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Insights from Tracking the Loss Sensitivity with Respect to the Net Load in Distribution Networks

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Abstract— **During the last years, the increase of the energy price and the growing concerns about the climate change effects led to a large installation of Distributed Energy Resources (DERs), which will play a crucial role in shaping the future electricity system. The penetration of renewable energy sources, the transformation from consumers to prosumers, and the diffusion of electric vehicles and battery storage systems into distribution networks, may have negative impacts on grid losses. The possible increase in the grid losses ends up in wasting of resources and imposes extra costs on utilities and users. The sensitivity of losses with respect to the loads has to be investigated in presence of DERs. This paper discusses the appropriateness of loss sensitivity analysis in distribution grids by determining the loss sensitivities starting from the equations of loss allocation to the loads in the grid nodes. The loss sensitivities are expressed based on the node current changes in the radial distribution network. Some tutorial case studies are proposed to investigate the sensitivity in case of reverse power flow in the grid with DERs in balanced and unbalanced distribution networks. The results of the loss sensitivity analysis provide insights on the location of distributed generators to mitigate the losses and the reverse power flow in the grid.**

Keywords—losses, loss analysis, loss sensitivity, distributed energy resources, renewable energy.

I. Introduction

The increasing diffusion of Distributed Energy Resources (DERs) in the distribution networks, with distributed generation (in particular, from renewable energy sources), demand response, management of flexible load, and energy storage, has led to a significant increase of active users or prosumers connected to the distribution system nodes [1]. In this context, comprehensive control of DERs enables achieving effective power flow management [2]. The system active power losses are an important indicator for assessing operating economy of active distribution networks [3]. Optimization based on the losses in active distribution networks is significant for improving the operation and planning of active distribution systems in the future [4].

The progressive growth in the size of DERs connected to the distribution networks has led to reaching conditions of reverse power flow (RPF), in which the whole distribution network becomes a generator seen from the supply point in some time periods, rather than a load as traditionally occurs. The RPF phenomenon could occur for example in mid hours of a bright day when the PV power generation in a grid-connected network exceeds the local load demand [5]. RPF is more likely to happen when there are relatively low peak loads [6].

When RPF conditions are reached, the currents in some branches in the network flow from the local generators to the supply point. In these conditions, the network losses could increase with respect to cases in which the local generation compensates the local load without inverting the direction of the current flowing to the supply point. A specific analysis of the network losses when the loads and local generations change can be carried out by resorting to the determination of the sensitivity of the network losses with respect to the network loads, in particular, considering the net load power (i.e., local load minus local generation) at each node.

In this paper, loss sensitivity analysis with respect to the net power load is applied to some tutorial cases to understand the implications of the loss sensitivity information in various situations, including RPF conditions. The loss sensitivities are calculated starting from the power flow solutions by considering progressive changes in the net load power. In this way, the sensitivities are tracked in various operational conditions of the distribution system, including RPF cases.

The novel contributions of this paper are as follows:

a) The formulation of the sensitivities of the total losses with respect to the net load currents, determined starting from the expression of the losses allocated to the network nodes taken from the literature, which already contains the load currents explicitly. In this way, the effects of the node voltages are automatically incorporated in the loss sensitivities, even though the loads and local generations are defined by their active and reactive power.

b) The tutorial discussion on the results of the parametric analysis carried out by tracking the loss sensivity information when a specific net load power is changed in the system. This discussion is helpful to interpret specific situations appearing in RPF cases, by linking the loss sensitivity information with the increase/decrease of the net load power to reduce the network losses and the RPF.

c) The tutorial examples include unbalanced circuits in which the RPF conditions could be reached on one or two phases only. The analysis of the loss sensitivities enables better understanding the effects of changing the net load in these conditions.

The next sections of this paper are organized as follows. Section II summarizes the literature review. Section III describes the loss sensitivity methodology developed in this work. Section IV presents the results obtained on tutorial test cases. Finally, the last section contains the conclusions.

