Non-equilibrium Strain and Elastic Hysteresis in Static and Dynamic Experiments in Sandstones

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NON-EQUILIBRIUM STRAIN AND ELASTIC HYSTERESIS IN STATIC AND DYNAMIC EXPERIMENTS IN SANDSTONES

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ABSTRACT

The physical origin of hysteretic elasticity in consolidated granular media is still debated. We show that hysteresis in quasi-static experiments (slow loading/unloading cycles) and dynamic acoustoelastic testing (fast perturbation of the sample obtained by a propagating or standing wave) can be seen as a consequence of slow dynamics, which also induces elastic anisotropy due to nonlinearity. Both conditioning (i.e., the evolution of the elastic properties towards a non-equilibrium steady state when a load is applied) and relaxation (i.e., the slow recovery of the original elastic properties when the strain is reduced) are described as the consequence of a non-equilibrium strain generated in the material by the applied load. Experiments performed span over five orders of magnitude in strain (from 10^{-7} to 10^{-2}) and we demonstrate that the proposed model captures well the observed phenomenology in the full strain range considered.

Keywords: *static and dynamic testing, sandstone, slow dynamics, non-equilibrium strains.*

1. INTRODUCTION

Consolidated granular media, and in particular sandstones, exhibit hysteresis and slow dynamics in their elastic response [1-3]. Hysteresis implies that the wave velocity and damping of the material vary depending on the strain amplitude, with distinct behaviors during loading and unloading. This phenomenon is noticeable in both static conditions (gradual increase or decrease of loads within a wide strain range) and dynamic scenarios (application of a rapidly changing sinusoidal wave with a small strain range) [4,5]. In the slow dynamic range (time scales of the orders of minutes), a time-dependence of velocity is observed when a constant dynamic strain is applied to the sample. Over time, the velocity evolves towards a non-equilibrium asymptotic value, which is influenced by the amplitude of the strain [6,7]. This effect is fully reversible.

Multiple experiments provide evidence of the correlation between slow dynamics and hysteresis, highlighting their significance in both static and dynamic ranges. Additionally, these effects exhibit anisotropy, with the most pronounced changes in velocity occurring when the wave is polarized and propagating parallel to the applied strain direction [8,9].

Here we present results from dynamic and static acoustoelastic testing experiments, aiming to measure velocity variation when a static load is applied and during conditioning and relaxation. We will show that results could be understood as the consequence of a non-equilibrium strain generated in the material by the applied load. Additional measurement mode was included in this analysis, expanding upon the experimental data already published in [9].

2. THEORY

The dependence of wave velocity on an applied longitudinal strain ε_{11} in classical nonlinear media is well described by the acoustoelastic theory [10]. Defining i and j the propagation and polarization directions indices we have

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$$\delta v_{ij} = \beta_{ij}\varepsilon_{11} + \gamma_{ij}\varepsilon_{11}^2, \tag{1}$$

where δv_{ij} is the relative velocity variation.
In order to introduce hysteresis in the acoustoelastic theory we assume the real strain to be given as the sum of the applied strain and a non-equilibrium strain [9]:

$$\bar{\varepsilon}_{11} = \varepsilon_{11} + \varepsilon_{\text{neq}}. \tag{2}$$

Here the non-equilibrium strain accounts for slow dynamics contribution to velocity and as such depends on time and maximum load. It follows

$$\delta v_{ij} = \beta_{ij}\left(\varepsilon_{11} + \varepsilon_{\text{neq}}\right) + \gamma_{ij}\left(\varepsilon_{11} + \varepsilon_{\text{neq}}\right)^2, \quad \varepsilon_{\text{neq}} = -\mu\varepsilon_{\max} f(t). \tag{3}$$

We use Eq. 3 to model the evolution of wave velocity (with any propagation and polarization directions) in static experiments conditioning and relaxation. In the three cases the definition of the strain is of course different:

a) Static loading: during loading $\varepsilon_{\max} = \varepsilon_{11}$ is coincident with the longitudinal applied load, while during unloading $\varepsilon_{\max}$ is the maximum strain achieved in the preceding loading phase;
b) conditioning: the sample is subject to an applied load of amplitude A and varying in a sinusoidal way. Thus $\varepsilon_{\max} = A$;
c) relaxation: no loading is applied and thus $\varepsilon_{\max} = A$ and $\varepsilon_{11} = 0$.

