POLITECNICO DI TORINO
Repository ISTITUZIONALE

Economic Nonlinear MPC for Conflicting Control Objectives: The Case of Adaptive Cruise Control

Original
Economic Nonlinear MPC for Conflicting Control Objectives: The Case of Adaptive Cruise Control / Calogero, Lorenzo; Pagone, Michele; Novara, Carlo; Rizzo, Alessandro. - ELETTRONICO. - (2025), pp. 3089-3094. (2025 IEEE 64th Conference on Decision and Control (CDC) Rio de Janeiro (Bra) December 9-12, 2025) [10.1109/CDC57313.2025.11312367].

Availability:
This version is available at: 11583/3002464 since: 2026-02-10T12:04:26Z

Publisher:
IEEE

Published
DOI:10.1109/CDC57313.2025.11312367

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright
IEEE postprint/Author's Accepted Manuscript

©2025 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

06 May 2026

Averaging Conflicting Objectives in Economic Nonlinear MPC for Adaptive Cruise Control

Lorenzo Calogero, *Graduate Student Member, IEEE*, Michele Pagone, *Member, IEEE*, Carlo Novara, *Senior Member, IEEE*, and Alessandro Rizzo, *Senior Member, IEEE*

***Abstract*— Optimizing the energy consumption of electric vehicles (EVs) during operation is a key factor in mitigating their overall environmental impact. Autonomous vehicle functions, such as Adaptive Cruise Control (ACC), typically disregard economic criteria such as energy optimization, being in general not trivial to conciliate tracking and economic control tasks. Within the domain of optimal control, Economic Nonlinear MPC (E-NMPC) is designed to deliver an economically optimal control action, optimizing the economic profit of the plant. However, E-NMPC does not allow to include additional adversarial tasks, such as tracking, and its closed-loop stability is not easy to guarantee. In this work, we propose a novel E-NMPC formulation for conflicting control objectives – such as tracking and economic tasks – that attains the optimal trade-off between them. Furthermore, we propose a constructive procedure to design stabilizing terms for E-NMPC, ensuring its closed-loop stability with minimal impact on the economic performance. We apply the proposed E-NMPC strategy to the ACC case study, proving its effectiveness in simulation: the E-NMPC-based ACC proficiently attains the conflicting tasks, delivering a higher economic profit than standard NMPC, while ensuring closed-loop stability.**

I. Introduction

In recent years, the escalating environmental impact of global warming and pollution has become a growing concern for both the public and the scientific community worldwide. In response, the automotive industry has accelerated its transition towards more sustainable alternatives to internal combustion engines. In this context, electric vehicles (EVs) have emerged as the leading solution, offering an effective way to reduce emissions and mitigate the environmental harm caused by traditional transportation methods [1].

Nonetheless, being EVs supplied with energy that is still largely generated through conventional means, eliminating emissions is not enough to minimize EV environmental impact: the key factor lies in optimizing the energy consumption of EVs during their operation. From a control perspective, energy optimization can be implemented alongside autonomous vehicle functions. A notable example is represented by Adaptive Cruise Control (ACC). ACC is a driver-assistance system designed to control the vehicle speed, ensuring safe driving conditions with respect to the preceding vehicle [2], [3]. ACC can either maintain a given velocity selected by the driver, or dynamically adjust it according to the leading vehicle behavior. As a result, the ACC control problem consists in regulating (or tracking) a given velocity profile. Alongside it, we would like to include the additional control task of mitigating EV energy consumption during ACC operation. We shall refer to the latter task as an "economic" control objective.

Conventional ACC control strategies typically only account for the tracking task, disregarding economic performance. Examples include the constant time gap (CTG) policy [4], [5], whose goal is to maintain a prescribed time-based headway gap between the ego and leading vehicle; such a gap equals the time needed by the ego vehicle to reach the leading one at its current speed. CTG, however, exhibits several drawbacks, among which non-optimality and inability to handle state and input constraints. Other widely-adopted ACC strategies include fuzzy logic control (FLC) [6], [7] and reinforcement learning (RL)-based control [8], [9]. FLC employs a rule-based decision-making approach, bearing the hallmarks of interpretability and fast execution, but lacking prediction and adaptation capability. RL-based control, on the other hand, leverages machine learning to deliver suitable control actions by learning from real driving data. RL, however, requires high-quality data and extensive training in diverse scenarios, while its lack of interpretability poses a challenge for ensuring safety.

In the framework of optimal control, a peculiar variant of Nonlinear MPC (NMPC), named Economic NMPC (E-NMPC), has established in the last two decades [10]–[13]. E-NMPC allows to steer the system under control towards its economically optimal equilibrium state – for a given economic criterion – while ensuring a profitable economic performance during the transient. This is achieved by directly employing the economic criterion as stage cost in the E-NMPC objective function [13].

The main drawback of E-NMPC is two-fold. First, it only accounts for the economic objective, thus not allowing to include any additional conflicting task, such as tracking. Second, employing the economic criterion as E-NMPC stage cost makes the latter, in general, non-minimal at the optimal economic equilibrium [11], [13]; this aspect does not allow to employ the optimal E-NMPC cost value as a Lyapunov function, rendering stability guarantees harder to ensure than

The authors are with the Department of Electronics and Telecommunications, Politecnico di Torino, 10129 Turin, Italy (e-mail: {lorenzo.calogero, michele.pagone, carlo.novara, alessandro.rizzo}@polito.it).

The work of L. Calogero was supported by the European Union NextGenerationEU (NGEU)–Piano Nazionale di Ripresa e Resilienza (PNRR) Project and the Italian Ministry of University and Research (MUR) under Grant 352/2022. The work of M. Pagone, C. Novara, and A. Rizzo was supported by the Sustainable Mobility National Research Center (MOST) and the European Union NGEU–PNRR Project (Mission 4, Component 2, Investment 1.4–D.D. 1033 17/06/2022) under Grant CN00000023.

