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Abstract

Motivated by theories and evidence from the social psychology literature, we propose a novel continuous-time mathematical model that captures the evolution of motivation and behavior of energy consumers in a social network. In our model, consumers are connected to the energy grid and their energy demand (which we shall refer to as their behavior) is affected by their personal motivation on reducing (or increasing) their energy consumption. The motivation-behavior dynamics of each consumer is modeled using a second-order continuous-time bilinear differential equation. Each consumer has the ability to share their motivation on a social network and observe the behavior of other consumers. Using the information gathered from their peers and their personal bias about the energy consumption behavior, consumers have the ability to revise their own motivation about energy consumption and, ultimately, their behavior. Moreover, we incorporate into the model an external control action that captures the implementation of external behavioral interventions that influence the weight each consumer assigns to their own bias. Then, we use the proposed framework to shed light on the collective motivation-behavior dynamics of all the consumers,

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establishing conditions for the existence of equilibria, characterizing them, and performing a sensitivity analysis of such equilibria with respect to variations in the steady-state interventions provided to each consumer.

Keywords: Energy networks, Opinion dynamics, Social dynamics, Multi-agent

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1. Introduction

The study of opinion dynamics as a highly interdisciplinary field of research has gained massive popularity in the last decade [1, 2, 3]. Many real-life phenomena including health, electoral and social media campaigns [4, 5], viral disease spreading [6, 7], and collective risk perception [8] can be elegantly studied and predicted using technical tools developed at the interface of mathematics, physics, system and control engineering, computer and data science. From a theoretical standpoint, the process of opinion formation and dissemination in complex networks has been investigated by a number of articles with the objective of influencing and optimizing collective behavior of the network, see, e.g., [9, 10, 11, 12, 13, 14, 15].

A promising research field in which opinion dynamics model can offer relevant insights is the study of collective energy consumption behavior of consumers. In fact, the energy network is an example of a complex system that involves humans-in-the-loop, who constantly make decisions on the amount of energy to buy from the grid. Each consumer makes decisions on the amount of energy to use depending on a complex decision-making process, which depends on their motivation. For instance, a consumer who is highly concerned about environmental safety might decrease their energy use. In contrast, a consumer with little concern for environmental issues can prioritize personal comfort and convenience, leading to behaviors that favor high energy consumption [16]. Hence, the motivation formation process in networks of consumers has a large impact on the energy network and thus should be integrated within realistic models of energy grids, to study effective incentivization strategies for the adoption of more sustainable behaviors [17, 18].

In this work, we take a first fundamental step towards this objective by formalizing and studying a novel mathematical model for energy consumption behavior of consumers, which is grounded in the theory of opinion dynamics. In particular, inspired by the framework laid out in [19], we develop a mathematical model to capture the energy consumption behavior of consumers within an energy network. In our model, consumers have behaviors in terms of their energy consumption that evolve in time to match their motivations on energy-related matters, such as environmental safety, climate change and their personal comfort. However, motivations are not static: they evolve over time, under the effect of factors known from the social psychology literature, such as social influence [20, 21, 22] and personal biases [18, 23].

Our model incorporates this dynamic nature of motivations, allowing for the possibility that a consumer's stance on energy consumption changes over time. For instance, a consumer who initially disregards climate change, over time, might become more environmentally conscious due to social influence or information campaigns. This change in motivation would subsequently lead to a shift in their energy consumption behaviors, possibly leading to the collective adoption of more sustainable practices. To incorporate such a feature, we couple the decision-making mechanism with a motivation dynamic model that describes the evolution of consumers' motivation, inspired by the recently-developed extension of opinion dynamics that accounts for the intertwined co-evolution of multiple dynamics on the social network [24, 25, 26, 27]. The coupled model accounts for two distinct layers of social influence via motivation exchange and observation of others' behaviors (e.g., the temperature set on their thermostats, the energy classes of their newly bought appliances, installation of solar panels on their roofs, etc.), respectively, and it is thus embedded on a two-layer network. Since the model employs motivation and behavior to capture consumer's actions for energy consumption, it holds potential for future research in domains, including traffic congestion management and electrical vehicle purchase and charging patterns, where individuals can share motivations (or opinions) and can observe each other's behavior [28, 29].

Technically, the motivation-behavior dynamics of each energy consumer is modeled using a continuous-time second-order differential equation. In the proposed model, the motivation of a consumer is affected by the motivation and behavior of their peers and their own bias towards energy-related matters. The exchange of information in the form of sharing opinions and observing behaviors with peers is modeled using a two-layer network, with the same set of nodes but different sets of edges. Each consumer assigns a nonnegative weight that dictates the influence of personal bias on their own motivation. This nonnegative weight can vary over time due to many factors, such as weather conditions, personal health, etc. The assigned weight can also be influenced by appropriate behavioral interventions such as financial incentives, and hence we treat this weight as an input, which imparts a bilinear nature to our model.

Our main contribution is threefold: i) we propose a novel model for consumers behavior in energy systems, tailored to the specific characteristics of energy-related decisions; ii) through the analysis of the model, we characterize its steady-state behavior, shedding light into how the motivation dynamics mechanisms affect the emergent behavior of the energy system; iii) we perform sensitivity analysis to study how changes in the input affect the equilibria of the dynamics for the case when the network has two biased consumers. The rest of this paper is organized in the following way. In Section 2, we present the notation and some preliminary results useful for the rest of the paper. Section 3 contains the model description. In Section 4, we present the steady-state analysis of the model along with a sensitivity analysis of equilibrium points. In Section 5 we validate our discussion in Section 4 using numerical examples. Section 6 concludes the paper.

2. Mathematical preliminaries

We denote the set of real, real positive, and complex numbers as $\mathbb{R}$, $\mathbb{R}_+$, and $\mathbb{C}$, respectively, and $\mathbb{C}^- = \{z \in \mathbb{C} : \mathrm{Re}(z) < 0\}$. Given a set V, $|V|$ denotes its cardinality. We use the notation $0_{n\times m}$ to denote a $n \times m$ all-0 matrix and

$\mathbf{1}_n$ to denote a n-dimensional all-1 vector. Dimensions are omitted when not necessary. Define $M_n(\mathbb{R}) := \{A : A \in \mathbb{R}^{n\times n}\}$ to be the set of all $n \times n$ square real matrices.

Definition 1. *Given a set of N-dimensional vectors $V = \{v_1, v_2, \dots v_m\}$, the convex hull of V is defined as*

$$\mathrm{Co}(V) = \{v \in \mathbb{R}^N : v = \sum_{i=1}^{m} \beta_i v_i,\, \beta_i \geq 0, \sum_{i=1}^{m} \beta_i = 1\}.$$

The following result gives the relationship between an irreducible invertible M–matrix and its inverse [30, Chapter 6, Theorem 2.3 and Theorem 2.7].