Experimental data (and some theoretical considerations) show that while the acoustoelastic coefficients are anisotropic, their ratio is independent on propagation/polarization indices: $\beta_{ij}/\gamma_{ij} = k$.
It follows that in each strain range (static, conditioning, relaxation) anisotropy is the same and could be defined as

$$m_{ij} = \frac{\delta v_{ij}}{\delta v_{22}} = \frac{\beta_{ij}}{\beta_{22}}. \tag{4}$$

3. EXPERIMENTAL SET-UP

Our goal is to measure the variation of propagation velocity under different load conditions. To this purpose, the set-up schematically reported in Fig. 1 is used. A mechanical testing system is used to induce controlled static load (in static experiments) and a couple of PZT transducer disks allow to propagate a sinusoidal excitation along the longitudinal direction (in conditioning experiments), while no longitudinal loading is applied in relaxation experiments.

During the full experiments, at successive times, the PZT transducers (shear and longitudinal) inject pulses in the transverse direction of the sample. Measurements of the time of flights (TOFs) from emitter to receiver allow to measure velocity as a function of time, which also corresponds to measurement of velocity vs. applied load. Furthermore, during relaxation experiment, measuring the resonant frequency of the first longitudinal mode using chirp signals transmitted by the PZT disks allows estimation of the velocity in the longitudinal direction.
The chosen set-up allows measuring at the same times (i.e. loading values) four velocity variations: longitudinal velocity in the transverse direction (indices 22), longitudinal velocity in the longitudinal direction (indices 11, measured only during relaxation) and shear velocities polarized orthogonal (indices 32) and parallel (indices 31) to the applied uniaxial load (indices 11).

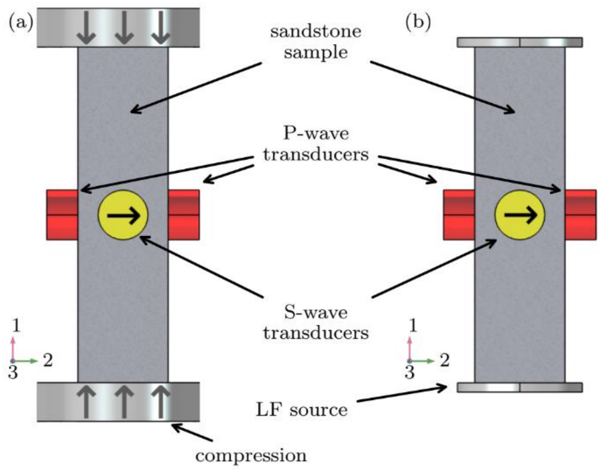

Figure 1. Experimental set-up. (a) Configuration for the SAET experiment; (b) configuration for the DAET (conditioning and relaxation) experiment.

4. RESULTS

4.1 Linear correlation between velocities with different polarizations

In each experiment (static, conditioning and relaxation), experiments allow obtaining the function describing the temporal evolution of velocities in different propagation directions and polarizations: $\delta v_{ij} = f(t)$. To obtain indications about anisotropy for shear waves polarized orthogonal or parallel to the uniaxial strain applied, we can

observe the linear correlation existing between δv_{3j}, δv_{11} and δv_{22}, as shown in Fig. 2.
It is remarkable to observe that the same linear correlation coefficient is found for static (subplot (a)) and dynamic experiments (relaxation in subplot (b)). The slope, as expected, is larger when the shear wave is polarized and the longitudinal wave is propagating in a direction parallel to the uniaxial strain (indices 31, 11), while the slope is 1 for orthogonal polarization. Indeed, in this case both the shear wave (32) and the longitudinal wave (22) are polarized in the same direction, thus the same velocity variation.