This manuscript reflects only the authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

© 2025 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

for conventional NMPC [11], [14]. Some works have tackled E-NMPC stability through dissipativity arguments [12], [13]. However, such approaches require a case-by-case analysis which, typically, holds only in some special cases [13].

In this work, we aim to fill the gaps represented by the mentioned E-NMPC drawbacks – namely, the inability to handle conflicting objectives and the non-triviality of stability guarantees – by proposing a novel E-NMPC control problem formulation. Our E-NMPC approach accounts for both economic and tracking tasks together and regulates the plant towards the optimal trade-off equilibrium between these two conflicting objectives. Moreover, we propose a general constructive procedure to design suitable stabilizing terms for E-NMPC, ensuring its closed-loop stability with minimal impact on the economic performance.

The proposed E-NMPC strategy is applied to ACC, in the scenario of velocity regulation. The effectiveness of the E-NMPC-based ACC is thoroughly assessed in simulation, encompassing E-NMPC control performance, trade-off between conflicting objectives, stability, and economic profit compared to standard NMPC. Results demonstrate the proficiency of E-NMPC in delivering the ACC control action, for both tracking and economic tasks. Also, E-NMPC attains a higher economic profit compared to NMPC, while ensuring its closed-loop stability.

Notation: $x = [x_i]_{i=1}^n \in \mathbb{R}^n$ is the vector with components x_i. $(x)_I$ is the vector collecting the components of x indexed by the set $I \subset \{1, \dots, n\}$. $\|x\|_p$ is the p-norm of x, $p \in \mathbb{R}_{\geq 1}$. $\|x\|_W = \sqrt{x^\top W x}$ is the weighted 2-norm of $x \in \mathbb{R}^n$, $W \in \mathbb{R}^{n \times n}$. Given $x, y \in \mathbb{R}^n$, any relation $x \lesseqqgtr y$ is considered component-wise, i.e., $(x)_i \lesseqqgtr (y)_i$, $\forall\, i \in \{1, \dots, n\}$. Finally, we employ the notations $[x^\top, y^\top]^\top = (x, y)$ interchangeably.

II. Problem Statement

Consider a discrete-time (DT) nonlinear dynamical system, defined as follows:

$$x_{k+1} = f(x_k, u_k), \quad k \in \mathbb{Z}_{\geq 0}, \tag{1}$$

where $x_k \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$ and $u_k \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ are the state and input vectors at time k, respectively; let $\mathcal{Z} = \mathcal{X} \times \mathcal{U}$.

Assumption 1: System (1) *is controllable [11], the state is available at each time k, and f is $\mathcal{C}^1$-smooth on $\mathcal{Z}$.*

Assumption 2: $\mathcal{X}$, $\mathcal{U}$ (and, thus, $\mathcal{Z}$) are convex polytopes.

Assumption 3: System (1) *admits a manifold $\mathcal{Z}_s$ of equilibrium points (x_s, u_s), i.e.,*

$$\mathcal{Z}_s = \{(x_s, u_s) \in \mathcal{Z} : x_s = f(x_s, u_s)\}. \tag{2}$$

Let us now consider a desired reference equilibrium point $(x_r, u_r) = z_r \in \mathcal{Z}_s$ toward which we want to stabilize system (1). However, we would also like that, in the transient phase of regulation, the system operates with a profitable economic performance. Such two control objectives, i.e., regulation and economic profit, are conflicting with each other. To quantify these two control tasks, let us define the performance-metric cost function ℓ as follows:

$$\begin{aligned} \ell(x, u) &= \ell_r(x, u) + \ell_e(x, u), \\ \ell_r(x, u) &= \|x - x_r\|_Q^2 + \|u - u_r\|_R^2. \end{aligned} \tag{3}$$

Here, the cost function ℓ encodes the two conflicting control objectives mentioned above: the term ℓ_r quantifies the regulation task towards the reference z_r; the term ℓ_e represents an economic criterion – which, typically, is non-quadratic and non-convex – to be optimized for the system to operate with a profitable economic performance. Therefore, ℓ is an economic cost function [13].

Assumption 4: The economic term ℓ_e (and, thus, ℓ) is locally Lipschitz continuous on $\mathcal{Z}$.

By minimizing ℓ_r and ℓ_e individually over $\mathcal{Z}_s$, they attain their minimum in z_r and $z_e = (x_e, u_e)$, respectively, with

$$\min_{(x,u)\in\mathcal{Z}_s} \ell_r(x, u) = 0, \tag{4a}$$

$$\min_{(x,u)\in\mathcal{Z}_s} \ell_e(x, u) = \ell_e(x_e, u_e). \tag{4b}$$

In general, being ℓ_e an economic criterion, there exist some transient points $(x, u) \in \mathcal{Z} \setminus \mathcal{Z}_s$ such that $\ell_e(x, u) < \ell_e(x_e, u_e)$, i.e., ℓ_e is not minimal in z_e [13].

Now, minimizing the whole cost ℓ over $\mathcal{Z}_s$, we obtain

$$\min_{(x,u)\in\mathcal{Z}_s} \ell(x, u) = \ell(x_s, u_s), \tag{5a}$$

$$\arg\min_{(x,u)\in\mathcal{Z}_s} \ell(x, u) = (x_s, u_s) = z_s. \tag{5b}$$

Here, z_s is a trade-off equilibrium between the two conflicting objectives, for which it holds that $\ell_r(x_s, u_s) > \ell_r(x_r, u_r)$ and $\ell_e(x_s, u_s) > \ell_e(x_e, u_e)$. The priority of one objective over the other can be set by properly tuning the weighting parameters of ℓ_r and ℓ_e. Also for ℓ, in general, it holds

$$\ell(x, u) < \ell(x_s, u_s) \;\text{ for some }\; (x, u) \in \mathcal{Z} \setminus \mathcal{Z}_s. \tag{6}$$

Given the above setting, our goal is to design an MPC-based optimal control strategy to regulate system (1) towards the trade-off equilibrium (x_s, u_s), while ensuring that:

- in the transient phase of regulation, the optimal trade-off between the two conflicting objectives (i.e., tracking and economic performance) is achieved;
- the stability of the closed-loop system is guaranteed.

