To be published in Optics Letters:

Title:	Metamaterial induced Bound State in Continuum via self-complementary Babinet principle
Authors:	Anar Ospanova,Mikhail Bukharin,dassano gianluca,Ladislau Matekovits,Alexey Basharin
Accepted:	29 March 25
Posted	01 April 25
DOI:	https://doi.org/10.1364/OL.559443

Metamaterial induced Bound State in Continuum via self-complementary Babinet principle

ANAR OSPANOVA[1], MIKHAIL BUKHARIN[1], GIANLUCA DASSANO[2], LADISLAU MATEKOVITS[2,3,4], AND ALEXEY BASHARIN[1,*]

[1] Center for Photonics Sciences, Department of Physics and Mathematics, University of Eastern Finland, Yliopistonkatu, 80140, Joensuu, Finland
[2] Department of Electronics and Telecommunications, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129, Turin, Italy
[3] Istituto di Elettronica e di Ingegneria dell'Informazione e delle Telecomunicazioni, National Research Council of Italy, 10129 Turin, Italy
[4] Department of Measurements and Optical Electronics, Politehnica University Timişoara, 300006 Timişoara, Romania.
[*] alexey.basharin@uef.fi

Compiled March 20, 2025

Achieving high-Q resonances in the photonics is crucial for applications such as sensors, filters, and emitters of electromagnetic radiation. One effective strategy for achieving such resonances involves the Bound state In the Continuum effect (BIC). In this paper, we demonstrate comprehensive study of the BIC in complementary planar Babinet metamaterials in a microwave regime. Symmetry protected BIC has been induced in planar metamaterial due to rotation of incident wave for small angles from normal incidence, thus, released the high Q-factor ($9.245 \cdot 10^3$ in theory and $1.606 \cdot 10^3$ in experiment) trapped mode at 6.625 GHz with localized components of electric dipoles and electric octopoles radiating exclusively in the plane of the metamaterial, which can be promising for planar lasers.

http://dx.doi.org/10.1364/ao.XX.XXXXXX

BIC is one of the most intriguing high-Q factor phenomena in photonics. Generally, BICs are defined as complete confinement of waves inside the continuous spectrum and coexist with radiating waves that constantly support energy leakage [1, 2]. Importantly, there is a fundamental difference between BICs and well-known resonances coupling to extended waves and leaking out[2]. Resonances, being leaky modes, can be defined as complex frequency $\omega = \omega_0 - i\gamma$, where ω_0 is the resonance frequency and γ is leakage rate that is key determining resonances. Since BICs are perfectly localized inside continuum spectrum, its leakage rate is considered to be $\gamma = 0$. Consequently, the Q-factor of the system tends to infinite values $Q = \omega_0/2\gamma$ and, on occasion, BICs are called as special case of trapped modes [3].

In general, BICs occur due to small changes of any parameter of the system and, depending on the variable value, referred to as symmetry-protected [4–10], parametric [11–14], accidentally BICs [15, 16]. Among them is the symmetry-protected BIC that occurs in high symmetry systems, especially in periodic structures – photonic crystals and metamaterials [17]. Metamaterials and metasurfaces are generally considered as specially designed arrays of subwavelength metallic and dielectric inclusions that exhibit unnatural scattering response [18, 19]. Every single inclusion, sometimes called metaatom, forms individual scattering response from local interaction with impinging wave and contributes to overall scattering response from metamaterial. Therefore, metamaterials are promising tool for wavefront engineering as well as beamshaping techniques whereas response of metaatoms is defined by their parameters [20]. Being a high-symmetry media, metamaterials are promising platform for the study of effects occurring due to small parametric changes which are also the BICs. It is noteworthy mentioning method of asymmetry introducing, for example, the defect of the system. A small fracture in the structure is considered as coupling parameter of bound state embedded in the continuous spectrum to the leakage channels of the continuous spectra that carries energy away [6]. There are plenty of studies of asymmetry introducing into periodic structures such as Fabry-Perot BICs [11, 13], separable BICs [21], BICs from inverse construction [22, 23] and others. The most notable example is symmetry protected BICs of (in-plane) asymmetry introduced by altering incidence angle of electromagnetic wave. Previously, Ref. [24] demonstrated such effect with the use of photonic crystal slab. In particular in Ref. [25], the transition from BIC to Fano resonance shown for photonics crystals. Importantly, Ref. [9] demonstrated theory allowing to investigate the BIC effect due to the change of in-plane asymmetry parameter $\alpha = sin\theta$, where θ is the angle of asymmetry. Another paramount study of BICs occurring by means of altered angle of incidence is given in Ref. [5], where authors have declared a photonic slab of arranged holes to demonstrate the high Q-factor resonances (up to 10^6) due to release of BIC mode by introducing significant in-plane asymmetry. For silicon nitrate thin film, it was demonstrated high Q-factor resonance at large tilt of angle of incidence of about 35 deg in optical range.

