \kappa\zeta^n, 0)\ , \\ \left(\xi^a_+,\ \xi_m,\ \xi^{mn},\ \xi_{mn},\ \xi^m,\ \xi^a_-\right) &= \left(\Xi^a, 0, \zeta^{mn}, \Xi_{[m}{}^k d_{n]kl}\zeta^l, -\tfrac{4}{3}\kappa\zeta^m, 0\right)\ , \end{aligned} \tag{2.27}$$

where d_{kmn} denotes the totally symmetric $E_{6(6)}$ invariant tensor. The tensors ζ^m, $\zeta^{[mn]}$ must be real eigenvectors under the action of Ξ according to

$$\begin{aligned} \delta_\Xi \zeta^m &\equiv -\Xi_n{}^m \zeta^n \overset{!}{=} \tfrac{4}{3}\kappa\zeta^m\ , \\ \delta_\Xi \zeta^{mn} &\equiv 2\Xi_k{}^{[m}\zeta^{n]k} \overset{!}{=} \tfrac{2}{3}\kappa\zeta^{mn}\ , \end{aligned} \tag{2.28}$$

which furthermore must satisfy the following set of polynomial constraints:

$$\zeta^k \zeta^l d_{mkl} \overset{!}{=} 0\ , \tag{2.29}$$

$$\zeta^k \zeta^{mn} d_{kml} \overset{!}{=} 0\ , \tag{2.30}$$

$$\zeta^{[k}\zeta^{mn]} \overset{!}{=} 0\ , \tag{2.31}$$

$$\left(t_a \cdot (\Xi + \tfrac{4}{3}\kappa I) \cdot (\Xi - \tfrac{2}{3}\kappa I)\right)_n{}^m \zeta^n \overset{!}{=} -\tfrac{1}{2}\zeta^{mk}\zeta^{ln} d_{klp} (t_a)_n{}^p\ . \tag{2.32}$$

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As we show in appendix B, this system of equations is equivalent to the original system of constraints (2.19)–(2.21). In contrast to the original system, solutions to (2.28)–(2.32) may easily be constructed.

A simple solution to the system (2.28)–(2.32) is given by setting $\zeta^m = 0 = \zeta^{mn}$. This leaves a non-trivial embedding tensor (2.27) parametrized by κ and an $E_{6(6)}$ generator Ξ. This solution satisfies a stronger version of the quadratic constraints: left and right hand sides of equations (2.19)–(2.21) vanish separately. In the limit $\kappa \to 0$ in which ϑ_M vanishes, this solution corresponds to the known gaugings induced by a Scherk-Schwarz reduction [31] from five dimensions parametrized by the choice of an $E_{6(6)}$ generator [14]. For non-vanishing κ, the higher-dimensional origin of these theories is a generalized Scherk-Schwarz reduction from five dimensions in which the fields are twisted by a linear combination of the $E_{6(6)}$ generator Ξ and the five-dimensional trombone symmetry. The form of the generators (2.7) shows that even for vanishing $\Xi = 0$, switching on κ corresponds to gauging a linear combination of the four-dimensional trombone generator $\mathbf{t}_0$ and a subset of $E_{7(7)}$ generators. More complicated solutions of the constraints involve non-vanishing zero-modes ζ^m, ζ^{mn}. While we defer the complete solution of the constraint system (2.28)–(2.32) to a separate publication, a typical example of such a solution will be discussed in section 2.5.

2.4 Invariants of the trombone

We can classify the inequivalent gaugings according to the $\mathrm{E}_{7(7)}$-invariants constructed out of the embedding tensor. In particular, the quadratic constraints can be regarded as conditions on the $\mathrm{E}_{7(7)}$-orbits of the embedding tensor. In terms of ϑ_M and $\Theta_M{}^\alpha$, several $\mathrm{E}_{7(7)}$-invariants can be constructed of which the simplest is the quartic invariant $I_4(\vartheta)$ depending only on the trombone component ϑ_M according to

$$\begin{aligned} I_4(\vartheta) &\equiv -2\,(t_\alpha)^{MN}(t^\alpha)^{PQ}\,\vartheta_M\,\vartheta_N\,\vartheta_P\,\vartheta_Q \\ &= -(\vartheta_\bullet\,\vartheta^\bullet + \vartheta_m\,\vartheta^m)^2 + 10 d_{mnp}\,d^{mrs}\vartheta_r\vartheta_s\,\vartheta^n\,\vartheta^p \\ &\quad -\tfrac{20}{3}\,\vartheta^\bullet\,d^{mnp}\,\vartheta_m\vartheta_n\vartheta_p + \tfrac{2}{3}\,\vartheta_\bullet\,d_{mnp}\,\vartheta^m\vartheta^n\vartheta^p\,. \end{aligned} \tag{2.33}$$

The different orbits of the 56-dimensional fundamental representation of $E_{7(7)}$ are characterized via this invariant as [32]:

- (i) $I_4(\vartheta) > 0$: the orbit is $\frac{\mathrm{E}_{7(7)}}{\mathrm{E}_{6(2)}}$;
- (ii) $I_4(\vartheta) < 0$: the orbit is $\frac{\mathrm{E}_{7(7)}}{\mathrm{E}_{6(6)}}$;
- (iii) $I_4(\vartheta) = 0$, $\frac{\partial I_4(\vartheta)}{\partial \vartheta_M} \neq 0$: the orbit is $\frac{\mathrm{E}_{7(7)}}{\mathrm{F}_{4(4)} \ltimes T_{26}}$;
- (iv) $I_4(\vartheta) = 0$, $\frac{\partial I_4(\vartheta)}{\partial \vartheta_M} = 0$, $t_{\alpha\,MN}\vartheta^M\,\vartheta^N \neq 0$: the orbit is $\frac{\mathrm{E}_{7(7)}}{\mathrm{SO}(6,5) \ltimes (T_{32} \times T_1)}$;
- (v) $t_{\alpha\,MN}\vartheta^M\,\vartheta^N = 0$: the orbit is $\frac{\mathrm{E}_{7(7)}}{\mathrm{E}_{6(6)} \ltimes T_{27}}$.

Inserting the solution (2.27) obtained in the previous section into (2.33), we find

$$I_4(\vartheta) \;=\; \frac{2}{3}\,\kappa^4\,d_{mnp}\,\zeta^m\zeta^n\zeta^p\,. \tag{2.34}$$

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From (2.29) it follows that $I_4(\vartheta) = 0 = \frac{\partial I_4(\vartheta)}{\partial \vartheta_M}$. Since $t_{\alpha\, MN}\vartheta^M\vartheta^N$ has a non-vanishing component $(t^m)^\bullet{}_n\,\vartheta_\bullet\vartheta^n \propto \kappa^2\,\zeta^m$, we conclude that ϑ_M belongs to the orbit (iv) of the above classification if ζ^k is non-vanishing, and otherwise to the orbit (v).

In both cases the gauge group G_g will then be a subgroup of the stability group of the corresponding orbit inside $\mathbb{R}^+ \times \mathrm{E}_{7(7)}$. In case (iv), for instance, we should have

$$G_g \subset \left[\mathbb{R}^+ \times \mathrm{SO}(6,5)\right] \ltimes (T_{32} \times T_1)\,, \tag{2.35}$$

where $\mathbb{R}^+$ is a suitable combination of the trombone symmetry and the $\mathrm{O}(1,1)^7$ symmetry inside $\mathrm{E}_{7(7)}$.

2.5 An explicit example

Here we present an example of a solution of the constraints (2.28)–(2.32). The quadratic condition (2.29) on ζ^m:

$$d_{mnp}\,\zeta^n\,\zeta^p = 0\,, \tag{2.36}$$

can be viewed as a kind of "$\mathrm{E}_{6(6)}$-pure spinor" constraint. It defines an orbit of the $\overline{\mathbf{27}}$ with stability group $\mathrm{SO}(5,5) \ltimes T_{16}$ (see [32]). This means that there exists an $\mathrm{SO}(5,5) \subset \mathrm{E}_{6(6)}$ with respect to which ζ^m is a singlet. If we decompose the adjoint and the fundamental representations of $\mathrm{E}_{6(6)}$ with respect to $\mathrm{O}(1,1) \times \mathrm{SO}(5,5) \subset \mathrm{E}_{6(6)}$ we find:

$$\begin{aligned} \mathbf{78} &\to \mathbf{1}^0 + \mathbf{45}^0 + \mathbf{16}_c^{+3} + \mathbf{16}_s^{-3}\,,\\ \overline{\mathbf{27}} &\to \mathbf{1}^{-4} + \mathbf{10}^{+2} + \mathbf{16}_c^{-1}\,. \end{aligned} \tag{2.37}$$

The stabilizer of ζ^m is thus generated by the $\mathbf{45}^0 + \mathbf{16}_s^{-3}$, while ζ^m corresponds to the $\mathbf{1}^{-4}$. We denote by $\mathbf{h}$ the $\mathrm{O}(1,1)$ generator, such that $\delta_{\mathbf{h}}\,\zeta^m = -4\,\zeta^m$. Eqs. (2.28), on the other hand, imply that $\delta_\Xi\,\zeta^m = -\Xi_n{}^m\,\zeta^n = \frac{4}{3}\,\kappa\,\zeta^m$. Since ζ^m is a simultaneous eigenvector of both Ξ and $\mathbf{h}$, we must have $\delta_{[\Xi,\,\mathbf{h}]}\,\zeta^m = 0$, namely that Ξ cannot have a component along the $\mathbf{16}_c^{+3}$:

$$\Xi \in \mathbf{1}^0 + \mathbf{45}^0 + \mathbf{16}_s^{-3}\,. \tag{2.38}$$

We conclude that Ξ consists of a component proportional to $\mathbf{h}$ plus an element Ξ_0 in the algebra of the little group of ζ^m:

$$\Xi = -\frac{1}{3}\,k\,\mathbf{h} + \Xi_0\,, \qquad \delta_{\Xi_0}\zeta^m = 0\,. \tag{2.39}$$

Let us consider the case in which Ξ_0 is a semisimple element of $\mathfrak{so}(5,5)$ and thus can be taken as an element of its Cartan subalgebra. One can show that in this case, taking $\zeta^{mn} = \zeta^{[m}\eta^{n]}$, with η^m in the $\mathbf{16}_c^{-1}$, all the constraints are satisfied. In particular the two sides of eq. (2.32) are separately zero. As we shall show in appendix B.2, this equation in particular implies that Ξ_0 should commute with an $\mathrm{SO}(4,4)$ subgroup of $\mathrm{SO}(5,5)$. The resulting gauge algebra $\mathfrak{g}_g$ is 21-dimensional and of the form:

$$\begin{aligned} \mathfrak{g}_g &= \mathfrak{o}(1,1) \oplus \mathfrak{so}(2,1) \oplus \mathfrak{l}^{(2\kappa)} \oplus \mathfrak{l}^{(4\kappa)}\,,\\ \dim(\mathfrak{l}^{(2\kappa)}) &= 16\,, \quad \dim(\mathfrak{l}^{(4\kappa)}) = 1\,, \end{aligned} \tag{2.40}$$

the gradings referring to the O(1,1)-generator. We can understand the embedding of the gauge group into the stability group $[\mathbb{R}^+ \times \mathrm{SO}(6,5)] \ltimes (T_{32} \times T_1)$ of the ϑ_M-orbit by decomposing SO(6,5) with respect to $\mathrm{SO}(2,1) \times \mathrm{SO}(4,4)$. Then the generators of $\mathbb{R}^+ \times \mathrm{SO}(2,1)$ provide the zero-grading part of the gauge algebra (2.40). The gauge generators can be written in a manifestly $\mathrm{SO}(2,1) \times \mathrm{SO}(4,4)$-covariant way. Let $A, B = 1, 2$ denote the SO(2,1)-doublet indices while $I, J = 1, \ldots, 8$ label the $\mathbf{8}_s$ of SO(4,4). Then let T_x, $x = 0, 1, 2, 3$, be the $\mathfrak{o}(1,1) + \mathfrak{so}(2,1)$ generators, $\mathfrak{l}^{(2\kappa)} = \mathrm{Span}(T_{AI})$ and $\mathfrak{l}^{(4\kappa)} = \mathrm{Span}(T)$. The relevant commutation relations between the gauge generators are:

$$[T_x,\, T_{AI}] = -(T_x)_A{}^B\, T_{BI}\ , \qquad [T_{AI},\, T_{BJ}] = \epsilon_{AB}\, C_{IJ}\, T\ , \tag{2.41}$$

where C_{IJ} is the symmetric invariant matrix in the product $\mathbf{8}_s \times \mathbf{8}_s$. In other words, with respect to $\mathrm{SO}(1,1) \times \mathrm{SO}(2,1) \times \mathrm{SO}(4,4)$ the generators $\{T_x\}$ are in the $(\mathbf{3},\mathbf{1})^0$, $\{T_{AI}\}$ in the $(\mathbf{2},\mathbf{8_s})^{2\kappa}$ while $\{T\}$ is in the $(\mathbf{1},\mathbf{1})^{4\kappa}$. In terms of the $\mathrm{E}_{7(7)}$-branching with respect to the $\mathrm{E}_{6(6)}$-subgroup, the T_{AI} consists of 8 generators from the $\overline{\mathbf{27}}$ and 8 from the **78**, while T originates from the $\overline{\mathbf{27}}$. This structure does not change either in the limit $\zeta^{mn} \to 0$, or in the limit $\zeta^m \to 0$. In the latter case the $\mathfrak{gl}(2)$ algebra of $\mathrm{SO}(1,1) \times \mathrm{SO}(2,1)$ contracts to a non-semisimple algebra of the form $\mathfrak{o}(1,1) + H_3$, where H_3 is a three-dimensional Heisenberg algebra spanned by nilpotent generators. Only if both the zero-modes vanish ($\zeta^{mn} \to 0$, $\zeta^m \to 0$) the T_{AI} generators which do not vanish become abelian, the last commutator in (2.41) becomes trivial and we retrieve the first example mentioned at the end of section 2.3, which will be discussed in section 6.

2.6 Cartan-Killing metric of the gauge group

In the previous sections we have been discussing the general solution to the quadratic constraints and worked out the corresponding gauge groups in certain examples. With the general solution given in section 2.3, the gauge group generators may be reconstructed from (2.12), putting together (B.1), (B.2) and (2.27). The explicit form of the generators $\{X_M\} = \{X_\bullet, X_m, X^m, X^\bullet\}$ in terms of the parameters κ, Ξ^a, ζ^m, and ζ^{mn} is given in (C.1) in the appendix. Via (2.22) these generators also encode the structure constants of the gauge algebra. We can compute the Cartan-Killing metric of the gauge group as $g_{MN} \equiv \mathrm{Tr}(X_M\, X_N)$. Its non-vanishing components are

$$\begin{aligned}
g_{\bullet\bullet} &= 64\kappa^2 + 2\,\Xi^a\Xi_a\ ,\\
g_\bullet{}^m &= (96\kappa^2 - 2\,\Xi^a\Xi_a)\,\zeta^m\ ,\\
g_{\bullet m} &= -6\,\Xi_l{}^n d_{nmk}\zeta^{kl}\ ,\\
g^{mn} &= (64\kappa^2 + 2\,\Xi^a\Xi_a)\,\zeta^m\zeta^n\ ,\\
g_{mn} &= -6\,\zeta^{kl}\zeta^{pq} d_{mkp} d_{nlq}\\
&\quad + \tfrac{2}{3}\,(80\kappa^2 - 3\,\Xi^a\Xi_a)\,d_{mnk}\,\zeta^k + 24\,d_{kl(m}\,(\Xi^2)_{n)}{}^k\zeta^l\ .
\end{aligned} \tag{2.42}$$

For $\zeta^m = 0 = \zeta^{mn}$ this shows that the semisimple part of the gauge algebra is one-dimensional in accordance with its origin as a Scherk-Schwarz reduction from five dimensions. If $\zeta^{mn} = 0$ but ζ^m is non-vanishing, a contraction of equation (2.32) implies that

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$\Xi^a\Xi_a = 32\kappa^2$, and the Cartan-Killing form accordingly reduces to

$$\begin{aligned} g_{\bullet\bullet} &= 128\kappa^2 \,, \\ g_{\bullet}{}^{m} &= 32\kappa^2\,\zeta^m \,, \\ g^{mn} &= 128\kappa^2\,\zeta^m\zeta^n \,, \\ g_{mn} &= -\tfrac{32}{3}\kappa^2\,d_{mnk}\,\zeta^k + 24\,d_{kl(m}\,(\Xi^2)_{n)}{}^k\zeta^l \,. \end{aligned} \tag{2.43}$$

2.7 Electric/magnetic gaugings

So far, we have discussed the structure of the gauge algebra by studying deformations that involve vector fields from the entire 56-dimensional fundamental representation of $E_{7(7)}$. It is well known [1], that only half of these vector fields are dynamical electric vector fields while the other half is given by their magnetic duals. Accordingly, only the former half appears in the action of the ungauged theory. Nevertheless, the connections of a general gauging may contain magnetic vector fields that are related by their first order duality equations to the electric fields of the theory. In [33] it has been shown how to elevate this construction to the level of an action by introducing additional auxiliary two-form tensor fields (which in turn are the magnetic duals to the scalar fields of the theory). The magnetic vector fields then do not possess a standard kinetic term but rather couple via a topological BF term to the two-form tensor fields. On the other hand, all standard gaugings of the theory [17] satisfy a symplectic locality condition that ensures the existence of a symplectic frame in which all the vector fields involved in the gauging live in the electric sector. In this sense even in presence of magnetic charges these theories remain electric gaugings in disguise which is in accordance with general results on the gauging of electric/magnetic duality [34, 35]. We shall see that this is no longer the case for the gaugings considered in this paper, related to the fact that these theories do no longer admit an action.