Being ℓ an economic cost function, we leverage Economic Nonlinear MPC (E-NMPC), as detailed in the next section.

III. Economic Model Predictive Control for Conflicting Control Objectives

A. Economic Nonlinear MPC Formulation

Let us consider the following economic cost function:

$$J(\hat{x}, \hat{u}) = \sum_{i=0}^{N_p-1} \varphi(\hat{x}_i, \hat{u}_i), \tag{7}$$

where $\hat{x} = \{\hat{x}_i\}_{i=0}^{N_p}$, $\hat{u} = \{\hat{u}_i\}_{i=0}^{N_p-1}$, and φ is the economic stage cost. The E-NMPC strategy, employed to control system (1), is given by the following optimal control problem (OCP), which is solved at each time instant $k \geq 0$:

$$\min_{\hat{x},\hat{u}} \quad J(\hat{x}, \hat{u}) \tag{8a}$$

$$\text{s.t.} \quad \hat{x}_0 = x_k, \quad \hat{x}_{i+1} = f(\hat{x}_i, \hat{u}_i), \tag{8b}$$

$$(\hat{x}_i, \hat{u}_i) \in \mathcal{Z}, \tag{8c}$$

$$\hat{x}_{N_p} \in \mathcal{X}_f, \tag{8d}$$

$$i = 0, \dots, N_p - 1.$$

Here, the decision variables $\hat{x}$ and $\hat{u}$ are state and input trajectories predicted i steps ahead at time k, respectively. Note that a terminal state constraint is enforced in Eq. (8d), with terminal set $\mathcal{X}_f$.

The optimal control input sequence $\hat{u}^*$, obtained by solving the OCP (8), is applied to system (1) through the receding horizon strategy: only the first sample $\hat{u}_0^*$ is applied to the system; the remainder of $\hat{u}^*$ is discarded. Then, the OCP (8) can be represented by a control policy π, as follows:

$$u_k = \hat{u}_0^* = \pi(x_k), \tag{9}$$

and the closed-loop system (1), (9) evolves as $x_{k+1} = f(x_k, \pi(x_k))$, $k \geq 0$.

Finally, for the OCP (8), let: i) $J^*(x_k)$ be the optimal cost value; ii) $\mathscr{U}_{N_p}(x_k) = \{\hat{u} : \exists\, \hat{x} \text{ satisfying Eqs. (8b)-(8d)}\}$ be the set of feasible input sequences; iii) $\mathcal{X}_{N_p} = \{x \in \mathcal{X} : \exists\, \hat{u} \in \mathscr{U}_{N_p}(x)\} \subseteq \mathcal{X}$ be the set of admissible states.

As stage cost φ in Eq. (7), we employ the function ℓ (3),

$$\begin{aligned}\varphi(x,u) &= \ell(x,u)\\ &= \|x-x_r\|_Q^2 + \|u-u_r\|_R^2 + \ell_e(x,u).\end{aligned} \tag{10}$$

With the above stage cost, system (1) is going to be regulated towards the trade-off equilibrium z_s (see Eq. (5)), achieving, during the transient, the optimal trade-off between the two conflicting control tasks (i.e., tracking and economic profit).

Now, a further challenge is posed by guaranteeing the stability in closed-loop of system (1), under the control of the E-NMPC policy (8), (9) with stage cost (10).

To this aim, in the next section we propose a constructive procedure to design stabilizing terms for the E-NMPC strategy (8). Moreover, we will analyze the trade-off between economic profit and stabilization introduced by such terms.

B. Design of Stabilizing Terms for Economic NMPC

In the Economic MPC framework, ensuring closed-loop stability is not as straightforward as in the classic MPC setting. This arises because the economic stage cost ℓ (10) is not minimal at the equilibrium z_s, as highlighted by Eq. (6). This aspect does not allow to employ the optimal cost value $J^*(x)$ of the OCP (8) as a Lyapunov function, since J^* may not be monotonically decreasing along the trajectories of the closed-loop system (1), (9), even if the latter is stable.

In the following, our goal is to introduce suitable ingredients into the E-NMPC problem (8) to enforce closed-loop stability. These stabilizing terms should also be tunable, enabling to set a trade-off between stabilization and the economic profit given by the original non-stabilized problem.

Theorem 1: Consider the E-NMPC OCP in Eq. (8) *and let Assumptions 1-4 hold.*

a) Let the cost function (7), (8a) *be*

$$J(\hat{x},\hat{u}) = \sum_{i=0}^{N_p-1} \overline{\ell}(\hat{x}_i, \hat{u}_i), \tag{11}$$

where $\overline{\ell}$ is an augmented stage cost, defined as

$$\overline{\ell}(x,u) = \ell(x,u) + \alpha(x,u), \tag{12}$$

where ℓ is given by Eq. (3), *$\alpha(z) = a\,\kappa(z-z_s)$, $\kappa : \mathcal{Z} \to \mathbb{R}_{\geq 0}$ is positive-definite, and $a \in \mathbb{R}_{>0}$.*

b) Let the terminal set in Eq. (8d) *be*

$$\mathcal{X}_f = \{x_s\}. \tag{13}$$

If the weighting parameter a satisfies

$$a > \frac{\ell(x_s,u_s) - \ell(x,u)}{\kappa(z-z_s)}, \;\; \forall\,(x,u) \in \mathcal{Z} \smallsetminus \{(x_s,u_s)\}, \tag{14}$$

then x_s is an asymptotically stable equilibrium of the closed-loop system (1), (9)*, with region of attraction $\mathcal{X}_{N_p}$.*

Remark 1: The optimal value of a that minimizes the influence of α on the original stage cost ℓ is given by

$$a^* = \sup_{(x,u)\in\mathcal{Z}\smallsetminus\{(x_s,u_s)\}} \frac{\ell(x_s,u_s) - \ell(x,u)}{\kappa(z-z_s)} + \epsilon, \tag{15}$$

where $\epsilon > 0$ is arbitrarily small.