Such a small parameter alteration of in-plane asymmetry is a promising basis for BICs realization. Although all previous approaches have demonstrated a high Q-factor, they rely on the implementation of highly resonant excitations in meta-atoms. These are challenging to control during the fabrication process and to tune within the metamaterial spectra. On the other hand, nonresonant systems can be promising platform for BIC excitation and for investigation of the occurrence of leakage channels upon it. Recently, we proposed a Babinet metamaterial demonstrating broadband transparency in microwave[26]. The metamaterial consists of two layers of self-complementary square cells of metal and void fillings (Fig.1). The transparency effect has established due to compensation of excited multipoles of first layer by multipoles with identical intensity but with opposite phase of the second layer[26].

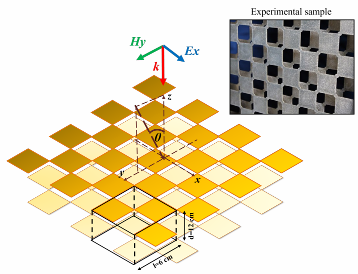

Fig. 1. Schematic illustration of Babinet metamaterial and rotation of incident wave for θ related to normal. Inset shows experimental sample.

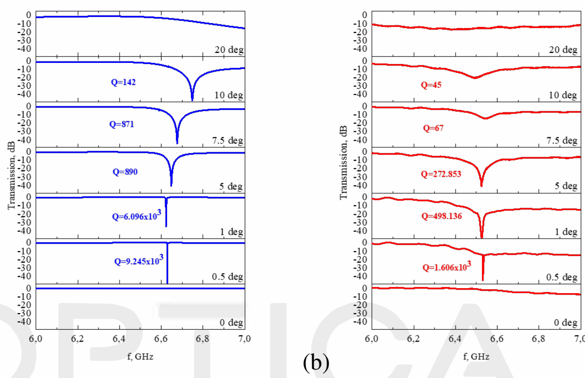

Fig. 2. Simulated (a) and measured (b) transmission characteristics of Babinet metamaterial in the dependence of rotation of incident wave angle θ. The slightest rotation θ = 0.5 deg reveals the quasi-BIC mode that degrades with the increase of θ.

Thus, the overall multipole intensity is zero, and the radiating losses are suppressed. This state closely resembles the anapole state; however, it does not originate from resonant mechanisms. As a small parameter, one can consider the tilt of angle θ of incident plane wave (Fig. 1). This tilt of angle θ acts as coupling parameter of strongly localized BIC wave with continuous spectrum of symmetric system. Generally, the smallest angle tilt θ is declared to release embedded BIC mode that couple to continuous spectrum and results in high Q-factor resonances.

In this paper, we demonstrate BIC excitation in microwave Babinet metamaterial through broad transmission spectra. We theoretically and experimentally demonstrate BIC excitation due to small changes of the incident electromagnetic wave direction from normal incidence. Furthermore we explore the nature of observed phenomenon from the perspective of multipole decomposition. For excitation of BIC in microwave regime, we consider a Babinet principle based two metallic layers metamaterial. Each metallic layer comprises metal and void squares arranged in checkerboard manner and these layers are oriented so that metallic square of first layer spatially coincides with void square of second one [26]. Here we consider the same structure of the same parameters, where unit cell length is l = 6 cm and the pattern is shown as insertion of Fig.1, the distance between layers is d = 12 cm, metal squares are supposed to be of a copper in experiment and Perfect electric conductor in simulations. The incident linearly polarized wave is normal to the surface of metamaterial. The electromagnetic properties of metamaterial are simulated by means of Maxwell equation solver CST Microwave Studio. Here periodic boundary condition was assumed. We observe a frequency range of 4.5-6.5 GHz that corresponds to simulated ultra-broadband transmission for θ=0 deg[26], Fig.2.

