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Physics of the Rupturing Mechanism for HP Memristor in Flux Mode

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From 1971 when Professor Leon O. Chua first theorized the memristor and the memristive behavior [1], reaserchers around the world started implementing devices able to perform such characteristics. Memristors nowdays are widely implemented in a wide spread range of applications. In 2008 Doctor Stanley Williams, at the HP laboratories, was able to develop the first industrial memristor [2] following the same properties of the one theorized by Chua more than 30 years before. In his paper he described the device as charged controlled, but from further calculations it can be established that the memristor can perform the same operations when flux controlled, since it is one of the base properties of memristors.

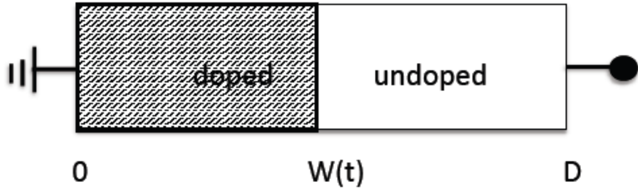

FIGURE 1. Schematic representation of Williams device. D is the complete length of the memristor and *W(t)* is the length of the doped area through time.

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Considering the device shown in Figure 1 it can be seen that this device can present three distinct states:

1) $W(t)=0$ then $R_{mem}=R_{off}$;
2) $W(t)=D$ then $R_{mem}=R_{on}$;
3) $0<W(t)<D$ then $R_{mem}=R_{on}(W(t)/D)+R_{off}(D-W(t))/D$.

The case of $R=R_{on}$ and sufficient negative flux should describe the rupturing or resetting of the device. As remarked already by Williams, his model as discussed before is based on purely ionic movement predicts that "*even a small negative bias will switch it back to the off state*". This questionable on general grounds, but is particulary troubling for the Williams model which assumes that ions are moving in the field of the doped region (i.e. with R_{on}). When the device is in the ON state, the entire oxide is conductive where the erroneous Williams device should apparently apply. But it doesn't as Williams has stated himself. The situation is much more consistent with our model since the ions are driven into or away from the un-doped region by the electric field of the un-doped region. Since, when device is in R_{on}, there is no un-doped region, our model naturally implies that it cannot be applied to this situation because the driving force is zero. Hence, there is a need for a physical mechanism responsible for the rupturing. Such a model is developed in the following and brought into such an analytical form that the constitutive relation can be expressed as an analytic function.

Applying a negative flux to the device when in an ON state generates a current flow through the device in the border between the doped TiO2 and the Pt electrode. This current at the starting time point can be considered uniform over all the electrode. Probabilistically it is highly improbable that the doped TiO2 will detach completely from the Pt border at the same time, thanks to the nature of the drift diffusive ionic transport. Asperities will generate on the TiO2 film, with regions that are not anymore in contact with the electrode, reducing considerably the conductive area. Thus, there will be a higher current density in the regions of the doped film still attached to the electrode. The higher current density will generate heating of the doped strand, and this heat will cause the rupture of the conductive filament. The rupturing point on the other hand is not to be considered on the Pt boundary, since its properties make the electrode a heat sink. The distance (*Δ*) is function then of the thermal conductivity of the

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undoped region. The higher this value is, the easier to transport heat away from the filament, thus the higher is the distance Δ from the Pt electrode. Also ambient temperature (T_0) in which the memristor is confined affects the value of Δ. The closer T_0 is to the critical filament rupturing temperature, the closer is the rupturing point to the Pt surface.

Taking into consideration what afore mentioned the rupture of the filament takes place when a critical temperature is reached through Joule's heating. In order to do so we expressed Joule's heating in terms of flux and a linear voltage ramp was considered to construct the final model (*V(t)=rt*, where *r* is the voltage ramp rate expressed in [V/s]). By these means we can express the flux $\varphi(t) = \frac{1}{2}rt^2$, the power dissipated $P(t) = \frac{r^2t^2}{R_{on}}$ and the energy $Q(t) = \frac{1}{3}\frac{r^2t^2}{R_{on}}$. So it can be seen that $P(t) \sim \varphi(t)$ and $Q(t) \sim 3/2\ \varphi(t)$. Power describes the momentary situation at a given time t and therefore does not represent any memory effect. Energy describes the integrated effect from t=0 until t=t'. Thus φ(t) would not describe any memory effect, only φ^n with $n \geq 3/2$ will describe the cumulative memory effect. Since Joule's heating is independent from the voltage polarity n should be even and larger than 2. In order to include the increased effect of local heating, the critical temperature is modeled as:

$$kT_{crit} = kT_0 + c\varphi_{crit}^n \quad \text{with } n>3/2 \tag{1}$$

where *k* is the Boltzmann constant is the specific heat of the doped region. From this the memristance was modeled as function of φ as:

$$R(\varphi) = R_{on}\big(1 - \exp\big(-A(\varphi)\big)\big) + R_\Delta \exp\left(-A(\varphi)\right)) \tag{2}$$

where

$$A(\varphi) = \frac{E_a}{k}\left(\frac{1}{T_0 + c\varphi^n} - \frac{1}{T_{crit}}\right) \tag{3}$$