Proposition 1: Consider Eq. (15) *and let $\beta(x,u) = \frac{\ell(x_s,u_s)-\ell(x,u)}{\kappa(z-z_s)}$. If*

$$\ell(z) = \ell(z_s) + O\big(\kappa(z-z_s)\big) \;\; \text{as} \;\; z \to z_s, \tag{16}$$

then a^* is finite.

Remark 2: In practice, since ℓ and κ are known, a^* can be computed by solving Eq. (15) *as a maximization problem over the set $\mathcal{Z}$.*

Corollary 1: If

$$\kappa(z-z_s) = \|z-z_s\|_1 = \|x-x_s\|_1 + \|u-u_s\|_1, \tag{17}$$

then a^* is finite.

To sum up, in Theorem 1 we have designed a cost term $\alpha(z) = a\,\kappa(z-z_s)$ that, if added to the stage cost ℓ (10) as in Eq. (12), guarantees the closed-loop stability of the E-NMPC scheme (1), (9). The optimal value for the weight a is given in Remark 1, which is ensured to be finite under the hypotheses of Proposition 1; if ℓ satisfies Assumption 4, Corollary 1 provides a suitable choice for the function κ.

The inclusion of α promotes stable regulation towards z_s but comes at the cost of reduced economic profit during the transient. While choosing $a = a^*$ (15) ensures stability with the minimal economic profit reduction, selecting lower values $0 \leq a < a^*$ further favors economic performance, at the price of (possibly) not guaranteeing closed-loop stability.

Remark 3: While the above stability guarantees have been proven for a constant reference z_s, they can be extended to slowly-varying piecewise-constant signals $z_{s,k}$ [15].

IV. The Case Study of Adaptive Cruise Control for Electric Vehicles

The proposed E-NMPC strategy for conflicting objectives is applied to the case study of Adaptive Cruise Control (ACC) in electric vehicles (EVs). As mentioned in the Introduction, the ACC is devoted to controlling the vehicle velocity, either to maintain a driver-defined value or to dynamically adjust it according to the leading vehicle behavior. This latter function relies on onboard sensors (e.g., radar and LiDAR) to measure the relative distance and velocity between the ego vehicle and the leading vehicle.

In addition to velocity regulation, the ACC case study is well-suited to include additional conflicting objectives. Among these, we especially care about minimizing the vehicle energy consumption. For EVs, this specifically involves battery consumption.

In the following, we first model the longitudinal vehicle dynamics, which acts as plant to control; then, we tailor the E-NMPC problem for the case study of ACC.

A. Longitudinal Vehicle Model

The longitudinal vehicle dynamics can be represented by the following continuous-time (CT) dynamical system:

$$\dot{x}(t) = f_c(x(t), u(t)), \quad x = v, \quad u = \tau, \quad t \in \mathbb{R}_{\geq 0}, \quad (18)$$

where x and u are the state and input, coinciding with the vehicle velocity v and the electric motor (EM) torque τ, respectively. The system dynamics (18) is governed by the following equation:

$$\begin{aligned} \dot{v} = a &= \frac{1}{M}(F - F_{\mathrm{vis}}(v) - F_{\mathrm{roll}}(v)) \\ &= \frac{1}{M}\left(\frac{1}{r_{\mathrm{w}}}\tau - \frac{1}{2}\rho A_{\mathrm{f}} C_{\mathrm{d}} v|v| - C_{\mathrm{r}} M g \,\mathrm{sign}(v)\right), \quad (19) \end{aligned}$$

where we define $\beta_{\mathrm{d}} = \frac{1}{2}\rho A_{\mathrm{f}} C_{\mathrm{d}}$ and $\beta_{\mathrm{r}} = C_{\mathrm{r}} M g$. From the above model, the total power P_{tot} requested by the vehicle is given by $P_{\mathrm{tot}} = F_{\mathrm{tot}} v$, where $F_{\mathrm{tot}} = F_{\mathrm{vis}} + F_{\mathrm{roll}} + Ma$ is the total thrust force acting on the vehicle. Such a force and power are provided by the EM, meaning that $P_{\mathrm{tot}} = P$ and $F_{\mathrm{tot}} = F = \frac{1}{r_{\mathrm{w}}}\tau$. Therefore, it holds that

$$P_{\mathrm{tot}} = P = \frac{1}{r_{\mathrm{w}}}\tau v. \quad (20)$$

It is worth noticing that system (18), (19) has an equilibrium manifold $\mathcal{Z}_s$ given by

$$\mathcal{Z}_s = \Big\{(v, \tau) : \tau = r_{\mathrm{w}}\beta_{\mathrm{d}} v|v| + r_{\mathrm{w}}\beta_{\mathrm{r}}\,\mathrm{sign}(v)\Big\}. \quad (21)$$

Notably, $(0, 0) \in \mathcal{Z}_s$. Henceforth, let $z = [v, \tau]^\top = (v, \tau)$.

