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Fast and relaxed vector fitting: an application to a fighter jet aircraft

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ABSTRACT

The Fast and Relaxed Vector Fitting (FRVF) is a dynamic System Identification technique for input-output measurement, recently proposed for applications in the fields of Aerospace, Civil, and Mechanical Engineering. Here, its capabilities are validated, for the first time, on experimental data from Ground Vibration Testing (GVT) under controlled laboratory conditions. More specifically, the case study of interest is a real-size, real-life decommissioned F-16 fighter jet aircraft, widely studied in the recent literature. The results of these tests are benchmarked against the identifications available in the published literature, showing very good comparability with different kinds of excitation.

Keywords: Fighter Jet Aircraft, Ground Vibration Testing, System Identification, Fast Relaxed Vector Fitting, Input-Output Testing, Experimental Modal Analysis

1. INTRODUCTION TO THE FAST AND RELAXED VECTOR FITTING (FRVF)

Linear System Identification (SI) techniques play a fundamental role in Structural Dynamics, enabling the extraction of the modal parameters from experimental vibration data. Among these techniques, Vector Fitting (VF) and its modified Fast and Relaxed version (FRVF) have been recently proposed for application in Civil Engineering [1], [2].

In this study, the FRVF algorithm is pioneeringly applied to the Ground Vibration Testing (GVT) data of a full-scale aircraft, an important setting for input-output SI commonly used in Aeronautical Engineering. In particular, a public dataset from a decommissioned F-16 fighter jet aircraft is specifically used, with different input excitations. his dataset, widely recognised in the scientific literature, presents a challenging application due to the presence of nonlinear effects originating from payload-wing mounting interfaces. Thus, the primary objective is to assess the capability of FRVF in extracting the underlying linear system characteristics from the experimental data while disregarding the influence of nonlinear distortions. For completeness, a brief recall of the theoretical aspects of VF and FRVF, as utilised here, are provided in this Section. However, due to length limitations, the interested reader can refer to [1], [2], and [3] for a more detailed and comprehensive description.

1.1. The Vector Fitting (VF) Iterative Algorithm

Vector Fitting (VF) is an iterative approach for approximating the Frequency Response Function (FRF) of a system using a rational function representation, first proposed by Gustavsen & Semlyen [4] in the field of Electrical Engineering. Considering $s = \sigma + i\omega$ as the complex-valued Laplace variable, the fundamental VF formulation can be expressed as:

$$H(s) = \sum_{n=1}^{N} \frac{C_n}{s - p_n} + d + se \qquad (1)$$

where C_n are the residues, p_n are poles, d and se are compensating terms for low and high-frequency residual effects, and $H(s)$ indicates a generic Transform Function of a given recorded frequency response, $f(s)$, both defined in the Laplace domain. The VF algorithm iteratively determines an optimal set of poles and residues, following two main steps:

1. Pole Identification: The algorithm initially assumes an arbitrary set of poles, then refines them iteratively by solving a least squares (LS) minimisation problem.
2. Residue Identification: Once the poles p_n are determined, the residues C_n are computed, using the updated pole values.

The algorithm proceeds iteratively, typically converging within a few iterations (less than five for most applications).

To ensure numerical stability, the scaling function $\sigma(s)$ is introduced, such that:

$$\sigma(s)f(s) \approx \sum_{n=1}^{N} \frac{C_n}{s - p_n} + d + se \qquad (2)$$

where $\sigma(s)$ itself is approximated using a rational function with shared poles, i.e.

$$\sigma(s) \approx \sum_{n=1}^{N} \frac{\tilde{C}_n}{s - p_n} + 1 \qquad (3)$$

This reformulation means that the problem can be rewritten, by combining Eq. (2) and (3), as

$$\sum_{n=1}^{N} \frac{C_n}{s - p_n} + d + se - \left(\sum_{n=1}^{N} \frac{\tilde{C}_n}{s - p_n} + 1\right) \approx f(s) \qquad (4)$$

This allows the LS problem to be well-conditioned and efficiently solvable.