Proposition 1. *Let $A = [a_{ij}] \in M_N(\mathbb{R})$ be such that $a_{ij} \leq 0$ for all $i \neq j$. Then, A is an invertible M-matrix if and only if all real eigenvalues of A are positive. Furthermore, A^{-1} is a positive matrix, i.e., each element of A^{-1} is positive, if and only if A is irreducible and invertible M-matrix.*

Graph Theory: A network (V, E) is defined by a node set $V = \{1, \dots, n\}$ with nodes indexed by positive integer numbers and an edge set $E \subseteq V \times V$, such that $(i, j) \in E$ if and only if there is a link from i to j. Networks may be characterized by multiple layers, i.e., multiple set of edges representing different types of interactions between the nodes. We refer to these networks as *multilayer networks*. In particular, a *two-layer network* is characterized by a set of nodes V and two sets of edges E_a and E_b; hence, we denote it using the triplet of sets (V, E_a, E_b). Given a network (or a layer of a multilayer network), we say that the network (or the layer) is strongly connected if, for any pair of nodes i and j, there is a sequence of links that connects i and j.

Given a network and, in the case of a multi-layer network, a specific layer, we denote by $N_i := \{j \in V : (i, j) \in E\}$ the set of neighbors of node i and by $d_i := |N_i|$ the degree of node i. Furthermore, we define the *adjacency matrix* $A \in \{0, 1\}^{n\times n}$ such that its generic entry $a_{ij} = 1$ if and only if $(i, j) \in E$. Given an adjacency matrix, we define the associated *Laplacian matrix* as $L := \mathrm{diag}(A\mathbf{1}_N) - A$, where $A\mathbf{1}_N$ is the N-dimensional degree vector. For multilayer networks, each node may have different sets of neighbors and degree on each layer. Hence, for multilayer networks, we will refer to the adjacency matrix of a specific layer.

3. Model

We consider a social network formed by N consumers of energy belonging to an energy grid. Consumers are labeled with positive integer numbers and are gathered in the set $V = \{1, 2, \ldots, N\}$. Each consumer in the network has a motivation regarding the energy usage with respect to factors such as climate change, environmental safety, self-comfort, etc. Driven by their own motivation, each consumer has a behavior in terms of energy demand from the electrical grid. For each consumer $i \in V$ and time $t \in \mathbb{R}_+$, the behavior and motivation of the ith consumer is denoted by $x_{b_i}(t)$ and $x_{m_i}(t)$, respectively. We assume that these two quantities are normalized between 0 and 1, i.e., $x_{b_i}(t) \in [0, 1]$ and $x_{m_i}(t) \in [0, 1]$. Large values of $x_{b_i}(t)$ means that consumer i requires large amount of energy from the grid; large values of $x_{m_i}(t)$ implies that the motivation of consumer i aligns with large energy consumption activities at time t. Consumers' behaviors and motivations are gathered in the vectors $x_b(t) = [x_{b_1}(t), x_{b_2}(t), \ldots, x_{b_N}(t)]^\top \in [0, 1]^N$ and $x_m(t) = [x_{m_1}(t), x_{m_2}(t), \ldots, x_{m_N}(t)]^\top \in [0, 1]^N$.

Remark 1. *In the field of social psychology, well-established theories emphasize that motivation acts as a crucial intermediary between actual energy consumption behavior of an individual, personal values and contextual influences (see, [23, 31]). For instance, we refer to [16, 19], where hedonic and biospheric values have been considered to link behavior, motivation, and values. In simple terms, someone who strongly endorses biospheric values deeply cares about the environment and is highly motivated to engage in energy-saving behaviors which is reflected in actions such as reducing heating use or investing in solar panels, even when these actions involve personal effort or sacrifice. In contrast, someone driven by hedonic values may find energy-saving measures inconvenient or*

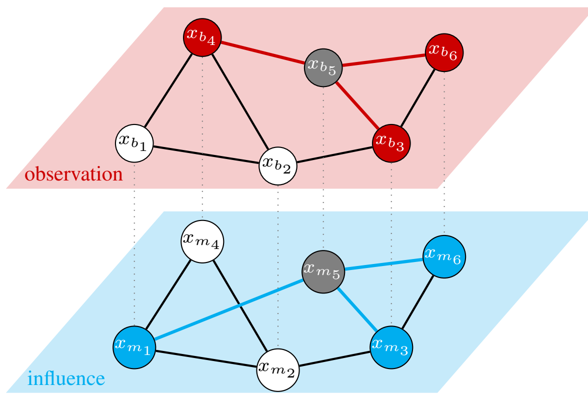

Figure 1: Two-layer network structure with the influence layer (below, in cyan) and the observation layer (above, in green). Consumer 5 (in gray) is influenced by the motivations shared by consumers 1, 3, and 6 (in cyan); and by the behavior observed of consumers 3, 4, and 6 (in red).

uncomfortable, reducing their motivation and likelihood to adopt such practices. External influences, such as educational interventions or peer-to-peer discussions, can also shape motivation. For instance, a consumer with strong hedonic values may be motivated to shift their energy consumption behavior when provided with information highlighting potential financial savings.

Consumers share their motivation and observe behaviors taken by their neighbors. In general, these two interactions occur on different layers, since the set of peers with whom consumer i discusses and shares their motivation can be different from the set of peers whose behaviors i observes. To model these two different sources of information, inspired by [27], we use a two-layer network (V, E_i, E_o), where the edge set E_i denotes the edges along which the consumer interact to exchange motivation with each other and E_o is the set of edges along which the consumers observe the behavior of their peers. We denote the two layers as the *influence* and *observation layer*, respectively. Namely $(i, j) \in E_i$ if consumer i is influenced by the motivation of j (i.e., $x_{m_j}(t)$); $(i, j) \in E_o$ if i observes the behavior of j (i.e., $x_{b_j}(t)$). Such a two layer network structure is illustrated in Fig. 1. The corresponding adjacency matrices are

$A_i = [\alpha_{ij}] \in \mathbb{R}^{N\times N}$ and $A_o = [\beta_{ij}] \in \mathbb{R}^{N\times N}$, respectively. Under this setting, the behavior and motivation of the generic ith consumer evolve according to the following dynamics:

$$\dot{x}_{b_i}(t) = a_i(x_{m_i}(t) - x_{b_i}(t)) \tag{1a}$$

$$\begin{aligned}\dot{x}_{m_i}(t) = &\sum_{j=1}^{N} \alpha_{ij}(x_{m_j}(t) - x_{m_i}(t)) + \sum_{j=1}^{N} \beta_{ij}(x_{b_j}(t) - x_{b_i}(t)) \\ &+ (v_i - x_{m_i}(t))u_i(t),\end{aligned} \tag{1b}$$

where $u_i(t) \in [0, u_{\max}]$, with $0 < u_{\max} < \infty$ is a control input, $a_i > 0$ is a time-constant that scales the relative velocity of the motivation dynamics with respect to the decision-making dynamics, and $v_i \in [0, 1]$ is a constant that captures the bias of ith consumer.

