Quasi-Dual Formulation of Homogenized Integral Equation for Metasurfaces

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***Abstract*—The impedance boundary condition (IBC) is a well known and convenient approximation of metasurfaces. When used with the Integral Equation formulation (MoM) it leads to the Electric Field Integral Equation-IBC (EFIE-IBC) format. While very convenient, this formulation has conditioning issues when the surface impedance has inductive values. In this communication, we present an approximate formulation that employs the dual of the EFIE and the admittance; this formulation is shown to improve the conditioning problem of the EFIE-IBC.**

***Index Terms*—Metasurfaces, Integral Equations, Impedance Boundary Conditions**

I. Introduction

Metasurfaces are widely used in a variety of field-manipulation devices because of their ability to generate specific behavior [1]. Their sub-wavelength patterning implies an intrinsically multiscale modeling problem to accurately represent both the unit-cells (sub-wavelength details) and the overall macro-scale structure. To simplify these models, surface Impedance Boundary Conditions (IBC) are commonly used at the macroscopic level. This is even more relevant in design issues, as the IBC allows decoupling the design at the macroscopic level (profile of the surface impedance) and at the microscopic, unit-cell level.

The EFIE-IBC for impenetrable metasurfaces (a.k.a. opaque, or one-sided) was shown to be severely ill-conditioned in [2] for inductive values of the impedance; the problem was solved there by employing penetrable impedance sheets (a.k.a. transparent, or two-sided), which allowed to safely address most metasurface antenna problems with a metasurface sheet on top of a grounded slab (for which the transparent sheet is capacitive to guide TM waves). However, the ill-conditioning remains for inductive sheets, e.g., in meta-screens [3]. The conditioning issue has been tackled by several authors. The work in [4] employs a self-dual approach that effectively solves the problem for impenetrable scatterers of finite thickness; the method hinges on the magnetic field integral operator, and thus its advantages are lost for thin planar structures. The case of thin planar structures has been addressed in [5] employing a spatial filtering approach.

Here, we address this problem via a more analytical approach, inspired by the work in [4]; recognizing the root of the problem in the inductive nature of the problematic impedance sheet, we formulate the problem in such a way as to use the corresponding admittance, whose imaginary part has an opposite sign with respect to the impedance.

In order to do so, we based our reasoning on the physical topological properties of an inductive screen for unit cells of non-resonant (electrically small) size; in this case, the inductive response implies holes in a metal (PEC) screen. We also consider an electrically large IBC plate; under the approximation that a (large) finite plate may be (for on-surface fields) be approximated as an infinite screen, the problem is conveniently phrased in terms of the usual integral equation for apertures in an infinite PEC screen, [6], [7, e.g.], usually called HFIE (the H instead of M is used to avoid confusion into the MFIE which is totally different). In the latter, the unknown is a magnetic current, and involves the same EFIE operator (H field radiated by magnetic currents); for an IBC, it will dually involve the surface admittance.

II. Formulation

In order to address the core of the problem, we consider here a single layer of transparent thin open planar structure (denoted Σ) in free space; also, we restrict our analysis to isotropic (scalar) metasurfaces. For a transparent textured surface composed of infinitely thin, perfectly conducting metals, the IBC approximation [2] is a local averaging resulting in the usual relationship between the electric field and the magnetic field jump,

$$\hat{n} \times \mathbf{E}_{avg}(\mathbf{r}) = \hat{n} \times \left(Z_s^e \cdot (\hat{n} \times (\mathbf{H}_+ - \mathbf{H}_-)) \right)(\mathbf{r}) \qquad \mathbf{r} \in \Sigma \tag{1}$$

where Z_s^e is called the surface impedance. Use of the standard integral equation process to this boundary condition leads to a modified Electric Field Integral Equation called EFIE-IBC for the equivalent surface electric current $\mathbf{J} = \hat{n} \times (\mathbf{H}_+ - \mathbf{H}_-)$,

$$\hat{n} \times \mathcal{L}(\eta_0 \mathbf{J})(\mathbf{r}) + z_s^e \hat{n} \times \eta_0 \mathbf{J}(\mathbf{r}) = \hat{n} \times \mathbf{E}^{inc}(\mathbf{r}) \qquad \mathbf{r} \in \Sigma \tag{2}$$

$$\mathcal{L}(X)(\mathbf{r}) = jk_0 \int_S \left(\mathcal{I} + \frac{1}{k_0^2} \nabla\nabla\cdot \right) \mathcal{G}_0(\mathbf{r}, \mathbf{r}') \cdot X(\mathbf{r}') d\mathbf{r}' \tag{3}$$

$$\mathcal{G}_0(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_0 R}}{4\pi R} \tag{4}$$

where z_s^e is the surface impedance normalized to the free space impedance, $Z_s^e = z_s^e \eta_0$. Only the Electric Field Integral Operator (EFIO) $\mathcal{L}$ is involved, as the Magnetic Field Integral Operator vanishes on planar surfaces.