For the solution of the consistency constraints of the embedding tensor discussed at the end of section 2.3, the left and right hand sides of equations (2.19)–(2.21) vanish separately. The gauge group generators thus satisfy the symplectic locality condition $\Omega^{MN}X_MX_N = 0$. I.e. as for the standard gaugings we can choose a symplectic frame $\{X^M\} \to \{X^\Lambda, X_\Lambda\}$ such that all X^Λ are identically zero. Indeed, in this case the explicit form of the generators (C.1) shows that $X_\bullet = 0 = X^k$. Accordingly, the gauging only involves electric vector fields $\{\mathcal{A}_\mu^\Lambda\} = \{\mathcal{A}_{\mu\bullet}, \mathcal{A}_\mu^k\}$. On the other hand, in the generic case the components ζ^m, ζ^{mn} in the embedding tensor are non-vanishing, such as in the example worked out in section 2.5. Then, equation (2.21) implies that $\Omega^{MN}X_MX_N \neq 0$, i.e. there is no symplectic frame in which the gauging involves only electric vector fields. We conclude that the general gaugings including the trombone generator are necessarily and genuinely dyonic!

3 Scalar coset space and the T-tensor

In this section, we discuss the structure of the scalar sector of the theory, discuss its interplay with the gauging defined in the previous section, and define the relevant scalar field dependent tensors (T-tensors) that enter in the field equations of the gauged supergravity.

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3.1 Coset space $E_{7(7)}/\mathrm{SU}(8)$

The scalar fields of $N = 8$ supergravity can be parametrized in terms of the 56-dimensional complex vectors $\mathcal{V}_M{}^{ij} = (\mathcal{V}_\Lambda{}^{ij}, \mathcal{V}^{\Sigma\,ij})$ and their complex conjugate $\mathcal{V}_{M\,ij} = (\mathcal{V}_{\Lambda\,ij}, \mathcal{V}^\Sigma{}_{ij})$, which together constitute a 56×56 matrix $\mathcal{V}$,

$$\mathcal{V}_M{}^{\underline{N}} = \Big(\mathcal{V}_M{}^{ij}, \mathcal{V}_{M\,kl}\Big) = \begin{pmatrix} \mathcal{V}_\Lambda{}^{ij} & \mathcal{V}_{\Lambda\,kl} \\ \mathcal{V}^{\Sigma\,ij} & \mathcal{V}^\Sigma{}_{kl} \end{pmatrix} . \tag{3.1}$$

Indices $M, N, \cdots = 1, \ldots, 56$, label the fundamental representation of $E_{7(7)}$, indices $i, j, \cdots = 1, \ldots, 8$ denote the fundamental complex **8** of SU(8).[4]

The underlined indices $\underline{M}, \underline{N}, \cdots = 1, \ldots, 56$, are a collective label for the **28** + $\bar{\mathbf{28}}$ of SU(8) . The matrix $\mathcal{V}_M{}^{\underline{N}}$ transforms under rigid $E_{7(7)}$ from the left and under local $SU(8)$ from the right. Strictly speaking, it does not constitute an element of $E_{7(7)}$, but it is equal to a constant matrix (to account for the different bases adopted on both sides) times a space-time dependent element of $E_{7(7)}$. We refer to [9, 17] for further details. In particular, the scalar matrix satisfies the properties

$$\begin{aligned} \mathcal{V}_M{}^{ij}\,\mathcal{V}_{N\,ij} - \mathcal{V}_{M\,ij}\,\mathcal{V}_N{}^{ij} &= \mathrm{i}\,\Omega_{MN}\,, \\ \Omega^{MN}\,\mathcal{V}_M{}^{ij}\,\mathcal{V}_{N\,kl} &= \mathrm{i}\,\delta^{ij}{}_{kl}\,, \\ \Omega^{MN}\,\mathcal{V}_M{}^{ij}\,\mathcal{V}_N{}^{kl} &= 0\,, \end{aligned} \tag{3.2}$$

reflecting the fact that $E_{7(7)}$ is embedded into Sp(56). The covariant scalar currents $\mathcal{Q}_{\mu i}{}^j$ and $\mathcal{P}_\mu{}^{ijkl}$ are defined by

$$\partial_\mu \mathcal{V}_M{}^{ij} - A_\mu^P X_{PM}{}^N\,\mathcal{V}_N{}^{ij} \equiv -\mathcal{Q}_{\mu\,k}{}^{[i}\,\mathcal{V}_M{}^{j]k} + \mathcal{P}_\mu{}^{ijkl}\,\mathcal{V}_{Mkl}\,, \tag{3.3}$$

with gauge group generators from (2.12), and satisfy

$$\mathcal{Q}_\mu{}^i{}_j = -\mathcal{Q}_{\mu\,j}{}^i\,, \qquad \mathcal{Q}_{\mu i}{}^i = 0\,, \qquad \mathcal{P}_\mu{}^{ijkl} = \tfrac{1}{24}\,\varepsilon^{ijklmnpq}\,\mathcal{P}_{\mu\,mnpq}\,, \tag{3.4}$$

as a consequence of $\mathcal{V}_M{}^{\underline{N}}$ being related to an $E_{7(7)}$ element by multiplication with a constant matrix. The integrability conditions of (3.3) yield the Cartan-Maurer equations,

$$\begin{aligned} \mathcal{F}(\mathcal{Q})_{\mu\nu\,i}{}^j \;\equiv\; 2\partial_{[\mu}\mathcal{Q}_{\nu]i}{}^j + \mathcal{Q}_{[\mu i}{}^k\,\mathcal{Q}_{\nu]k}{}^j &= \tfrac{4}{3}\,\mathcal{P}_{[\mu}{}^{jklm}\,\mathcal{P}_{\nu]iklm} - \tfrac{2}{3} i\,\mathcal{H}^M_{\mu\nu}\,(\check{X}_M)^{PQ}\,\mathcal{V}_{P\,ik}\mathcal{V}_Q{}^{jk} \\ \mathcal{D}_{[\mu}\mathcal{P}_{\nu]}{}^{ijkl} &= -\tfrac{1}{2} i\,\mathcal{H}^M_{\mu\nu}\,(\check{X}_M)^{PQ}\,\mathcal{V}_P{}^{ij}\mathcal{V}_Q{}^{kl}\,, \end{aligned} \tag{3.5}$$

with the SU(8) covariant derivative $\mathcal{D}_\mu$ and the covariant field strength $\mathcal{H}^M_{\mu\nu}$ from (2.13). Note that its part carrying the two-forms $B_{\mu\nu\,\alpha}$ drops from (3.5) due to the orthogonality relation (2.23).

[4] Earlier, in section 2.3 we have used indices $m, n, \ldots$ in a different context labeling the 27 dimensional fundamental representation of $E_{6(6)}$. We hope that this does not lead to extra confusion.

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3.2 The T-tensor

Following [9, 17] we define the T-tensor as the gauge group generator (2.12) dressed with the scalar vielbein

$$(T_{ij})^{klmn} \equiv \tfrac{1}{2}\,(\mathcal{V}^{-1})_{ij}{}^{M}(\mathcal{V}^{-1})^{klN}\,(X_M)_N{}^{K}\,\mathcal{V}_K{}^{mn}\,, \qquad \text{etc.}\,. \tag{3.6}$$

The various components of this tensor will show up in the modified field equations of the gauged theory and parametrize the new couplings. The linear constraints (2.8) satisfied by the embedding tensor can be made explicit by parametrizing the T-tensor in terms of the irreducible SU(8) tensors A^{ij}, $A_i{}^{jkl}$, B^{ij}, transforming in the **36**, **420**, and the **28**, respectively,[5] according to

$$\begin{aligned}
(T_{ij})_{kl}{}^{mn} &= \tfrac{1}{2}\delta^{[m}_{[k}A^{n]}{}_{l]ij} + \delta^{mn}_{[i[k}A_{l]j]} - \tfrac{1}{6}(8\,\delta^{mn}_{[i[k}B_{l]j]} + \delta^{mn}_{kl}B_{ij}) - \tfrac{1}{2}\delta^{mn}_{kl}B_{ij}\,,\\
(T_{ij})^{rs}{}_{pq} &= -\tfrac{1}{2}\delta^{[r}_{[p}A^{s]}{}_{q]ij} - \delta^{rs}_{[i[p}A_{q]j]} + \tfrac{1}{6}(8\,\delta^{rs}_{[i[p}B_{q]j]} + \delta^{rs}_{pq}B_{ij}) - \tfrac{1}{2}\delta^{rs}_{pq}B_{ij}\,,\\
(T_{ij})_{kl\,pq} &= \tfrac{1}{24}\epsilon_{klpqrstu}\delta^{r}_{[i}A_{j]}{}^{stu} + \tfrac{1}{12}\epsilon_{klpqijtu}B^{tu}\,,\\
(T_{ij})^{rs\,mn} &= \delta^{[r}_{[i}A_{j]}{}^{smn]} + 2\delta^{[rs}_{ij}B^{mn]}\,.
\end{aligned} \tag{3.7}$$

The tensors A^{ij}, $A_i{}^{jkl}$ together with their complex conjugates fill the **912** representation $\Theta_M{}^{\alpha}$ of the embedding tensor and carry the structure of the standard gaugings. The tensor B^{ij} is related to the new components ϑ_M of the embedding tensor according to

$$\vartheta_M = \mathcal{V}_M{}^{ij}B_{ij} + \mathcal{V}_{M\,ij}B^{ij}\,, \tag{3.8}$$

and contains all the new contributions due to the gauging of the trombone generator. Together, the tensors A and B will describe the scalar couplings of the gauged theory. From their definition (3.7) and (3.6) one derives the differential relations

$$\mathcal{D}_\mu A^{ij} = \tfrac{1}{3}A^{(i}{}_{klm}\mathcal{P}_\mu{}^{j)klm}\,, \tag{3.9}$$

$$\mathcal{D}_\mu A_i{}^{jkl} = 2A_{im}\mathcal{P}_\mu{}^{mjkl} + 3\mathcal{P}_\mu{}^{mn[jk}A^{l]}{}_{imn} + \mathcal{P}_\mu{}^{mnp[j}\delta^{k}_{i}A^{l]}{}_{mnp}\,, \tag{3.10}$$

$$\mathcal{D}_\mu B_{ij} = -\mathcal{P}_{\mu\,ijkl}B^{kl}\,, \tag{3.11}$$

where again $\mathcal{D}_\mu$ refers to the SU(8) covariant derivative with the composite connection $\mathcal{Q}_{\mu\,i}{}^{j}$ from (3.3).

For the supersymmetry algebra it will also be useful to compute the tensor $Z^M{}_{KL}$ upon dressing with $(\mathcal{V}^{-1})_{kj}{}^{K}(\mathcal{V}^{-1})^{ijL}$:

$$\begin{aligned}
Z^M{}_{kj}{}^{ij} = &-\frac{3}{2}(\mathcal{V}^{-1\,in\,M}A_{nk} + \mathcal{V}^{-1}{}_{kl}{}^{M}A^{ni}) + \frac{3}{4}(\mathcal{V}^{-1\,mn\,M}A^{i}{}_{kmn} + \mathcal{V}^{-1}{}_{mn}{}^{M}A_{k}{}^{imn})\\
&+4(\mathcal{V}^{-1\,in\,M}B_{nk} + \mathcal{V}^{-1}{}_{kl}{}^{M}B^{ni}) + \frac{1}{2}\delta^{i}_{k}(\mathcal{V}^{-1\,mn\,M}B_{mn} + \mathcal{V}^{-1}{}_{mn}{}^{M}B^{mn})\,.
\end{aligned} \tag{3.12}$$

Dressing the quadratic constraints (2.19)–(2.21) (or alternatively (2.22)) with the scalar vielbein (3.1) induces a plethora of relations bilinear in the tensors A, B. In appendix D,

[5] I.e. $A^{ij} = A^{(ij)}$, $A_i{}^{jkl} = A_i{}^{[jkl]}$, $A_i{}^{ikl} = 0$, $B^{ij} = B^{[ij]}$, and complex conjugates $(A^{ij})^* = A_{ij}$, etc.

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we have collected a number of such identities which are important in the subsequent calculations. Here, we only give two examples of such identities. A linear combination of the constraints (D.1) transforming in the **63** of SU(8) shows that the traceless part of the hermitean tensor defined by

$$\Pi^i{}_j \equiv 6A^{ik}A_{jk} - \tfrac{1}{3}A^i{}_{mnk}A_j{}^{mnk} + \tfrac{4}{3}\left(A_j{}^{imn}B_{mn} + A^i{}_{jmn}B^{mn}\right) - \tfrac{256}{9}B^{ik}B_{jk} \,,$$

vanishes

$$\Pi^i{}_j = \tfrac{1}{8}\delta^i_j\,\Pi^k{}_k \,. \tag{3.13}$$

Another useful identity is given by the self-duality equation

$$\begin{aligned} &\Pi_{ijkl} = \tfrac{1}{24}\epsilon_{ijklmnpq}\,\Pi^{mnpq} \,, \\ \text{for}\quad &\Pi_{ijkl} \equiv A^m{}_{[ijk}A_{l]m} - \tfrac{3}{4}A^m{}_{p[ij}A^p{}_{kl]m} + 2A^m{}_{[ijk}B_{l]m} - 8B_{[ij}B_{kl]} \,, \end{aligned} \tag{3.14}$$

which is obtained as a linear combination of the constraints (D.3) transforming in the **70** of SU(8).

