Direct data-driven controller design from bounded errors-in-variables measurements

V. Cerone S. M. Fosson S. Pirrera D. Regruto

***Abstract*— he work focuses on direct data-driven controller design using input-output measurements corrupted by bounded additive noise. The main goal is to tackle the data-driven model reference control problem. To this end, we employ a set-membership framework and define the feasible controller parameter set, i.e., the set of parameters consistent with the noise bounds and the model-matching condition. We determine the controller parameters as the Chebyshev center of this set. We also provide a data-driven condition that is sufficient for establishing stability. This condition is also robust against the presence of bounded noise. The approach utilizes polynomial optimization and achieves global optimality via semidefinite relaxation techniques. A simulation example demonstrates the effectiveness of the proposed approach.he work focuses on direct data-driven controller design using input-output measurements corrupted by bounded additive noise. The main goal is to tackle the data-driven model reference control problem. To this end, we employ a set-membership framework and define the feasible controller parameter set, i.e., the set of parameters consistent with the noise bounds and the model-matching condition. We determine the controller parameters as the Chebyshev center of this set. We also provide a data-driven condition that is sufficient for establishing stability. This condition is also robust against the presence of bounded noise. The approach utilizes polynomial optimization and achieves global optimality via semidefinite relaxation techniques. A simulation example demonstrates the effectiveness of the proposed approach.T**

I. Introduction

Direct controller design from data is a topic attracting growing attention from the control and system identification communities, motivated by the increased complexity of modern applications, where obtaining an accurate model of the plant under control is often too expensive, time-consuming or even practically infeasible.

Direct data-driven control (DDDC) offers several advantages when compared with the traditional indirect approach where the controller is designed on the basis of a plant model identified from experimental data. One of the key benefits is the avoidance of unrealistic model assumptions, which can often lead to suboptimal results. This confidence in the direct approach is further reinforced by the fact that the indirect approach may be suboptimal because there is no separation principle between identification and control, see, e.g., [1] for a detailed discussion.

The scientific literature is rich in several contributions within the DDDC framework. Iterative feedback tuning [2] consists of optimizing the parameters of a given initial controller using closed-loop data collected during operation. However, the main drawback of such a method is the need to perform multiple experiments, which can be time-consuming and resource-intensive. This has led the community to dedicate much effort to developing non-iterative methods. Among them, we can find the non-iterative correlation-based tuning (NCbT) [3] and the virtual reference feedback tuning [4].

One of the main challenges related to DDDC is to guarantee stability when the data are corrupted by noise. To address this issue, [5] considers the case where the noise is not correlated with the input and proposes incorporating approximated stability constraints into the optimization problem to be solved for NCbT design. The authors then prove that the proposed formulation is asymptotically convergent to a stabilizing controller.

The main drawback of the solution proposed in [5] is the need for an infinite amount of data, which is infeasible in practice. Recent works focus on methods that guarantee stability with finite and noisy data to overcome this limitation. Works along this direction are [6], [7], [8], and [9]. In [7], the authors cast the problem in the set-membership framework and provide an algorithm for controller certification when ℓ_∞ bounded noise corrupts the finite-length input-output data. The other mentioned works consider a state-space perspective and solve the problem of designing a static state-feedback controller $u = Kx$. In [6], the authors propose a method that guarantees the finding of a stabilizing K if a certain signal-to-noise ratio condition is met. The works [8] and [9] consider the presence of noise affecting the state and the input measurements; this is referred to as the errors-in-variables (EIV) noise structure. In [9], the authors assume ℓ_2 or ℓ_∞ norm bounded EIV noise and provide a method to design K by solving a semidefinite programming (SDP) problem. In contrast, in [8], the authors consider ℓ_∞ norm bounded EIV noise and propose a method based on sum-of-squares programming to design superstabilizing controllers K.