To match the DT formulation in Eq. (1), the CT system (18) is converted to DT by employing a temporal discretization method of choice (e.g., Euler, Runge-Kutta 4, etc.) with time step T_s, yielding

$$x_{k+1} = f(x_k, u_k), \quad (22)$$

where $\star_k = \star(kT_s)$ for the generic quantity $\star(t)$. The equilibrium manifold $\mathcal{Z}_s$ (21) is the same for both systems (18) and (22).

Together with the longitudinal dynamics, we also take into account the dynamics of the electric battery, which defines the time evolution of the battery state of charge $SOC = \zeta$ as follows:

$$\begin{aligned} \dot{\zeta} = f_{\mathrm{b}}(\zeta, P_{\mathrm{b}}) = -\frac{\eta_{\mathrm{b},1}(P_{\mathrm{b}})}{Q_{\mathrm{nom}}}\frac{1}{2R^{\mathrm{o}}_{\mathrm{b}}(\zeta)}\Big(V^{\mathrm{oc}}_{\mathrm{b}}(\zeta) - \\ \sqrt{V^{\mathrm{oc}}_{\mathrm{b}}(\zeta)^2 - 4R^{\mathrm{o}}_{\mathrm{b}}(\zeta)\,\eta_{\mathrm{b},2}(P_{\mathrm{b}})\,P_{\mathrm{b}}}\Big), \quad (23) \end{aligned}$$

where P_{b} is the power requested to the battery. Refer to [1] for a detailed description of the battery model (23). To a first approximation, the dependence of f_{b} on ζ can be removed by replacing it with a suitable constant value $\tilde{\zeta}$ [1], yielding

$$\dot{\zeta} \approx f_{\mathrm{b}}(\tilde{\zeta}, P_{\mathrm{b}}) = \tilde{f}_{\mathrm{b}}(P_{\mathrm{b}}). \quad (24)$$

The total power P_{tot}, provided by the EM, is entirely requested to the battery, meaning that $P_{\mathrm{tot}} = P_{\mathrm{b}}$. From this, we can relate $\dot{\zeta}$ to the vehicle state and input as follows:

$$\dot{\zeta} = \tilde{f}_{\mathrm{b}}(P_{\mathrm{b}}) = \tilde{f}_{\mathrm{b}}(P_{\mathrm{tot}}) = \tilde{f}_{\mathrm{b}}\left(\frac{1}{r_{\mathrm{w}}}\tau v\right) = \ell_{\mathrm{b}}(x, u). \quad (25)$$

B. Economic NMPC Formulation for ACC

The E-NMPC cost function (7), (8a) for ACC is constructed according to Sections II and III.

The regulation term ℓ_r matches Eq. (3), i.e.,

$$\ell_r(v, \tau) = \|v - v_r\|^2_Q + \|\tau - \tau_r\|^2_R, \quad (26)$$

where v_r is the ACC reference velocity (either driver-defined or dependent on the leading vehicle) and τ_r is the corresponding reference torque such that $(v_r, \tau_r) \in \mathcal{Z}_s$.

As economic criterion ℓ_e, we select battery saving, which takes into account the SOC variation over time $\dot{\zeta}$ and is described by two objectives: minimize $\dot{\zeta}$ when $P > 0$ (battery discharge); maximize $\dot{\zeta}$ when $P < 0$ (battery recharge). The expression for $\dot{\zeta}$ as function of (v, τ) is given by Eq. (25); thus, we construct ℓ_e as follows:

$$\ell_e(v, \tau) = Q_e\,\mathrm{sign}(\tau v)|\ell_{\mathrm{b}}(v, \tau)|. \quad (27)$$

Eq. (27) comprises the absolute value of $\dot{\zeta}$, which is going to be minimized or maximized according to the sign of P, since, by Eq. (20), $\mathrm{sign}(\tau v) = \mathrm{sign}(P)$; $Q_e \geq 0$ is the weighting parameter of ℓ_e.

The stabilizing term is designed according to Section III-B. Specifically, leveraging Corollary 1, we define $\kappa(z - z_s) = \|z - z_s\|_1 = |v - v_s| + |\tau - \tau_s|$ as in Eq. (17), where $z_s = (v_s, \tau_s)$ is the trade-off equilibrium given by Eq. (5). Then,

$$\alpha(v, \tau) = a\big(|v - v_s| + |\tau - \tau_s|\big). \quad (28)$$

where, by Eq. (15), $a \geq a^*$ to ensure stability.

Finally, as in Eq. (12),

$$\bar{\ell}(v, \tau) = \ell_r(v, \tau) + \ell_e(v, \tau) + \alpha(v, \tau), \quad (29)$$

and the E-NMPC OCP is formulated according to Eqs. (8), (11)-(13).

V. Simulations and Results

The proposed E-NMPC strategy for conflicting control objectives is validated in simulation on the case study of Adaptive Cruise Control (ACC) for velocity regulation.

1) Implementation Details: The E-NMPC OCP (8), (11)-(13) is formulated with CasADi [16] and solved with the interior-point solver Ipopt. The optimal weight a^* for the stabilizing term α is computed by solving the maximization problem (15) with a global metaheuristic solver (Genetic Algorithm) [1], as in Remark 2.

For numerical implementation, a smooth approximation of the 1-norm has been employed, i.e., $\|x\|_1 \approx \sum_{i=1}^{n_x}\sqrt{x_i^2 + \epsilon^2} - n_x\epsilon$, $\epsilon \gtrapprox 0$. This ensures that the function κ (17) given by Corollary 1, the economic term ℓ_e (27), and the augmented stage cost $\bar{\ell}$ (29) are $\mathcal{C}^1$-smooth on $\mathcal{Z}$.

Simulations are performed in MATLAB® 2023b on a 13th Gen Intel® Core™ i7 CPU at 1.7 GHz.