3.3 Vector fields

As mentioned above, only half of the 56 vector fields $\mathcal{A}^M_\mu$ enter the Lagrangian of the ungauged theory. This corresponds to selecting a symplectic frame, such that the vector fields split according to $\{\mathcal{A}^M_\mu\} \to \{\mathcal{A}^\Lambda_\mu, \mathcal{A}_{\mu\,\Lambda}\}$ into electric and magnetic fields. Accordingly, we define the electric field strengths $\mathcal{H}^\Lambda_{\mu\nu}$ via (2.13) as the curvature of $\mathcal{A}^\Lambda_\mu$ while their magnetic duals are defined as functions of the electric vector fields according to

$$\mathcal{G}^+_{\mu\nu\,\Lambda} \equiv \mathcal{N}_{\Lambda\Sigma}\,\mathcal{H}^{+\,\Sigma}_{\mu\nu} \;+\text{fermions} \,, \tag{3.15}$$

with the complex matrix $\mathcal{N}_{\Lambda\Sigma}$ defined by $\mathcal{V}^{\Sigma\,ij}\,\mathcal{N}_{\Lambda\Sigma} \equiv -\mathcal{V}_\Lambda{}^{ij}$, and where the superscript $^\pm$ refers to the (anti-)selfdual part of the field strength. The fermionic part of (3.15) is explicitly given in [1, 9, 17]. We define the full symplectic vector $\mathcal{G}^M_{\mu\nu} \equiv (\mathcal{H}^\Lambda_{\mu\nu}, \mathcal{G}_{\mu\nu\,\Lambda})$, which will in particular enter the fermionic field equations and supersymmetry transformation rules. By construction, it allows the decomposition

$$\mathcal{G}^M_{\mu\nu} = (\mathcal{V}^{-1})^{ij\,M}\mathcal{G}^+_{\mu\nu\,ij} + (\mathcal{V}^{-1})_{ij}{}^M\mathcal{G}^{-\,ij}_{\mu\nu} \;+\text{fermions} \,, \tag{3.16}$$

into its selfdual and anti-selfdual part. In contrast, we introduce the field strengths $\mathcal{H}^{ij}_{\mu\nu}$ and $\mathcal{H}_{\mu\nu\,ij}$ as the dressed version

$$\mathcal{H}^M_{\mu\nu} = (\mathcal{V}^{-1})^{ij\,M}\mathcal{H}_{\mu\nu\,ij} + (\mathcal{V}^{-1})_{ij}{}^M\mathcal{H}^{ij}_{\mu\nu} \,, \tag{3.17}$$

of the covariant non-abelian field strengths introduced in (2.13), that combine electric and magnetic vector fields. Note that $\mathcal{H}^\Lambda_{\mu\nu} = \mathcal{G}^\Lambda_{\mu\nu}$ is identically satisfied, whereas $\mathcal{H}_{\mu\nu\,\Lambda} = \mathcal{G}_{\mu\nu\,\Lambda}$ describes the first order duality relation between electric and magnetic vector fields.

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4 Supersymmetry algebra

Before deriving the full set of supersymmetric equations of motion, we establish the supersymmetry transformation rules by verifying the supersymmetry algebra. Under supersymmetry, the bosonic fields transform as

$$\begin{aligned}
\delta_\epsilon e_\mu{}^a &= \bar\epsilon^i\gamma^a\psi_{\mu i} \;+\; \bar\epsilon_i\gamma^a\psi_\mu{}^i \;, \\
\delta_\epsilon \mathcal{V}_M{}^{ij} &= 2\sqrt{2}\,\mathcal{V}_{Mkl}\left(\bar\epsilon^{[i}\chi^{jkl]} + \tfrac{1}{24}\varepsilon^{ijklmnpq}\,\bar\epsilon_m\chi_{npq}\right), \\
\delta_\epsilon \mathcal{A}_\mu^M &= -i\,\Omega^{MN}\mathcal{V}_N{}^{ij}\left(\bar\epsilon^k\,\gamma_\mu\,\chi_{ijk} + 2\sqrt{2}\,\bar\epsilon_i\,\psi_{\mu j}\right) \;+\; \text{c.c.}\;, \\
\delta_\epsilon B_{\mu\nu\,\alpha} &= \tfrac{2}{3}\sqrt{2}\,(t_\alpha)^{PQ}\left(\mathcal{V}_{P\,ij}\mathcal{V}_{Q\,kl}\,\bar\epsilon^{[i}\,\gamma_{\mu\nu}\,\chi^{jkl]} + 2\sqrt{2}\,\mathcal{V}_{P\,jk}\mathcal{V}_Q{}^{ik}\,\bar\epsilon_i\,\gamma_{[\mu}\,\psi_{\nu]}{}^j \;+\; \text{c.c.}\right) \\
&\quad + 2(t_\alpha)_{MN}\,\mathcal{A}^M_{[\mu}\,\delta\mathcal{A}^N_{\nu]}\;.
\end{aligned} \tag{4.1}$$

while the transformation of the fermions is given by

$$\begin{aligned}
\delta_\epsilon\psi^i_\mu &= 2\mathcal{D}_\mu\epsilon^i + \frac{\sqrt{2}}{4}\mathcal{G}^-_{\rho\sigma}{}^{ij}\gamma^{\rho\sigma}\gamma_\mu\epsilon_j + \sqrt{2}A^{ij}\gamma_\mu\epsilon_j - 2\sqrt{2}B^{ij}\gamma_\mu\epsilon_j\;, \\
\delta_\epsilon\chi^{ijk} &= -2\sqrt{2}\mathcal{P}_\mu{}^{ijkl}\gamma^\mu\epsilon_l + \frac{3}{2}\mathcal{G}^-_{\mu\nu}{}^{[ij}\gamma^{\mu\nu}\epsilon^{k]} - 2A_l{}^{ijk}\epsilon^l - 4B^{[ij}\epsilon^{k]}\;,
\end{aligned} \tag{4.2}$$

up to higher order fermion terms. Except for the respective last terms in the fermionic transformation rules (carrying the tensor B_{ij}), these supersymmetry transformations are known from [9, 17]. The structure of the new terms follows from the SU(8) representation content, their factors are determined from the closure of the supersymmetry algebra. This algebra is given by

$$[\delta(\epsilon_1),\delta(\epsilon_2)] = \xi^\mu\hat D_\mu + \delta_{\rm Lor}(\Omega^{ab}) + \delta_{\rm susy}(\epsilon_3) + \delta_{\rm SU(8)}(\Lambda^i{}_j) + \delta_{\rm gauge}(\Lambda^M) + \delta_{\rm gauge}(\Xi_{\mu\alpha})\,. \tag{4.3}$$

The first term refers to a covariantized general coordinate transformation with diffeomorphism parameter

$$\xi^\mu = 2\,\bar\epsilon_2{}^i\gamma^\mu\epsilon_{1\,i} + 2\,\bar\epsilon_{2\,i}\gamma^\mu\epsilon_1{}^i\;, \tag{4.4}$$

and including terms of order g induced by the gauging. The last two terms refer to gauge transformations (2.11), (2.14) and (2.15), with parameters

$$\begin{aligned}
\Lambda^N &= -4i\sqrt{2}\,\Omega^{NP}\,(\mathcal{V}_P{}^{mn}\bar\epsilon_{2\,m}\epsilon_{1\,n} - \mathcal{V}_{P\,mn}\bar\epsilon_2^m\epsilon_1^n)\;, \\
\Xi_{\mu\alpha} &= -\tfrac{8}{3}(t_\alpha)^{PQ}\,\mathcal{V}_{P\,ik}\mathcal{V}_Q{}^{jk}\left(\bar\epsilon_2{}^i\gamma_\mu\epsilon_{1j} + \bar\epsilon_{2j}\gamma_\mu\epsilon_1{}^i\right)\;,
\end{aligned} \tag{4.5}$$

respectively. Up to the contributions from the new terms in the supersymmetry transformation rules, the supersymmetry algebra has been verified in [9, 17]. In presence of B_{ij}, B^{ij}, the commutator (4.3) evaluated on the vielbein acquires the additional terms

$$[\delta_{\epsilon_1},\delta_{\epsilon_2}]\,e_\mu{}^a = \ldots \quad - 4\sqrt{2}\,(B^{mn}\bar\epsilon_{2\,m}\epsilon_{1\,n} + B_{mn}\bar\epsilon_2^m\epsilon_1^n)\,e_\mu{}^a\;. \tag{4.6}$$

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These precisely reproduce the action of a scaling gauge transformation with parameter (4.5) on the vielbein

$$\begin{aligned}\delta_\Lambda e_\mu{}^a &= \Lambda^M \vartheta_M \, \mathbf{t}_0 \cdot e_\mu{}^a \\ &= -4i\sqrt{2}\, \Omega^{MN} \vartheta_M (\mathcal{V}_N{}^{mn} \bar\epsilon_{2\,m} \epsilon_{1\,n} - \mathcal{V}_{N\,mn} \bar\epsilon_2^m \epsilon_1^n)\, e_\mu{}^a \\ &= -4\sqrt{2}\, (B^{mn} \bar\epsilon_{2\,m} \epsilon_{1\,n} + B_{mn} \bar\epsilon_2^m \epsilon_1^n)\, e_\mu{}^a \,,\end{aligned} \tag{4.7}$$

where we have used (3.2) and (3.8). Similarly, one may check that the terms carrying the scalar tensors (3.7) in the supersymmetry commutator on the scalar fields combine into

$$\begin{aligned}\mathcal{V}^{-1\,ij\,M} [\delta_{\epsilon_1}, \delta_{\epsilon_2}] \mathcal{V}_M{}^{kl} &= \ldots \; - 8\sqrt{2} \left(T^{mnijkl}\, \bar\epsilon_{2\,m} \epsilon_{1\,n} + T_{mn}{}^{ijkl}\, \bar\epsilon_2^m \epsilon_1^n \right) \\ &= \ldots \; + \mathcal{V}^{-1\,ij\,M}\, \Lambda^N (X_N)_M{}^K\, \mathcal{V}_K{}^{kl} \,, \qquad \text{etc.}\end{aligned}$$

and consistently reproduce the action of a gauge transformation with parameter (4.5). In checking the supersymmetry algebra on the vielbein and the scalar fields, we have fixed all the new factors in the supersymmetry transformation rules (4.2). As a consistency check, one may further verify that the algebra also closes on the vector and the tensor gauge fields.

5 Equations of motion

5.1 Einstein equations

Having established the supersymmetry algebra, we can now determine the deformed equations of motion by requiring covariance under the new supersymmetry transformation rules. As there is no longer an action underlying the gauged theory, we have to work directly on the level of the equations of motion. This derivation of the supersymmetric field equations is based on reference [36]. We will start from the gravitino equations of motion for which we use the following ansatz

$$\begin{aligned}0 \;=\; (\mathcal{E}_\psi)^\mu{}_i \equiv\; & -e^{-1} \varepsilon^{\mu\nu\rho\sigma} \gamma_\nu \mathcal{D}_\rho \psi_{\sigma\,i} - \tfrac{\sqrt{2}}{6} \gamma^\nu \gamma^\mu \chi^{jkl}\, \mathcal{P}_{\nu\,jkli} - \tfrac{\sqrt{2}}{4} \mathcal{G}^{+\,\rho\sigma}{}_{ij} \gamma^{[\mu} \gamma_{\rho\sigma} \gamma^{\nu]} \psi_\nu{}^j \\ & + \tfrac{1}{8} \mathcal{G}^{-\,\rho\sigma jk} \gamma_{\rho\sigma} \gamma^\mu \chi_{jki} + \sqrt{2} A_{ij} \gamma^{\mu\nu} \psi_\nu{}^j + \tfrac{1}{6} A_i{}^{jkl} \gamma^\mu \chi_{jkl} \\ & + \lambda \sqrt{2} B_{ij} \gamma^{\mu\nu} \psi_\nu{}^j + \zeta\, B^{kl} \gamma^\mu \chi_{ikl} \,.\end{aligned} \tag{5.1}$$

Except for the last two terms, these are the equations obtained from variation of the Lagrangian [9, 17] of the gauged theory. While the SU(8) structure of these two additional terms is fully determined by the representation content, we will in the following determine their unknown coefficients λ and ζ by compatibility with supersymmetry.

E.g. vanishing of the $\mathcal{D}_\mu \epsilon$ terms in the supersymmetry variation of (5.1) imposes

$$2\sqrt{2}\, B_{ij} \left(\lambda \gamma^{\mu\nu} - e^{-1} \varepsilon^{\mu\nu\rho\sigma} \gamma_{\rho\sigma} \right) \mathcal{D}_\nu \epsilon^j = 0 \,, \tag{5.2}$$

from which we deduce $\lambda = -2$. Vanishing of the terms linear in $B\mathcal{G}^{\pm}\epsilon$ further determines $\zeta = -5/3$, but we will for the moment keep the parameters in the formulas so as to allow for further consistency checks.

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Let us concentrate on the part of the supersymmetry variation of (5.1) which is bilinear in the scalar tensors A, B which originate from variation of its last four terms. We obtain

$$\begin{aligned}\Big\{&6A_{ik}A^{jk} - \frac{1}{3}A_i{}^{klm}A^j{}_{klm} + 12A_{ik}B^{jk} + 6\lambda B_{ik}A^{jk} \\ &- \frac{2}{3}A_i{}^{jlm}B_{lm} - 2\zeta A^j{}_{ikl}B^{kl} + \left(12\lambda + \frac{8\zeta}{3}\right)B_{ik}B^{jk} - \frac{4\zeta}{3}\delta_i^j B_{kl}B^{kl}\Big\} \times \gamma^\mu\epsilon_j \ .\end{aligned} \tag{5.3}$$

Only the (ij)-trace of the braced expression can be absorbed into a modification of the Einstein equations. In particular its anti-hermitean part must vanish. Indeed, this follows from the first of the bilinear constraint relations (D.1) provided that $\lambda = 2\zeta + \frac{4}{3}$, which is satisfied for our above choice of constants. With this value for λ, the expression (5.3) reduces to its hermitean part

$$\begin{aligned}\Big\{&6A_{ik}A^{jk} - \frac{1}{3}A_i{}^{klm}A^j{}_{klm} + (10+6\zeta)(A_{ik}B^{jk} + B_{ik}A^{jk}) \\ &- \big(\tfrac{1}{3}+\zeta\big)(A_i{}^{jlm}B_{lm} + A^j{}_{ikl}B^{kl}) + \left(16 + \frac{80\zeta}{3}\right)B_{ik}B^{jk} - \frac{4\zeta}{3}\delta_i^j B_{kl}B^{kl}\Big\} \times \gamma^\mu\epsilon_j \ .\end{aligned} \tag{5.4}$$

Finally, we observe that with the above value $\zeta = -5/3$ all coefficients precisely reproduce the linear combination appearing in the quadratic constraint (3.13), such that the full expression reduces to its trace part

$$\left\{\frac{3}{4}A_{kl}A^{kl} - \frac{1}{24}A_n{}^{jkl}A^n{}_{jkl} - \frac{4}{3}B_{kl}B^{kl}\right\} \times \gamma^\mu\epsilon_i \ , \tag{5.5}$$

which can be absorbed into the modified Einstein equations. Another important ingredient in the calculation is the evaluation of the commutator

$$\begin{aligned}\gamma^{\mu\nu\rho}\,[\mathcal{D}_\nu,\mathcal{D}_\rho]\,\epsilon_i &= \gamma^{\mu\nu\rho}\left(\tfrac{1}{2}\mathcal{F}(\mathcal{Q})_{\nu\rho\,i}{}^j\,\epsilon_j - \tfrac{1}{4}\widehat{\mathcal{R}}_{\nu\rho}{}^{ab}\gamma_{ab}\,\epsilon_i - \tfrac{1}{2}\vartheta_M\mathcal{H}^M_{\nu\rho}\,\epsilon_i\right) \\ &= \left(\tfrac{1}{2}g^{\mu\nu}\widehat{\mathcal{R}} - \widehat{\mathcal{R}}^{(\mu\nu)}\right)\gamma_\nu\epsilon_i + \vartheta_M\left(\mathcal{H}^{-\,\mu\nu M} - 3\mathcal{H}^{+\,\mu\nu M}\right)\gamma_\nu\epsilon_i \\ &\quad + \tfrac{1}{2}\gamma^{\mu\nu\rho}\,\mathcal{F}(\mathcal{Q})_{\nu\rho\,i}{}^j\epsilon_j \ ,\end{aligned} \tag{5.6}$$

with the modified Riemann tensor and the curvature of the SU(8) connection from (2.4) and (3.5), respectively