This work addresses the problem of DDDC design for single-input single-output (SISO) systems in the presence of bounded EIV noise; it can be considered an extension of the results presented in [7], where only the output data are assumed to be corrupted by bounded noise (OE noise structure). Differently from works [8] and [9], which also consider the EIV noise structure, our contribution takes an input-output perspective on the problem by assuming that only the output is available for measurements, not the whole state. We assume ℓ_q-norm bounded noise, with $q = 1, 2, \infty$. We cast the problem in the set-membership framework and

* The authors are with the Dipartimento di Automatica e Informatica, Politecnico di Torino, corso Duca degli Abruzzi 24, 10129 Torino, Italy; e-mail: vito.cerone@polito.it, sophie.fosson@polito.it, simone.pirrera@polito.it, diego.regruto@polito.it;

design the controller parameters as the Chebyshev center of the feasible controller parameters set. To compute an approximate solution, we apply convex relaxation techniques based on the theory of moments and sum of squares (SOS) to relax the polynomial optimization problems to convex SDPs. Concerning stability, similarly to [7], we propose a data-driven controller certification approach. More precisely, we first design the controller and then compute the upper bound on the $\mathcal{H}_\infty$-norm of a suitably defined transfer function, which plays a role in defining a sufficient condition for the stability of the feedback loop. Such an upper bound is the worst-case for any allowed noise realization, and we compute it by exploiting the results in [10].

We organize the remainder of this paper as follows. Section II states the DDDC design problem through model matching. Section III casts the problem in the set-membership framework, provides a characterization of the optimization problems to be solved, and some considerations on controller order selection. Section IV proposes a data-driven method to check whether the controller obtained through the procedure described in Section III stabilizes the unknown plant. Section V provides a numerical example to demonstrate the effectiveness of the proposed approach, and Section VI concludes the paper.

II. Problem formulation

Let us consider the feedback control scheme in Fig. 1, where $G(q^{-1})$ is the unknown transfer function of the SISO plant under control, $K(\rho, q^{-1})$ is the controller we want to design, ρ is the vector of controller parameters and q^{-1} is the backward-shift operator. Let the controller be a discrete-time (DT) linear time-invariant (LTI) system of order n described by the following transfer function in the q^{-1} operator:

$$K(\rho) = \frac{\sum_{j=0}^{n} b_{1+j} q^{-j}}{1 + \sum_{j=1}^{n} a_j q^{-j}}. \tag{1}$$

The objective of the contribution is to propose a method to compute the controller parameter vector $\rho = [a^\top, b^\top]^\top \in \mathbb{R}^{2n+1}$ such that the complementary sensitivity function of the closed-loop system in Fig. 1, defined as

$$T(q^{-1}) = \frac{K(\rho, q^{-1})G(q^{-1})}{1 + K(\rho, q^{-1})G(q^{-1})}, \tag{2}$$

matches a given reference model $M(q^{-1})$ describing the desired behavior of the controlled system.

Concerning the reference model, we take the following standard assumption, as adopted in, e.g., [7], [11], [5].

Assumption 1 (Appropriateness): The reference model M is appropriate, i.e., the transfer function $1 - M$ includes the same non-minimum phase zeros as the plant, if any.

Under such an assumption, the ideal controller, i.e., the controller that achieves perfect matching for the actual plant, $K_{\text{id}} = M/(P(1 - M))$, stabilizes the unknown plant P.

Let us now introduce the following definition.

Definition 1 (Model-matching error transfer function): According to [7], we define the model-matching error transfer function $E(\rho, q^{-1})$ as the difference between the reference model and the complementary sensitivity function, i.e.,

$$E(\rho, q^{-1}) = M(q^{-1}) - \frac{G(q^{-1})K(\rho, q^{-1})}{1 + G(q^{-1})K(\rho, q^{-1})}. \tag{3}$$

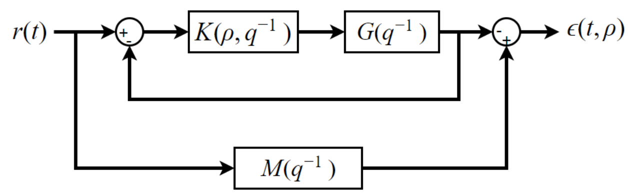

Fig. 1. Feedback control system to be designed compared with the reference model $M(q^{-1})$.

The problem of computing the controller parameters cannot be solved by simply imposing $E(\rho, q^{-1}) = 0$ since the plant transfer function $G(q^{-1})$ is unknown. However, thanks to Result 1 in [7, Section II-A], we know that $E(\rho, q^{-1}) = 0$ is equivalent to the following data-driven condition:

$$\frac{M}{1 - M} u(t) = K(\rho) y(t) \tag{4}$$

where $u(t)$ is a persistently exciting input applied to the plant and $y(t)$ is the corresponding noise-free output sequence.