2) Simulation Data: The following data is shared by all simulations. For the longitudinal vehicle model (18): $M = 1.2 \times 10^3\,\mathrm{kg}$, $r_{\mathrm{w}} = 0.3\,\mathrm{m}$, $\beta_{\mathrm{d}} = 0.4043\,\mathrm{kg\,m^{-1}}$, $\beta_{\mathrm{r}} = 117.72\,\mathrm{N}$. For the battery model (23), refer to [1].

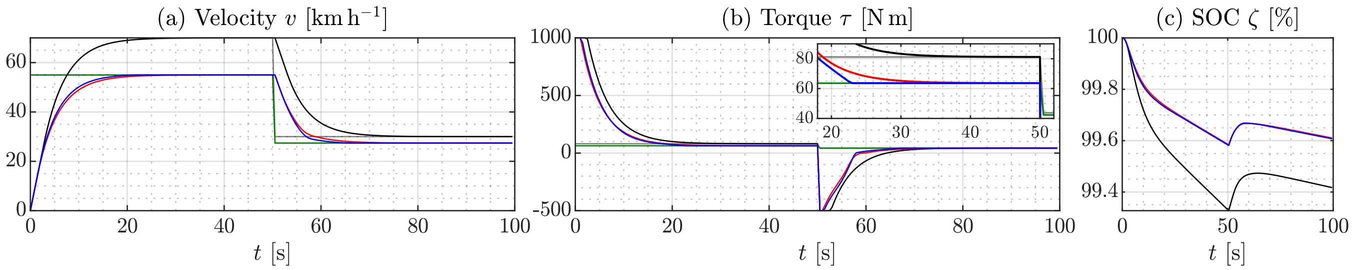

Fig. 1. ACC closed-loop trajectories (velocity profile 1): (a) ego vehicle velocity v; (b) EM torque τ, with detail of the time interval $[18, 52]$ s; (c) battery state of charge ζ (economic criterion); NMPC ($\bar{\ell}_1$) ——; E-NMPC ($\bar{\ell}_2$) ——; Stabilized E-NMPC ($\bar{\ell}_3$) ——; reference equilibria $z_r = (v_r, \tau_r)$ ——; trade-off equilibria $z_s = (v_s, \tau_s)$ ——.

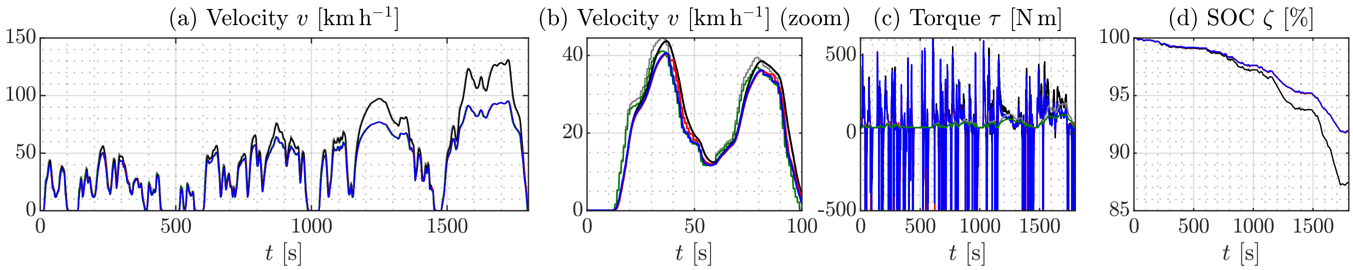

Fig. 2. ACC closed-loop trajectories (velocity profile 2, WLTC-3b driving cycle): (a), (b) ego vehicle velocity v, with detail of the time interval $[0, 100]$ s; (c) EM torque τ; (d) battery state of charge ζ (economic criterion); NMPC ($\bar{\ell}_1$) ——; E-NMPC ($\bar{\ell}_2$) ——; Stabilized E-NMPC ($\bar{\ell}_3$) ——; reference equilibria $z_r = (v_r, \tau_r)$ ——; trade-off equilibria $z_s = (v_s, \tau_s)$ ——.

A. *ACC for Velocity Regulation*

Let $v_{r,k}$ be the ACC reference velocity (either driver-defined or equal to the leading vehicle velocity at each $k \geq 0$) and related torque $\tau_{r,k}$ (such that $(v_{r,k}, \tau_{r,k}) \in \mathcal{Z}_s$). For $v_{r,k}$, two velocity profiles are considered:

1) a step transition between two constant velocity values (Fig. 1a);
2) the Worldwide Harmonized Light Vehicles Test Cycle–Class 3b (WLTC-3b) (Fig. 2a).

By Remark 3, both velocity profiles are shaped as piecewise-constant signals.

With reference to Section IV-B, to assess control performance, stability, and trade-off between conflicting objectives, we compare the following three stage costs:

$$\bar{\ell}_1(v, \tau) = \ell_r(v, \tau), \tag{30a}$$

$$\bar{\ell}_2(v, \tau) = \ell_r(v, \tau) + \ell_e(v, \tau), \tag{30b}$$

$$\bar{\ell}_3(v, \tau) = \ell_r(v, \tau) + \ell_e(v, \tau) + \alpha(v, \tau), \tag{30c}$$

where $\bar{\ell}_1$ comprises the velocity regulation term (26) only (hence, is a standard tracking NMPC)[1], $\bar{\ell}_2$ adds the battery saving economic term (27) (E-NMPC), and $\bar{\ell}_3$ adds the stabilizing term (28).

1) Trade-Off Equilibrium and Stage Cost Minimizer: Let us consider velocity profile 1 (Fig. 1a). It consists of two constant velocity values $v_r = \{70, 30\}\,\mathrm{km\,h^{-1}}$; the related torques are, by Eq. (21), $\tau_r = \{81.17, 43.74\}\,\mathrm{N\,m}$.