Putting all this together, a somewhat lengthy calculation shows that the full supersymmetry variation of the Rarita-Schwinger equation (5.1) eventually takes the form

$$\delta_\epsilon(\mathcal{E}_\psi)^\mu{}_i = (\mathcal{E}_{\text{Einstein}})^{\mu\nu}\,\gamma_\nu\epsilon_i - 2\sqrt{2}\,(\mathcal{E}_{\text{vector}})^\mu{}_{ij}\,\epsilon^j \ , \tag{5.7}$$

where $(\mathcal{E}_{\text{Einstein}})^{\mu\nu}$ and $(\mathcal{E}_{\text{vector}})^\mu{}_{ij}$ denote the modified Einstein and the vector field equations of motion, respectively. In bringing the supersymmetry variation into this form, we have in particular made use of the equations

$$X_M\left(\mathcal{G}^M_{\mu\nu} - \mathcal{H}^M_{\mu\nu}\right) \ = \ 0 \ = \ \vartheta_M\left(\mathcal{G}^M_{\mu\nu} - \mathcal{H}^M_{\mu\nu}\right) \ . \tag{5.8}$$

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For a purely electric gauging, these equations are identically satisfied. In presence of magnetic charges, these equations represent the first order duality equations between electric and magnetic vector fields. The second order field equations obtained in (5.7) read

$$(\mathcal{E}_{\text{Einstein}})^{\mu\nu} = \widehat{\mathcal{R}}^{(\mu\nu)} - \tfrac{1}{2}g^{\mu\nu}\widehat{\mathcal{R}} + \tfrac{1}{6}\,\mathcal{P}^{(\mu}_{ijkl}\mathcal{P}^{\nu)ijkl} - \tfrac{1}{12}g^{\mu\nu}\,\mathcal{P}^{\rho}_{ijkl}\mathcal{P}_{\rho}^{ijkl} - \mathcal{G}^{+}{}_{\rho}{}^{(\mu}{}_{jk}\,\mathcal{G}^{\nu)\rho-\,jk}$$
$$+g^{\mu\nu}\left(\tfrac{3}{4}A_{kl}A^{kl} - \tfrac{1}{24}A_{n}{}^{jkl}A^{n}{}_{jkl} - \tfrac{4}{3}B_{kl}B^{kl}\right)\,. \tag{5.9}$$

and

$$(\mathcal{E}_{\text{vector}})^{\mu}{}_{ij} = \mathcal{D}_{\nu}\mathcal{G}^{+\nu\mu}{}_{ij} + \mathcal{P}_{\nu\,ijkl}\,\mathcal{G}^{-\,\nu\mu\,kl} - \tfrac{1}{3}\left(A_{[i}{}^{nkl}\mathcal{P}^{\mu}{}_{j]nkl} + 4B^{kl}\mathcal{P}^{\mu}{}_{ijkl}\right)\,. \tag{5.10}$$

The modified Einstein equations show that the presence of the tensor B_{ij} induces a positive contribution to the effective cosmological constant. This is a typical feature of the theories with trombone gauging [19–24, 37].

5.2 Scalar field equations

We start from the following ansatz for the equations of motion for the spin-1/2 fermion fields χ_{ijk}

$$0 \;=\; (\mathcal{E}_{\chi})_{ijk} \equiv -\tfrac{1}{6}\gamma^{\mu}\mathcal{D}_{\mu}\chi_{ijk} - \tfrac{\sqrt{2}}{6}\gamma^{\nu}\gamma^{\mu}\psi_{\nu}{}^{l}\mathcal{P}_{\mu\,ijkl} + \tfrac{1}{8}\gamma_{\rho}\gamma_{\mu\nu}\psi^{\rho}{}_{[k}\mathcal{G}^{+\mu\nu}{}_{ij]}$$
$$+\tfrac{\sqrt{2}}{288}\epsilon_{ijklmnpq}\gamma^{\mu\nu}\chi^{lmn}\mathcal{G}^{-}{}_{\mu\nu}{}^{pq} - \tfrac{1}{6}A^{l}{}_{ijk}\gamma^{\mu}\psi_{\mu\,l} + 2A_{ijk,\,lmn}\chi^{lmn}$$
$$-\tfrac{1}{3}\gamma^{\mu}\psi_{\mu[k}B_{ij]} + \tfrac{\sqrt{2}}{36}\epsilon_{ijklmnpq}\chi^{lmn}B^{pq}\,, \tag{5.11}$$

with the scalar tensor

$$A_{ijk,lmn} = \tfrac{1}{144}\sqrt{2}\,\epsilon_{ijkpqr[lm}\,A_{n]}{}^{pqr}\,. \tag{5.12}$$

Again, up to the last two terms whose structure is determined by SU(8), equations (5.11) follow from varying the Lagrangian of [9, 17]. The new coefficients are fixed by compatibility with supersymmetry and follow as in the last section by imposing the vanishing of the linear terms of the form $B\mathcal{D}\epsilon$ and $B\mathcal{P}\epsilon$ in the supersymmetry variation of (5.11).

Again we first focus on the part of the supersymmetry variation of $(\mathcal{E}_{\chi})_{ijk}$ which is bilinear in the scalar tensors A, B and find

$$-\frac{\sqrt{2}}{3}\left\{2A^{r}{}_{ijk}A_{lr} + 4A^{r}{}_{ijk}B_{lr} + 4B_{[ij}A_{k]l} - 8B_{[ij}B_{k]l} - \frac{1}{9}\epsilon_{ijklrmnp}A_{q}{}^{rmn}B^{pq}\right.$$
$$\left. + \frac{1}{12}\epsilon_{ijkrmnpq}A_{l}{}^{pqs}\left(A_{s}{}^{rmn} + \tfrac{8}{3}\delta^{r}_{s}B^{mn}\right) + \frac{1}{3}\epsilon_{ijklmnpq}B^{mn}B^{pq}\right\} \times \epsilon^{l}\,. \tag{5.13}$$

Upon adding a proper linear combination of the two quadratic constraints (D.4) in the **378**, this expression reduces to

$$-\frac{\sqrt{2}}{3}\left\{2A^{r}{}_{[ijk}A_{l]r} + 4A^{r}{}_{[ijk}B_{l]r} - 8B_{[ij}B_{kl]}\right.$$
$$\left. + \frac{1}{16}\epsilon_{ijklmnpq}\left(A_{r}{}^{pqs}A_{s}{}^{mnr} + \tfrac{16}{3}B^{mn}B^{pq}\right)\right\} \times \epsilon^{l}\,, \tag{5.14}$$

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which is manifestly antisymmetric in $[ijkl]$. Finally, the combination of quadratic constraints (3.14) can be used to simplify this expression to the manifestly self-dual expression

$$-\frac{\sqrt{2}}{3}\left\{\mathcal{C}_{ijkl}+\tfrac{1}{24}\epsilon_{ijklmnpq}\mathcal{C}^{mnpq}\right\}\times\epsilon^{l}\,, \tag{5.15}$$

with the tensor

$$\mathcal{C}_{ijkl}=A^{m}{}_{[ijk}A_{l]m}+\tfrac{3}{4}A^{m}{}_{n[ij}A^{n}{}_{kl]m}+2A^{m}{}_{[ijk}B_{l]m}\,. \tag{5.16}$$

This expression will be part of the modified scalar field equations. After some more calculation, and using the first order field equations (5.8), the full supersymmetry variation of the fermionic field equation $(\mathcal{E}_\chi)_{ijk}$ eventually takes the form

$$\delta_\epsilon(\mathcal{E}_\chi)_{ijk}=\tfrac{\sqrt{2}}{3}\,(\mathcal{E}_{\rm scalars})_{ijkl}\,\epsilon^{l}-\gamma_\mu\,(\mathcal{E}_{\rm vector})^{\mu}{}_{[ij}\,\epsilon_{k]}\,. \tag{5.17}$$

with the vector field equations from (5.10) and the full scalar field equations given by

$$\begin{aligned}(\mathcal{E}_{\rm scalars})_{ijkl}=\;&\mathcal{D}^{\mu}\mathcal{P}_{\mu\,ijkl}-\tfrac{3}{2}\mathcal{G}^{+}_{\mu\nu\,[ij}\mathcal{G}^{+\,\mu\nu}{}_{kl]}-\tfrac{1}{16}\epsilon_{ijklpqrs}\mathcal{G}^{-\mu\nu\,pq}\mathcal{G}^{-}{}_{\mu\nu}{}^{rs}\\&-\mathcal{C}_{ijkl}-\tfrac{1}{24}\epsilon_{ijklpqrs}\,\mathcal{C}^{pqrs}\,.\end{aligned} \tag{5.18}$$

We note that the term bilinear in the tensor B_{ij} has dropped out from the scalar field equations. Also this is a characteristic feature for theories with trombone gauging.[6] As a consequence, for pure trombone gaugings ($\Theta_M{}^{\alpha}=0$, implying that $A_{ij}=0=A_i{}^{jkl}$) a simple solution to the bosonic field equations is given by a de Sitter geometry with all scalar and vector fields vanishing. We shall discuss this solution in more detail below. Let us finally note, that due to the presence of the mixed term $A^{m}{}_{[ijk}B_{l]m}$ the scalar field equations (5.18) cannot be integrated up to a scalar potential.

5.3 Yang-Mills equations

The vector field equations of motion (5.10) can be rewritten equivalently as

$$\mathcal{V}_M{}^{ij}\,\mathcal{D}_{[\mu}\,\mathcal{G}_{\nu\rho]}{}^{M}=-\tfrac{1}{9}e\varepsilon_{\mu\nu\rho\sigma}\left\{A^{[i}{}_{nkl}\mathcal{P}^{\sigma\,j]nkl}+4B_{kl}\mathcal{P}^{\sigma\,ijkl}\right\}\,, \tag{5.19}$$

or

$$\mathcal{D}_{[\mu}\,\mathcal{G}_{\nu\rho]}{}^{M}=\tfrac{1}{9}e\varepsilon_{\mu\nu\rho\sigma}Z^{M\alpha}(t_\alpha)^{KL}\mathcal{V}_K{}^{ij}\mathcal{V}_L{}^{kl}\mathcal{P}^{\sigma}{}_{ijkl}\,, \tag{5.20}$$

with the tensor $Z^{M\alpha}$ from (2.9). Since in the derivation of the field equations we have also come across the first order duality equations (5.8) for the vector fields, an immediate question is the compatibility of these equations with the second order field equations. Upon contracting equation (5.20) with ϑ_M or X_M, its r.h.s. vanishes by virtue of (2.23) while on the l.h.s. the first order duality equations (5.8) allow to replace $\mathcal{G}^{M}_{\nu\rho}$ by the covariant field strength $\mathcal{H}^{M}_{\nu\rho}$. Then also the l.h.s. vanishes by virtue of the Bianchi identity (2.16)

[6] Let us correct a misprint in reference [24]: the last term in equation (4.44) of that reference is in fact absent in accordance with the equation obtained by dimensional reduction of (5.18).

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and (2.23). Both sets of equations are thus compatible. As another consistency check, we observe that upon applying the operator $\varepsilon^{\mu\nu\rho\tau}\mathcal{D}_\tau$, the l.h.s. of (5.20) reduces to

$$-\tfrac{1}{2}\varepsilon^{\mu\nu\rho\tau}\,\mathcal{G}^K_{\mu\nu}\mathcal{G}^L_{\rho\tau}\,X_{KL}{}^M = -Z^{M\alpha}\,(t_\alpha)_{KL}\,\left(\mathcal{G}^{+\,K}_{\mu\nu}\mathcal{G}^{+\mu\nu\,L}+\mathcal{G}^{-\,K}_{\mu\nu}\mathcal{G}^{-\mu\nu\,L}\right)\,, \tag{5.21}$$

whereas the r.h.s. contains the scalar field equation (5.18). This provides a number of important non-trivial consistency checks on the set of bosonic field equations that we have obtained from supersymmetry variation, but not derived from an action.

5.4 Maximally symmetric solutions and mass matrices

According to (5.18), a solution to the field equations with constant scalar and vanishing vector fields requires the anti-selfduality condition

$$\mathcal{C}_{ijkl}+\tfrac{1}{24}\varepsilon_{ijklpqrs}\,\mathcal{C}^{pqrs}=0\,, \tag{5.22}$$

for the scalar field dependent tensor $\mathcal{C}_{ijkl}$ from (5.16). For the standard gaugings, this is precisely the condition for an extremal point of the scalar potential [9]. In presence of the local scaling symmetry, however, we recall that the scalar field equations can in general not be integrated up to a scalar potential. Any solution to (5.22) yields a solution to the field equations with maximally symmetric four-dimensional spacetime and cosmological constant

$$\Lambda=-\tfrac{3}{4}A_{kl}A^{kl}+\tfrac{1}{24}A_n{}^{jkl}A^n{}_{jkl}+\tfrac{4}{3}B_{kl}B^{kl}\,. \tag{5.23}$$

The spectrum of the theory around this solution can be obtained by linearizing the field equations. Using (D.2), linearization of the scalar field equations (5.18) around a solution of (5.22) yields to lowest order

$$\Box\,\phi_{ijkl}=\mathcal{M}_{ijkl}{}^{mnpq}\,\phi_{mnpq}+\mathcal{O}(\phi^2)\,, \tag{5.24}$$

with self-dual scalar fields $\phi_{ijkl}=\frac{1}{24}\varepsilon_{ijklpqrs}\,\phi^{pqrs}$ and the scalar mass matrix $\mathcal{M}_{ijkl}{}^{mnpq}$ whose symmetric part is given by

$$\begin{aligned}\mathcal{M}_{ijkl}{}^{mnpq}\,\phi^{ijkl}\phi_{mnpq}=\;&6\left(A_m{}^{ijk}A^l_{ijn}-\tfrac{1}{4}A_i{}^{jkl}A^i_{jmn}-A_m{}^{ikl}B_{in}-A^k{}_{imn}B^{il}\right)\phi^{mnpq}\phi_{klpq}\\&+\left(\tfrac{5}{24}\,A_i{}^{jkl}A^i{}_{jkl}-\tfrac{1}{2}A_{ij}A^{ij}\right)\phi^{mnpq}\phi_{mnpq}\\&-\tfrac{2}{3}\,A_i{}^{jkl}A^m{}_{npq}\,\phi^{inpq}\phi_{jklm}\,,\end{aligned} \tag{5.25}$$

while its antisymmetric part reads

$$\begin{aligned}\mathcal{M}_{ijkl}{}^{mnpq}\,\phi^{ijkl}_{[1}\phi_{2]\,mnpq}=\;&\tfrac{8}{3}\left(B_{rs}A_i{}^{rsm}-B^{rs}A^m{}_{rs[i}\right)\phi^{ijkl}_{[1}\phi_{2]\,mjkl}\\&-4\left(A_i{}^{mnp}B_{jk}-A^m{}_{ijk}B^{np}\right)\phi^{ijkl}_{[1}\phi_{2]\,mnpl}\,.\end{aligned} \tag{5.26}$$

The calculation of (5.25), (5.26) makes use of identities for self-dual tensors, such as those given in [38] as well as of the quadratic constraints derived in appendix D. For $B^{ij}=0$,

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the mass matrix consistently reduces to the expression given in [39] and its antisymmetric part vanishes.

