Novel Computational Strategies for the Inverse Source Problem in Electromagnetics

Supervisor:
Prof. Francesco Paolo Andriulli

Candidate: Paolo Ricci

Summary

This thesis presents doctoral research activities on the analysis of electromagnetic integral equations discretized using the boundary element method, with a focus on the solution of the inverse source problem. When it is coupled with the equivalence principle, this problem consists in estimating current distributions over an equivalent surface given the fields that they produce. The inverse source strategies can be divided in single and double current solutions. The first group includes schemes that reconstruct only one family between electric and magnetic currents. These approaches are appealing because of the reduced dimensions of the linear systems to be solved and because of their reduced null-space. Nevertheless, they require additional care in the solution process unless further physical constraints are used to ensure a simple relationship between equivalent currents and fields. On the other hand, the double current formulations evaluate both electric and magnetic currents on the chosen surface to reconstruct the fields. This approach results in systems with a large null-space, that is often addressed selecting a particular solution. Another feature of interest among inverse source schemes is their capacity to find equivalent currents that satisfy the Love's condition, i.e., that enforce a null field everywhere inside the equivalent surface. The Love currents can be obtained by adding further constraints to double current formulations or by filtering any of the solution via Calderón projection.

Part of this work is dedicated to the study of a single current formulation able to enforce the Love's condition despite a reduced-in-size system to solve. The scheme utilizes a Steklov-Poincaré mapping and relies on a stable discretization using dual elements. In addition, the low-frequency breakdown that affects the formulation is cured by leveraging a novel frequency stabilization of the Steklov-Poincaré operator via quasi-Helmholtz projectors. The spurious internal resonances of the proposed scheme are also addressed by means of the derivation of a formulation of the Steklov-Poincaré operator based on the combined field integral operator, which is employed to obtain a family of resonance-free formulations for the inverse source problem.

Finally, this thesis includes a preliminary analysis of a process to solve the inverse source problem with the assistance of an advanced machine learning

technique. This strategy relies on the construction of a weighted generalized inverse based on the information extracted from previously computed simulations. A graph neural network is employed to gather information from the geometry of the problem and to help in the construction of the matrix responsible of the selection of the desired particular solution of the problem.