1.2. Variant for Fast Computation with Relaxed Constraints (FRVF)

The Fast Relaxed Vector Fitting (FRVF) method, originally proposed in [5] and [6], extends the conventional VF of the previous subsection to improve its convergence speed and robustness against noise. The main modifications include:

1. Relaxation of the Scaling Function: The high-frequency constraint ($H_\infty = 1$) is relaxed, allowing more flexibility in pole relocation. Therefore, the revised formulation of Eq. (3) introduces an additional non-unitary free term $\tilde{d}$, becoming

$$\sigma(s) \approx \sum_{n=1}^{N} \frac{\tilde{C}_n}{s - p_n} + \tilde{d} \tag{5}$$

which prevents over-constraining the solution, enhancing stability in noisy conditions (e.g., real-world vibrational data from mechanical systems).

2. Improved Numerical Stability: To avoid trivial solutions, an additional constraint ensures that the sum of the real parts of $\sigma(s)$ remains equal to the number of frequency samples N_s:

$$\Re\left\{\sum_{w=1}^{N_s}\left(\sum_{n=1}^{N}\frac{\tilde{C}_n}{s-p_n}+\tilde{d}\right)\right\} = N_s \tag{6}$$

Basically, this modification requires the sum of the real parts of $\sigma(s)$ to be strictly equal to N_s. As the problem converges, this is identical to constrain $\sigma(s) = 1\ \forall s$, similarly to the original VF formulation. However, in the first and intermediate iterations, the frequency samples are not fixed to any specific value, thus allowing much more flexibility to the search in the s domain for pole relocation.

3. **Faster Convergence:** The computational efficiency of FRVF is enhanced using a QR decomposition of the LS problem matrix, leveraging sparse matrix techniques:

$$\boldsymbol{LS} = \boldsymbol{Q}\,\boldsymbol{R} \tag{7}$$

where $\boldsymbol{Q}$ is an orthogonal matrix and $\boldsymbol{R}$ is an upper triangular matrix. This well-known property of QR decomposition reduces computational complexity while maintaining an unaltered accuracy.

This concludes the very brief and very general recall of FRVF; the reader can refer to the aforementioned sources for much more details, and to [7] for a comparison of this approach with other similar input-output algorithms with applications to aeronautical structures.

2. CASE STUDY

The General Dynamics F-16 Fighting Falcon is a US-produced military aircraft, built in the thousands since 1976 and operated by dozens of air forces throughout the world. The experimental data used in this study were acquired on a full-scale F-16 aircraft on the occasion of the Siemens LMS Ground Vibration Testing Master Class, held in September 2014 at the Saffraanberg military basis, Sint-Truiden, Belgium [8]. Two dummy payloads were mounted at the wing tips during the test campaign, to simulate the mass and inertia properties of real devices typically equipping an F-16 in flight. For input-output GVT, one shaker was attached underneath the right wing to apply input signals (Fig. 1).

Since its first publication online, this experimental dataset has captivated the interest of researchers in the field of Structural Dynamics, especially for the high level of nonlinear distortions measured by the output accelerometers – see e.g. the works of [8], [9], [10], [11], [12]. In particular, the dominant source of nonlinearity in the dynamics of structures originates from the mounting interfaces of the two payloads; more specifically, the back connection of the right-wing-to-payload interface was found to be the predominant source of nonlinear distortions in the aircraft dynamics [9].

In this work, however, the aim will not be on addressing these nonlinearities, but rather to establish the potentialities of the FRVF algorithm to identify the underlying linear systems of the aircraft. In this context, the nonlinear noise-like distortions included in the signals will be considered as a confounding term, which should be intentionally omitted in the identification of the linear terms.

Figure 1. Ground Vibration Testing setup of the F-16 aircraft. Retrieved from [9].