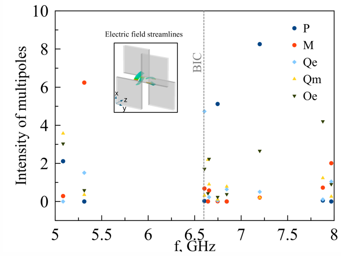

Fig. 3. Multipole decomposition for eigenmodes of single unit cell. Inset shows electric field streamlines on BIC mode at 6.55 GHz.

The ideal BIC is an Eigenmode of the structure, then we use the eigenfrequency solver of CST Microwave Studio for simulation of modes of single unit-cell. We analyze ten first modes and their multipoles decompositions up to Magnetic 16-pole [27]. The essential BIC mode is trapped mode on 6.55 GHz (Fig.3). The response of the system is defined by multipoles radiated only in xy-plane, i.e. in the plane of unit-cell (Fig.4). Q^e- electric quadrupole possesses xy-component and O_e octupole possesses zzz-component. Such multipole decomposition does not comprise radiating components in z-direction. Beside this frequency, the multipole decomposition consist of multipoles radiating in z-direction as well (inset on Fig.3). BIC state is confirmed by electric steam-lines distribution of electric field. The electric field is almost localized between layers of metamolecules. We expect that the metasurface organized by such type of unit-cells can support quasi-BIC excitation. Due to components of multipoles, which are radiated only in x, y plane of unit-cell, the excitation of BIC mode is impossible by normal incident wave.

As for metamaterial, according to symmetry protected BICs approach, the smallest change of any parameter in this kind high-symmetry structure (Fig.1) could bring to excitation of leakage channels in the continuous spectrum. If we consider the angle of incidence θ as a parameter of asymmetry, we anticipate the slightest tilt of angle from normal incidence to induce a redistribution of energy within the system resulting in the emergence of a narrow high-Q quasi-BIC with Q=$9.245 \cdot 10^3$ at θ=0.5 deg (Fig.2a). We rotate metamaterial around electric field vector (Fig.1). Indeed, absolute transmission observed at normal incidence (θ=0 deg) challenges very narrow high efficient resonance at θ=0.5 deg obviously corresponding to quasi-BIC manifestation (Fig.2). The further increase of θ=1 deg and up to θ=20 deg leads to damage of BIC state and broadening of resonance. Moreover, in Fig.2a, we demonstrate the results of experimental metamaterial transmission spectra at the same angles of incidence presented by red lines as in simulations by blue lines. The simulated and measured results are in good agreement (Fig.2). For experimental study of metamaterial in microwave range, firstly, experimental sample of given parameters has been fabricated (Fig.1, inset). Checkerboard patterned two copper layers were fabricated by means of laser cutting facility XTOOL MXF-K001-LG4, which offers tolerances up to 0.1 mm. To improve the connection between individual squares of the structure, radius 2 mm disk-shaped connectors were used.

For measurement procedure, we use an anechoic chamber equipped with ECCOSORB absorbers. For measurement of transmission properties two-horn antenna method has been utilized. For this reason, the chamber has been equipped with two broadband horn antennas. They are positioned at 1.5 meter distance on both side from experimental sample. The transmission coefficient S21 of the electromagnetic waves through the metamaterial sample was measured by a Vector network analyzer Rohde Schwarz SVB20 at frequencies 2–8 GHz, Fig.2b. In simulation, we used infinite boundary conditions. However, in ex-

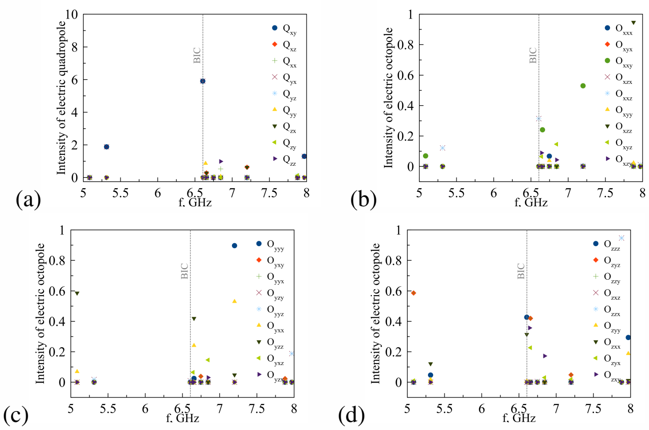

Fig. 4. Multipole decomposition of eigenmodes of single unit cell. (a) - intensities of components of the electric quadrupole, (b) - intensities of x-components of electric octupole, (c)- intensities of y-components of electric octupole, (d) - intensities of z-components of electric octupole