The problem of designing a reference model $M(q^{-1})$ that satisfies a set of quantitative performance requirements is out of the scope of this paper. We refer the interested reader to the work [12].

We assume to collect experimental measurements $\{\tilde{u}(t), \tilde{y}(t)\}$, $t = 1, \ldots, N$, by performing an open loop experiment on the plant. We further suppose that unknown but bounded noise sequences $\eta(t)$ and $\epsilon(t)$ corrupt the collected data according to the following equation

$$\begin{aligned} \tilde{u}(t) &= u(t) + \epsilon(t), \quad \|\epsilon\|_q \leq \Delta_\epsilon \\ \tilde{y}(t) &= y(t) + \eta(t), \quad \|\eta\|_q \leq \Delta_\eta \end{aligned} \tag{5}$$

where $q = 1, 2$ or ∞.

III. Set-membership estimation of controller parameters

Based on equations (4) and (5), we can define the set of all controllers compatible with the collected data, the *a-priori* information on the noise and the considered controller structure. In the set-membership framework, such a set is called the *feasible controller set* (FCS) (see, e.g., [7]). The FCS, for the problem considered in this paper, can formally be defined as follows.

Definition 2 (Feasible controller set): The feasible controller set (FCS) is defined as

$$\mathcal{D}_K = \{K(\rho, q^{-1}) : \frac{M(q^{-1})}{1 - M(q^{-1})} u(t) = K(\rho, q^{-1}) y(t), \tag{6a}$$

$$\|\tilde{u} - u\|_q \leq \Delta_\epsilon, \quad \|\tilde{y} - y\|_q \leq \Delta_\eta.\} \tag{6b}$$

To simplify notation, we drop the backward shift operator q^{-1} from the equations in the rest of the manuscript.

Let us consider the following description for the transfer function $M/(1-M)$:

$$\frac{M}{1-M} = \frac{\sum_{f=0}^{n_M} \beta_{1+f} q^{-f}}{1+\sum_{f=1}^{n_M} \alpha_f q^{-f}}. \tag{7}$$

Using (7) , we can rewrite (6a) as

$$\begin{aligned} &\left(\sum_{f=0}^{n_M} \beta_{1+f} q^{-f}\right)\left(1+\sum_{j=1}^{n} a_j q^{-j}\right) u(t) = \\ &= \left(1+\sum_{f=1}^{n_M} \alpha_f q^{-f}\right)\left(\sum_{j=0}^{n} b_{1+j} q^{-j}\right) y(t), \end{aligned} \tag{8}$$

then, by expanding the products, we obtain

$$\begin{aligned} &\sum_{f=0}^{n_M} \beta_{1+f} u(t-f) + \sum_{j=1}^{n}\sum_{f=0}^{n_M} \beta_{1+f} a_j u(t-j-f) = \\ &= \sum_{j=0}^{n} b_{1+j} y(t-j) + \sum_{j=0}^{n}\sum_{f=1}^{n_M} \alpha_f b_{j+1} y(t-j-f). \end{aligned} \tag{9}$$

Equation (9), for $t = 1, \ldots, N$, provides a set of constraints that need to be satisfied by the parameter ρ of the controllers in the FCS $\mathcal{D}_K$. According to set-membership estimation theory, see, e.g., [7], [13], we define the *extended feasible controller parameter set* (EFCPS) as the set of all the values of the parameter ρ and the uncertain variables $y(t) = \tilde{y}(t) - \eta(t)$ and $u(t) = \tilde{u}(t) - \epsilon(t)$ which satisfies constraints (9), $\forall t = 1, \ldots, N$.

The EFCPS is formally defined as follows.