II. LITERATURE REVIEW

An extended overview on the state of the art in load sensitivity analysis (LSA) for modern distribution systems is presented in [7]. The authors summarized the theoretical formulations of existing LSA methods and highlighted the applications of LSA in distribution systems. At the end, the potential issues found in the literature review and the future path of regarding this topic are discussed. Luo et al. [8] propose one power loss optimization model and solution process by sensitivity analysis (SA) algorithm, on the basis of the traditional optimal power flow (OPF) model. The model and solution process are simulated on the 34-node distribution system in North China. A fast and useful formula to calculate loss sensitivity for any slack bus is proposed in [9]. According to the authors, this formula is based on the results from the distributed slack without repeating the traditional power flow calculation. An analytical power loss sensitivity analysis in distribution systems is proposed in [10] to tackle the slow performance and lack of flexibility of classical loss sensitivity methods. Their approach is tested on the IEEE 69-bus distribution system and verified against classical load flow-based method. A novel sensitivity analysis using two loss sensitivity indices (LSIs) based on active power to select the most candidate capacitor's locations in a radial distribution network (RDN) is proposed in [11]. A correlation parameter for each sensitivity type is suggested. In this work, the total active power losses, bus voltages and installation costs are considered. The validity of the proposed methodology is confirmed using the standard IEEE-34 test bus system. An application of loss sensitivity factor (LSF) to obtain Battery Energy Storage System (BESS) optimal placement is proposed by Widjaja et al. [12]. The aim is to minimize the BESS losses by means of optimization techniques. The results obtained confirm that this approach minimizes significantly the grid losses compared to the conventional LSF approach. Recent studies have shown that locational marginal pricing (LMP) model can be extended to the electric distribution system to create better economic and operational signals [13]. A novel approach of nodal loss sensitivities calculates the losses belonging to each node. A quadratic sensitivity model of distributed generation output or load to line loss and a linear sensitivity model of distributed generation output or load to node voltages are established based on one initial power flow calculation [14]. Using these models, the variety of line losses and node voltages in distribution networks can be directly determined with the variation of Distributed Generator (DG) output or load demand. An approach to place single and multiple energy storage units in a distribution network with photovoltaic units considering energy losses is proposed by Sardi et al. [15]. The approach proposed, based on Average Loss Sensitivity factor ALSFmax, is defined as the difference between the minimum and maximum Loss Sensitivity Factor (LSF) values. Results show that the performance of the proposed ALSFmax approach in reducing the losses and voltage profiles can be more efficient than other methods such as average LSF, centralized energy storage (i.e., with energy storage located at the substation) and energy storage located near the PV units. An analytical method for optimal DG placement is presented in [16] with the definition of a sensitivity index denoted as Loss-Voltage Sensitivity Index (LVSI), which combines the traditional Voltage Sensitivity Index (VSI) and the Loss Sensitivity Index (LSI). VSI and LSI are utilized to find the optimal location of DG in the grid. Loss sensitivity analysis is utilized to improve power losses when DGs are present in the grid [17]. The loss sensitivity analysis approach leads to the reduction of total losses and to finding the optimal displacement of DGs.

Different approaches concerning loss sensitivity are proposed in the literature to reduce the complexity of the calculations and providing the capacitors, storage systems, loads, DG placements. Despite the importance of RPF in the electrical grid with DERs, there is no approach investigating the role and the possible negative impacts of RPF on the quality of grid services. The view proposed in this paper underlines the outcomes of loss sensitivity analysis under RPF conditions and provides insights on the location and values of loads and DGs on the RPF on the whole network. The discussions provided in the tutorial cases for balanced and unbalanced distribution systems highlight the meaning of the sensitivity information in different operating conditions.

III. METHODOLOGY

Let us consider a distribution network with radial configuration, supplied by the slack bus (denoted as node 0), consisting of K nodes and B branches. In the distribution system, the loads in the nodes are characterized by their net active and reactive power, both calculated as load power minus local generated power. In the analysis for a period of time T with time steps $t = 1, \dots, T$, the power at each time step is considered as constant (average power) for that time step.

After the execution of the power flow calculations for the radial distribution network, considering the resistance R_b of each branch $b = 1, \dots, B$ belonging to the set $\mathbb{B}_k$, the total average power losses at time step t are calculated as:

$$L_{\mathrm{tot},t} = \sum_{b=1}^{B} R_b I_{b,t}^2 \qquad (1)$$

In a balanced distributed network, the losses allocated to the distribution system nodes are calculated with the method developed in [18]. The current absorbed by the load connected to node k at time step t is denoted as $\bar{I}_{\mathrm{L}k,t}$ and the current of each branch at the corresponding time step is $\bar{I}_{b,t}$. The losses allocated to node k are determined as:

$$\lambda_{k,t} = \mathcal{R}e\left\{ \bar{I}_{\mathrm{L}k,t} \sum_{b\in\mathbb{B}_k} \left(R_b\, \bar{I}_{b,t}^{*} \right) \right\} \qquad (2)$$

The formulation (2) guarantees that the total allocated losses are equal to the total network losses in the operating conditions in which there is no RPF, such that:

$$L_{\mathrm{tot},t} = \sum_{k=1}^{K} \lambda_{k,t} \qquad (3)$$

The previous formulas are valid in case of balanced distribution networks. In case of unbalanced distribution networks with four conductors (including the neutral conductor) the mutual relationship between the conductor impedances impacts the formulation of the loss allocation.