In the following, we discuss the different terms present in Eq. (1). The dynamics in Eq. (1a) captures each consumer's natural tendency to align their behavior $x_{b_i}(t)$ with their motivation $x_{m_i}(t)$, with the parameter $a_i > 0$ capturing the velocity of such an alignment process: the larger is a_i, the faster is the behavior alignment process. The dynamics of the motivation $x_{m_i}(t)$ in Eq. (1b) accounts for three different terms. The first term, inspired by classical motivation dynamics models [1, 32], models the effect of peer interaction on the motivation of a consumer when others share motivations via the influence layer. The second term captures phenomena such as social pressure and internalization of descriptive norms well-known in the social psychology literature [33, 34] and is associated with behaviors observed on the observation layer. Finally, the third term accounts for bias: In particular, a high value of v_i implies that motivation aligns with comfort and self-prioritization, while a low value of v_i implies that the motivation is aligned towards sustainable energy usage at the cost of self-comfort. The input $u_i(t)$ is a time-varying parameter that determines the weight of bias v_i in the evolution of $x_{m_i}(t)$. In real-world applications, the values of $u_i(t)$ model the effect of an external behavioral intervention on the consumer. A schematic illustrating the dynamic mechanism of behaviors and motivations is illustrated in Fig. 2.

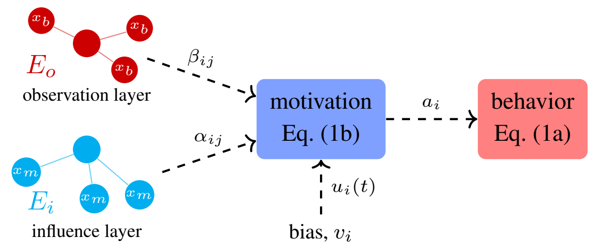

Figure 2: Schematic of the update rule for a generic consumer i.

The dynamics in Eq. (1) can be written in a compact form using the Laplacian matrices L_i and L_o associated with the influence and observation layer of the consumer network, obtaining the following bilinear dynamical system:

$$\begin{bmatrix} \dot{x}_b(t) \\ \dot{x}_m(t) \end{bmatrix} = \begin{bmatrix} -A_d & A_d \\ -L_o & -L_i \end{bmatrix} \begin{bmatrix} x_b(t) \\ x_m(t) \end{bmatrix} + \begin{bmatrix} 0_{N\times N} \\ v_d - x_{m_d}(t) \end{bmatrix} u(t), \tag{2}$$

with $u(t) = [u_1(t), \ldots, u_N(t)]^\top$, $v_d = \text{diag}\{v_1, \ldots v_N\}$, $x_{m_d}(t) = \text{diag}(x_{m_i}(t))$ and $A_d = \text{diag}\{a_1, \ldots, a_N\}$.

Remark 2. *The model in Eq. (1) is an extension of the model presented in [19], where the motivation of consumers is influenced by the motivation of their peers. In terms of social psychology, such an influence is attributed to the injunctive norms of an individual [35]. Additionally, the motivation of consumers can also be influenced by the behavior of their peers, and such an influence is associated with the descriptive norms of individuals [36]. In this model, we encapsulate both the injunctive and descriptive norms into the dynamics. The model considered in this work is substantially different from the model in [19]. In particular, the model in [19] is linear and does not incorporate descriptive norms. On the other hand, in this work, we consider a bilinear model and incorporate both descriptive and injunctive norms. However, note that [19] takes into account the effect of hedonic and biospheric values, see also Remark 1. To keep the analysis simple, in this work we combine the different personal values in a single parameter, to which we refer as the bias v_i in Eq. (1b).*

Remark 3. *The model in Eq. (1) differs from the models defined in [37], [38],* *and [39] as these works utilize first-order dynamics with agents interacting on a single network. For studying the energy consumption behavior of consumers, the second-order model presented in our work is consistent with concepts from social psychology and is technically relevant as the state x_{b_i} can be used to interconnect the behavioral-motivation system with a physical system like a power grid, see [19, Section III]. Differently from the aforementioned works, here the bias v_i can be different for different consumers and can take any value in $[0, 1]$. Moreover, while the weights associated with consumers' biases in [19] and [39] are fixed, yielding to linear models, here we introduce bilinearity through weights that can change over time. Finally, in contrast to [39], we assume the biases to be constant, as they represent inherent characteristics of an individual that typically evolve on a much slower timescale than the other system dynamics.*

4. Steady-state analysis

In this section, we analyze the asymptotic behavior of the system defined in Eq. (2). Our analysis will focus on characterizing properties of the equilibrium points and studying the dependence of the steady-state of the system on the steady-state input. First, we denote the spectrum of L_i and L_o by $\sigma(L_i) = \{\lambda_1, \lambda_2, \dots \lambda_N\}$ and $\sigma(L_o) = \{\mu_1, \mu_2, \dots \mu_N\}$, respectively, and make the following standing assumption.

Assumption 1. *The layers E_i and E_o are undirected and at least one of the two layers E_i or E_o is strongly connected.*

Let x_{b_e}, x_{m_e} and u_e be the collective steady-state behavior, motivation and input for Eq. (2). Using $\dot{x}_{b_e}(t) = 0_N$, we get $x_{b_e} = x_{m_e}$, which along with $\dot{x}_{m_e}(t) = 0$ implies $(L_o+L_i+u_{e_d})x_{m_e} = u_{e_d}v$. Here $u_{e_d} = \text{diag}\{u_{1_e}, u_{2_e}, \dots, u_{N_e}\}$ and u_{i_e} is the steady-state input for consumer $i \in V$. Note that when $u_{i_e} = 0$ for all $i \in V$, the network is said to be in consensus as $x_{m_e} = \alpha \mathbf{1}_N^\top$ with scalar $\alpha \in \mathbb{R}$ that depends on the initial condition. Such a scenario implies the presence of consensus among the consumers, as in the steady state, the bias of all the consumers vanishes. However, in social networks, consumers have diverse motivations, internal biases and external influences, and so consensus of motivation is a rare phenomenon [39]. Hence, the equilibrium point of the zero-input system, $x_{m_e} = \alpha \mathbf{1}_N^\top$, is not of much practical interest. Therefore, we first state conditions on u_e for the existence of other non-trivial equilibrium points (x_{b_e}, x_{m_e}) for our system.