Definition 3 (Extended feasible controller parameter set): The extended feasible controller parameter set (EFCPS) $\mathcal{D}_{\rho,u,y}$ is

$$\begin{aligned} \mathcal{D}_{\rho,u,y} = \{&\rho \in \mathbb{R}^{2n+1}, u \in \mathbb{R}^N, y \in \mathbb{R}^N : \sum_{f=0}^{n_M} \beta_{1+f} u(t-f) + \\ &+ \sum_{j=1}^{n}\sum_{f=0}^{n_M} \beta_{1+f} a_j u(t-j-f) = \\ &= \sum_{j=0}^{n} b_{1+j} y(t-j) + \sum_{j=0}^{n}\sum_{f=1}^{n_M} \alpha_f b_{j+1} y(t-j-f), \\ &\text{for } t \in [n_M + n + 1, N] \\ &\|\tilde{u} - u\|_q \leq \Delta_\epsilon, \quad \|\tilde{y} - y\|_q \leq \Delta_\eta.\}. \end{aligned} \tag{10}$$

A. Central controller

One of the most interesting features of $\mathcal{D}_{\rho,u,y}$ is that the *ideal controller* is guaranteed by construction to belong to $\mathcal{D}_{\rho,u,y}$ if the controller order n is correct. Therefore, we propose to pick from $\mathcal{D}_{\rho,u,y}$ the value of ρ that minimizes the distance, in the ℓ_∞-norm, from the farthest point in the set. By definition, this value is the ℓ_∞-Chebyshev center of $\mathcal{D}_{\rho,u,y}$, which is given by

$$\rho^c = \arg\min_{\rho' \in \mathbb{R}^{2n+1}} \max_{(\rho,u,y) \in \mathcal{D}_{\rho,u,y}} \|\rho' - \rho\|_\infty. \tag{11}$$

.

As is well known, see, e.g., [13], the ℓ_∞-Chebyshev center can be computed as

$$\rho_i^c = \frac{1}{2}(\overline{\rho}_i + \underline{\rho}_i) \tag{12}$$

where

$$\underline{\rho}_i = \min_{(\rho,u,y) \in \mathcal{D}_{\rho,u,y}} \rho_i \tag{13a}$$

$$\overline{\rho}_i = \max_{(\rho,u,y) \in \mathcal{D}_{\rho,u,y}} \rho_i. \tag{13b}$$

Remark 1: When comparing the EFCPS in (10) with the one defined in [7, Section III-A], we highlight two fundamental differences:

1) due to the presence of input and output noise, the number of the variables describing the set is $2n+1+2N$, while in [7] is only $2n+1+N$.
2) in [7], thanks to the absence of input noise, it is possible to evaluate the signal

$$L(t) \doteq \frac{M}{1-M} u(t) \tag{14}$$

through simulation and use it to easily obtain the mathematical description of the constraints defining the EFCPS. In contrast, in the EIV problem considered here, the presence of input noise does not make it possible to compute $L(t)$ by simulation. Consequently, as discussed in Section III, we need to explicitly embed the transfer function $M/(1-M)$ in the constraints defining the EFCPS and in the optimization problems to be solved.

B. Feasibility and controller order selection

When $\mathcal{D}_{\rho,u,y}$ is empty, we cannot solve optimization problems (13) because of infeasibility. Such a result provides a numerical certificate of the fact that no controller in the selected model class (1) can solve the model-matching problem. In this case, we propose progressively increasing the controller order n until we obtain feasibility. If the plant under study is a finite-dimensional LTI system, this procedure leads to a non-empty EFCPS for a finite value of n.

C. Convex relaxation

Optimization problems (13) are polynomial optimization problems (POPs) due to the presence of products between optimization variables ρ, $u(t)$, and $y(t)$. Therefore, problems (13) are non-convex and NP-hard.

To get a tractable solution, we rely on semidefinite programming (SDP) relaxation according to the theory of moments and sum of squares. This approach allows us to construct a hierarchy of SDP problems whose solution is guaranteed to converge to the global optimum of the original POP for a sufficiently large order of relaxation (see [14] for a detailed discussion). The main drawback of such relaxation techniques is the computational effort required when the number of data, optimization variables and constraints, grows. However, by resorting to peculiar sparsity patterns in the optimization problems, it is possible to reduce the computational effort considerably and potentially solve large-scale problems. More specifically, problems (13) enjoy both correlative sparsity [15] and term sparsity [16], [17].

IV. Stability certification

This section proposes a method to check whether the controller obtained through the procedure described in Section III stabilizes the unknown plant. The method is based on data and does not require the knowledge of a mathematical model of the plant. This section assumes that an estimate of the ℓ_∞-Chebyshev center ρ^c is available.