E_a is the ionic activation energy fron Arrhenius law and:

$$R_\Delta = R_{on}\left(1 - \frac{D-\Delta}{D}\right) + R_{off}\left(\frac{D-\Delta}{D}\right) \tag{4}$$

This model is valid for all φ smaller or equal to φ_{crit} , when T_{crit} is reached and when the device settles to $R(W=(D-\Delta)/D)$.

Observing that when $\varphi=0$, $R(\varphi)=0$ and when $\varphi=\varphi_{crit}$, $R=R_\Delta$, we were able to obtain:

$$\frac{dq}{d\varphi} = \frac{1}{R_{on}(1-\exp\left(-A(\varphi)\right)) + R_\Delta(\exp\left(-A(\varphi)\right))} \tag{5}$$

and by integrating this function, by constructing secondary function which is continuous, infinitely differentiable and numerically shows the same behavior of (5) which is impossible to integrate analytically it was possible to compute:

$$q(\varphi) = \frac{\varphi}{R_{on}} + \frac{R_\Delta - R_{on}}{R_\Delta R_{on}}(\varphi_{crit} - \varphi)\exp\left(-A(\varphi)\right) \tag{6}$$

As shown in Figure 2 it is possible to observe that for small fluxes the slope of the curve is $1/R_{on}$ and for $\varphi>\varphi_{crit}$ the slope rate is $1/R_\Delta$. From the enlargement in Figure 2 it is possible to see that the transition of the charge is smooth and not abrupt. The exponent n in (1) actually describes the abruptness of the transition (rupture) accounting the effects described before. The higher *n*, the more abrupt is the transition.

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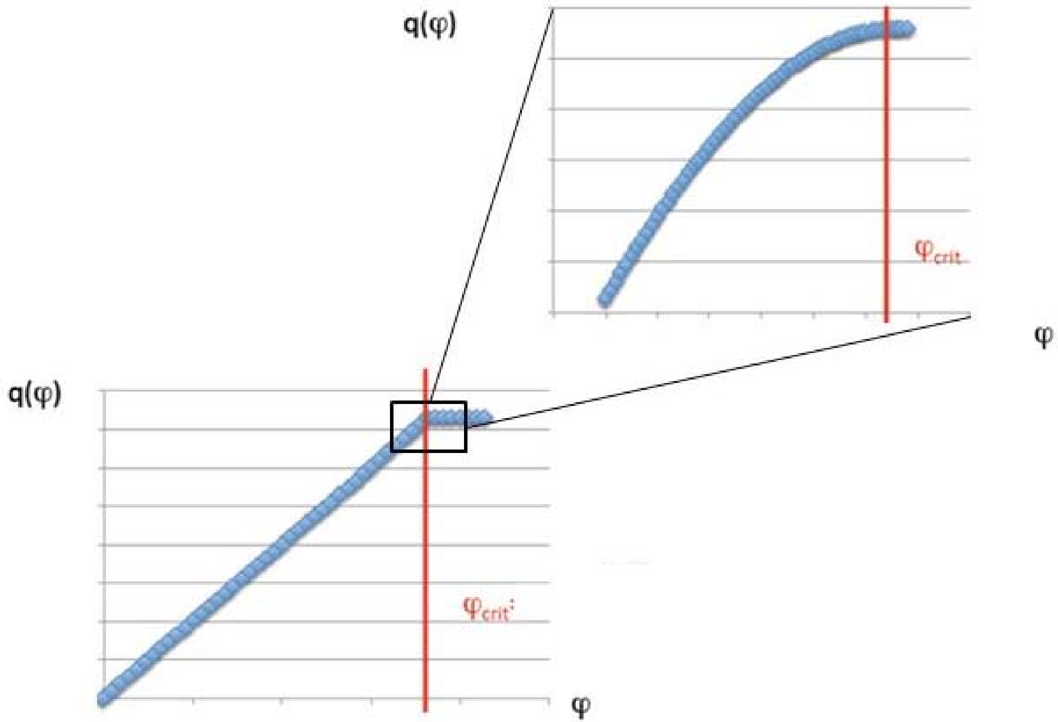

FIGURE 2. Charge transition in the Williams device due to rupturing of the doped filament.

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With this rupturing model the entire analytical model for the Williams device is completed.

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