The Carson equations [19] are used to obtain the self-impedances and the mutual impedances of the multi-conductor grid. Under specific assumptions about the voltage at the terminals of the neutral conductor [20], the impedance matrix of each branch can be reduced to a 3x3 square matrix referring to the phases (a,b,c), indicated as $\mathbf{Z}_{\mathrm{abc},b}$, using the Kron reduction technique. The load currents in the three phases are included in the vector $\mathbf{i}_{\mathrm{L}k,t} = \left[\bar{I}_{\mathrm{L}k\mathrm{a},t}, \bar{I}_{\mathrm{L}k\mathrm{b},t}, \bar{I}_{\mathrm{L}k\mathrm{c},t} \right]^{\mathrm{T}}$. The branch currents in the three phases are included in the vector $\mathbf{i}_{b,t} = [\bar{I}_{\mathrm{ba},t}, \bar{I}_{\mathrm{bb},t}, \bar{I}_{\mathrm{bc},t}]^{\mathrm{T}}$. Dealing with the active power losses, the real part of the impedance matrix is extracted as $\mathbf{R}_{\mathrm{abc},b} = \mathcal{R}e\{\mathbf{Z}_{\mathrm{abc},b}\}$. The losses allocated to each phase of the three-phase load are determined as in [21] and are included in the vector:

$$\boldsymbol{\lambda}_{k,t} = \begin{bmatrix} \lambda_{k\mathrm{a},t} \\ \lambda_{k\mathrm{b},t} \\ \lambda_{k\mathrm{c},t} \end{bmatrix} = \mathcal{R}e\{ \mathbf{i}_{\mathrm{L}k,t} \otimes \sum_{b \in \mathbb{B}_k} (\mathbf{R}_{\mathrm{abc},b}\, \bar{I}^*_{b,t})\} \qquad (4)$$

where $\otimes$ represents the component-by-component (Hadamard) vector product. The total losses are calculated as:

$$L_{\mathrm{tot},t} = [1 \quad 1 \quad 1] \sum_{k=1}^{K} \boldsymbol{\lambda}_{k,t} \qquad (5)$$

The variation of active power losses with respect to the net load power is defined as the general sensitivity of the total active power losses with respect to the net load power. In more detail, the loss sensitivity coefficient $\sigma_{k,t}^{(P)}$ represents the variation of the total active power losses with respect to the variation of the local load $P_{k,t}^{(\mathrm{d})}$ *or* of the local generation $P_{k,t}^{(\mathrm{g})}$ at a given node k and time step t [21].

Let us consider the reference power flow solution (denoted as "ref"). The power flow solution (denoted as "new") obtained after the variation of the local load or local generation, which explicitly represents the entries that contribute to the net load power variation, leads to expressing the loss variation as:

$$L_{\mathrm{tot},t}\Big|_{\mathrm{new}} - L_{\mathrm{tot},t}\Big|_{\mathrm{ref}} \cong \sigma_{k,t}^{(P)} \left(P_{k,t}^{(\mathrm{d})}\Big|_{\mathrm{new}} - P_{k,t}^{(\mathrm{d})}\Big|_{\mathrm{ref}} - P_{k,t}^{(\mathrm{g})}\Big|_{\mathrm{new}} + P_{k,t}^{(\mathrm{g})}\Big|_{\mathrm{ref}} \right) \qquad (6)$$

The loss sensitivity coefficient can be either positive or negative and represent the expected trend of the total losses when *small* variations in the generation or load are introduced. The calculated loss sensitivity values introduce some important concepts concerning the possibility of reducing the total power losses in different ways:

1) with a positive loss sensitivity coefficient, the losses can be reduced by a small reduction of the demand *or* a small increase of the local generation;

2) with a negative loss sensitivity coefficient, the losses can be reduced by a small increase of the demand *or* a small reduction of the local generation.

On the other side, there are the losses allocated to the nodes, determined as shown in (2) for balanced systems and in (4) for the three phases in unbalanced systems. The allocated losses are formally not equivalent to sensitivities. However, it would be of interest to establish a consistent qualitative link between the losses allocated to the network nodes and the loss sensitivity values computed at the same nodes.

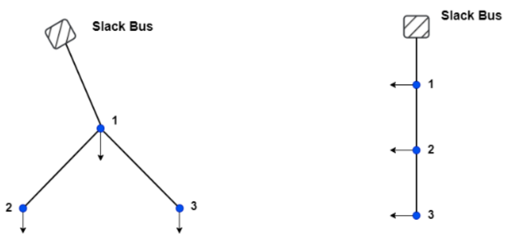
For the allocated losses, there is a conceptual limit that refers to the situation with RPF. In distribution systems, the slack bus is not considered for the loss allocation, as it is the main supply point of the network and does not correspond to a physical entity to which the losses in the distribution system losses have to be allocated. Practically, the loss allocation mechanism used in distribution systems corresponds to connecting the slack node to the reference [22]. This situation is viable when the power flows from the slack node to the network. However, if there is an RPF, the situation changes, and the reasoning of loss allocation has to be revisited.