2.1. Available signals and target values

The publicly available data consisted of sweep-sine and multisine random tests. In particular, multisine tests were performed with both a full frequency grid and an odd one. Most of the published literature on this dataset focuses on results from multisine tests; nevertheless, for completeness, it was decided to opt for all three available options. In any case, a sampling frequency of $f_s = 400$ Hz was applied on all cases.

Different levels of input excitations were tested by the authors of the original dataset in all cases. As this research work focuses on the identification of the F-16 linearised model, only one input amplitude level was considered, the lowest (and thus, less nonlinear) one. However, as mentioned in [9], all input excitation levels for odd frequency multisine tests entail nonlinear oscillations. This was confirmed by the analyses of the data performed here.

The available data include one input (the shaker located beneath the right wing [9]) and three output channels, more specifically, (1) at the excitation location, (2) on the right wing next to the nonlinear interface of interest, and (3) on the payload next to the same interface [9].

For the sake of this work, wherever not specified differently, all identifications were performed as single-input single-output (SISO), using only the first output channels (the one closer to the input source).

3. RESULTS

Table 1 summarises the results obtained for three Ground Vibration Tests, performed with both odd and full frequency grids, as discussed in the previous Section. Importantly, most of the available sources reported exclusively on the flexible vibration modes, omitting the rigid body motions at lower frequencies. Regarding the specific settings for linear System Identification (SI), Noel et al 2014 [10] and Dossogne et al. 2015 [12] used the PolyMAX method [13], while Csurcsia et al. [11] estimated the modal parameters using a Best Linear Approximation (BLA) framework.

Noel et al 2014 [10] reports that the aircraft was instrumented with 45 uniaxial and 40 triaxial accelerometers and that two shakers were attached underneath the wings to excite both wing bending and torsion modes, but it does not explicitly remark which (and how many) input and output channels were used for the SI results in their Table 1. Their analysis was upper bounded to 10 Hz. Similarly, Dossogne et al. 2015 [12] bandlimited their SI in the 2-14 Hz frequency range, while not explicitly stating if a SISO, SIMO, or MIMO approach was followed (the paper mentions 145 measured degrees of freedom, suggesting a different test campaign or slightly reduced instrumentation). Csurcsia et al. [11] investigated the 1-15 Hz range, using a single input source (the shaker underneath the right wing) and a single output channel.

Table 1. Comparison between FRVF SISO identifications and values reported in the literature.

Mode	FRVF			benchmark values		
	Sweep sine with a linear, negative rate of 0.05 Hz/s, from 15 Hz to 2 Hz, level 1 (4.8 N) [Hz]	Multisine excitation with a full frequency grid, level 1 (12.4 N RMS) [Hz]	Multisine excitation with odd frequencies, level 1 (12.2 N RMS) [Hz]	Noel et al 2014 [10] root-mean-squared (RMS) force amplitude of 8 N, one realisation [Hz]	Dossogne et al. 2015 [12] low-level random test (RMS value of 10N), one realisation [Hz]	Csurcsia et al. 2020 [11] odd multisine, three different input levels measured at 12.2, 49.0, and 97.1 N RMS; nine realizations per excitation level [Hz]
1		2.06; 2.10		2.50 (First rigid-body mode, roll movement)	n.d.	n.d. (rigid body motions below 5 Hz)
	3.47	3.50	3.52			
	3.95	3.97		All sources mention more than one rigid body motion in the frequency range before the first flexible mode.		
2	5.20	5.20	5.17	4.82 (First wing bending mode)	5.16 (First wing bending mode)	5.20 (First wing bending mode)
3	6.65	6.62	6.94	6.18 (Antisymmetric payload rotation mode)	6.61 (Antisymmetric payload rotation mode)	n.d.
4	7.32	7.27 and 7.33	7.34	6.95 (Symmetric wing torsión mode)	7.31 (Symmetric wing torsion mode)	7.30 (Symmetric wing torsion mode)
5	unidentified	unidentified	unidentified	7.78 (Antisymmetric wing torsion mode)	n.d.	n.d.
6	unidentified	unidentified	8.56	8.85 (Antisymmetric wing torsion and vertical tail bending mode)	n.d.	n.d.
7	9.17	9.18	9.18	n.d.	9.14 (Antisymmetric wing bending mode)	n.d.
8	unidentified	unidentified	13.11	n.d	n.d	n.d
9	unidentified	unidentified	14.22	n.d	n.d	n.d
10	unidentified	unidentified	14.74	n.d	n.d	n.d