For the vector field equations (5.10) we find the linearized form

$$\Box\,\mathcal{A}_{\mu\,ij} = \tfrac{2}{3}\left(A_{[i}{}^{nkl} - 4\delta^n_{[i}B^{kl}\right)(T^{pq})_{j]nkl}\,\mathcal{A}_{\mu\,pq} + \tfrac{2}{3}\left(A_{[i}{}^{nkl} - 4\delta^n_{[i}B^{kl}\right)(T_{pq})_{j]nkl}\,\mathcal{A}^{pq}_{\mu}\,,$$

from which using (3.6), (3.7) we read off the vector mass matrix

$$\mathcal{M}_{\rm vec} = \begin{pmatrix} \mathcal{M}_{ij}{}^{kl} & \mathcal{M}_{ijkl} \\ \mathcal{M}^{ijkl} & \mathcal{M}^{ij}{}_{kl} \end{pmatrix}\,, \tag{5.27}$$

with

$$\begin{aligned}
\mathcal{M}_{ij}{}^{kl} &= -\tfrac{1}{6}A_{[i}{}^{npq}\delta^{[k}_{j]}A^{l]}{}_{npq} + \tfrac{1}{2}A_{[i}{}^{pq[k}A^{l]}{}_{j]pq} + \tfrac{2}{3}\delta^{[k}_{[i}A_{j]}{}^{l]pq}B_{pq} - \tfrac{2}{3}A_{[i}{}^{nkl}B_{j]n} \\
&\quad -\tfrac{4}{3}\delta^{[k}_{[i}A^{l]}{}_{j]pq}B^{pq} + \tfrac{4}{3}A^{[k}{}_{nij}B^{l]n} - \tfrac{8}{9}\delta^{kl}_{ij}B^{pq}B_{pq} - \tfrac{8}{9}B^{kl}B_{ij} + \tfrac{32}{9}B^{n[k}\delta^{l]}_{[i}B_{j]n}\,, \\
\mathcal{M}_{ijkl} &= \tfrac{1}{36}A_{[i}{}^{pqr}\epsilon_{j]pqrmns[k}A_{l]}{}^{mns} - \tfrac{1}{18}\epsilon_{klmnpqr[i}A_{j]}{}^{pqr}B^{mn} \\
&\quad +\tfrac{1}{9}\epsilon_{ijpqrmn[k}A_{l]}{}^{pqr}B^{mn} - \tfrac{2}{9}\epsilon_{ijklmnpq}B^{mn}B^{pq}\,.
\end{aligned} \tag{5.28}$$

Finally, the gravitino and fermion mass matrices are directly read off from (5.1) and (5.11), respectively and take the form

$$\begin{aligned}
\mathcal{M}_{\psi}{}^{ij} &= \sqrt{2}\left(A^{ij} - 2B^{ij}\right)\,, \\
\mathcal{M}_{\chi}{}^{ijk,lmn} &= \tfrac{1}{12}\sqrt{2}\left(\epsilon^{ijkpqr[lm}\,A^{n]}{}_{pqr} + 2\epsilon^{ijklmnpq}B_{pq}\right)\,,
\end{aligned} \tag{5.29}$$

where the first matrix carries the information about the breaking of supersymmetry and the latter matrix has to be evaluated after projecting out the fermions that are eaten by the massive gravitinos.

6 Example: de Sitter geometry and mass spectrum

We have in the previous section derived the full set of fermionic and bosonic field equations of the gauging in presence of the trombone generator and shown that they transform into each other under supersymmetry. In absence of an action, these equations capture the full dynamics of the theory. As a simple example and application of the construction, in this section we analyze in more detail the gauging discussed at the end of section 2.3, parametrized by κ and Ξ^a. In particular, we show that this theory admits a de Sitter solution with constant scalar fields and work out its mass spectrum by linearizing the equations of motion around the vacuum solution. In the absence of the trombone gauging, i.e. for $\kappa = 0$, the theory is characterized by an $E_{6(6)}$ generator Ξ^a and corresponds to the Scherk-Schwarz reduction from five dimensions first analyzed in [40, 41] and revisited in the context of four-dimensional gaugings in [14].

As a first step, we calculate for this theory the value of the tensors A_{ij}, $A_i{}^{jkl}$ and B^{ij} at the origin, i.e. for all scalar fields vanishing. Since at the origin, the group $E_{6(6)}$ is broken

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down to its maximally compact subgroup, the values of these tensors will be expressed in terms of the USp(8) building blocks $(\kappa, \xi^{ij}, \xi^{ijkl})$, transforming in the 1, 36, and 42 of USp(8), respectively, of which the latter two compose the $E_{6(6)}$ generator Ξ^a. The indices $i, j, \cdots = 1, \ldots 8$, here label the fundamental representation of USp(8). Explicitly, these tensors satisfy the relations

$$\xi^{ij} = \xi^{ji} \ , \quad \xi^{ijkl} = \xi^{[ijkl]} \ , \quad \xi^{ijkl}\omega_{kl} = 0 \ , \tag{6.1}$$

with the USp(8)-invariant symplectic matrix ω_{ij}, and the reality properties

$$(\xi^{ij})^* = \xi_{ij} = \omega_{ik}\omega_{jl}\xi^{kl} \ , \qquad \text{etc.} \tag{6.2}$$

At the origin, the scalar tensors A_{ij}, $A_i{}^{jkl}$ and B^{ij} take the form

$$A^{ij} = \tfrac{1}{\sqrt{2}}\xi^{ij} \ , \quad A_i{}^{jkl} = -\tfrac{3}{\sqrt{2}}\omega_{im}\xi^{m[j}\omega^{kl]} + \omega_{im}\xi^{mjkl} \ , \quad B^{ij} = \tfrac{1}{\sqrt{2}}\kappa\,\omega^{ij} \ . \tag{6.3}$$

The condition for extremality (5.22) coming from the scalar field equations splits into the equations

$$\kappa\,\xi^{ijkl} = \sqrt{2}\omega_{mn}\xi^{m[i}\xi^{jkl]n} \ , \qquad \xi^{ijkl}\xi_{ijkl} = 0 \ . \tag{6.4}$$

Obviously, even for non-vanishing parameter κ these equations leave no other solution than $\xi^{ijkl} = 0$, i.e. induce a Scherk-Schwarz gauging with a compact generator of $E_{6(6)}$. On the other hand this shows that choosing $\xi^{ijkl} = 0$ suffices to guarantee that the scalar field equations (5.18) are solved by setting all scalar fields to zero. For the cosmological constant (5.23), we obtain

$$\Lambda = \tfrac{3}{8}\xi^{ijkl}\xi_{ijkl} + \tfrac{16}{3}\kappa^2 = \tfrac{16}{3}\kappa^2 \ , \tag{6.5}$$

i.e. the Einstein field equations (5.9) are solved by a Minkowski space for the standard gaugings and by a de Sitter geometry for non-vanishing κ.

The fermionic mass spectrum for this solution is obtained by linearizing the fermionic field equations (5.1), (5.11) around the de Sitter background with the mass matrices given by (5.29). For the eight gravitino masses we obtain

$$m_{\text{grav}} : \pm\sqrt{m_i^2 + 4\kappa^2} \ , \qquad i = 1, \ldots, 4 \ , \tag{6.6}$$

where we have denoted by im_i the eigenvalues of the anti-hermitean matrix $\xi_i{}^j$. For non-vanishing κ thus all supersymmetries are broken, as is required by the de Sitter geometry and 8 Goldstinos are found among the spin-1/2 fermions. For vanishing κ on the other hand, supersymmetry is broken according to the number of non-vanishing eigenvalues of $\xi_i{}^j$, in accordance with the results of [40, 41].

The remaining fermion masses are given by

$$m_{\text{ferm}} : \pm\sqrt{m_i^2 + 4\kappa^2} \ , \quad \pm\sqrt{(m_i \pm m_j \pm m_k)^2 + 4\kappa^2} \ , \ \ (i < j < k) \ . \tag{6.7}$$

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We find that the effect of the additional trombone gauging is a shift in all the fermion masses. Likewise, we may calculate the scalar mass matrix (5.25), (5.26), with (6.3) and obtain

$$\mathcal{M}_{ijkl}{}^{mnpq} = \mathbb{P}_{42}\left(\xi^r{}_s\xi^s{}_r\,\delta^{mnpq}_{ijkl} - 24\xi^{[m}{}_{[i}\,\xi^n{}_j\,\delta^{pq]}_{kl]} + 32\,\kappa\,\xi^{[m}{}_{[i}\,\delta^{npq]}_{jkl]}\right)\mathbb{P}_{42}\,, \tag{6.8}$$

where $\mathbb{P}_{42}$ refers to the projector onto the 42 scalars in the decomposition

$$\mathbf{70} \longrightarrow \mathbf{1} + \mathbf{27} + \mathbf{42}\,, \tag{6.9}$$

of SU(8) under USp(8), i.e. all other scalars come with zero mass. In (6.8), the last term lives entirely in the antisymmetric part of the mass matrix. In terms of the eigenvalues of $\xi^m{}_n$, diagonalization of (6.8) leads to the following spectrum

$$\begin{aligned}
0 \;||\; & 30\times \\
(m_i \pm m_j)^2 + 4i\kappa|m_i \pm m_j| \;||\; & i<j \\
(m_i \pm m_j)^2 - 4i\kappa|m_i \pm m_j| \;||\; & i<j \\
(m_1 \pm m_2 \pm m_3 \pm m_4)^2 + 4i\kappa|m_1 \pm m_2 \pm m_3 \pm m_4| \;||\; & \\
(m_1 \pm m_2 \pm m_3 \pm m_4)^2 - 4i\kappa|m_1 \pm m_2 \pm m_3 \pm m_4| \;||\; & \,,
\end{aligned} \tag{6.10}$$

for the masses of the scalar fields. For vanishing κ, we precisely reproduce the mass spectrum of [40]. Upon switching on κ, all non-vanishing mass-eigenvalues degenerate according to $m^2 \to m^2 \pm 4i\kappa m$. The fact that most of the mass eigenvalues come out to be imaginary is due to the antisymmetric contributions to the mass matrix. Finally, the vector mass matrix (5.27) takes the form

$$\begin{aligned}
\mathcal{M}_{ij}{}^{kl} &= -\xi^{[k}{}_{[i}\xi^{l]}{}_{j]} + \xi^{[k}{}_n\delta^{l]}_{[i}\,\xi^n{}_{j]} + 4\kappa\delta^{[k}_{[i}\xi^{l]}{}_{j]} - \tfrac{16}{9}\kappa^2\delta^{kl}_{ij} - \tfrac{4}{9}\kappa^2\,\omega_{ij}\omega^{kl}\,, \\
\mathcal{M}_{ijkl} &= \xi_{i[k}\xi_{l]j} - \tfrac{1}{2}\omega_{i[k}\xi^n{}_{l]}\xi_{jn} + \tfrac{1}{2}\omega_{j[k}\xi^n{}_{l]}\xi_{in} - 2\kappa\omega_{i[k}\xi_{l]j} + 2\kappa\omega_{j[k}\xi_{l]i} \\
&\quad -\tfrac{8}{9}\kappa^2\omega_{ij}\omega_{kl} - \tfrac{16}{9}\kappa^2\omega_{i[k}\omega_{l]j}\,.
\end{aligned} \tag{6.11}$$

In terms of the eigenvalues of the matrix $\xi^m{}_n$, we find the following mass spectrum

$$\begin{aligned}
0 \;||\; & 28\times \\
(m_i \pm m_j)^2 + 4i\kappa|m_i \pm m_j| - \tfrac{32}{9}\kappa^2 \;||\; & i<j \\
(m_i \pm m_j)^2 - 4i\kappa|m_i \pm m_j| - \tfrac{32}{9}\kappa^2 \;||\; & i<j \\
-\tfrac{32}{9}\kappa^2 \;||\; & 3\times \\
-\tfrac{32}{3}\kappa^2 \;||\; & 1\times\,.
\end{aligned} \tag{6.12}$$

For non-vanishing κ thus all 28 vector fields become massive. The associated massless Goldstone bosons can be identified in the scalar spectrum (6.10) which provides a strong consistency check of the result. The matrix (6.11) has half-maximal rank in accordance with the fact that this gauging is purely electric and involves only 28 of the vector fields.

To summarize, we have shown that for the theory discussed at the end of section 2.3, a de Sitter geometry with constant scalar and vector fields provides a solution to the full

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set of field equations. While the a non-vanishing κ in the fermionic sector simply induces a shift in all the fermion masses, we find that in the bosonic sector, most of the modes have imaginary mass square eigenvalues. This is due to the fact that the equations of motion do not descend from an action and may be a sign of an instability of this solution in de Sitter space. Actually the imaginary shift in the mass squared of the bosonic fluctuations reminds of the Breit-Wigner formula[7] for the propagator of an unstable particle, which has a characteristic imaginary shift in the pole proportional to the particle decay width Γ:

$$\frac{1}{p^2 - m^2 + im\Gamma} \,. \tag{6.13}$$

From this point of view our results seem to suggest that the bosonic fluctuations "die off" at some characteristic time $\delta t \sim m\Gamma/E$ proportional to the trombone parameter κ. It would be interesting to understand the implications of this feature for the stability properties of the background.

A particular limit of this theory is the case of a 'pure trombone gauging', i.e. $\xi^m{}_n = 0$ with vanishing mass parameters m_i. It follows from the above formulas, that in this case all scalar fields remain massless while all vector fields appear with negative mass squared, again a sign of an instability of the solution. It will be interesting to analyze in more detail, if this instability is a generic feature of the theories with local scaling invariance or if some classes of theories among the more complicated gaugings constructed in this paper eventually admit stable vacuum solutions.

7 Conclusions

In this paper, we have derived the most general couplings of four-dimensional supergravity with a maximal number of supercharges. With a gauge group embedded in the $E_{7(7)} \times \mathbb{R}$ global symmetry group of the Cremmer-Julia theory, the gauge generators are parametrized in terms of an embedding tensor, carrying $56 + 912$ parameters, subject to a set of bilinear algebraic consistency constraints. After suitable parametrization, we find that the latter reduces to the system (2.29)–(2.32) which allows to construct simple solutions. The standard gaugings whose gauge group is a subgroup of $E_{7(7)}$ correspond to an embedding tensor in the irreducible **912** representation. Additional non-vanishing components in the **56** representation define theories in which local scaling invariance $\mathbb{R}$ (the so-called trombone symmetry) is part of the gauge group.

We have determined the general form of the gauge groups and worked out the full set of modified field equations of these gauged $N = 8$ supergravities. As a particular feature of these theories, we have found that a gauging of the trombone generator leads to an additional positive contribution to the effective cosmological constant. Moreover, it turns out that gaugings with local scaling symmetry are generically dyonic, i.e. involve simultaneously electric and magnetic vector fields. We have analyzed in detail the simplest example of such a theory which has its higher-dimensional origin as a generalized Scherk-Schwarz reduction from five dimensions. We have shown that this theory admits a de Sitter

[7] We are grateful to Riccardo D'Auria for pointing out this analogy.

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solution with constant scalar fields and determined its mass spectrum indicating that the solution is not stable.

While in this paper we have analyzed only a single example within the new class of theories, which describes a one-parameter deformation of the known Scherk-Schwarz gaugings from five dimensions, it would be highly interesting to generalize this analysis to other examples and to perform a systematic study of the possibilities of deformations of the known gaugings. In particular, starting from a theory with supersymmetric AdS vacuum, the additional positive contribution to the effective cosmological constant may lift the space-time geometry to Minkowski or de Sitter upon inclusion of the trombone generator. In this context it would be important to investigate if the instabilities that we have found in the bosonic spectrum of our example are due to the simple structure of that example or if they persist to more complex situations and represent a generic feature of these theories.

Another interesting aspect for further study is the dyonic structure of the constructed gaugings. Whereas the appearance of magnetic vector fields is not new and has shown up in previously constructed gaugings in four dimensions [17, 33], in the standard theories there is always a symplectic frame in which all magnetic vector fields drop from the action and the field equations. This frame can be reached in a systematic way by integrating out the two-forms from the action. In contrast, we have found that for the gaugings constructed in this paper there is in general no symplectic frame in which all gauge fields would be electric. These gaugings are of genuinely dyonic nature. This does not contradict the no-go results on the gauging of electric/magnetic dualities [34, 35], as the resulting theories do no longer admit an action. It would be highly interesting to study the structure of such dyonic theories in more detail.