In this paper, a discussion on the evaluation of loss sensitivities is proposed, based on the load and branch currents, in particular considering the occurrence of RPF at some time in the system. The approach used for the discussion is to track the variation of the loss sensitivity with respect to the net load variation. The calculations are carried out in an analytic way, by taking the derivatives of the total losses to the magnitude of the load currents, considering the fixed nature of the loads (e.g., constant phase displacement between voltages and currents at the same phase).

Analyzing loss sensitivities using the proposed methodology, the distribution system operator (DSO) could receive further information on the need to increase or decrease loads or local generations in specific nodes and send indications to the passive and active users (with DG) about the suggested direction of net power variation.

IV. Test Case Application and Results

To discuss the basic aspects of loss sensitivity calculations in the distribution system, some simple tutorial grid examples are provided in the balanced (represented as a single-phase equivalent circuit) and three-phase unbalanced cases. In both cases, two configurations that start from the slack bus and contain three branches and three nodes with loads have been defined, as shown in Fig. 1, namely, a "lambda"-like structure and a feeder-like structure. In the three-phase unbalanced case, two loads are single-phase loads connected to different phases of different nodes, while one load is three-phase unbalanced. In the current work, all the simulations are carried out using the OpenDSS power flow solver linked with Matlab for sensitivity calculations.



(a) (b)
Fig. 1. Tutorial grid example topologies: (a) “lambda”-like and (b) feeder.

A. Balanced Network Tutorial Case

Starting from the total loss formulation (3), with the load current $\bar{I}_{\mathrm{L}k,t} = I_{\mathrm{L}k,t} \mathrm{e}^{\mathrm{j}\gamma_{I_{\mathrm{L}k,t}}}$ at node k = 1, …, K, and the currents $\bar{I}_{b,t} = I_{b,t} \mathrm{e}^{\mathrm{j}\gamma_{I_{b,t}}}$ flowing in the branches b = 1, …, B with resistance R_b, the losses in the “lambda”-like case are calculated as follows:

$$L_{\mathrm{tot}} = \mathcal{R}e\{ (\bar{I}_{\mathrm{L1},t} \; R_1 \bar{I}_{1,t}^*) + \bar{I}_{\mathrm{L2},t} (R_2 \bar{I}_{2,t}^* + R_1 \bar{I}_{1,t}^*) + \bar{I}_{\mathrm{L3},t} (R_3 \bar{I}_{3,t}^* + R_1 \bar{I}_{1,t}^*) \} = \mathcal{R}e\left\{ \bar{I}_{\mathrm{L1},t} \; R_1 (\bar{I}_{1,t}^* + \bar{I}_{2,t}^* + \bar{I}_{3,t}^*) + \bar{I}_{\mathrm{L2},t} \left(R_2 \bar{I}_{2,t}^* + R_1 (\bar{I}_{1,t}^* + \bar{I}_{2,t}^* + \bar{I}_{3,t}^*) \right) + \bar{I}_{\mathrm{L3},t} (R_3 \bar{I}_{3,t}^* + R_1 (\bar{I}_{1,t}^* + \bar{I}_{2,t}^* + \bar{I}_{3,t}^*) \right\} \quad (7)$$

The formulation is further simplified using the complex number properties, obtaining:

$$L_{\mathrm{tot}} = R_1 I_{L1,t}^2 + R_1 (I_{\mathrm{L1},t} I_{\mathrm{L2},t} \cos(\gamma_{I_{\mathrm{L1},t}} - \gamma_{I_{\mathrm{L2},t}}) + I_{\mathrm{L1},t} I_{\mathrm{L3},t} \cos(\gamma_{I_{\mathrm{L1},t}} - \gamma_{I_{\mathrm{L3},t}})) + R_1 I_{L2,t}^2 + R_1 (I_{\mathrm{L2},t} I_{\mathrm{L1},t} \cos(\gamma_{I_{\mathrm{L2},t}} - \gamma_{I_{\mathrm{L1},t}}) + I_{L2,t}^2 + I_{\mathrm{L2},t} I_{\mathrm{L3},t} \cos(\gamma_{I_{\mathrm{L2},t}} - \gamma_{I_{\mathrm{L3},t}})) + R_3 I_{L3,t}^2 + R_1 (I_{\mathrm{L3},t} I_{\mathrm{L1},t} \cos(\gamma_{I_{\mathrm{L3},t}} - \gamma_{I_{\mathrm{L1},t}}) + I_{L3,t}^2 + I_{\mathrm{L3},t} I_{\mathrm{L2},t} \cos(\gamma_{I_{\mathrm{L3},t}} - \gamma_{I_{\mathrm{L2},t}})) \quad (8)$$