To certify closed-loop stability, we rely on the following result

Theorem 1 (Closed-loop stability): Let

$$\Delta(\rho^c, z) = M(z) - (1 - M(z))K(\rho^c, z)G(z). \tag{15}$$

The closed-loop system in Figure 1 is stable if the following two conditions are met.

(a) $\Delta(\rho^c, z)$ is BIBO stable;
(b) $\|\Delta(\rho^c, z)\|_\infty < 1$, where $\|\cdot\|_\infty$ denotes the H_∞ norm of a transfer function.

Theorem 1 is a consequence of the small-gain theorem and provides a sufficient condition for closed-loop stability. This result was initially proposed in [5, Section III-A] and later used in [7, Section V].

Concerning condition (a), Assumption 1 ensures that the *ideal controller* stabilizes the unknown plant. Moreover, $\Delta(\rho^c, z)$ is BIBO stable if $K(\rho^c)$ is stable or contains as many integrators as $1 - M$; see [5, Section III] for more details.

To check condition (b), we rely on the results presented in [10] to estimate an upper bound on the Bode plot of $|\Delta(e^{i\omega})|$. If the computed bound is less than one for all ω, stability is certified. Thanks to the results in [10], we can compute the Bode envelopes of transfer functions of the form

$$\frac{\sum_{i=0}^{\nu_b} \delta_i z^i}{1 + \sum_{i=1}^{\nu_a} \gamma_i z^i} \tag{16}$$

where the parameters δ and γ are assumed to belong to an arbitrary semialgebraic set $\mathcal{S}$. In the following, we characterize $\mathcal{S}$ for the specific problem considered in this paper.

Applying the same input signal $u(t)$ to both sides of (15), we derive

$$\begin{aligned} &\Delta(q^{-1})u(t) = \\ &= M(q^{-1})u(t) - (1 - M(q^{-1}))K(\rho^c, q^{-1})Gu(t) \\ &= M(q^{-1})u(t) - (1 - M(q^{-1}))K(\rho^c, q^{-1})y(t). \end{aligned} \tag{17}$$

Since Δ is an LTI dynamical system, we can describe it by means of the following transfer function in the q^{-1} operator

$$\Delta(q^{-1}) = \frac{N_\Delta(q^{-1})}{D_\Delta(q^{-1})} = \frac{\sum_{j=0}^{n_\delta} \delta_j q^{-j}}{1 + \sum_{j=1}^{n_M + n_G + n} \gamma_j q^{-j}} \tag{18}$$

where $n_\delta \leq n_M + n + n_G$ by the definition of Δ. Moreover, we denote

$$M(q^{-1}) = \frac{N_M(q^{-1})}{D_M(q^{-1})} = \frac{\sum_{j=0}^{n_M} \mu_j q^{-j}}{1 + \sum_{j=1}^{n_M} \nu_j q^{-j}} \tag{19}$$

$$K(\rho^c, q^{-1}) = \frac{N_K(q^{-1})}{D_K(q^{-1})} = \frac{\sum_{j=0}^{n} b_j^c q^{-j}}{1 + \sum_{j=1}^{n} a_j^c q^{-j}}. \tag{20}$$

where $\rho^c = [a^{c,\top}, b^{c,\top}]^\top$. Based on (19) and (20), we rewrite (17) as

$$\begin{aligned} &D_M D_K N_\Delta u(t) = \\ &\quad = N_M D_K D_\Delta u(t) - (D_M - N_M) N_K D_\Delta y(t) \end{aligned} \tag{21}$$

which corresponds to

$$\begin{aligned} &\sum_{i=0}^{n_M} \sum_{j=0}^{n} \sum_{\kappa=0}^{n_\delta} (\nu_i - \mu_i) b_j^c \gamma_\kappa y(t - i - j - \kappa) = \\ &= \sum_{i=0}^{n_M} \sum_{j=0}^{n} \sum_{\kappa=0}^{n_\delta} (\nu_i \gamma_\kappa - \mu_i \delta_\kappa) a_j^c u(t - i - j - \kappa) \end{aligned} \tag{22}$$

where we set $\gamma_0 = \nu_0 = a_0^c = 1$ to simplify the notation.