E-NMPC (8), (11)-(13) data is as follows[2]: $T_s = 0.5\,\mathrm{s}$, $N_p = 10$, $Q = 10$, $R = 10$, $Q_e = 10$, $\mathcal{X} = \{x \in \mathbb{R}^{n_x} : x_{\mathrm{lb}} \leq x \leq x_{\mathrm{ub}}\}$, $x_{\mathrm{lb}} = -50\,\mathrm{km\,h^{-1}}$, $x_{\mathrm{ub}} = 150\,\mathrm{km\,h^{-1}}$, $\mathcal{U} = \{u \in \mathbb{R}^{n_u} : u_{\mathrm{lb}} \leq u \leq u_{\mathrm{ub}}\}$, $u_{\mathrm{lb}} = -500\,\mathrm{N\,m}$, $u_{\mathrm{ub}} = 1 \times 10^3\,\mathrm{N\,m}$.

With the latter data, for each value of $z_r = (v_r, \tau_r)$, we can compute the trade-off equilibria $z_s = (v_s, \tau_s)$ of $\bar{\ell}_2$ through Eq. (5), obtaining $v_s = \{54.99, 27.40\}\,\mathrm{km\,h^{-1}}$, $\tau_s = \{63.61, 42.34\}\,\mathrm{N\,m}$.

It is also worth computing, for each z_r, the minimizers z^* of $\bar{\ell}_2$, i.e., $z^* = (v^*, \tau^*) = \arg\min_{z \in \mathcal{Z}} \bar{\ell}_2(z)$, obtaining $v^* = \{150, 117.03\}\,\mathrm{km\,h^{-1}}$, $\tau^* = \{-500, -500\}\,\mathrm{N\,m}$. As expected from Eq. (6), $z^* \neq z_s$; specifically, the values of z^* correspond to transient points for which $P < 0$ (i.e., the battery is recharging), meaning that the economic term ℓ_e has a dominant influence over the tracking term ℓ_r.

Now, for each z_r, we compute the optimal weights a^* (15) for the stabilizing term α in $\bar{\ell}_3$, obtaining $a^* = \{7.43, 3.20\}$ (which are finite values, consistently with Corollary 1). Now, computing the minimizers z^* of $\bar{\ell}_3$, i.e., $z^* = (v^*, \tau^*) = \arg\min_{z \in \mathcal{Z}} \bar{\ell}_3(z)$, we obtain $v^* = \{54.99, 27.40\}\,\mathrm{km\,h^{-1}}$, $\tau^* = \{63.38, 42.34\}\,\mathrm{N\,m}$, which are equal to z_s, consistently with Theorem 1.

All the above observations hold equivalently also for velocity profile 2 (Fig. 2a). E-NMPC data stays the same, expect for the following: $T_s = 0.25\,\mathrm{s}$, $Q = 100$, $Q_e = 100$.

2) Control Performance and Stage Costs Comparison: Simulation results for both velocity profiles are reported in Figures 1 and 2, respectively. Focusing on velocity profile 1, we observe that regulation is achieved by both NMPC (towards z_r) and E-NMPC (towards z_s). With the addition of the stabilization term, the tracking behavior of E-NMPC is slightly favored during transients (see time intervals $[0, 30]$ s and $[50, 70]$ s) but, importantly, the closed-loop trajectories reach z_s in finite time and evolve under the optimal input τ_s for all subsequent time instants (see the detail in Figure 1b).

[1]For the stage cost $\bar{\ell}_1$ (30a), we set $\mathcal{X}_f = \{x_r\}$ in Eq. (8d).

[2]Since the quantities in Eqs. (19) and (23) differ in magnitude by some orders, suitable rescaling factors have been included in the stage costs (30).

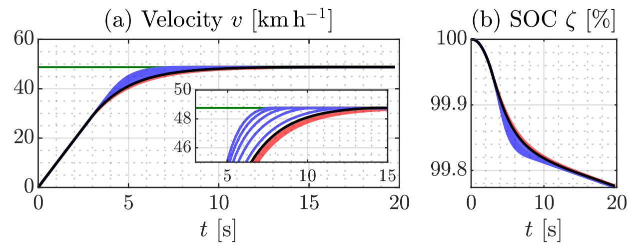

Fig. 3. E-NMPC trade-off between stabilization and economic profit: (a) ego vehicle velocity v; (b) battery state of charge ζ; $a = a^*$ (black line); $a \in [0, a^*)$ (red line); $a \in (a^*, 10\,a^*]$ (blue line); trade-off equilibria v_s (green line).

Economic profit is quantified by the *SOC* evolution in Figure 1c: as expected, E-NMPC attains around $33\,\%$ less battery consumption compared to NMPC. Stabilized E-NMPC consumes slightly more battery than E-NMPC but, interestingly, achieves better battery recharging ($[50, 60]\,\mathrm{s}$), thanks to its enhanced tracking behavior when the requested power is negative.

Considering now velocity profile 2, tracking is attained by both NMPC (towards $z_{r,k}$) and E-NMPC (towards $z_{s,k}$), with a slight error due to the velocity profile not varying too slowly. From Figure 2b, we observe that stabilized E-NMPC better tracks z_s compared to E-NMPC. Finally, from Figure 2c, we see that, as expected, E-NMPC attains a higher economic profit that NMPC.

3) Trade-Off Between Stabilization and Economic Profit: Now, we evaluate the trade-off between stabilization and economic profit, given by the value of a (28). We consider a constant velocity $v_r = 70\,\mathrm{km\,h^{-1}}$ and we let a vary in the interval $[0, 10\,a^*]$, where a^* is given by Eq. (15). Results are reported in Figure 3: higher values of a lead to a slight decrease in the economic profit (Fig. 3b); for $a \geq a^*$, the closed-loop trajectories reach v_s in finite time.