The loss sensitivities are calculated as the derivatives of total losses with respect to current flowing in the load, as shown in (9):

$$\sigma_{k,t} = \frac{\partial L_{\mathrm{tot}}}{\partial I_{\mathrm{L}k,t}} \quad (9)$$

In the balanced case, the loss sensitivities with respect to the node current magnitudes are calculated using (9):

$$\sigma_{1,t} = 2R_1 I_{\mathrm{L1},t} + 2R_1 I_{\mathrm{L2},t} \cos(\gamma_{I_{\mathrm{L1},t}} - \gamma_{I_{\mathrm{L2},t}}) + 2R_1 I_{\mathrm{L3},t} \cos(\gamma_{I_{\mathrm{L1},t}} - \gamma_{I_{\mathrm{L3},t}}) \quad (10)$$

$$\sigma_{2,t} = 2R_1 I_{\mathrm{L1},t} \cos(\gamma_{I_{\mathrm{L1},t}} - \gamma_{I_{\mathrm{L2},t}}) + 2 (R_1 + R_2) I_{\mathrm{L2},t} + 2R_1 I_{\mathrm{L3},t} \cos(\gamma_{I_{\mathrm{L2},t}} - \gamma_{I_{\mathrm{L3},t}}) \quad (11)$$

$$\sigma_{3,t} = 2R_1 I_{\mathrm{L1},t} \cos(\gamma_{I_{\mathrm{L1},t}} - \gamma_{I_{\mathrm{L3},t}}) + 2 R_1 I_{\mathrm{L2},t} \cos(\gamma_{I_{\mathrm{L2},t}} - \gamma_{I_{\mathrm{L3},t}}) + 2 (R_1 + R_3) I_{\mathrm{L3},t} \quad (12)$$

In the example presented here, the active power is P_{L1} = 20 kW (load at node 1), P_{L3} = 20 kW (load at node 3), P_{L2} at node 2 variable from -50 kW (local generation) to 50 kW (local load), and all reactive to active power ratios are 0.5. The branches are represented with their resistances only, with equal values 0.02116Ω. The power flow and the loss sensitivity coefficients are calculated for each value of P_{L2} (with fixed reactive to power ratio). The resulting node voltages are shown in Fig. 2. The analytical loss sensitivities are calculated by using the load currents referring to power and voltage taken from the power flow solutions. The load current magnitudes and angles are shown in Fig. 3 and the branch current magnitudes are represented in Fig. 4. The changes of slope of the current magnitudes reported in Fig. 4 correspond to the “turning points” of the currents in branch 2 (where the net load power is equal to zero and separates the condition of prevailing local load and prevailing local generation at node 2) and at branch 1 (when the RPF with current flowing from the network to the supply point is created, corresponding to the most negative values of P_{L2} represented in the figure).

The total network losses are depicted in Fig. 5. The minimum loss condition corresponds to the case in which, having the same branch resistances, all branch current magnitudes are equal, following the known geometrical property for which the sum of the squares of a set of numbers is minimum when all numbers are equal. In the “lambda”-like configuration this condition is reached when there is a generation at node 2 that compensates half of the total load current (i.e., about half of the total load power, because of the different node voltages), with the other half of the load supplied by the network through the slack node. By definition, in this case the minimum network losses condition also corresponds to the loss sensitivity $\sigma_{2,t} = 0$.

The loss sensitivities with respect to the node current magnitudes are shown in Fig. 6. The almost linear behaviours are consistent with the equations (10)-(12) and with the two possibilities of positive or negative net load at node 2. Null values of the loss sensitivity appear at the two “turning points” indicated before. At the turning point on branch 1, the loss sensitivity becomes negative for the most negative values of P_{L2} presented in the figure, meaning that to reduce the total losses the load at node 1 should be increased.

Loss sensitivities could be considered as an indication for locating loads and generations in the electrical grid. When the loss sensitivities are negative in a range of values of the parameter considered (P_{L2} in the case shown), the load increase in the corresponding node ends up in the reduction of absolute value of the total losses. The slope of the loss sensitivity curve shown in Fig. 6, together with the sign of the loss sensitivity, indicate how the load (or local generation) can be changed to determine a positive or negative variation of the total losses. For example, when P_{L2} changes in the range from -50 kW to -40 kW, RPF occurs in the branch connected to the slack node. To reduce RPF, it is possible to increase the load either in node 1 (with negative loss sensitivity and positive slope) or in node 2 (with positive loss sensitivity and negative slope). Conversely, increasing the load in node 3 causes an increase of the losses, because after the load increase the magnitude of the current in branch 3 is higher than the magnitude of the (reverse) current in branch 1, increasing the losses (which depend on the square of the current magnitude).