From the Method of Moment solution method, we discretize our unknown $\mathbf{J}$ into a linear combination of Rao-Wilton-Glisson (RWG) functions [8], and we use the classical Galerkin test method to define the standard matrix problem

$$\left[\mathbf{L}^{\mathbf{\Lambda}} + z_s^e \mathbf{G}^{\mathbf{\Lambda}} \right] \cdot [\eta_0 \mathbf{J}^{\mathbf{\Lambda}}] = V^E \tag{5}$$

where, $\mathbf{L}$ is the tested EFIE operator, G^{Λ} is the Gram matrix associated to the RWG basis, and V^E denotes the right-hand vector of the linear system.

An observation of the evolution of EFIE-IBC matrix problem conditioning (in figure 1) illustrates the impact of Z_s^e adding into the system. As alluded in the Introduction, to address this problem we consider the dual of our IBC plate, under the approximation of being electrically large. It results into an infinitely PEC screen with an aperture of the size of the plate, represented by an equivalent magnetic current. For this, using the standard HFIE procedure [6], [7] the homogenized boundary condition in (1) becomes the following modified HFIE

$$\left[\mathbf{L}^{\mathbf{\Lambda}} + y_s^m \mathbf{G}^{\mathbf{\Lambda}}\right] \cdot [\mathbf{M}_a] = \eta_0 V^H \tag{6}$$

where $Y_s^m = y_s^m / \eta_0$ is the (magnetic) equivalent admittance resulting from the condition (1). This equation will be called HFIE-ABC in the following, where ABC stands for Admittance Boundary Condition. Its duality to the EFIE-IBC is apparent.

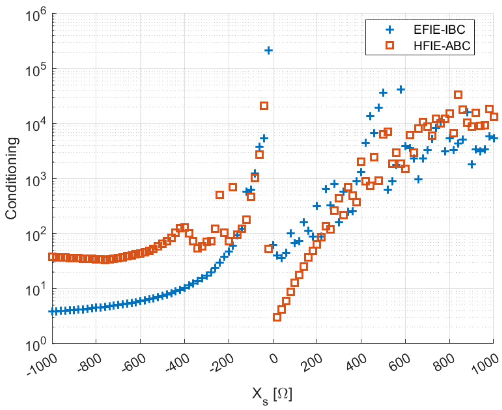

Fig. 1: Conditioning analysis of the EFIE-IBC and HFIE-ABC matrices problem, for a certain range of pure reactive isotropic IBC values $Z_s = jX_s$

In the standard HFIE the equivalence theorem application ("shorting" of holes) implies that the total scattered field includes the reflection from the screen; the standard approximation is the PO windowing of the incident field. Here we improve that by adding the field reflected by a finite size PEC plate.

III. Application Example

To illustrate our model, we use as an example a square XY IBC plate of length side $2\lambda_0 \times 2\lambda_0$, surrounded by vacuum and illuminated by an incident plane wave The surface patterning is represented by an isotropic inductive IBC with $Z_s^e = 33j$; this value corresponds to a realizable structure, an inductive grid with circular holes with cell size of 1/6 wavelength.

The structure is meshed into 1152 triangles, that will represent both the plate, for the standard EFIE-IBC, and the aperture computation for (6). The results of our approach are presented in figure 2 for both the E and H-planes.

Acknowledgment

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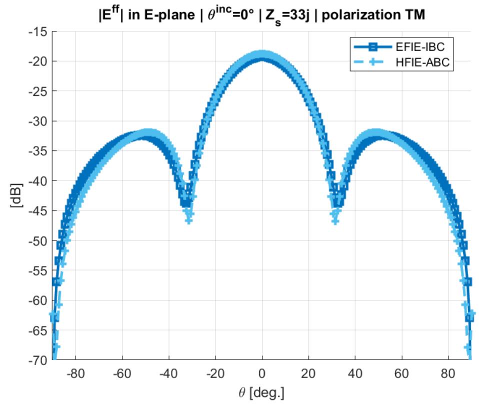
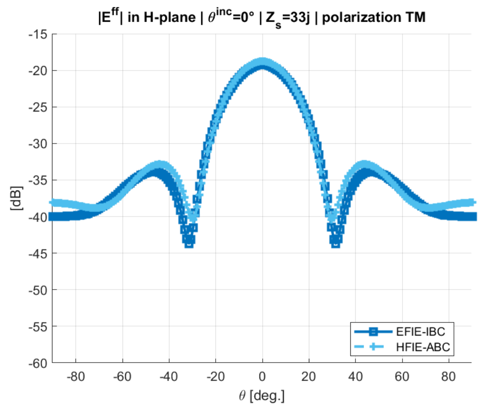


(a)



(b)

Fig. 2: Total scattered field for Far-Field comparison between standard EFIE-IBC and the new HFIE-ABC formulation for normal incidence and TM polarization; (a) E-plane, (b) H-plane.

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