It is certainly remarkable that maximal supersymmetry in four dimensions not only admits the standard gaugings with gauge groups inside $E_{7(7)}$, described by an embedding tensor in the **912** representation [9, 16, 17], but moreover allows for yet another non-trivial deformation of the field equations described by 56 additional components of the embedding tensor. On the other hand, this may be viewed as another sign of the underlying symmetry structure of extended supergravity theories: upon dimensional reduction to two dimensions, the global symmetry group of maximal ungauged supergravity is the affine group $E_{9(9)}$ [42] while its gaugings are parametrized by an embedding tensor $\Theta_{2-\mathrm{dim}}$ transforming in the basic representation of that group [43]. This infinite-dimensional highest-weight representation thus captures all deformation parameters of the two-dimensional theory. Decomposition w.r.t. the finite-dimensional subgroup $E_{7(7)} \times \mathrm{SL}(2)$ gives rise to its lowest level components

$$\Theta_{2-\mathrm{dim}} \quad \longrightarrow \quad \begin{array}{c|c} +1 & (1,2) \\ \hline +2 & (56,2) \\ \hline +3 & (133,2)+(1,2) \\ \hline +4 & (912,1)+(56,1)+(56,3) \\ \hline \cdots & \cdots \end{array} \quad , \tag{7.1}$$

from which the higher-dimensional origin of these theories may be inferred. E.g. the theories

described by parameters in the first two rows correspond to torus reductions from four to two dimensions, in which the KK vector field and the two-dimensional vector fields acquire non-vanishing flux components along the two-torus, with the corresponding deformation parameters transforming in the $(1,2)$ and the $(56,2)$, respectively. Parameters in the third row describe Scherk-Schwarz reduction from four to two dimensions, including twists with the four-dimensional trombone generator. The $(912,1)$ in the fourth row corresponds to theories obtained by dimensional reduction from the standard gaugings in four dimensions, while the $(56,1)$ describes the dimensional reduction of the theories with local scaling symmetry constructed in this paper. This shows that after dimensional reduction both the standard gaugings as well as trombone gaugings and combinations of the two are described on equivalent footing by parameters residing within a single irreducible representation of the affine global symmetry group. In this sense, the new gaugings constructed in this paper may be viewed as obtained by $E_{9(9)}$ rotation from the standard gaugings in four dimensions. Moreover, the infinite tail of higher level parameters in (7.1) still advocates the tempting possibility of discovering yet other maximally supersymmetric couplings in four dimensions which must however be of genuinely different nature than the present constructions.

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A Some algebra

A.1 Useful E_7 relations

We denote by $(t_\alpha)_M{}^N$ the $E_{7(7)}$ generators in the fundamental representation, i.e. the index α runs over $1,\ldots,133$ and $M,N=1,\ldots,56$. We raise and lower adjoint indices with the invariant metric $\kappa_{\alpha\beta}\equiv \mathrm{Tr}\,[t_\alpha t_\beta]$, which is a rescaled Cartan-Killing metric. Fundamental indices are raised and lowered with the symplectic matrix Ω^{MN} using north-west south-east conventions: $X^M=\Omega^{MN}X_N$, etc.. We note the following two useful algebraic identities:

$$(t^\alpha)_M{}^K(t_\alpha)_N{}^L = \tfrac{1}{24}\delta^K_M\delta^L_N + \tfrac{1}{12}\delta^L_M\delta^K_N + (t^\alpha)_{MN}\,(t_\alpha)^{KL} - \tfrac{1}{24}\Omega_{MN}\,\Omega^{KL}\,, \qquad (A.1)$$

and

$$(t^\alpha)_{KL}(t_\alpha)_{MN} = \tfrac{1}{12}\Omega_{K(M}\Omega_{N)L} + C_{KLMN}\,, \qquad (A.2)$$

with the quartic E_7 invariant $C_{KLMN}\equiv (t_\alpha)_{(KL}(t^\alpha)_{MN)}$.

A.2 Breaking $E_{7(7)}$ to SU(8)

Upon breaking $E_{7(7)}$ to its maximal compact subgroup SU(8), the fundamental and the adjoint representation break according to

$$\mathbf{56}\to\mathbf{28}+\overline{\mathbf{28}}\,,\qquad \mathbf{133}\to\mathbf{63}+\mathbf{70}\,, \qquad (A.3)$$

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respectively. We label the fundamental representation of SU(8) by indices $i, j, \cdots = 1, \ldots 8$. Then, the $E_{7(7)}$ generators $(t_\alpha)_M{}^N$ break according to

$$\begin{aligned}(t_i{}^j)_{mn}{}^{kl} &= -\delta^j_{[m}\,\delta^{kl}_{n]i} - \tfrac{1}{8}\delta^j_i\,\delta^{kl}_{mn} = -(t_i{}^j)^{kl}{}_{mn}\,,\\ (t_{ijkl})_{mnpq} &= \tfrac{1}{24}\,\epsilon_{ijklmnpq}\,, \qquad (t_{ijkl})^{mnpq} \;=\; \delta^{mnpq}_{ijkl}\,,\end{aligned} \tag{A.4}$$

and the rescaled Cartan-Killing metric $\kappa_{\alpha\beta} \equiv \mathrm{Tr}\,[t_\alpha t_\beta]$ breaks into

$$\kappa_m{}^n{}_,{}_p{}^q = 3\left(\delta^q_m\delta^n_p - \tfrac{1}{8}\delta^n_m\delta^q_p\right)\,, \qquad \kappa_{ijkl,mnpq} \;=\; \tfrac{1}{12}\epsilon_{ijklmnpq}\,. \tag{A.5}$$

A.3 Breaking $E_{7(7)}$ to $E_{6(6)} \times \mathrm{SO}(1,1)$

Upon breaking $E_{7(7)}$ to its maximal subgroup $E_6 \times \mathrm{SO}(1,1)$, its lowest dimensional representations decompose according to

$$\begin{aligned}\mathbf{56} &\to \mathbf{1}^{+3} + \mathbf{27}^{+1} + \bar{\mathbf{27}}_{-1} + \mathbf{1}^{-3}\,,\\ \mathbf{133} &\to \mathbf{1}^0 + \mathbf{78}^0 + \mathbf{27}^{-2} + \bar{\mathbf{27}}^{+2}\,,\\ \mathbf{912} &\to \mathbf{78}^{+3} + \mathbf{78}^{-3} + \mathbf{27}^{+1} + \bar{\mathbf{27}}^{-1} + \mathbf{351}^{-1} + \bar{\mathbf{351}}^{+1}\,,\\ \mathbf{1539} &\to \mathbf{1}^0 + \mathbf{78}^0 + \mathbf{650}^0 + \mathbf{27}^{-2} + \mathbf{27}^{+4} + \bar{\mathbf{27}}^{+2} + \bar{\mathbf{27}}^{-4} + \mathbf{351}^{+2} + \bar{\mathbf{351}}^{-2}\,,\\ \mathbf{8645} &\to 2\cdot\mathbf{78}^0 + \mathbf{650}^0 + \mathbf{2925}^0 + \mathbf{27}^{-2} + \bar{\mathbf{27}}^{+2} + \mathbf{351}^{+2} + \mathbf{351}^{-4} + \bar{\mathbf{351}}^{-2} + \bar{\mathbf{351}}^{+4}\\ &\quad + \mathbf{1728}^{-2} + \bar{\mathbf{1728}}^{+2}\,,\end{aligned} \tag{A.6}$$

with the superscript indicating the $\mathrm{SO}(1,1)$ charge. We use the explicit notation

$$\begin{aligned}\mathbf{56}: X_M \;&\to\; (X_\bullet,\, X_m,\, X^m,\, X^\bullet)\,,\\ \mathbf{133}: \;X_\alpha \;&\to\; (X_{\mathrm{o}},\, X_a,\, X_m,\, X^m)\,,\end{aligned} \tag{A.7}$$

with indices $m = 1, \ldots, 27$ and $a = 1, \ldots, 78$ labeling the fundamental and the adjoint representation of $E_{6(6)}$, respectively. The symplectic matrix Ω_{MN} and the rescaled Cartan-Killing metric $\kappa_{\alpha\beta}$ break according to

$$\Omega_{MN} \to (\Omega_\bullet{}^\bullet = 1,\; \Omega_m{}^n = \delta^n_m,\; \Omega^\bullet{}_\bullet = -1,\; \Omega^m{}_n = -\delta^m_n)\,, \tag{A.8}$$

and

$$\kappa_{\alpha\beta} \to (\kappa_{\mathrm{oo}} = 72\,, \qquad \kappa_{ab} = 2\eta_{ab}\,, \qquad \kappa_m{}^n = 12\delta^n_m)\,. \tag{A.9}$$

The $E_{7(7)}$ generators $(t_\alpha)_M{}^N$ decompose as

$$\begin{aligned}&(t_{\mathrm{o}})_\bullet{}^\bullet = 3\,, \qquad (t_{\mathrm{o}})_m{}^n = \delta^n_m\,, \qquad (t_{\mathrm{o}})^m{}_n = -\delta^m_n\,, \qquad (t_{\mathrm{o}})^\bullet{}_\bullet = -3\,,\\ &(t_a)_m{}^n = -(t_a)^n{}_m\,,\\ &(t_m)_\bullet{}^n = -(t_m)^n{}_\bullet = \delta^n_m\,, \qquad (t^m)^\bullet{}_n = -(t^m)_n{}^\bullet = -\delta^m_n\,,\\ &(t_m)_{nk} = d_{mnk}\,, \qquad (t^m)^{nk} = 10\,d^{mnk}\,.\end{aligned} \tag{A.10}$$

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and the decomposition of the structure constants $f_{\alpha\beta}{}^{\gamma}$ can be read off from the algebra

$$\begin{aligned}
&[t_{\rm o}, t_m] = 2t_m \ , && [t_{\rm o}, t^m] = -2t^m \ , \\
&[t_a, t_m] = -t_{am}{}^n t_n \ , && [t_a, t^m] = t_{an}{}^m t^n \ , \\
&[t_m, t^n] = \frac{1}{3}\,\delta^n_m\, t_{\rm o} - 6(t^a)_m{}^n\, t_a \ . &&
\end{aligned} \tag{A.11}$$

Here, d_{mnk} denotes the totally symmetric tensor of $E_{6(6)}$, and $t_{am}{}^n$ denotes the $E_{6(6)}$ generators in the adjoint representation. Adjoint indices are raised and lowered with the rescaled Cartan-Killing metric $\eta_{ab} \equiv \mathrm{Tr}\,[t_a t_b]$.

A.4 Useful E_6 relations

We denote by d_{mnk} and d^{mnk} the totally symmetric tensors of $E_{6(6)}$ in the fundamental **27** and $\overline{\mathbf{27}}$, respectively. We choose a relative normalization such that

$$d^{mnp} d_{mnq} = \delta^p_q \ . \tag{A.12}$$

In the following, we give a list of useful algebraic relations that be be shown by various contractions and/or by using an explicit realization of the $E_{6(6)}$ generators:

$$d_{mrs}\, d^{spt}\, d_{tnu}\, d^{urq} = \tfrac{1}{10}\,\delta_{(mn)}^{(pq)} - \tfrac{2}{5}\, d_{mnr}\, d^{pqr} \ , \tag{A.13}$$

$$d_{mps}\, d^{sqt}\, d_{tru}\, d^{upv}\, d_{vqw}\, d^{wrn} = -\tfrac{3}{10}\,\delta_m{}^n \ , \tag{A.14}$$

$$(t^a)_m{}^k (t_a)_n{}^l = \tfrac{1}{18}\,\delta^k_m\delta^l_n + \tfrac{1}{6}\,\delta^l_m\delta^k_n - \tfrac{5}{3}\, d_{mnp}\, d^{klp} \ , \tag{A.15}$$

$$(t_a)_r{}^p (t_b)_s{}^q\, d^{mrs} d_{npq} = -\tfrac{1}{30}\,\eta_{ab}\delta^m_n + \tfrac{2}{5}(t_{(a}t_{b)})_n{}^m \ , \tag{A.16}$$

$$(t_a t_b)_r{}^q\, d^{mrs} d_{nqs} = \tfrac{1}{30}\,\eta_{ab}\delta^m_n - \tfrac{1}{5}(t_a t_b)_n{}^m + \tfrac{3}{10}(t_b t_a)_n{}^m \ , \tag{A.17}$$

$$d_{pqr} d^{p(kl} d^{m)qs} = \tfrac{1}{30} d^{klm}\delta^s_r + \tfrac{1}{10} d^{s(kl}\delta^{m)}_r \ . \tag{A.18}$$

These play a key role in reducing the the $E_{7(7)}$ system of constraints (2.19)–(2.21) for the embedding tensor to the system (2.29)–(2.32) for its $E_{6(6)}$ components.

B Solution of the quadratic constraints

Our strategy for solving the quadratic constraints for the embedding tensor follows the analysis of [24] for the pure trombone gaugings. We make use of the fact that under breaking to $E_{6(6)}$ the tensor ϑ_M contains a singlet which (if invertible) allows to explicitly solve all the quadratic equations. Decomposing under $E_{6(6)}$ according to (A.6), we label the components ϑ_M and $\Theta_M{}^\alpha$ of the embedding tensor according to

$$\vartheta_M \ \to \ \ (\vartheta_\bullet,\ \vartheta_m,\ \vartheta^m,\ \vartheta^\bullet) \ . \tag{B.1}$$

and

$$\Theta_M{}^\alpha = \begin{pmatrix} 0 & \xi^a_+ & \xi_n & 0 \\ -\frac{1}{3}\xi_m & \frac{3}{2}t^a{}_m{}^n\xi_n + 3t^a{}_p{}^q d_{rqm}\xi^{pr} & \frac{1}{2}d_{mnp}\xi^p + \xi_{mn} & -t_{am}{}^n\xi^a_+ \\ -\frac{1}{3}\xi^m & \frac{3}{2}t^a{}_n{}^m\xi^n - 30t^a{}_p{}^q d^{rpm}\xi_{qr} & -t_{an}{}^m\xi^a_- & -5d^{mnp}\xi_p + \xi^{mn} \\ 0 & \xi^a_- & 0 & \xi^n \end{pmatrix} , \tag{B.2}$$

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respectively, in terms of various $E_{6(6)}$ tensors. The relative coefficients among the various terms within $\Theta_M{}^\alpha$ are determined by the fact that $\Theta_M{}^\alpha$ is constrained to live in the **912** representation of $E_{7(7)}$, i.e. satisfies the relations (2.8).

B.1 Determining the components of the embedding tensor

To solve the quadratic constraints in a systematic way, we start from the equation with the highest SO(1, 1) grading. From (A.6) is follows that this is a $\mathbf{27}^{+4}$ representation inside **1539**, which thus corresponds to evaluating equations (2.20) for $MN = {}_{m\bullet}$. Explicitly, this leads to

$$0 \equiv \xi_m \vartheta_\bullet + t_{am}{}^n \xi^a_+ \vartheta_n \,. \tag{B.3}$$

Without loss of generality we may assume $\vartheta_\bullet$ to be non-vanishing (which can always be achieved by change of basis in case ϑ_M is not identically zero) and from (B.3) express ξ_m (one of the $\mathbf{27}^{+1}$ components of the embedding tensor) in terms of the unconstrained parameters $(\vartheta_\bullet, \vartheta_m, \xi^a_+)$ transforming in the $\mathbf{1}^{+3} + \mathbf{27}^{+1} + \mathbf{78}^{+3}$. For convenience, we parametrize the latter as

$$\vartheta_\bullet \equiv \kappa \,, \qquad \vartheta_m \equiv \kappa \lambda_m \,, \qquad \xi^a_+ \equiv \Xi^a \,, \tag{B.4}$$

and solve equation (B.3) as

$$\xi_m = -\Xi^a t_{am}{}^n \lambda_n \;\equiv\; -\delta_\Xi \lambda_m \,. \tag{B.5}$$

By similar computations, the remaining parts of the embedding tensor can be determined from other components of the constraint equations. Evaluating equation (2.21) for $\alpha\beta = {}^{mn}$ (the $\mathbf{3\bar{5}1}^{+4}$ equation) yields

$$0 \equiv \vartheta_\bullet \, \xi^{mn} - 3\, \Xi^a \, t_{ak}{}^{[m} \, \xi^{n]k} - 10\, \xi^a_+ \, t_{ap}{}^{[m} \, d^{n]pq} \, (\vartheta_q + \tfrac{3}{2} \xi_q) \,, \tag{B.6}$$

which upon plugging in (B.5) reduces to

$$\mathcal{O}_{2/3} \cdot \xi^{mn} \equiv 10\, \Xi^a \, t_{ak}{}^{[m} \, d^{n]kl} \, \mathcal{O}_{2/3} \cdot \lambda_l \,, \tag{B.7}$$

where we have defined the operator

$$\mathcal{O}_{2/3} \equiv \delta_\Xi - \tfrac{2}{3} \kappa \,. \tag{B.8}$$

As the same operator appears on both sides of equation (B.7), the general solution for the component ξ^{mn} can be given in polynomial form as

$$\xi^{mn} = 10\, \Xi^a t_{ak}{}^{[m} \, d^{n]kl} \, \lambda_l + \zeta^{mn} \,, \tag{B.9}$$

where ζ^{mn} denotes a (real) zero mode of the operator $\mathcal{O}_{2/3}$. It corresponds to an eigenvector of the action of the $\mathfrak{e}_{6(6)}$ generator defined by Ξ^a with the particular real eigenvalue $\frac{2}{3}\kappa$. In particular, such zero-modes only exist for non-compact choice of Ξ^a and some very particular values of κ.