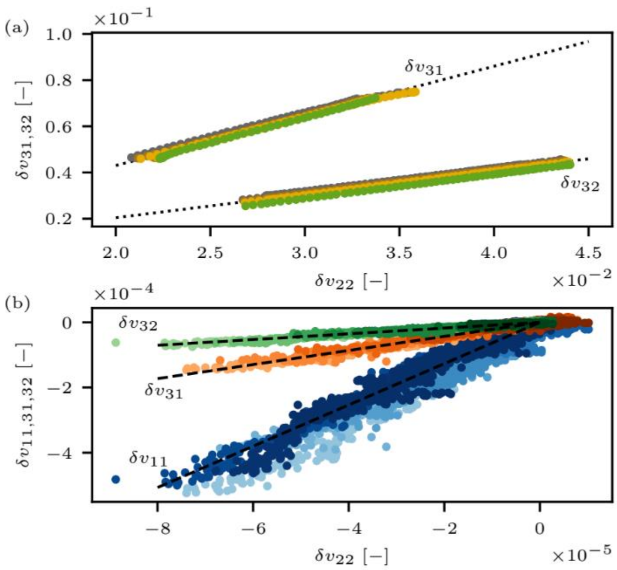

Figure 2. Linear correlation between velocities variation in different directions. (a) Static experiment. Colors correspond to four repeated loading cycles. (b) Relaxation (DAET) experiment. Different tints correspond to various conditioning amplitudes. See Table 1 for mean slopes.

4.2 Anisotropy

The slopes of the curves in Fig. 2 define the anisotropy coefficient for shear waves in orthogonal or parallel polarization to the uniaxial strain and results are shown in Tab. 1.
In static experiments, four loading cycles were measured and anisotropy coefficients were obtained in both loading and unloading branches of the protocol. High repeatability and independence from the branch are observed, with anisotropy 1 and 2 for the two shear waves polarizations considered.
In relaxation experiments, the relative velocity variations were analyzed for different amplitudes of the conditioning uniaxial strain. We obtain the same anisotropy degree as found in static experiments and as predicted by the theory. Furthermore, anisotropy is independent from conditioning amplitude.
Finally, results of our experiments are given in Tab. 1, where all measured anisotropy coefficients are reported. We observe anisotropy to be the same in the three experiments; SAET, conditioning and relaxation, as expected by our theory. It is remarkable to observe that in the three experiments strain ranges from 10^{-2} in SAET to 10^{-7} in conditioning, thus over about 5 orders of magnitude. The different anisotropy for the two polarizations is confirmed and experiments are repeatable, as shown by the data reported for a second sample (same geometry and material).

Table 1. Anisotropy coefficients (slopes in Fig. 2) measured in SAET and DAET. Coefficients are derived from four cycles (SAET) or five conditioning amplitudes (DAET) and averaged. Two samples are reported for DAET experiment. For conditioning phases, all measurements were treated as one dataset thus errors are not provided.

s.	regime	orthogonal	parallel
I	loading	1.01 ± 0.01	2.09 ± 0.04
	unloading	1.03 ± 0.01	2.21 ± 0.01
	conditioning	0.90	2.20
	relaxation	0.88 ± 0.15	2.15 ± 0.11
II	conditioning	1.02	2.09
	relaxation	0.96 ± 0.01	2.14 ± 0.10
		longitudinal parallel	
I	relaxation	6.37 ± 1.72	

5. CONCLUSIONS

We have presented an experiment aiming to measure loading induced anisotropy of shear waves with different polarization in sandstones. By analyzing TOFs of pulses propagating in a direction orthogonal to an external uniaxial strain, we observed the evolution of velocity vs. time when either a quasi-static load is applied (SAET, with a very slowly increasing loading/unloading protocol), a dynamic load is

applied (conditioning DAET with a sinusoidal load applied) and no load is applied (relaxation DAET).
We observed the same anisotropy measured in all the three cases, despite the strain ranges from 10^{-7} to 10^{-2}, thus over about 5 orders of magnitude. Also, we have proposed a model to understand the observations based on introducing a non-equilibrium strain which evolves towards an amplitude-dependent steady state.

Our future work aims to establish a nonlinear model for the evolution of the non-equilibrium strain (differential equation), which could then be coupled with elastic wave propagation equations. The final goal would be to demonstrate that the same concept of evolving non equilibrium strain could explain some of the results observed in propagation/standing wave conditions, such as in Nonlinear Resonance Ultrasound Spectroscopy (NRUS).

6. ACKNOWLEDGMENTS

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