For the feeder-like structure, considering equal branch resistances with the same values and the same loads indicated for the “lambda”-like structure (with the same P_{L2} variability)

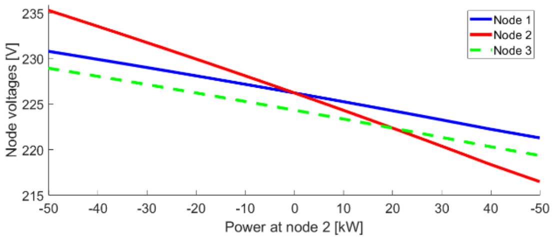


Fig. 2. Node voltages ("lambda"-like configuration).

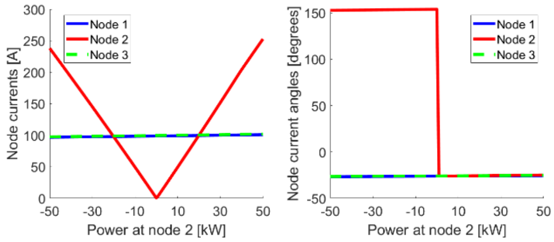


Fig. 3. Node current magnitude and angle ("lambda"-like configuration).

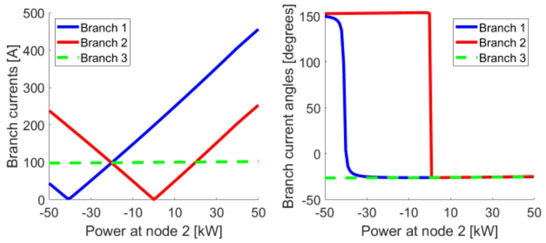


Fig. 4. Branch current magnitudes and angles ("lambda"-like configuration).

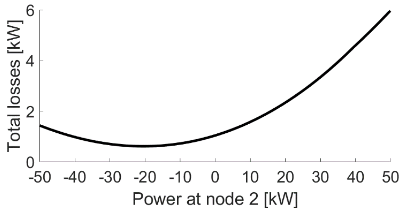


Fig. 5. Total losses ("lambda"-like configuration).

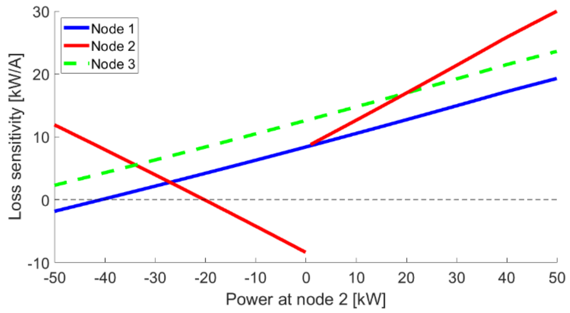


Fig. 6. Loss sensitivities in the "lambda"-like configuration.

and following the same analytical developments, the loss sensitivities with respect to the load current magnitudes are calculated from (9), obtaining:

$$\sigma_{1,t} = 2R_1\, I_{L1,t} + 2R_1\, I_{L2,t}\, \cos(\gamma_{I_{L1,t}} - \gamma_{I_{L2,t}}) + 2R_1\, I_{L3,t}\, \cos(\gamma_{I_{L1,t}} - \gamma_{I_{L3,t}}) \quad (14)$$

$$\sigma_{2,t} = 2R_1\, I_{L1,t}\, \cos(\gamma_{I_{L1,t}} - \gamma_{I_{L2,t}}) + 2\,(R_1 + R_2)\, I_{L2,t} + 2\,(R_1 + R_2)\, I_{L3,t}\, \cos(\gamma_{I_{L2,t}} - \gamma_{I_{L3,t}}) \quad (15)$$

$$\sigma_{3,t} = 2R_1\, I_{L1,t}\, \cos(\gamma_{I_{L1,t}} - \gamma_{I_{L3,t}}) + 2\,(R_1 + R_2)\, I_{L2,t} \cos(\gamma_{I_{L2,t}} - \gamma_{I_{L3,t}}) + 2(R_1 + R_2 + R_3)\, I_{L3,t} \quad (16)$$

Fig. 7 shows the total losses and Fig. 8 the loss sensitivities. The discontinuity of the loss sensitivities at node 2 is again due to the transition from generation to load when the power P_{L2} at node 2 is null. The main difference with respect to the "lambda"-like case, using the same load data, is that the loss sensitivity curves for node 1 and node 3 pass from zero in the same condition (corresponding to P_{L2} = -40 kW) and the slopes of the curves are different.