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Going down the grading, the next constraint equations live in the $\bar{\mathbf{27}}^{+2}$, of which there are four different ones. The relevant ones are obtained from (2.19) for $\alpha = {}^{m}$ and from (2.20) for $MN = {}^{m}{}_{\bullet}$, respectively, leading to

$$\begin{aligned}\left(\xi^m + \tfrac{8}{3}\vartheta^m\right)\vartheta_\bullet + \Xi^a\, t_{an}{}^m\,\vartheta^n &= \xi^{mn}\vartheta_n + 5\, d^{mpq}\vartheta_p\left(\xi_q + \tfrac{8}{3}\vartheta_q\right)\,,\\ \xi^m\vartheta_\bullet - \Xi^a\, t_{an}{}^m\,\vartheta^n &= \xi^{mn}\vartheta_n - 15\, d^{mpq}\vartheta_p\,\xi_q\,.\end{aligned} \tag{B.10}$$

Using the explicit form of (B.5), (B.9), these two equations determine the components ξ^m and ϑ^m in terms of the free parameters according to

$$\begin{aligned}\vartheta^m &= 5\kappa\, d^{mkl}\,\lambda_k\lambda_l + \kappa\zeta^m\,,\\ \xi^m &= 5\,\Xi^a t_{an}{}^m\, d^{nkl}\,\lambda_k\lambda_l + \zeta^{mn}\lambda_n - \tfrac{4}{3}\kappa\zeta^m\,,\end{aligned} \tag{B.11}$$

up to the constant vector ζ^m which is a zero mode of the operator $\mathcal{O}_{4/3} \equiv \delta_\Xi - \frac{4}{3}\kappa$. Again, such zero modes exist only for very particular values of κ. Next, we evaluate the constraint (2.20) for $MN = {}_{mn}$ (the equation transforming in the $\mathbf{351}^{+2}$) to obtain

$$0 \equiv \tfrac{1}{2}\xi_{[m}\vartheta_{n]} - 5\, d_{mkp}d^{prq}d_{qln}\,\xi^{kl}\vartheta_r + t_{a[m}{}^k d_{n]pk}\,\Xi^a\vartheta^p - \xi_{mn}\vartheta_\bullet\,, \tag{B.12}$$

which uniquely determines the $\mathbf{351}^{-1}$ component ξ_{mn} of the embedding tensor. Explicitly, after some computation and using the relations obtained above, we find

$$\begin{aligned}\xi_{mn} &= 2\,\Xi^a t_{a[m}{}^k\,\lambda_{n]}\lambda_k - 5\,\Xi^a t_{a[m}{}^p d_{n]pq}d^{qkl}\,\lambda_k\lambda_l\\ &\quad -5\, d_{mkp}d^{prq}d_{qln}\,\zeta^{kl}\lambda_r + t_{a[m}{}^k d_{n]pk}\,\Xi^a\zeta^p\,.\end{aligned} \tag{B.13}$$

The singlet equation $\mathbf{1}^0$ from evaluating (2.19) for $\alpha = \mathrm{o}$ yields

$$\tfrac{4}{3}\vartheta_\bullet\vartheta^\bullet = \tfrac{1}{3}\left(\xi^m\vartheta_m - \xi_m\vartheta^m\right) - \tfrac{4}{9}\,\vartheta^m\vartheta_m\,, \tag{B.14}$$

which allows to express the singlet $\mathbf{1}^{-3}$ component $\vartheta^\bullet$ in terms of the other fields

$$\vartheta^\bullet = -\tfrac{5}{3}\,\kappa\, d^{klm}\,\lambda_k\lambda_l\lambda_m - \kappa\zeta^m\lambda_m\,. \tag{B.15}$$

Evaluating (2.19) for $\alpha = a$ finally yields

$$\begin{aligned}0 &= \tfrac{3}{2}t^a{}_m{}^n(\xi^m\vartheta_n - \xi_n\vartheta^m) - 3t^a{}_p{}^q(d_{rqn}\xi^{pr}\vartheta^n + 10d^{rpn}\xi_{qr}\vartheta_n)\\ &\quad +\xi^a_-\vartheta_\bullet - \Xi^a\vartheta^\bullet + 16t^a{}_m{}^n\vartheta^m\vartheta_n\,.\end{aligned} \tag{B.16}$$

which yields the $\mathbf{78}^{-3}$ component ξ^a_-

$$\begin{aligned}\xi^a_- &= 30\,\xi^b_+(t_bt^a)_n{}^m\, d^{nkl}\,\lambda_m\lambda_k\lambda_l - \tfrac{5}{3}\,\Xi^a\, d^{klm}\,\lambda_m\lambda_k\lambda_l\\ &\quad +\Xi^a\lambda_k\zeta^k - 6\Xi^b(t^at_b)_k{}^l\lambda_l\zeta^k - 8\kappa\, t^a{}_k{}^l\lambda_l\zeta^k\\ &\quad -6t^a{}_m{}^l\lambda_k\lambda_l\zeta^{km} + 15t^a{}_n{}^q d^{klp}d_{mpq}\lambda_k\lambda_l\zeta^{mn}\,.\end{aligned} \tag{B.17}$$

We have thus determined all the components of the embedding tensor (B.1), (B.2) in terms of the parameters κ, λ_m, Ξ^a, ζ^m, ζ^{mn}, of which the latter two are particular eigenvectors

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under the $E_{6(6)}$ action of Ξ. In the following, we need to check that this solution indeed satisfies all the constraint equations (2.19)–(2.21). In particular, the remaining constraint equations may impose further restriction on these tensors. E.g. evaluating equation (2.19)) for $\alpha = {}_m$ implies that

$$d_{mkl}\zeta^k\zeta^l = 0\,, \tag{B.18}$$

i.e. ζ^k is not only zero mode of $\mathcal{O}_{4/3}$ but also satifies a 'pure-spinor type' condition of $E_{6(6)}$. In the following, we evaluate all the remaining constraint equations.

B.2 Evaluating the remaining equations

We can now check all the remaining equations upon using the solution obtained above. To this end, let us first calculate the invariant $I_4(\vartheta)$ from (2.33) quartic in the trombone parameters ϑ_M for the explicit solution (B.4), (B.11), (B.15). As a result, we obtain

$$I_4(\vartheta) = \frac{2}{3}\,\kappa^4\, d_{mnp}\,\zeta^m\zeta^n\zeta^p\,, \tag{B.19}$$

i.e. the quartic invariant does not depend on the parameters λ_m. This shows that all λ_m can be set to zero by an $E_{7(7)}$ transformation and therefore do not induce inequivalent gaugings. For simplicity, we will thus in the following set $\lambda_m = 0$. The solution found in the previous section then reduces to the solution (2.27) given in the main text. In this section we will evaluate all remaining constraint equations for this solution. The calculation is rather tedious and has been performed using mathematica and cadabra [44, 45]. As a result, we find that all remaining constraint equations of the system (2.19)–(2.21) are satisfied provided, the parameters ζ^k, ζ^{mn} obey the following set of identities

$$\zeta^k\zeta^l d_{mkl} = 0\,, \tag{B.20}$$

$$\zeta^k\zeta^{mn} d_{kml} = 0\,, \tag{B.21}$$

$$\zeta^{[k}\zeta^{mn]} = 0\,, \tag{B.22}$$

$$\big(t_a\cdot\big(\Xi+\tfrac{4}{3}\kappa I\big)\cdot\big(\Xi-\tfrac{2}{3}\kappa I\big)\big)_n{}^m\,\zeta^n = -\tfrac{1}{2}\zeta^{mk}\zeta^{ln}d_{klp}(t_a)_n{}^p\,, \tag{B.23}$$

with the matrix Ξ given by $\Xi_m{}^n \equiv \Xi^a(t_a)_m{}^n$ and '$\cdot$' denoting the matrix product. The third equation comes from the constraint (2.21) evaluated for $[\alpha\beta] = [ab]$; the fourth equation comes from the same constraint evaluated for $[\alpha\beta] = {}^{am}$. We have explicitly verified that *all other constraints* are satisfied as a consequence of the ansatz and the relations (B.23). In particular, the constraint obtained from (2.21) evaluated for $[\alpha\beta] = {}^a{}_m$ follows after some computation from the fourth equation of (B.23) and the other constraints.

The fourth equation of (B.23) is linear in ζ^n. In particular, contracting this equation with $(t^a)_m{}^q$ implies

$$(32\,\kappa^2 - \Xi^a\Xi_a)\,\zeta^m = \tfrac{15}{2}\,\zeta^{kl}\zeta^{rs}d_{krp}d_{lsq}d^{pqm}\,, \tag{B.24}$$

i.e. for $\Xi^a\Xi_a \neq 32\,\kappa^2$, we can express ζ^m as a bilinear in ζ^{kl}. Note that this is consistent, as the r.h.s. of (B.24) is indeed eigenvector of Ξ associated to an eigenvalue which is twice

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the one of ζ^{kl}. Still, plugging this expression for ξ^m into the four equations (B.23) will lead to a set of nontrivial constraints polynomial in ζ^{mn}.

As illustrated in section 2.5, the "$E_{6(6)}$-pure spinor" constraint (B.20) on ζ^m singles out an $O(1,1) \times SO(5,5)$ subgroup of $E_{6(6)}$ in which $SO(5,5)$ is part of the little group of its solution. In particular ζ^m coincides with the singlet $\mathbf{1}^{-4}$ in the branching (2.37) of the $\overline{\mathbf{27}}$ relative to this $O(1,1) \times SO(5,5)$, and Ξ should have the form in eq. (2.39). Let us first consider the simple case in which Ξ_0 is a semisimple element of $\mathfrak{so}(5,5)$ and thus can be considered as an element of its Cartan subalgebra. Let us also consider the case in which the two sides of eq. (2.32) are separately zero. The constraint (B.23) is then satisfied if Ξ_0 commutes with an $\mathfrak{so}(4,4)$ subalgebra of $\mathfrak{so}(5,5)$ and if its norm is $\mathrm{Tr}(\Xi_0 \cdot \Xi_0) = 24\,\kappa^2$. The last requirement is easily understood by observing that Ξ_0 and Ξ_1 are mutually orthogonal and that $\mathrm{Tr}(\Xi_1 \cdot \Xi_1) = 8\,\kappa^2$, since (B.23) implies that $\mathrm{Tr}(\Xi \cdot \Xi) = 32\,\kappa^2$. This fixes the normalization of Ξ_0. We can branch the relevant $E_{6(6)}$ representations with respect to its $SO(1,1)^2 \times SO(4,4)$ subgroup, where the $SO(1,1)^2$ factor is generated by $\Xi_1 + \Xi_0$. The $\overline{\mathbf{27}}$ then branches as follows:

$$\begin{aligned}
\overline{\mathbf{27}} &\to \mathbf{1}^{(\frac{4}{3},0)} + \mathbf{1}^{(-\frac{2}{3},2)} + \mathbf{1}^{(-\frac{2}{3},-2)} + \mathbf{8}_v^{(-\frac{2}{3},0)} + \mathbf{8}_s^{(\frac{1}{3},1)} + \mathbf{8}_c^{(\frac{1}{3},-1)}\,, \\
\mathbf{78} &\to (\mathbf{28}+\mathbf{1}+\mathbf{1})^{(0,0)} + \mathbf{8}_s^{(1,-1)} + \mathbf{8}_c^{(1,1)} + \mathbf{8}_s^{(-1,1)} + \mathbf{8}_c^{(-1,-1)} + \mathbf{8}_v^{(0,2)} + \mathbf{8}_v^{(0,-2)}\,,
\end{aligned}$$

where the gradings are the eigenvalues of Ξ_1/κ, Ξ_0/κ. The "pure spinor" ζ^m corresponds to the $\mathbf{1}^{(\frac{4}{3},0)}$ representation. Consider now the vector $\zeta \cdot t^a \equiv (\zeta^m\, t^a{}_m{}^n)$. The constraint (B.23) reads:

$$\left(\delta_\Xi + \frac{2}{3}\,\kappa\right)\left(\delta_\Xi - \frac{4}{3}\,\kappa\right)\zeta \cdot t^a = 0\,. \tag{B.25}$$

Let us analyze the relevant cases:

- If $t^a \in \mathfrak{so}(5,5) + \mathfrak{so}(1,1)$, $\zeta \cdot t^a$ is still in the $\mathbf{1}^{(\frac{4}{3},0)}$ and is thus annihilated by $\delta_\Xi - \frac{4}{3}\,\kappa$;
- If $t^a \in \mathbf{8}_s^{(1,-1)} + \mathbf{8}_c^{(1,1)}$, $\zeta \cdot t^a = \mathbf{0}$ and the constraint is satisfied;
- If $t^a \in \mathbf{8}_s^{(-1,1)} + \mathbf{8}_c^{(-1,-1)}$, $\zeta \cdot t^a \in \mathbf{8}_s^{(\frac{1}{3},1)} + \mathbf{8}_c^{(\frac{1}{3},-1)}$. The component in $\mathbf{8}_s^{(\frac{1}{3},1)}$ is a zero-mode of $\delta_\Xi - \frac{4}{3}\,\kappa$, while the second component is a zero mode of $\delta_\Xi + \frac{2}{3}\,\kappa$ and the constraint is still satisfied;

Consider now the case in which Ξ_0 has a nilpotent component Ξ_{n} in $\mathbf{16}_s^{-3}$, so that Ξ has a semisimple and a nilpotent component $\Xi = \Xi_{\mathrm{ss}} + \Xi_{\mathrm{n}}$. The constraint (B.23) is still satisfied provided the following condition holds: $[\Xi_{\mathrm{ss}}, \Xi_{\mathrm{n}}] = 2\,\kappa\,\Xi_{\mathrm{n}}$.

C Gauge group generators in E_6 components

Here, we give the gauge group generators $(X_M)_N{}^K$ for the general solution (2.27) of the quadratic constraints as obtained from (2.12), (B.1), (B.2), and (2.27). They are

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parametrized by κ, Ξ^a, ζ^m, and ζ^{mn}, which are subject to the identities (B.23).

$$
\begin{aligned}
X_\bullet &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Xi_m{}^n - \frac{2}{3}\kappa\delta_m^n & 0 & 0 \\ 0 & 0 & -\Xi_n{}^m - \frac{4}{3}\kappa\delta_n^m & 0 \\ 0 & 0 & 0 & -2\kappa \end{pmatrix} , \\
X_k &= \begin{pmatrix} 0 & -\Xi_k{}^n + \frac{2}{3}\kappa\delta_k^n & 0 & 0 \\ \Xi_{[k}{}^p\zeta^q d_{m]pq} & \frac{1}{2}\zeta^{np}d_{kpm} + 5\zeta^{pq}d_{kpr}d_{qsm}d^{rsn} & \frac{2}{3}\kappa d_{kmn} - \Xi_k{}^p d_{pmn} & 0 \\ 0 & 10\Xi_{[k}{}^p\zeta^q d_{r]pq}d^{rmn} & 5\zeta^{pq}d_{npr}d_{kqs}d^{rsm} - \frac{1}{2}\zeta^{mp}d_{kpn} & \Xi_k{}^m - \frac{2}{3}\kappa\delta_k^m \\ 0 & 0 & -\Xi_{[k}{}^p\zeta^q d_{n]pq} & 0 \end{pmatrix} , \\
X^k &= \begin{pmatrix} 0 & \zeta^{kn} & 0 & 0 \\ 0 & X^k{}_m{}^n & \zeta^{kp}d_{pmn} & 0 \\ 0 & 0 & X^{k\,m}{}_n & \zeta^{mk} \\ 0 & 0 & 0 & -2\kappa\,\zeta^m \end{pmatrix} , \\
X^\bullet &= \begin{pmatrix} 0 & -2\kappa\zeta^n & 0 & 0 \\ 0 & 0 & -2\kappa\zeta^p d_{pmn} & 0 \\ 0 & 0 & 0 & 2\kappa\zeta^m \\ 0 & 0 & 0 & 0 \end{pmatrix} ,
\end{aligned}
\tag{C.1}
$$

where

$$
\begin{aligned}
X^k{}_m{}^n &= 2\Xi_m{}^{[k}\zeta^{n]} - \tfrac{4}{3}\kappa\zeta^k\delta_m^n - \tfrac{2}{3}\kappa\zeta^n\delta_m^k + \tfrac{20}{3}\kappa\zeta^p d_{pqm}d^{qkn} - 10\Xi_p{}^k\zeta^q d_{mqr}d^{rpn} , \\
X^{k\,m}{}_n &= 2\Xi_n{}^{[m}\zeta^{k]} + \tfrac{4}{3}\kappa\delta_n^{[k}\zeta^{m]} - \tfrac{20}{3}\kappa\zeta^p d_{pqn}d^{qkm} + 10\Xi_p{}^k\zeta^q d_{nqr}d^{rpm} .
\end{aligned}
$$

Via (2.22) these generators also encode the structure constants of the gauge algebra.