Lemma 1. *Under Assumption 1, there exists a unique equilibrium point* (x_e, w_e) *for the system in Eq. (2) if there exists a* $i \in V$ *such that* $u_{i_e} \in (0, u_{\max}]$.

Proof. Without loss of generality, let the layer E_o be strongly connected, implying that $0 \in \sigma(L_o)$ with algebraic multiplicity of one and the associated eigenvector is $\mathbf{1}_N^\top$. Since L_i and L_o are positive semi-definite matrices, the eigenvalues $\lambda_i \geq 0$ and $\mu_i \geq 0$ are real for each $i \in V$. Furthermore, if $u_{i_e} \neq 0$ for at least one $i \in V$, then the spectrum of the matrix $A = L_i + L_o + u_{e_d}$ satisfies $\sigma(A) \subset \mathbb{R}^+$ and A is invertible. Indeed, assume that $0 \in \sigma(A)$ and let z_0 be the eigenvalue associated with zero eigenvector of A which implies that $L_i z_0 + L_o z_0 + u_{e_d} z_0 = 0$. On multiplying the previous equation by $z_0^\top$ from the left-hand side, we get $z_0^\top A z_0 = 0$, which holds if and only if the terms on the left-hand side are each and all equal to zero, since $z_0^\top L_i z_0 \geq 0$, $z_0^\top L_o z_0 \geq 0$ and $z_0^\top u_{e_d} z_0 \geq 0$. Since E_o is connected, $L_o z_0 = 0$ implies that $z_0 = \alpha \mathbf{1}_N^\top$ for some non-zero constant α. This choice of z_0 also lies in the null-space of L_i. Hence, $z_0^\top u_{e_d} z_0 = 0$, which cannot happen since at least one diagonal entry of u_{e_d} is positive. Hence, we obtain that $0 \notin \sigma(A)$ and $\sigma(A) \subset \mathbb{R}^+$ follows from the symmetric and positive semi-definite property of A. So, using inverse of A, a non-trivial equilibrium point of Eq. (2) is

$$\begin{bmatrix} x_{b_e} \\ x_{m_e} \end{bmatrix} = \underbrace{\begin{bmatrix} A_d & -A_d \\ L_o & L_i + u_{e_d} \end{bmatrix}^{-1}}_{:=W} \underbrace{\begin{bmatrix} 0_{N\times 1} \\ u_{e_d} v \end{bmatrix}}_{:=d} \tag{3}$$

which depends on the individual bias and the steady-state input to each consumer. $\square$

In the next result, we establish the stability of the equilibrium point in Eq. (3) for constant inputs.

Theorem 1. *Under Assumption 1, the equilibrium point* (x_{b_e}, x_{m_e}) *defined in Eq. (3) is stable if there exists a* $i \in V$ *such that the steady-state input to consumer* i *is nonzero, i.e.,* $u_{i_e} \in (0, u_{\max}]$.

Proof. Fix a $i \in V$ and let u_e be such that $u_{i_e} \neq 0$ for $i \in V$. For any fixed nonzero value u_{i_e}, the system in Eq. (2) is a linear system. Indeed, Eq. (2) can be written as $y(t) = Wy(t) + d$, where

$$W = \begin{bmatrix} -A_d & A_d \\ -L_o & -(L_e + u_{e_d}) \end{bmatrix}, \; d = \begin{bmatrix} 0_{N\times N} \\ u_{e_d} v \end{bmatrix}, \; y(t) = \begin{bmatrix} x_b(t) \\ x_m(t) \end{bmatrix}$$

and the claim follows by transforming the state y to $\tilde{y} = y + W^{-1}d$. Hence, the nature of the equilibrium points (x_{b_e}, x_{m_e}) is decided by the location of the eigenvalues of W.

First, we show that $0 \notin \sigma(W)$ using proof by contradiction. Let $0 \in \sigma(W)$ and $z = [z_1^\top, z_2^\top]^\top \in \mathbb{R}^{2N}$ be the eigenvector associated with the zero eigenvalue. With $Wz = 0$, we get $A_d z_1 = A_d z_2$ and $L_o z_1 + (L_i + u_{e_d}) z_2 = 0$. Since A_d is invertible, $z_1 = z_2 \neq 0_N$. So $(L_o + L_i + u_{e_d}) z_1 = 0$ and 0 is an eigenvalue of $(L_o + L_i + u_{e_d})$, which is a contradiction, see the discussion above Eq. (3). Next, we prove $\sigma(W) \subset \mathbb{C}^-$. Let the (k, j) element of L_i and W be $L_{i_{kj}}$ and W_{kj}, respectively. Since the sum of all elements in any row of matrices A_d, L_o and L_i is zero, we obtain

$$\sum_{\substack{j=1 \\ j\neq k}}^{N} |W_{kj}| = \begin{cases} a_k \quad \text{if} \quad k \in V, \\ u_{(k-N)_e} + L_{i_{(k-N)(k-N)}} \quad \text{if} \quad k - N \in V. \end{cases}$$

Here $u_{(k-N)_e} = 0$ if $k - N \neq i$ and $u_{(k-N)_e} = u_{i_e} > 0$, otherwise. Using Geršgorin theorem [40, Theorem 6.1.1], the eigenvalues of W lie in union of $2N$ disks with center at $-a_k$ and radius a_k if $k \in V$, and center at $-L_{i_{(k-N)(k-N)}} - u_{(k-N)_e}$ with radius $-L_{i_{(k-N)(k-N)}}$ if $k - N \in V$. It is clear that the disk corresponding to $k - N = i \in V$ lies entirely in $\mathbb{C}^-$ and remaining all disks lie in the region $\mathbb{C}^- \cup \{0\}$. Since $0 \notin \sigma(W)$, all the eigenvalues of W lie in $\mathbb{C}^-$. □

Building on Theorem 1, we derive the following corollary.

Corollary 1. *Under Assumption 1, the equilibrium point* (x_{b_e}, x_{m_e}, u_e) *of the system in Eq. (2) is locally asymptotically stable.*

Proof. First, define the error state and input as $e_m = x_m - x_{m_e}$, $e_b = x_b - x_{b_e}$ and $e_u = u - u_e$ to obtain the shifted dynamics

$$\dot{e}_b(t) = -A_d e_b(t) + A_d e_m(t) \tag{4a}$$

$$\dot{e}_m(t) = -L_o e_b(t) - (L_i + u_{e_d}) e_m(t) + (v_d - x_{m_{e_d}} - e_{m_d}(t)) e_u(t). \tag{4b}$$

Then, note that the state matrix of the linearized version of the aforementioned error dynamics is W, defined in the proof of Theorem 1, satisfying $\sigma(W) \subset \mathbb{C}^-$, which yields the claim. $\square$

Note that $\sigma(W) \subset \mathbb{C}^-$ also ensures that the system in Eq. (2) has strong integral input-to-state stability, see [41, Definition 2], as highlighted in the following result.