Equation (22) defines the relationship between variables $u(t), y(t), \delta$ and γ, while a^c, b^c, μ, ν are known. Using information on the noise bounds (5), we conclude that the parameters δ, γ defining Δ belong to the set

$$\begin{aligned} \mathcal{S} = \{&u, y \in \mathbb{R}^N, \delta \in \mathbb{R}^{n_\delta + 1}, \gamma \in \mathbb{R}^{n_\delta} : \\ &\sum_{i=0}^{n_M} \sum_{j=0}^{n} \sum_{\kappa=0}^{n_\delta} (\nu_i - \mu_i) b_j^c \gamma_\kappa y(t - i - j - \kappa) = \\ &= \sum_{i=0}^{n_M} \sum_{j=0}^{n} \sum_{\kappa=0}^{n_\delta} (\nu_i \gamma_\kappa - \mu_i \delta_\kappa) a_j^c u(t - i - j - \kappa), \\ &\quad \text{for } t \in [n_\delta + n_M + n + 1, N] \\ &\|\tilde{u} - u\|_q \leq \Delta_\epsilon, \quad \|\tilde{y} - y\|_q \leq \Delta_\eta.\} \end{aligned} \tag{23}$$

The set (23) is semi-algebraic because (22) are bilinear equations in the considered variables, and we can recast the ℓ_1, ℓ_2, and ℓ_∞ norms constraints on u and y into polynomial constraints. By applying the procedure in [10] to the set (23), we can compute the upper bounds on $|\Delta(e^{i\omega})|$, which we use to check if condition (b) holds.

Remark 2: If the stability condition is not met, we propose to collect more data to improve the quality of the central estimate ρ^c and reduce the eventual conservativeness of the upper bound on $|\Delta(e^{i\omega})|$.

Remark 3: The method proposed in this section

- requires a single open-loop experiment;
- provides a worst-case analysis for all admissible noise sequences;
- certifies stability using a finite amount of data.

The method proposed in [5] does not enjoy the above features. Furthermore, compared with the method proposed in [7], the approach proposed in this paper is computationally cheaper since it does not require additional optimization variables representing the SVD decomposition of Δ.

V. Simulation Example

This section provides a simulation example to demonstrate the effectiveness of the proposed approach.

Consider the LTI plant described by the following transfer function:

$$P(s) = \frac{0.1(s+0.5)}{s^2+2s+1}. \tag{24}$$

By simulating the system, we generate $N = 500$ synthetic data with a sampling period of $30\,\mathrm{ms}$. The input signal is a uniformly distributed random sequence in $[-1, 1]$. ℓ_∞-norm bounded additive noises ϵ, η, with bounds $\Delta_\epsilon = 0.005$ and $\Delta_\eta = 0.001$, corrupt the input and output collected samples. The resulting signal-to-noise ratios are $\mathrm{SNR}_u = 20\log_{10}(\|\tilde{u}\|_2/\|\epsilon\|_2) = 46.2\,\mathrm{dB}$ and $\mathrm{SNR}_y = 20\log_{10}(\|\tilde{y}\|_2/\|\eta\|_2) = 33.2\,\mathrm{dB}$, respectively. We use the entire dataset to carry out controller design and stability certification. In principle, using more data allows handling lower signal-to-noise ratio noise at the expense of an increased computational effort.

We select as a reference model the second-order transfer function

$$M(z) = \frac{0.00478(z+0.957)}{z^2-1.869z+0.878}. \tag{25}$$

This model is the zero-order-hold discretization of the continuous-time reference model $M_{\mathrm{ct}}(s) = 1/(s^2+1.3s+1)$, obtained by following the approach proposed in [12] to fulfil the following performance requirements: rise time $t_r \leq 3.5\,\mathrm{s}$, overshoot $\hat{s} \leq 8\%$, and attenuation of sinusoidal disturbances on the feedback path $M_T^{\mathrm{HF}} \leq -35\,\mathrm{dB}$ in the frequency range $\omega \in [10\,,\, +\infty)$ rad/s.

To guarantee zero steady-state tracking error in the presence of constant reference and disturbances, we enforce the presence of one integrator in the controller transfer function. Regarding the controller's order, we pick $n = 2$ because selecting $n = 1$ leads to the infeasibility of optimization problems (13). Consequently, the selected controller structure is

$$K(z) = \frac{\rho_1 z^2 + \rho_2 z + \rho_3}{(z-1)(z+\rho_4)}. \tag{26}$$

By solving optimization problems (13), we obtain the parameters' bounds and the central estimate reported in Table I. The central controller is then given by

$$K(z) = \frac{0.5151z^2 + 0.0214z - 0.4558}{z^2 - 1.941z + 0.9411}. \tag{27}$$

Figure 2 shows the step response of the designed feedback control system compared to the response of the reference model M_{ct}.