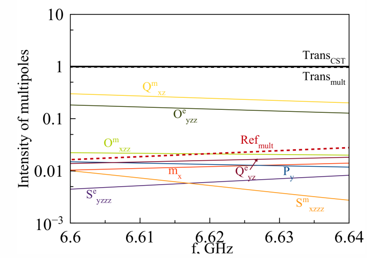

Fig. 5. Simulated intensities of multipoles in the vicinity of 6.62 GHz at normal incidence $\theta = 0$. Dashed black and red curves stands for transmission and reflection coefficients.

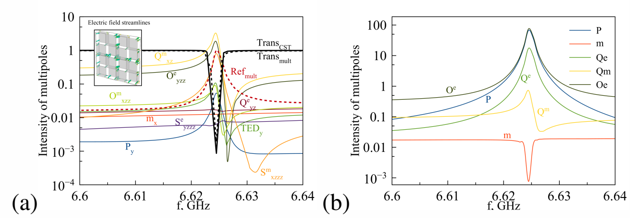

Fig. 6. (a) Simulated intensities of multipoles in the vicinity of 6.62 GHz at incidence $\theta = 0.5$ deg radiating in $-z$-direction. Dashed black and red curves stands for transmission and reflection coefficients. (b) Multipole decomposition of all components (x, y, z) in the vicinity of 6.62 GHz at incidence $\theta = 0.5$ deg

periment the metamaterial has array of 20x20 unit-cells as well as disk-shaped connectors of the radius 2 mm were used. The Q-factor is maximized in the case of an infinite metamaterial but decreases with smaller array of unit cells due to the excitation of surface waves reflected from the metamaterial borders[28, 29]. A second reason is scattering from the anechoic chamber's equipment, such as pylons and antenna edges. We normalized the metamaterial transmission spectra against free-space spectra to eliminate significant artifacts in the anechoic chamber.

In order to explain the origin of BIC effect in metamaterial sample we use method of multipole decomposition. Firstly, we analyze the excited multipoles in metamaterial inclusions by normal incident plane wave, $\theta = 0$ deg, Fig.5. The response of the system is defined by suppressing all excited multipoles in the system, up to magnetic 16-pole, $S^{(\mathrm{m})}$, Fig.5. We analyze here only multipoles, which contribute to metamaterial response only in z-direction and form the reflection r and transmission t coefficients excluded from multipoles[27]. If the system is illuminated by a normal incidence x-axis polarized plane wave propagating along the z-axis, then due to translational symmetry, all unit-cells in the metamaterial have the same multipole moments and the transmission $\mathrm{Trans}_{mult} = |t|^2$ and reflection $\mathrm{Ref}_{mult} = |r|^2$ coefficients can be approximated as Eq.1[27]:

$$
\begin{aligned}
r = & \frac{ik_s}{2S_L E_0 \varepsilon_0 \varepsilon_s}\left(P_x - \frac{1}{v_s} m_y + \frac{ik_s}{6} Q_{xz} - \frac{ik_s}{2v_s} M_{yz}\right. \\
& \left. - \frac{k_s^2}{6} O^{(\mathrm{e})}_{xzz} + \frac{k_s^2}{6v_s} O^{(\mathrm{m})}_{yzz} - \frac{ik_s^3}{24} S^{(\mathrm{e})}_{xzzz} + \frac{ik_s^3}{24v_s} S^{(\mathrm{m})}_{yzzz}\right), \\
t = 1 + & \frac{ik_s}{2S_L E_0 \varepsilon_0 \varepsilon_s}\left(P_x + \frac{1}{v_s} m_y - \frac{ik_s}{6} Q_{xz} - \frac{ik_s}{2v_s} M_{yz}\right. \\
& \left. - \frac{k_s^2}{6} O^{(\mathrm{e})}_{xzz} - \frac{k_s^2}{6v_s} O^{(\mathrm{m})}_{yzz} + \frac{ik_s^3}{24} S^{(\mathrm{e})}_{xzzz} + \frac{ik_s^3}{24v_s} S^{(\mathrm{m})}_{yzzz}\right).
\end{aligned}
\tag{1}
$$