Since branch 3 serves a constant load at node 3, the power flow to node 3 is always in the same direction, with no turning point for the branch 3 current. As such, the losses on branch 3 are approximately the same in all conditions of changing P_{L2} (not exactly the same, due to the variation of the node voltages). Loss minimization is then substantially a matter of what happens in branch 1 and branch 2. The minimum total losses condition corresponds to the loss sensitivity $\sigma_{2,t} = 0$ and occurs when the currents in branch 1 and branch 2 are equal (again, following the geometrical property for which the sum of the squares of a set of numbers is minimum when all numbers are equal). In the case analysed, this condition corresponds to node 2 that behaves as a generator to compensate half of the load at node 1, i.e., with a power generation of about 30 kW, i.e., $P_{\mathrm{L}2}$ = -30 kW.

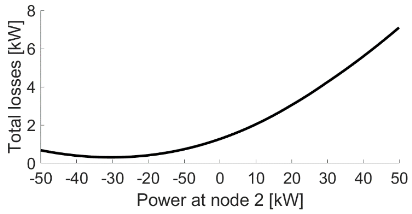


Fig. 7. Total losses in the feeder configuration.

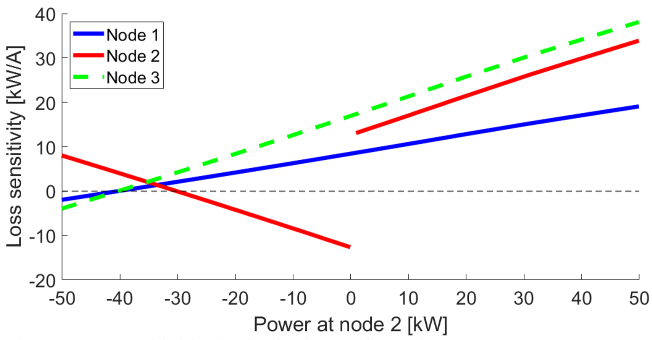


Fig. 8. Loss sensitivities in the feeder configuration.

The RPF condition corresponds to $P_{\mathrm{L}2}$ < -40 kW, because with the generation of about 40 kW at node 2 all the network loads are served by the generator located at node 2 and no power is withdrawn from the slack bus.

The results referring to the loss sensitivity $\sigma_{3,t}$ at node 3 are interesting. Considering the case with $P_{\mathrm{L}2}$ = -50 kW (shown at the left-hand side in Fig. 7 and Fig. 8), in which there is an excess of generation from node 2 with respect to the total load, leading to RPF, the increase of the load at node 3 causes a reduction of the RPF, being the loss sensitivity $\sigma_{3,t} < 0$. Likewise, reduction of the RPF occurs by increasing the load at node 2 with $\sigma_{2,t} > 0$ and negative slope of the loss sensitivity, as well as increasing the load at node 1 with $\sigma_{3,t} < 0$ and positive loss sensitivity. This kind of behaviour is observed for $P_{\mathrm{L}2}$ < -40 kW, but also limitedly to node 2 for $P_{\mathrm{L}2}$ from -40 kW to the minimum total losses condition (-30 kW) with $\sigma_{2,t} > 0$ and negative slope of the loss sensitivity.

B. *Unbalanced Network Tutorial Case*

In the three-phase unbalanced test case, the branch variables are different with respect to the balanced case, due to the presence of the mutual impedances of the conductors, handled during the formation of the 3x3 impedance matrix referring to the three phases, built by using the Kron technique as indicated in Section III. The 3x3 matrix, equal for all the branches, is diagonal (0.02116) Ω.

The loads are assigned as follows. At node 1 there is a single-phase load of 25 kW in phase a. At node 2 there is a variable load (from -50 kW to 50 kW) in phase a, a single-phase load of 50 kW at phase b, and a single-phase load of 20 kW at phase c. At node 3 there is a single-phase load of 50 kW in phase b. For all loads, the reactive power to active power ratio is 0.5.

The analytical expression of the total losses is taken from (5). The loss sensitivities are determined with respect to the load currents at each node and at each phase, following the same conceptual scheme presented in Section IV.A for the balanced case. The analytical expressions are not shown here for space reasons.

The parametric analysis, organized in the same way as the one presented in Section IV.A, has been carried out also for the unbalanced system, taking the power at phase "a" of node 2, denoted as $P_{\mathrm{L}2\mathrm{a}}$, as the parameter that is changed from -50 kW to 50 kW. Some results are presented below for the feeder-line structure.