Corollary 2. *Let Assumption 1 hold and suppose there exists* $i \in V$ *such that the steady-state input satisfies* $u_{i_e} \neq 0$. *Then, the system in Eq. (2) is strongly integral input-state stable (iISS) with respect to the equilibrium point* (x_{b_e}, x_{m_e}, u_e).

Proof. In order to establish the strong integral input-state stability of Eq. (2) with respect to (x_e, u_e), we establish the strong iISS property of Eq. (4). To this aim, we denote $e_x = [e_b^\top \, e_m^\top] \in \mathbb{R}^{2N}$ and rewrite Eq. (4) as

$$\dot{e}_x(t) = W e_x(t) + B e_u(t) - \sum_{i=1}^{N} e_{u_i}(t) B_i e_x(t), \tag{5}$$

where W is defined in Eq. (3) and

$$B = \begin{bmatrix} 0_{N \times N} \\ v_d - x_{m_{e_d}} \end{bmatrix}, B_i = \begin{bmatrix} 0_{N \times N} \\ E_i \end{bmatrix},$$

such that the i^{th} diagonal element of matrix $E_i \in \mathbb{R}^{N \times N}$ is one and rest all the elements are zero. Finally, using the result in [41, Corollary 2], we obtain that Eq. (5) is strongly iISS with respect to origin and consequently Eq. (2) is strongly iISS with respect to (x_{b_e}, x_{m_e}, u_e), yielding the claim. $\square$

Note that the strong iISS property of Eq. (2) straightforwardly implies that the system is also integral input-to-state stable, see [42, Theorem 5].

Consider the case when $u_i(t) \equiv 0$ at all times for each $i \in \{2, \dots N\}$. Then, the steady-state input is $u_e = [u_{1_e}, 0, 0, \dots, 0]^\top$ and the corresponding equilibrium state is $x_{m_e} = x_{b_e} = -(L_o + L_i + u_{e_d})^{-1} u_{1_e} v_1$. In this case, the equilibrium state is decided by v_1 and u_{1_e}, and the rate of convergence to the steady-state is determined by the smallest eigenvalue of the matrix $(L_o + L_i + u_{e_d})$. A sophisticated scenario occurs when the steady-state input is nonzero for multiple consumers. In such cases, the network fails to achieve consensus, and the steady-state motivation and behavior of each consumer lies in a convex hull generated by the individual bias v_i of each consumer. Below in Lemma 2, we establish properties of $(L_o + L_i + u_{e_d})$ which will be used to analyze the steady-state behavior of Eq. (2), see Proposition 1 and Theorem 2. Let $\Omega \subset V$ be a nonempty set such that $u_i(t) = 0$ for all $i \in V \setminus \Omega$ and for all $t \in \mathbb{R}^+$ with $|\Omega| = N_1 \leq N$. Denote $L = L_i + L_o$ and $L_u = L + u_{e_d}$. Denote the (i, j) element of L_u^{-1} and L by γ_{ij} and L_{ij}, respectively. The following result is an extension of the result in [39, Appendix A.1] for the case when $u_{i_e} \in (0, u_{\max}]$ for all $i \in \Omega$.

Lemma 2. *Suppose Assumption 1 holds and assume that $u_{i_e} \neq 0$ for all $i \in \Omega$. Then, the matrix $L_u^{-1} \in M_N(\mathbb{R})$ is symmetric, positive, and*

$$\sum_{i \in \Omega} u_{i_e} \gamma_{ik} = 1 \qquad \forall k \in \{1, 2, \dots N\}. \tag{6}$$

Proof. The symmetric nature of L_u^{-1} follows from the fact that L_u is a symmetric and non-singular matrix. Without loss of generality, assume that the layer E_o is strongly connected and hence L_o is an irreducible matrix, see [30, Theorem 2.7]. It also follows that the matrix $L = L_i + L_o$ is irreducible since addition of L_i does not alter the connectivity of the layer $E_i \cup E_o$. Since L is irreducible, $L + u_{e_d}$ is irreducible as no matrix of the form $P u_{e_d} P^{-1}$, with P being a permutation matrix, can destroy the connectivity between different nodes of the layer $E_i \cup E_o$.

Furthermore, the smallest eigenvalue of L_u is positive, see [43, Lemma 1]. Hence L_u is an irreducible invertible M-Matrix and using Proposition 1 we get that L_u^{-1} is a positive matrix. Next, we compute the sum of elements of $L_u L_u^{-1}$ over the k^{th} column and use $(L + u_{e_d})L_u^{-1} = I_N$ to obtain

$$\sum_{i=1}^{N} \Big(\sum_{j=1}^{N} L_{ij}\gamma_{jk} + \sum\nolimits_{j=1}^{N} u_{e_{d_{ij}}} \gamma_{jk} \Big) = 1$$

for each $i, k \in \{1, 2, \dots N\}$. Since $\sum_{i=1}^{N} L_{ij} = 0$ and

$$\sum\nolimits_{j=1}^{N} u_{e_{d_{ij}}} \gamma_{jk} = \begin{cases} 0 & \text{if} \quad i \in V \setminus \Omega, \\ u_{i_e}\gamma_{ik} & \text{if} \quad i \in \Omega, \end{cases}$$

then we obtain Eq. (6). □

In the next result, we show that $x_{m_{i_e}}$ for each $i \in V$ always lies in a convex-hull formed by individual biases of the consumers. The location of $x_{m_{i_e}}$ in the convex hull depends on the choices of u_e, see also [43] for a similar result with $u_i(t) \equiv 1$ for each $i \in \Omega$. Denote $v_\Omega \in \mathbb{R}^N$ whose i^{th} element is v_i if $i \in \Omega$ and is zero otherwise.