The multipoles definitions are presented in Supplimentary material. The metamaterial is almost transparent in the frequency range 6.6-6.64 GHz. In some sense, such a Babinet metamaterial supports a compound anapole state, which arises due to destructive interference between the multipoles excited in each layer[26, 30, 31]. We should note, that the anapole state is a perspective platform for BIC excitation. Indeed, one can destroy the anapole state, as in our case by introduction of nonzero incident angle, $\theta = 0.5$ deg. In this case, we provide a leakage channel for excitation of BIC eigenstates of unit-cell. We can still use the equations for the metasurface, Eq.1, because the angle of incident $\theta = 0.5$ deg is very small. At the frequency f=6.625 GHz we observe very narrow resonance with Q-factor $9.245 \cdot 10^3$ in theory and $1.606 \cdot 10^3$ in experiment, Fig. 2. The transmission and reflection spectra of the metamaterial are determined by its magnetic quadrupole Q^m_{xz} and electric octopole O^e_{yzz} components on this resonance frequency, Fig.6a. Moreover, we reconstructed a transmission coefficient based on the multipoles and compared it with the CST-simulated transmission coefficient for both normal incidence and a 0.5 deg incidence angle. We observe excellent agreement between $Trans_{CST}$ and $Trans_{mult}$, indicating that the multipole decomposition was chosen correctly. These multipoles have components radiating in $-z, +z$ directions. However, the total multipoles (Fig.6b), with taken into account multipoles radiated in all directions is determed mainly by electric dipole moment P_z, electric quadropole Q^e_{xy} and electric octopole O^e_{zzz}, Fig.7. These components radiate only in xy- plane of the metamaterial, the streamlines of electric fields are distributed between layers and alongated along z-direction, inset on Fig.6b. This trapped mode is similar to eigenstate of single unit-cell, Fig.3. However, the multipole decomposition is different. Due to arrangement of unit-cells in array, the electric qudropole is supressed and main response of trapped mode is defined by electric dipole P_z, electric octopole O^e_{zzz} multipoles in the case of array. We note that the z components of the electric dipole and electric octopole do not radiate in the z direction; consequently, they are not observed in the spectrum of the metamaterial, Fig.6a. Thus, the trapped mode at 6.625 GHz is quasi-BIC mode corresponds to eigenmode for single unit cell due to similar set of multipoles and distribution of electric field streamlines. The curiosity is that the incident wave is propagating along the z-axis, while the electric component of the wave can be oriented along the x- or y-directions, hence, external plane wave could not have excited z-component of the electric field. Based on this, we can conclude that the resulting z components of electric dipole and electric octopoles is the quasi-BIC phenomenon embedded in the transmission

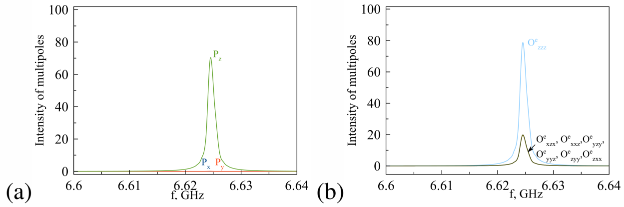

Fig. 7. (a) Components of electric dipole in the vicinity of BIC resonance. (b) Decomposition of electric octupole in the vicinity of BIC resonance.

spectrum excited due to their coupling with magnetic quadropole Q^m_{xz} and electric octopole O^e_{yzz}, (Fig.6a). The observed effect aligns with the "trapped mode" paradigm in photonics. It is defined as a localized electromagnetic state where electromagnetic wave is effectively "confined" in a specific region of the structure, while coexisting with a leakage channels. In the fields of metamaterials and nanophotonics, trapped modes are often achieved by breaking symmetry in resonators or periodic structures[3]. This results in the suppression of radiative losses and the emergence of high-quality resonances. In general sense, BIC is the special case of trapped mode.