Despite the ideal controller providing perfect matching is of order 3, the proposed procedure leads to a feasible central controller of order 2. We explain this fact by the presence of noise, which enlarges the feasible controller parameter set and eventually introduces feasible controllers with reduced order. However, we point out that the designed reduced order controller fulfils all the performance requirements used to design the reference model, although it does not provide a perfect matching of M.

Finally, we apply the procedure proposed in Section IV to check if the controller (27) stabilizes the unknown plant. Figure 3 shows the estimated upper bound on $|\Delta(e^{i\omega})|$ compared to its actual value. The upper bound is well below $0\,\mathrm{dB}$ at all frequencies; therefore, stability is certified for the worst-case noise realization.

-	$\underline{\theta}_i$	θ_i^c	$\overline{\theta}_i$
ρ_1	0.497	0.515	0.532
ρ_2	0.0060	0.0214	0.0367
ρ_3	-0.478	-0.455	-0.432
ρ_4	-0.923	-0.941	-0.959

TABLE I

Example: controller parameter bounds and central estimate.

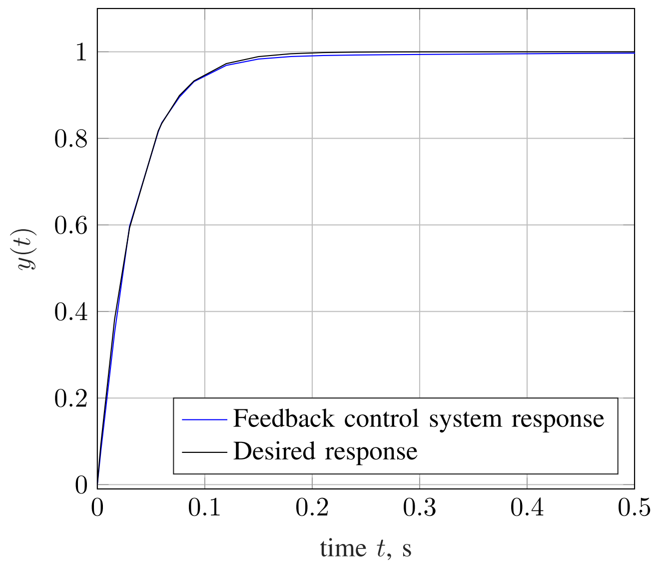

Fig. 2. Example: response of the feedback control system.

VI. Conclusions

This paper addresses the problem of designing controllers directly from input-output data affected by error-in-variables noise. We assume the noise to be unknown but bounded in the ℓ_1, ℓ_2 or ℓ_∞ norm. Unlike previous works on direct data-driven control using error-in-variables measurements, we do not look for state feedback controllers. Instead, we aim to solve a model-matching problem by looking for general dynamic linear-time-invariant controllers, and we design the controller parameters as the Chebyshev center of the set containing all candidate solutions compatible with the noise bounds. We rely on semidefinite programming relaxation to solve the resulting polynomial optimization problems.

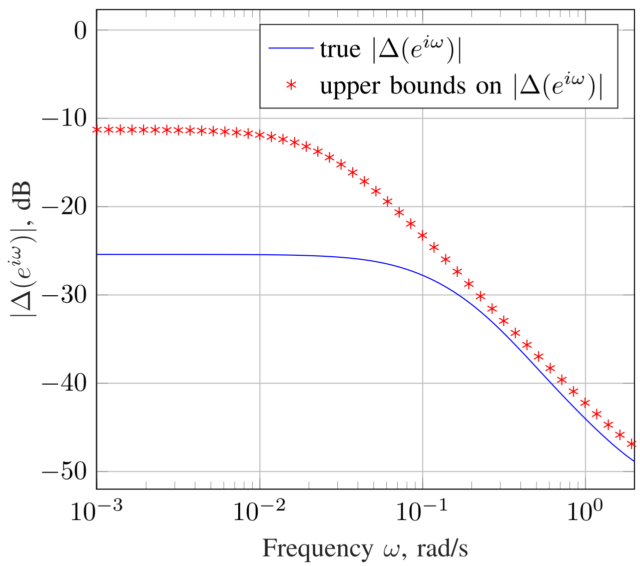

Fig. 3. Example: estimated bounds on $|\Delta(e^{i\omega})|$.