Proposition 2. *Under Assumption 1, the steady-state motivation of each consumer stays in the convex hull $Co(v_\Omega)$.*

Proof. Using the symmetric nature of L_u^{-1}, the steady-state motivation of the i^{th} consumer is

$$x_{m_{i_e}} = \sum\nolimits_{j \in \Omega} \gamma_{ji} u_{j_e} v_j. \tag{7}$$

With $\gamma_{ji} u_{i_e} \geq 0$ for each $j \in \Omega$, all $i \in V$ and Eq. (6) we have $x_{m_{i_e}} \in \text{Co}(v_\Omega)$ for each $i \in V$. □

Furthermore, we also get $\sum_{i=1}^{N_1} u_{i_e} x_{m_{i_e}} = \sum_{i=1}^{N_1} u_{i_e} v_i$ by pre-multiplying $\mathbf{1}^\top$ to $(L_o + L_i + u_{e_d})w_e = u_{e_d} v$ and using the property that $\mathbf{1}_N^\top (L_o + L_i) = 0$. It follows from the previous discussion that the weighted average steady-state motivation of the biased consumers equals the weighted average of their biases.

Next, we study the variation of $x_{m_{i_e}}$ for each $i \in V$ with respect to u_{i_e} when only a subset of the consumers have biases. For $\Omega \subset V$, the motivation-behavior dynamics of each consumer in the social network can be written, using Eq. (1b), as

$$\dot{x}_{m_i}(t) = \sum_{j=1}^{N} \alpha_{ij}(x_{m_j}(t) - x_{m_i}(t)) + \sum_{j=1}^{N} \beta_{ij}(x_{b_j}(t) - x_{b_i}(t)) + (v_i - x_{m_i}(t))u_i(t), \tag{8}$$

for each $i \in \Omega$ and

$$\dot{x}_{m_i}(t) = \sum_{j=1}^{N} \alpha_{ij}(x_{m_j}(t) - x_{m_i}(t)) + \sum_{j=1}^{N} \beta_{ij}(x_{b_j}(t) - x_{b_i}(t)) \tag{9}$$

for every $i \in V \setminus \Omega$. Hence, for each $i \in \Omega$, the i^{th} diagonal element of u_{e_d} is $u_{i_e} \in (0, u_{\max}]$, and rest all of its elements are zero. In the steady-state, we have $x_{m_e} = L_u^{-1} u_{e_d} v$ and

$$\frac{\partial x_{m_{k_e}}}{\partial u_{i_e}} = \gamma_{ki}(v_i - x_{m_{i_e}}) \quad \forall i \in \Omega, \quad \forall k \in V. \tag{10}$$

Since L_u^{-1} is a positive matrix, for any $i \in \Omega$ and any $k \in V$, whether $x_{m_{k_e}}$ increases or decreases with increasing values of u_{i_e} depends entirely on the sign of $v_i - x_{m_{i_e}}$. Hence, for each k, $x_{m_{k_e}}$ is either a strictly increasing function of u_{i_e} if $v_i - x_{m_{i_e}} > 0$ or $x_{m_{k_e}}$ is strictly decreasing function of u_{i_e} if $v_i - x_{m_{i_e}} < 0$. Note that the magnitude of variation in $x_{m_{k_e}}$ with respect to u_{i_e} depends also on the entry $\gamma_{ki}(u_e) > 0$ of L_u^{-1}. Furthermore, on taking derivative of Eq. (10) w.r.t u_{i_e},

$$\frac{\partial^2 x_{m_{k_e}}}{\partial^2 u_{i_e}} = -2\gamma_{ki}^2(v_i - x_{m_{i_e}}) \qquad \forall i \in \Omega, \quad \forall k \in V. \tag{11}$$

Combining Eq. (10) and Eq. (11) we get that with increasing u_{i_e}, $i \in \Omega$, the amount of variation of $x_{m_{k_e}}$ decreases. In the next result, we formalize the aforementioned discussion when the social network has two biased consumers.

Theorem 2. *For the social network in Eq. (1a) and Eqs. (8–9) with $\Omega = \{p, q\} \subset V$ and $v_p \neq v_q$, the steady-state $x_{m_{k_e}}$ is strictly monotone with respect to u_{p_e} and u_{q_e} for each $k \in V$.*

Proof. For $v_p \neq v_q$, we claim that $x_{m_{i_e}} \neq v_i$ for any $i \in \Omega$ and any $u_{p_e}, u_{q_e} \in (0, u_{\max}]$. Suppose that the claim is not true for some u_{p_e} and u_{q_e}. WLOG, let $x_{m_{p_e}} = v_p$. Then, from Eq. (7) with $\Omega = \{p, q\}$ and $x_{m_{p_e}} = v_p$, we have $v_p = u_{p_e}\gamma_{pp}v_p + u_{q_e}\gamma_{pq}v_q$. This, using the symmetric property of L_u^{-1} and Eq. (6), implies that $v_p = v_q$ which is a contradiction. Next, for any initial condition $(u^0_{p_e}, u^0_{q_e}) \in (0, u_{\max}]$ of (u_{p_e}, u_{q_e}), we can write that $w^0_{i_e} = u^0_{p_e}\gamma_{pi}(u^0_{p_e}, u^0_{q_e})v_p + u^0_{q_e}\gamma_{qi}(u^0_{p_e}, u^0_{q_e})v_q$. Using Eq. (10) and Eq. (11), when u_{i_e} is increased continuously from $u^0_{i_e}$ to $u_{\max}$, the sign of $(v_i - x_{m_{i_e}})$ remains unchanged for each $i \in \Omega$. This implies that $x_{m_{k_e}}$ either strictly increases or strictly decreases with respect to u_{i_e} depending on whether $v_i - w^0_{i_e}$ is positive or negative, respectively. □

5. Numerical examples

In this section, we provide analysis of the behavior-motivation model in Eq. (2) using two numerical examples. In the first example, we focus on the steady-state behavior of the network, and in the second example, we present a simulation to show the effect of time-varying $u_i(t)$ on the overall network behavior.

5.1. Example 1

In this example, we study the steady-state behavior of Eq. (2) with respect to variations in steady-state inputs for a network of four consumers. We consider the Laplacian matrices

$$L_i = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad L_o = \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & 2 & -1 & -1 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix},$$

with layer E_o not connected but a connected layer E_i. We consider two scenarios of interest.