In some sense, the excitation an ideal BIC is the requirement to selectively excite a single mode, known as the "trapped mode", while simultaneously suppressing all other modes in the system, essentially suppressed them by anapoles. Thus, we presented here a Babinet system in which both anapole and trapped mode must be excited simultaneously due to the inherent asymmetry of the metamaterial. The high-Q-factor of BIC metamaterial shows great promise for laser applications. The general strategy for realizing such lasers is to effectively confine light and enhance light–matter interaction by embedding gain materials into high-Q-factor cavities. The BIC mode involves fields that are largely confined to the metamaterial surface and concentrated between its layers. An active region can be introduced between these layers—for instance, by using quantum dots in optical systems or non-Foster elements (operational amplifiers) in microwave systems. Because the BIC metamaterial exhibits an effectively infinite Q-factor, the incident wave is excited and amplified within the metamaterial.

In this paper, we introduce the concept of bound states in the continuum resonances in complementary metamaterials at microwave regime. We note, that the metamaterial can be easily scaled to higher THz and optical frequencies. In particular, chemical etching or sputtering methods can be used for fabrication of metamaterials. To avoid using lossy dielectric substrates, bridging connectors between the free-standing metamaterial layers should be utilized. While metal losses (especially at optical frequencies) must be taken into account, the fields in the BIC mode is largely confined between the layers, consequently these losses are not expected to play a crucial role. We achieved a high-quality resonance facilitated by the BIC concept, which allows the generation of resonances through asymmetries. The planarity of the proposed free-standing metamaterials is noteworthy; this approach does not necessitate the development of intricate three-dimensional structures typically required for high-Q resonances. Moreover, The scalability of the solution is a considerable advantage. We demonstrated a BIC resonance with a Q-factor of approximately $9.245 \cdot 10^3$ in theory and $1.606 \cdot 10^3$ in experiment, and we anticipate that scaling this solution to higher frequency ranges will prove beneficial in various applications from microwave to optics.

Funding. Research Council of Finland Flagship Programme, Photonics Research and Innovation (PREIN), decision number 346518.

Acknowledgment. We thank Grigorii Matveev for his help with the simulations.

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

REFERENCES

1. V. I. Kukulin, V. Krasnopolsky, and J. Horácek, *Theory of resonances: Principles and Applications*, vol. 3 (Springer Science & Business Media, 2013).
2. C. W. Hsu, B. Zhen, A. D. Stone, *et al.*, Nat. Rev. Mater. **1**, 1 (2016).
3. V. Fedotov, M. Rose, S. Prosvirnin, *et al.*, Phys. review letters **99**, 147401 (2007).
4. J. Lee, B. Zhen, S.-L. Chua, *et al.*, Phys. review letters **109**, 067401 (2012).
5. C. W. Hsu, B. Zhen, J. Lee, *et al.*, Nature **499**, 188 (2013).
6. S. Campione, S. Liu, L. I. Basilio, *et al.*, Acs Photonics **3**, 2362 (2016).
7. K. Koshelev, A. Bogdanov, and Y. Kivshar, Sci. Bull. **64**, 836 (2019).
8. D. Marinica, A. Borisov, and S. Shabanov, Phys. review letters **100**, 183902 (2008).
9. K. Koshelev, S. Lepeshov, M. Liu, *et al.*, Phys. review letters **121**, 193903 (2018).
10. J. Jin, X. Yin, L. Ni, *et al.*, Nature **574**, 501 (2019).
11. W. Suh, M. Yanik, O. Solgaard, and S. Fan, Appl. physics letters **82**, 1999 (2003).
12. A. F. Sadreev, D. N. Maksimov, and A. S. Pilipchuk, J. Physics: Condens. Matter **27**, 295303 (2015).
13. V. Liu, M. Povinelli, and S. Fan, Opt. express **17**, 21897 (2009).
14. M. V. Rybin, K. L. Koshelev, Z. F. Sadrieva, *et al.*, Phys. review letters **119**, 243901 (2017).
15. Y. Plotnik, O. Peleg, F. Dreisow, *et al.*, Phys. review letters **107**, 183901 (2011).
16. S. Weimann, Y. Xu, R. Keil, *et al.*, Phys. review letters **111**, 240403 (2013).
17. J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, Princet. Univ. Press. Princeton, NJ [ua] (2008).
18. D. R. Smith, J. B. Pendry, and M. C. Wiltshire, Science **305**, 788 (2004).
19. A. Lagarkov and V. Kissel, Phys. Rev. Lett. **92**, 077401 (2004).
20. X. You, C. Fumeaux, and W. Withayachumnankul, J. Appl. Phys. **131**, 061101 (2022).
21. M. Robnik, J. Phys. A: Math. Gen. **19**, 3845 (1986).
22. M. I. Molina, A. E. Miroshnichenko, and Y. S. Kivshar, Phys. review letters **108**, 070401 (2012).
23. A. Jain and C. Shastry, Phys. Rev. A **12**, 2237 (1975).
24. B. Zhen, C. W. Hsu, L. Lu, *et al.*, Phys. review letters **113**, 257401 (2014).
25. L. Pilozzi, M. Missori, and C. Conti, Opt. letters **48**, 2381 (2023).
26. A. Ospanova, M. Cojocari, P. Lamberti, *et al.*, Appl. Phys. Lett. **122** (2023).
27. I. Allayarov, A. B. Evlyukhin, and A. Calà Lesina, Opt. Express **32**, 5641 (2024).
28. M. V. Cojocari, A. K. Ospanova, V. I. Chichkov, *et al.*, Phys. Rev. B **104**, 075408 (2021).
29. V. A. Fedotov, N. Papasimakis, E. Plum, *et al.*, Phys. review letters **104**, 223901 (2010).
30. A. Ospanova, M. Cojocari, and A. Basharin, Phys. Rev. B **107**, 035156 (2023).
31. A. A. Basharin, E. Zanganeh, A. K. Ospanova, *et al.*, Phys. Rev. B **107**, 155104 (2023).