D T-identities

Upon dressing the quadratic constraints (2.19)–(2.21) with the scalar vielbein and using the definitions (3.6), (3.7), one obtains a large number of SU(8) identities bilinear in the tensors A and B. Here we collect those identities that are important for the calculations in the main text. Working out all quadratic constraints that transform in the **63** of SU(8), we find the following relations

$$
\begin{aligned}
\mathbf{1539}: \quad 0 &= 6A^{ik}B_{jk} - 6B^{ik}A_{jk} + A_j{}^{imn}B_{mn} - A^i{}_{jmn}B^{mn} , \qquad (D.1) \\
\mathbf{133}: \quad 0 &= 2A^{ik}B_{jk} + 2B^{ik}A_{jk} - A_j{}^{imn}B_{mn} - A^i{}_{jmn}B^{mn} + \tfrac{32}{3}B^{ik}B_{jk} \; - \; \tfrac{1}{8}\delta_j^i\,\text{trace} , \\
\mathbf{133}: \quad 0 &= 12A^{ik}A_{jk} - A^i{}_{mnk}A_j{}^{mnk} + 3A^k{}_{mnj}A_k{}^{mni} \\
&\quad +12\left(2A^{ik}B_{jk} + 2B^{ik}A_{jk} - A_j{}^{imn}B_{mn} - A^i{}_{jmn}B^{mn}\right) \; - \; \tfrac{1}{8}\delta_j^i\,\text{trace} , \\
\mathbf{8645}: \quad 0 &= -240A^{ik}A_{jk} + 11A^i{}_{mnk}A_j{}^{mnk} + 21A^k{}_{mnj}A_k{}^{mni} \\
&\quad -12\left(10A^{ik}B_{jk} + 10B^{ik}A_{jk} + A_j{}^{imn}B_{mn} + A^i{}_{jmn}B^{mn}\right) \; - \; \tfrac{1}{8}\delta_j^i\,\text{trace} ,
\end{aligned}
$$

descending from the various irreducible $E_{7(7)}$ contributions of (2.19)–(2.21) as indicated. The $E_{7(7)}$ origin of these constraints can also be confirmed by calculating the action of the quadratic Casimir operator upon using the $E_{7(7)}$ transformation properties

$$
\begin{aligned}
\delta A^{ij} &= \tfrac{1}{3}A^{(i}{}_{klm}\Sigma^{j)klm} , \\
\delta A_i{}^{jkl} &= 2A_{im}\Sigma^{mjkl} + 3\Sigma^{mn[jk}A^{l]}{}_{imn} + \Sigma^{mnp[j}\delta_i^k A^{l]}{}_{mnp} , \\
\delta B^{ij} &= -\Sigma^{ijkl}B_{kl} ,
\end{aligned}
\qquad (D.2)
$$

with Σ^{ijkl} satisfying $\Sigma_{ijkl} = \frac{1}{24}\epsilon^{ijklmnpq}\Sigma_{mnpq}$.

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Similarly, we can deduce we can deduce two constraints in the **70** of SU(8):

$$\begin{aligned}
\mathbf{133}: \quad 0 &= A^{m}{}_{[jkl}B_{n]m} + 4B_{[jk}B_{ln]} - \tfrac{1}{24}\epsilon_{jklnmqrs}\left(A_p{}^{mqr}B^{sp} + 4B^{mq}B^{rs}\right) \\
\mathbf{133}: \quad 0 &= 4A^{m}{}_{[jkl}A_{n]m} - 3A^{m}{}_{p[jk}A^{p}{}_{ln]m} + 16A^{m}{}_{[jkl}B_{n]m} \\
&\quad -\tfrac{1}{24}\epsilon_{jklnmpqr}\left(4A_s{}^{mpq}A^{rs} - 3A_t{}^{ump}A_u{}^{qrt} + 16A_u{}^{mpq}B^{ru}\right) ,
\end{aligned} \tag{D.3}$$

and two constraints in the **378** of SU(8):

$$\begin{aligned}
\mathbf{1539}: \quad 0 &= 4A_{j[k}B_{ln]} + A^{m}{}_{j[kl}B_{n]m} + A^{m}{}_{kln}B_{jm} \\
&\quad -\tfrac{1}{9}\epsilon_{rsmpqnkl}A_j{}^{mpq}B^{rs} + \tfrac{1}{18}\epsilon_{mjqrsnkl}A_p{}^{qrs}B^{mp} ,
\end{aligned} \tag{D.4}$$

$$\begin{aligned}
\mathbf{8645}: \quad 0 &= -18A^{m}{}_{nkl}A_{jm} - 54A^{m}{}_{j[kl}A_{n]m} + 60A_{j[k}B_{ln]} - 9A^{m}{}_{j[kl}B_{n]m} - 9A^{m}{}_{kln}B_{jm} \\
&\quad +\epsilon_{mpqrsnkl}A_j{}^{urs}\left(A_u{}^{mpq} - \tfrac{1}{3}\delta^{m}_{u}B^{pq}\right) - \tfrac{3}{4}\epsilon_{jnklpmrs}A_u{}^{rst}\left(A_t{}^{mup} - \tfrac{2}{9}\delta^{m}_{t}B^{up}\right) .
\end{aligned}$$

If the components $\Theta_M{}^{\alpha}$, ϑ_M are chosen such as to satisfy the quadratic constraints (2.19)–(2.21), the relations (D.1)–(D.4) among the scalar tensors A, B, follow as an immediate consequence. For $B^{ij} = 0$, all these identities consistently reduce to the quadratic identities given in [9, 17].

References

[1] E. Cremmer and B. Julia, *The* SO(8) *Supergravity*, *Nucl. Phys.* **B 159** (1979) 141 [SPIRES].

[2] N. Arkani-Hamed, F. Cachazo and J. Kaplan, *What is the Simplest Quantum Field Theory?*, *JHEP* **09** (2010) 016 [arXiv:0808.1446] [SPIRES].

[3] Z. Bern, J.J. Carrasco, L.J. Dixon, H. Johansson and R. Roiban, *The Ultraviolet Behavior of* $N = 8$ *Supergravity at Four Loops*, *Phys. Rev. Lett.* **103** (2009) 081301 [arXiv:0905.2326] [SPIRES].

[4] R. Kallosh and T. Kugo, *The footprint of* $E_{7(7)}$ *in amplitudes of* $N = 8$ *supergravity*, *JHEP* **01** (2009) 072 [arXiv:0811.3414] [SPIRES].

[5] J. Broedel and L.J. Dixon, R^4 *counterterm and* $E_{7(7)}$ *symmetry in maximal supergravity*, *JHEP* **05** (2010) 003 [arXiv:0911.5704] [SPIRES].

[6] H. Elvang and M. Kiermaier, *Stringy KLT relations, global symmetries and* $E_{7(7)}$ *violation*, *JHEP* **10** (2010) 108 [arXiv:1007.4813] [SPIRES].

[7] G. Bossard, C. Hillmann and H. Nicolai, $E_{7(7)}$ *symmetry in perturbatively quantised* $N = 8$ *supergravity*, *JHEP* **12** (2010) 052 [arXiv:1007.5472] [SPIRES].

[8] N. Beisert et al., $E_{7(7)}$ *constraints on counterterms in* $N = 8$ *supergravity*, *Phys. Lett.* **B 694** (2010) 265 [arXiv:1009.1643] [SPIRES].

[9] B. de Wit and H. Nicolai, $N = 8$ *Supergravity*, *Nucl. Phys.* **B 208** (1982) 323 [SPIRES].

[10] C.M. Hull, *More gaugings of* $N = 8$ *supergravity*, *Phys. Lett.* **B 148** (1984) 297 [SPIRES].

JHEP04(2011)079

[11] C.M. Hull, *Noncompact gaugings of $N = 8$ supergravity*, *Phys. Lett.* **B 142** (1984) 39 [SPIRES].

[12] C.M. Hull, *A new gauging of $N = 8$ supergravity*, *Phys. Rev.* **D 30** (1984) 760 [SPIRES].

[13] F. Cordaro, P. Fré, L. Gualtieri, P. Termonia and M. Trigiante, *$N = 8$ gaugings revisited: An exhaustive classification*, *Nucl. Phys.* **B 532** (1998) 245 [hep-th/9804056] [SPIRES].

[14] L. Andrianopoli, R. D'Auria, S. Ferrara and M.A. Lledó, *Gauging of flat groups in four dimensional supergravity*, *JHEP* **07** (2002) 010 [hep-th/0203206] [SPIRES].

[15] C.M. Hull, *New gauged $N = 8$, $D = 4$ supergravities*, *Class. Quant. Grav.* **20** (2003) 5407 [hep-th/0204156] [SPIRES].

[16] B. de Wit, H. Samtleben and M. Trigiante, *On Lagrangians and gaugings of maximal supergravities*, *Nucl. Phys.* **B 655** (2003) 93 [hep-th/0212239] [SPIRES].

[17] B. de Wit, H. Samtleben and M. Trigiante, *The maximal $D = 4$ supergravities*, *JHEP* **06** (2007) 049 [arXiv:0705.2101] [SPIRES].

[18] E. Cremmer, H. Lü, C.N. Pope and K.S. Stelle, *Spectrum-generating symmetries for BPS solitons*, *Nucl. Phys.* **B 520** (1998) 132 [hep-th/9707207] [SPIRES].

[19] P.S. Howe, N.D. Lambert and P.C. West, *A new massive type IIA supergravity from compactification*, *Phys. Lett.* **B 416** (1998) 303 [hep-th/9707139] [SPIRES].

[20] I.V. Lavrinenko, H. Lü and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, *Class. Quant. Grav.* **15** (1998) 2239 [hep-th/9710243] [SPIRES].

[21] E. Bergshoeff, T. de Wit, U. Gran, R. Linares and D. Roest, *(Non-)Abelian gauged supergravities in nine dimensions*, *JHEP* **10** (2002) 061 [hep-th/0209205] [SPIRES].

[22] J. Kerimo and H. Lü, *New $D = 6$, $N = (1, 1)$ gauged supergravity with supersymmetric Minkowski$_4 \times S^2$ vacuum*, *Phys. Lett.* **B 576** (2003) 219 [hep-th/0307222] [SPIRES].

[23] J. Kerimo, J.T. Liu, H. Lü and C.N. Pope, *Variant $N = (1, 1)$ supergravity and (Minkowski)$_4 \times S^2$ vacua*, *Class. Quant. Grav.* **21** (2004) 3287 [hep-th/0401001] [SPIRES].

[24] A. Le Diffon and H. Samtleben, *Supergravities without an Action: Gauging the Trombone*, *Nucl. Phys.* **B 811** (2009) 1 [arXiv:0809.5180] [SPIRES].

[25] F. Riccioni, *Local E_{11} and the gauging of the trombone symmetry*, *Class. Quant. Grav.* **27** (2010) 125009 [arXiv:1001.1316] [SPIRES].

[26] F. Riccioni, *The very-extended trombone*, *JHEP* **04** (2010) 032 [arXiv:1001.1619] [SPIRES].

[27] F. Riccioni and P.C. West, *The E_{11} origin of all maximal supergravities*, *JHEP* **07** (2007) 063 [arXiv:0705.0752] [SPIRES].

[28] E.A. Bergshoeff, I. De Baetselier and T.A. Nutma, *E_{11} and the embedding tensor*, *JHEP* **09** (2007) 047 [arXiv:0705.1304] [SPIRES].

[29] B. de Wit and H. Samtleben, *Gauged maximal supergravities and hierarchies of nonabelian vector-tensor systems*, *Fortsch. Phys.* **53** (2005) 442 [hep-th/0501243] [SPIRES].

[30] B. de Wit, H. Nicolai and H. Samtleben, *Gauged Supergravities, Tensor Hierarchies and M-theory*, *JHEP* **02** (2008) 044 [arXiv:0801.1294] [SPIRES].

[31] J. Scherk and J.H. Schwarz, *How to Get Masses from Extra Dimensions*, *Nucl. Phys.* **B 153** (1979) 61 [SPIRES].

JHEP04(2011)079

[32] S. Ferrara and M. Günaydin, *Orbits of exceptional groups, duality and BPS states in string theory*, *Int. J. Mod. Phys.* **A 13** (1998) 2075 [hep-th/9708025] [SPIRES].

[33] B. de Wit, H. Samtleben and M. Trigiante, *Magnetic charges in local field theory*, *JHEP* **09** (2005) 016 [hep-th/0507289] [SPIRES].

[34] C. Bunster and M. Henneaux, *Can (Electric-Magnetic) Duality Be Gauged?*, *Phys. Rev.* **D 83** (2011) 045031 [arXiv:1011.5889] [SPIRES].

[35] S. Deser, *No local Maxwell duality invariance*, *Class. Quant. Grav.* **28** (2011) 085009 [arXiv:1012.5109] [SPIRES].

[36] A. Le Diffon, *Supergravités jaugées et symétrie locale d'échelle* (in French), Ph.D. Thesis, École Normale Supérieure de Lyon, Lyon France (2010).

[37] A. Chamblin and N.D. Lambert, *de Sitter space from M-theory*, *Phys. Lett.* **B 508** (2001) 369 [hep-th/0102159] [SPIRES].

[38] B. de Wit, *Properties of* SO(8) *extended supergravity*, *Nucl. Phys.* **B 158** (1979) 189 [SPIRES].

[39] B. de Wit and H. Nicolai, *The parallelizing S^7 torsion in gauged $N = 8$ supergravity*, *Nucl. Phys.* **B 231** (1984) 506 [SPIRES].

[40] E. Cremmer, J. Scherk and J.H. Schwarz, *Spontaneously Broken $N = 8$ Supergravity*, *Phys. Lett.* **B 84** (1979) 83 [SPIRES].

[41] E. Sezgin and P. van Nieuwenhuizen, *Renormalizability properties of spontaneously broken $N = 8$ supergravity*, *Nucl. Phys.* **B 195** (1982) 325 [SPIRES].

[42] B. Julia, *Infinite Lie algebras in physics*, in the proceedings of *Johns Hopkins Workshop on Current Problems in Particle Theory*, Baltimore U.S.A. (1981).

[43] H. Samtleben and M. Weidner, *Gauging hidden symmetries in two dimensions*, *JHEP* **08** (2007) 076 [arXiv:0705.2606] [SPIRES].

[44] K. Peeters, *A field-theory motivated approach to symbolic computer algebra*, *Comput. Phys. Commun.* **176** (2007) 550.

[45] K. Peeters, *Introducing Cadabra: A symbolic computer algebra system for field theory problems*, hep-th/0701238 [SPIRES].

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