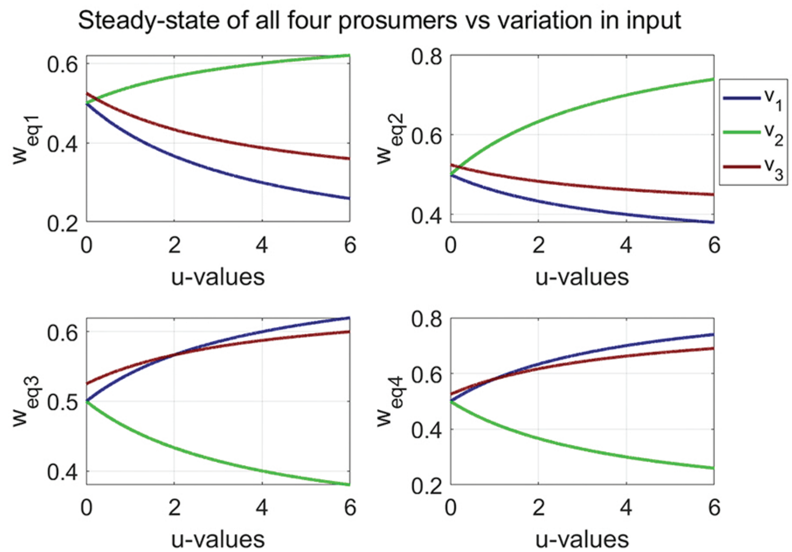

Figure 3: Simulations for Scenario I of Example 1. Plot of steady-state motivation of all four consumers, when $E = E_i \cup E_o$ is a complete graph.

Scenario I: We fix $u_{e_i} = u_{ss} \in [0.001, 6]$ and $a_i = 1$ for each $i \in V$ and perform simulations for three different bias vectors given by $v_1 = [0.1, 0.3, 0.7, 0.9]^\top$, $v_2 = [0.7, 0.9, 0.3, 0.1]^\top$ and $v_3 = [0.25, 0.4, 0.65, 0.8]^\top$. From Fig. 3, we see that when u_{ss} is close to zero, the steady-state motivations of the consumers are close. In fact, $x_{m_{i_e}}$ is in a small neighborhood around $\alpha \mathbf{1}_N^\top$, where $\alpha \in \mathbb{R}$ is determined by the initial conditions. However, as u_{ss} increases, the individual bias of each consumer starts dictating the steady-state motivation and hence we witness a monotone increase or decrease in the steady-state motivation for each consumer, which eventually approaches their own biases.

Scenario II: We fix $v = [0.9, 0.5, 0.65, 0.1]^\top$ and $u_e = [u_{1_e}, 0, u_{3_e}]^\top$ and vary u_{1_e} and u_{3_e}, in $[0, 2]$. In Fig. 4, when u_{1_e} is close to zero, $x_{m_{i_e}}$ remains close to $v_4 = 0.1$ for all $i \in V$. Similarly, in Fig. 5 with u_{4_e} close to zero, $x_{m_{i_e}}$ is close to $v_1 = 0.9$ for $i \in V$. By changing u_{1_e} and u_{4_e}, it is possible to steer $x_{m_{i_e}}$ towards a wide range of values. For instance, by giving suitable intervention to consumers 1 and 4, w_{2_e} can be steered towards values that are

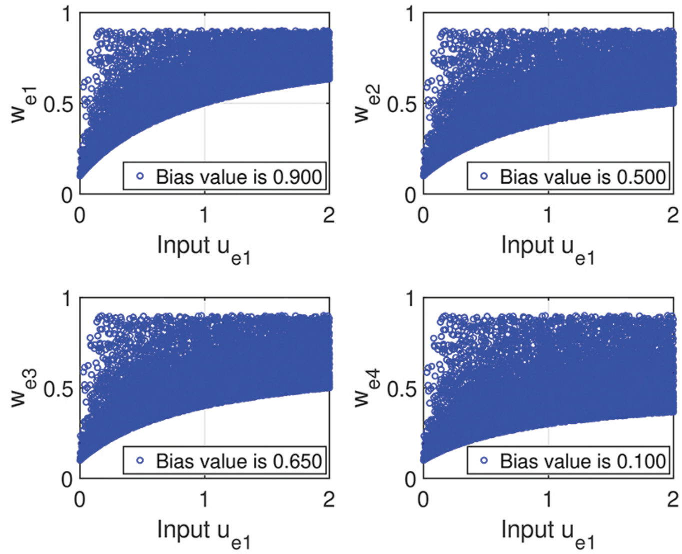

Figure 4: Simulations for Scenario II of Example 1. Plot of steady-state motivation of all four consumers with respect to steady-state input u_1 for each $i = 1, 2, 3, 4$.

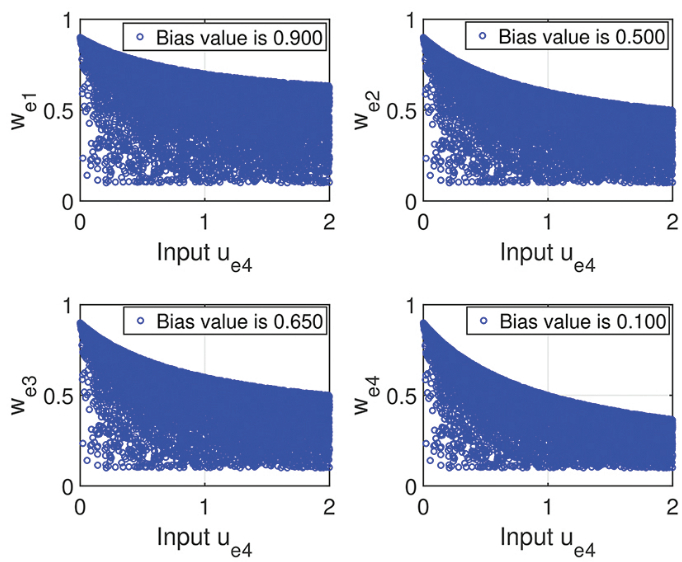

Figure 5: Simulations for Scenario II of Example 1. Plot of steady-state motivation of all four consumers with respect to steady-state input u_4 for each $i = 1, 2, 3, 4$.

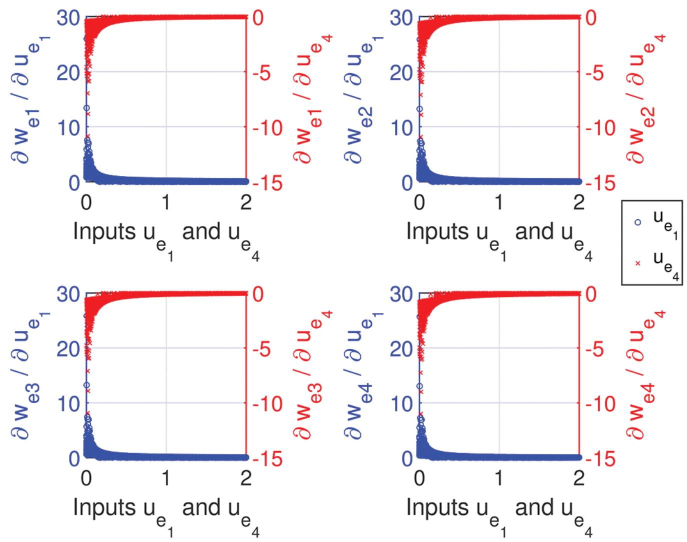

Figure 6: Simulations for Scenario II of Example 1. Plot of rate of change of steady-state motivation of all four consumers with respect to steady-state input u_{e_1} (in blue) and with u_{e_4} (in red).