Remark 4: To ensure real-time feasibility in solving the E-NMPC problem (8), (11)-(13) *and computing a^* via Eq.* (15)*, one may rely on two complementary measures: adopting a fast, embeddable NMPC solution method from the many available in recent literature (e.g., [1], [17]); precomputing a^* offline, over a sufficiently dense set of reference points $z_r \in \mathcal{Z}_s$, and interpolating these precomputed values, to be used in real time according to a suitable scheduling policy.*

VI. Conclusions

In this paper, we presented a novel formulation of Economic Nonlinear MPC (E-NMPC) that integrates economic criteria with additional conflicting control tasks, such as tracking. The proposed E-NMPC approach regulates the plant towards the optimal trade-off equilibrium state, averaging the given conflicting objectives. Furthermore, we proposed a general constructive procedure to design stabilizing terms for E-NMPC, ensuring its closed-loop stability with minimal impact on the economic performance.

We applied our methodology to the case study of autonomous electric vehicles (EVs), with specific interest to Adaptive Cruise Control (ACC). The E-NMPC strategy is tasked to attain the velocity regulation task while optimizing an economic criterion that consists in minimizing the vehicle energy consumption. The E-NMPC-based ACC has been validated with an extended simulation campaign, demonstrating its proficient control action and effectiveness in averaging the conflicting tasks, while ensuring closed-loop stability.

The promising preliminary results presented in this paper pave the way for several avenues of future research. First, effort should be placed in extending the proposed E-NMPC strategy within the domain of adaptive control, to deal with time-varying conflicting objectives. Second, the ACC economic criterion could be augmented to also optimize the electric motor efficiency.

References

[1] L. Calogero, M. Pagone, F. Cianflone, E. Gandino, C. Karam, and A. Rizzo, “Neural Adaptive MPC with Online Metaheuristic Tuning for Power Management in Fuel Cell Hybrid Electric Vehicles,” *IEEE Trans. Autom. Sci. Eng.*, 2025.

[2] S. Vasebi, Y. M. Hayeri, and A. M. Saghiri, “A Literature Review of Energy Optimal Adaptive Cruise Control Algorithms,” *IEEE Access*, vol. 11, pp. 13 636–13 646, Jan. 2023.

[3] J. Zhao, Z. Wang, Y. Lv, J. Na, C. Liu, and Z. Zhao, “Data-Driven Learning for H_∞ Control of Adaptive Cruise Control Systems,” *IEEE Trans. Veh. Technol.*, vol. 73, no. 12, pp. 18 348–18 362, 2024.

[4] J. Wang and R. Rajamani, “Should adaptive cruise-control systems be designed to maintain a constant time gap between vehicles?” *IEEE Trans. Veh. Technol.*, vol. 53, no. 5, pp. 1480–1490, 2004.

[5] H. Rezaee, T. Parisini, and M. M. Polycarpou, “Leaderless Cooperative Adaptive Cruise Control Based on the Constant Time-Gap Spacing Policy,” *IEEE Trans. Autom. Control.*, vol. 69, no. 1, pp. 659–666, 2023.

[6] H. Asere, C. Lei, and R. Jia, “Cruise Control Design Using Fuzzy Logic Controller,” in *Proc. IEEE Int. Conf. Syst. Man Cybern.*, 2015, pp. 2210–2215.

[7] M. Zahra, Y. Z. Ashkan, K. Hamid, M. Rammohan, and J. Ali, “Fuzzy adaptive cruise control with model predictive control responding to dynamic traffic conditions for automated driving,” *Eng. Appl. Artif. Intell.*, vol. 136, Oct. 2024.

[8] M. Brosowsky, F. Keck, J. Ketterer, S. Isele, D. Slieter, and M. Zöllner, “Safe Deep Reinforcement Learning for Adaptive Cruise Control by Imposing State-Specific Safe Sets,” in *Proc. IEEE Intell. Veh. Symp.*, 2021, pp. 488–495.

[9] M. Acquarone, F. Miretti, D. Misul, and L. Sassara, “Cooperative Adaptive Cruise Control Based on Reinforcement Learning for Heavy-Duty BEVs,” *IEEE Access*, vol. 11, pp. 127 145–127 156, Nov. 2023.

[10] J. B. Rawlings, D. Bonne, J. B. Jorgensen, A. N. Venkat, and S. B. Jorgensen, “Unreachable Setpoints in Model Predictive Control,” *IEEE Trans. Autom. Control.*, vol. 53, no. 9, pp. 2209–2215, 2008.

[11] M. Diehl, R. Amrit, and J. B. Rawlings, “A Lyapunov Function for Economic Optimizing Model Predictive Control,” *IEEE Trans. Autom. Control.*, vol. 56, no. 3, pp. 703–707, 2011.

[12] D. Angeli, R. Amrit, and J. B. Rawlings, “On Average Performance and Stability of Economic Model Predictive Control,” *IEEE Trans. Autom. Control.*, vol. 57, no. 7, pp. 1615–1626, 2012.

[13] J. B. Rawlings, D. Angeli, and C. N. Bates, “Fundamentals of Economic Model Predictive Control,” in *Proc. IEEE Conf. Decis. Control*, 2012, pp. 3851–3861.

[14] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.

[15] D. Limon, A. Ferramosca, I. Alvarado, and T. Alamo, “Nonlinear MPC for Tracking Piece-Wise Constant Reference Signals,” *IEEE Trans. Autom. Control.*, vol. 63, no. 11, pp. 3735–3750, 2018.

[16] J. A. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl, “CasADi: a software framework for nonlinear optimization and optimal control,” *Math. Program. Comput.*, vol. 11, pp. 1–36, Mar. 2019.

[17] L. Calogero, M. Boggio, C. Novara, and A. Rizzo, “A General Analytical Framework for Fast Solving Nonlinear MPC Problems in the Linear Koopman Space,” in *Proc. Eur. Control Conf.*, 2025, pp. 743–748.