Furthermore, we provide a condition to ensure that the designed controller stabilizes the unknown plant. This condition is data-driven and robust; it relies on experimentally collected data and provides a worst-case condition with respect to all the possible noise realizations within the specified bound.

Future research will focus on the extension of the presented results to the multi-input, multi-output case and the application to real-world processes.

References

[1] F. Dörfler, "Data-driven control: Part two of two: Hot take: Why not go with models?" *IEEE Control Syst. Mag.*, vol. 43, no. 6, pp. 27–31, 2023.

[2] H. Hjalmarsson, S. Gunnarsson, and M. Gevers, "A convergent iterative restricted complexity control design scheme," in *IEEE Conf. Decis. Control (CDC)*, vol. 2. IEEE, 1994, pp. 1735–1740.

[3] A. Karimi, K. Van Heusden, and D. Bonvin, "Non-iterative data-driven controller tuning using the correlation approach," in *Eur. Control Conf. (ECC)*. IEEE, 2007, pp. 5189–5195.

[4] M. C. Campi, A. Lecchini, and S. M. Savaresi, "Virtual reference feedback tuning: a direct method for the design of feedback controllers," *Automatica*, vol. 38, no. 8, pp. 1337–1346, 2002.

[5] K. Van Heusden, A. Karimi, and D. Bonvin, "Data-driven model reference control with asymptotically guaranteed stability," *Int. J. Adapt. Control Signal Process.*, vol. 25, no. 4, pp. 331–351, 2011.

[6] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 909–924, 2019.

[7] M. Abuabiah, V. Cerone, S. Pirrera, and D. Regruto, "A non-iterative approach to direct data-driven control design of MIMO LTI systems," *IEEE Access*, vol. 11, pp. 121 671–121 687, 2023.

[8] J. Miller, T. Dai, and M. Sznaier, "Data-driven stabilizing and robust control of discrete-time linear systems with error in variables," 2023. [Online]. Available: https://arxiv.org/abs/2210.13430

[9] A. Bisoffi, L. Li, C. De Persis, and N. Monshizadeh, "Controller synthesis for input-state data with measurement errors," *IEEE Control Syst. Lett.*, pp. 1571 – 1576, 2024.

[10] V. Cerone, S. M. Fosson, D. Regruto, and A. Salam, "Bode envelope bounds for linear time-invariant systems affected by semialgebraic parametric uncertainty," in *IEEE Int. Conf. Control Autom. (ICCA)*. IEEE, 2020, pp. 247–252.

[11] F. Valderrama and F. Ruiz, "Limited-complexity controller tuning: A set membership data-driven approach," *European Journal of Control*, vol. 58, pp. 82–89, Mar. 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0947358019304455

[12] V. Cerone, M. Abuabiah, and D. Regruto, "An H_∞ method to design the reference model in direct data-driven control design," in *IEEE Conf. Decis. Control (CDC)*. IEEE, 2020, pp. 6004–6009.

[13] V. Cerone, D. Piga, and D. Regruto, "Set-membership error-in-variables identification through convex relaxation techniques," *IEEE Trans. Autom. Control*, vol. 57, no. 2, pp. 517–522, 2012.

[14] J.-B. Lasserre, *Moments, Positive Polynomials and Their Applications*, 10 2009.

[15] H. Waki, S. Kim, M. Kojima, and M. Muramatsu, "Sums of squares and semidefinite program relaxations for polynomial optimization problems with structured sparsity," *SIAM J. Optim.*, vol. 17, no. 1, pp. 218–242, 2006. [Online]. Available: https://doi.org/10.1137/050623802

[16] J. Wang, V. Magron, and J.-B. Lasserre, "Chordal-TSSOS: A Moment-SOS hierarchy that exploits term sparsity with chordal extension," *SIAM J. Optim.*, vol. 31, pp. 114–141, 01 2021.

[17] ——, "TSSOS: A Moment-SOS hierarchy that exploits term sparsity," *SIAM J. Optim.*, vol. 31, pp. 30–58, 01 2021.