significantly higher (or lower) than its own bias $v_2 = 0.5$. This observation has potentially strong implications in the design of intervention policies to control motivation (and, eventually, behavior) in energy systems. In Fig. 6, it is seen that $\partial x_{m_{i_e}} / \partial u_{k_e}$ for $k = 1, 4$ has the same profile for all $i \in V$. For each $i \in V$, $\partial x_{m_{i_e}} / \partial u_{1_e} > 0$ while $\partial x_{m_{i_e}} / \partial u_{4_e} < 0$ with higher rate of variation when u_{i_e} is small and vice-versa. This observation suggests that, while it is possible to influence $x_{m_{i_e}}$ of each consumer by making small changes in u_{i_e}, a behavioral intervention that corresponds to a large value for any of the u_{i_e} will drive the motivation of each consumer towards the bias of the consumer whose behavior is being intervened.

5.2. Example 2

The setup of this example is motivated by practical considerations. In most of the energy consumption networks, there exists a centralized entity termed energy regulator. One of the tasks of the energy regulator is demand-side management (DSM), i.e., influencing consumer behavior to align the aggregate energy demand with the available energy supply. A more explored approach for DSM involves providing direct incentives to individual consumers in the network, encouraging them to shift or reduce consumption during peak periods. However, this approach can be costly and requires extensive monitoring and infrastructure. An alternative strategy involves leveraging the structure of the underlying social network. Instead of incentivizing all the consumers in the network, the regulator can identify and incentivize a small set of influential consumers whose behavior and motivation significantly impact others in the network. This indirect method relies on social influence to propagate desirable consumption patterns, offering a potentially cost-effective solution.

With regard to the discussion above, we consider a network of five consumers with $v_1 = 0.1$, $v_2 = 0.1$, $v_3 = 0.2$, $v_4 = 0.8$ and $v_5 = 0.9$, and $a_i = 1$ for all $i \in \{1, 2, \ldots, 5\}$. We assume that $u_1(t)$ can vary, while we fix $u_i(t) = 1$ for $i \in \{2, 3, 4, 5\}$. In particular, we set

$$u_i(t) = \begin{cases} 4 & t \in [0, 20), \\ 3 & t \in [20, 40), \\ 2 & t \in [40, 60), \\ 1 & t \in [60, 80), \\ 0 & t \in [80, 100]. \end{cases} \tag{12}$$

In this example, we study the effect of a decreasing $u_1(t)$ on the steady-state behavior of the remaining four agents. We assume the influence and observation layer of the overall network to be a star graph with Consumer 1 being the root node. Hence,

$$L_i = L_o = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The network is made of two clusters: Consumers 2 and 3 have low-bias

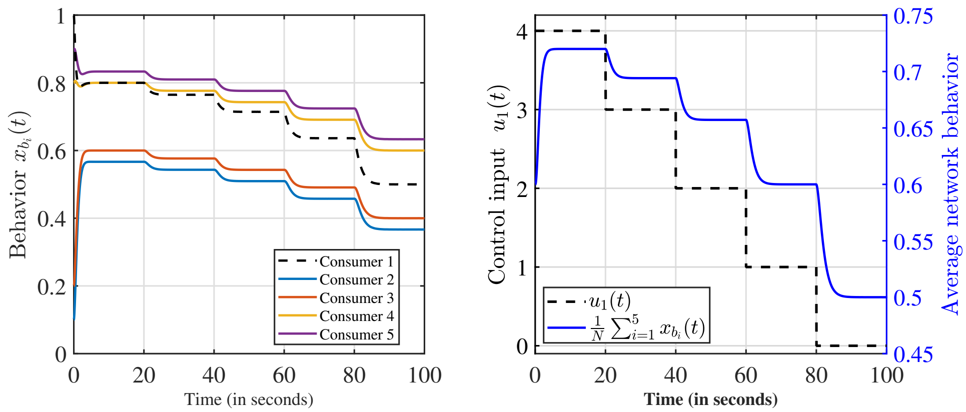

Figure 7: Simulations for Example 2. The left plot represents the behavior (x_{b_i}) for each consumer. The plot on the right depicts the average network behavior, $\frac{1}{N}\sum_{i=1}^{N} x_{b_i}(t)$.

values, Consumers 1, 4, and 5 have high-bias values. We initialize the system by setting $x_{b_i}(0) = x_{m_i}(0) = v_i$ for all $i \in V$. From Fig. 7 it can be seen that the behavior of others consumers x_{b_i} decreases, guided by the aforementioned variation in $u_1(t)$. Hence, By changing the input to consumer 1, the average energy demand of the whole network $\frac{1}{N}\sum_{i=1}^{N} x_{b_i}(t)$ can conveniently shaped, as reported in the right-hand side plot in Fig. 7, suggesting that targeted control interventions may be implemented by public authorities in order to incentivize the adoption of a desired collective behavior.

6. Conclusion

In this paper, we presented a novel dynamical system to model the intertwined evolution of motivation and behavior related to the energy consumption of consumers in an energy network. Rooted in opinion dynamics and its recent extensions, our model accounts for personal biases, social influence through observation and peer discussion, and their impact on collective energy behavior. Through a steady-state analysis of the model, we established a framework to study how different intervention policies shape the collective energy behavior of a population.

The preliminary results presented here open several research directions. First, the steady-state analysis should be extended to explore different network structures and external disturbances, leveraging matrix L_u^{-1} analysis as in [39]. Second, while grounded in social psychology, the model requires empirical validation through questionnaires or experiments, as done in [44]. In this regard, it is also crucial to design surveys or experiments that can help in obtaining reliable estimates of the system parameters. Finally, once validated, it will be integrated with the dynamical model of the power grid, [19] to study and optimize the cyber-physical-human energy system.

CRediT authorship contribution statement

Vaibhav Kumar Singh: Conceptualization, Methodology, Formal analysis, Investigation, Software, Validation, Data Curation, Visualization, Writing — Original Draft

Lorenzo Zino: Conceptualization, Methodology, Investigation, Writing — Review & Editing, Supervision, Project administration

Gabriel Muinos: Conceptualization, Writing — Review & Editing, Supervision

Jacquelien M.A. Scherpen: Resources, Writing — Review & Editing, Supervision, Funding acquisition

Michele Cucuzzella: Conceptualization, Methodology, Resources, Writing — Review & Editing, Supervision, Project administration, Funding acquisition

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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