Similarly to the last case, as mentioned before, all identifications reported here are SISO, obtained from a single output channel Please remind as well that, due to the lack of reliable mode shapes due to the few output channels, all pairings with the modes reported in the three benchmark sources are merely based on educated guesses, made by the Authors, according to the closest natural frequency.

The resulting FRVF approximations of the swept-down sine, multisine-full, and multisine-odd signals will be briefly commented here. These correspond to the values of the second, third, and fourth columns of Table 1, in the same order. In all cases, a model order 10 was arbitrarily selected, based on an estimate of the number of modes expected on the frequency range of interest.

Fort he swept-down sine, only six out of ten modes fell into the bandpassed range. Nevertheless, these identifications align very well with the peaks visible to the naked eye, showcasing also a good agreement with the values reported in the papers used as benchmarks.

For the multisine-full signal, only nine modes were identified in the 2-15 Hz range of interest, and at least two cases (2.10 Hz and 7.33 Hz) are very likely double identifications. It must also be said that the double identification at 2.06 / 2.10 Hz is very close to the lower bound of the bandpass filter and clearly affected by it.

Finally, despite the use of the lowest level of input energy available, the experimental FRFs obtained from the multisine test with odd frequency are severely affected by the nonlinearities of the mechanical systems, making the phase measurements almost unusable. Despite so, the algorithm still manages to identify what can likely be the rigid and flexible modes of the F-16. However, as mentioned multiple times earlier, the lack of detailed mode shapes does not allow for a more methodical double-check with the results detailed in Noel et al 2014 [10] and Dossogne et al. 2015 [12].

The identifications at 13.11 Hz, 14.22 Hz, and 14.74 Hz cannot be confirmed from the available sources and are very likely spurious identifications due to model overfitting over data affected by noise-like nonlinear distortions at higher frequencies. On the other hand, the identification at 3.52 Hz most likely corresponds to a physical mode, very probably a rigid body motion. Finally, the frequency identified at 8.56 Hz seems to be consistent with the results of Noel et al. 2014 [10]; however, the lack of it in the other three signals, which are overall much less affected by nonlinearities and, therefore, closer to the underlying linear model (as also suggested by the deviations, highlighted in green, which is at least one order of magnitude lower), suggests that it could also be a misidentification. Indeed, this mode is not reported in the other two sources, and the peculiarity of its mode shape may indicate a local mode of the vertical tail, unobservable without accelerometers located in its proximity, as in the case of this layout, with the channels near the right-wing-to-payload interface.

4. CONCLUSIONS

This short contribution aimed at validating the Fast Relaxed Vector Fitting (FRVF) algorithm for linear System Identification (SI) purposes on experimental data with non-negligible nonlinear distortions. This is a quite common characteristic of many mechanical assemblies, due to the dry friction between surfaces and other input amplitude-dependent phenomena. In this case, an application of aeronautical interest was considered: the Ground Vibration Test (GVT) data of a real-size F-16 fighter jet.

The results of the identifications performed via the FRVF algorithm, when benchmarked with target values retrieved from the existing scientific literature, show how the proposed methodology compares effectively with other state-of-the-art approaches, which still struggle to properly identify the underlying linear dynamic behaviour of such complex systems. In this regard, future works will include the use of FRVF to track and investigate nonlinear terms for increasing force input levels.

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