Fig. 9, Fig. 10 and Fig. 11 show the node voltages, node currents and branch currents, respectively, in the three phases of each node. From Fig. 11, the RPF condition occurs in phase a of the line that supplies node 1 before the zero-crossing point, i.e, for $P_{\mathrm{L}2\mathrm{a}}$ < -25 kW. Fig. 12 shows the total network losses. The minimum total losses condition is reached when $P_{\mathrm{L}2\mathrm{a}}$ = -12 kW.

The loss sensitivities calculated analytically with respect to the load currents at each phase of each node are presented in Fig. 12. By applying the same reasoning used in the previous examples, the cases in which increasing the loads leads to reduce the losses are the ones with negative loss sensitivity and positive slope of the loss sensitivity curve (node 1 phase a), as well as positive loss sensitivity and negative slope of the loss sensitivity curve (node 2 phase a). Both cases correspond to RPF towards the supply system at phase a. Moreover, in this example, increasing the load at node 2 phase a leads to reduce the losses in the branch at phase a from node 1 to node 2.

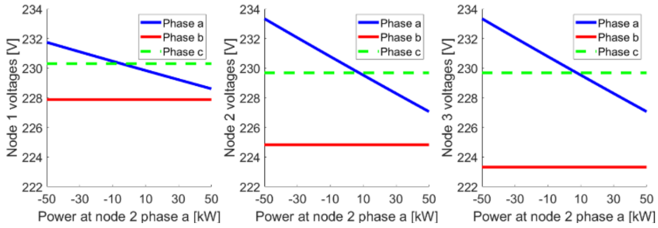


Fig. 9. Node voltages in the feeder-like structure, unbalanced.

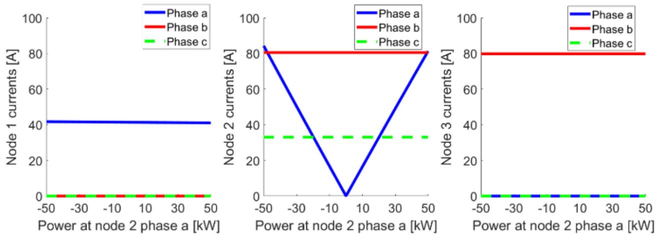


Fig. 10. Node currents in the feeder-like structure, unbalanced.

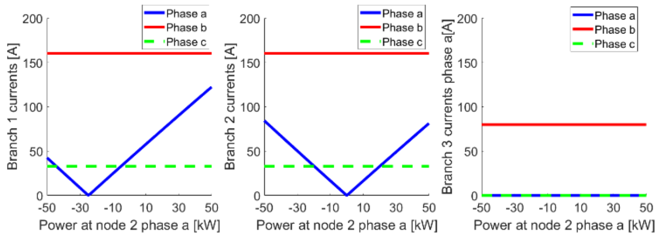


Fig. 11. Branch currents in the feeder-like structure, unbalanced.

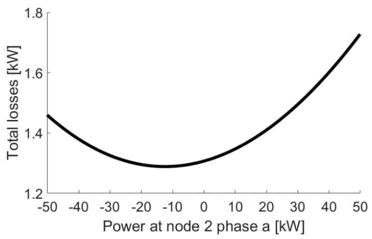


Fig. 12. Total losses in the feeder-like structure, unbalanced.

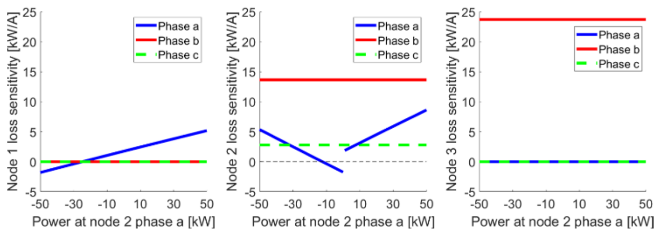


Fig. 13. Node loss sensitivities in the feeder-like structure, unbalanced.

V. Conclusions

RPF has become a phenomenon that can occur in modern distribution networks in the presence of DERs. This paper has illustrated with tutorial examples how considering the total loss sensitivities with respect to the current magnitudes of the loads is effective in detecting RPF in the grid. It has been established that there is a link between the presence of the RPF and the product between the total loss sensitivity with respect to the load at the ending node of the first branch and the slope of the curve that represents the variation of the same loss sensitivity with respect to the variation of a load parameter. The negative slope and positive values (or positive slope with negative values) of loss sensitivities indicate the need for increasing the loads to reduce the total losses. In summary, in a balanced system, when RPF occurs this product is negative. In a three-phase unbalanced system, RPF could only occur in one or two phases and again corresponds to the negative product between the loss sensitivity at a specific node and phase and the slope of the loss sensitivity curve.

These considerations lead to determining when RPF could occur through a set of power flow simulations. The results obtained may be effective for establishing the impact of the variations of DERs and loads within the system.

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