FULL REFERENCES

1. V. I. Kukulin, V. Krasnopolsky, and J. Horácek, *Theory of resonances: Principles and Applications*, vol. 3 (Springer Science & Business Media, 2013).
2. C. W. Hsu, B. Zhen, A. D. Stone, *et al.*, "Bound states in the continuum," Nat. Rev. Mater. **1**, 1–13 (2016).
3. V. Fedotov, M. Rose, S. Prosvirnin, *et al.*, "Sharp trapped-mode resonances in planar metamaterials with a broken structural symmetry," Phys. review letters **99**, 147401 (2007).
4. J. Lee, B. Zhen, S.-L. Chua, *et al.*, "Observation and differentiation of unique high-q optical resonances near zero wave vector in macroscopic photonic crystal slabs," Phys. review letters **109**, 067401 (2012).
5. C. W. Hsu, B. Zhen, J. Lee, *et al.*, "Observation of trapped light within the radiation continuum," Nature **499**, 188–191 (2013).
6. S. Campione, S. Liu, L. I. Basilio, *et al.*, "Broken symmetry dielectric resonators for high quality factor fano metasurfaces," Acs Photonics **3**, 2362–2367 (2016).
7. K. Koshelev, A. Bogdanov, and Y. Kivshar, "Meta-optics and bound states in the continuum," Sci. Bull. **64**, 836–842 (2019).
8. D. Marinica, A. Borisov, and S. Shabanov, "Bound states in the continuum in photonics," Phys. review letters **100**, 183902 (2008).
9. K. Koshelev, S. Lepeshov, M. Liu, *et al.*, "Asymmetric metasurfaces with high-q resonances governed by bound states in the continuum," Phys. review letters **121**, 193903 (2018).
10. J. Jin, X. Yin, L. Ni, *et al.*, "Topologically enabled ultrahigh-q guided resonances robust to out-of-plane scattering," Nature **574**, 501–504 (2019).
11. W. Suh, M. Yanik, O. Solgaard, and S. Fan, "Displacement-sensitive photonic crystal structures based on guided resonance in photonic crystal slabs," Appl. physics letters **82**, 1999–2001 (2003).
12. A. F. Sadreev, D. N. Maksimov, and A. S. Pilipchuk, "Gate controlled resonant widths in double-bend waveguides: bound states in the continuum," J. Physics: Condens. Matter **27**, 295303 (2015).
13. V. Liu, M. Povinelli, and S. Fan, "Resonance-enhanced optical forces between coupled photonic crystal slabs," Opt. express **17**, 21897–21909 (2009).
14. M. V. Rybin, K. L. Koshelev, Z. F. Sadrieva, *et al.*, "High-q supercavity modes in subwavelength dielectric resonators," Phys. review letters **119**, 243901 (2017).
15. Y. Plotnik, O. Peleg, F. Dreisow, *et al.*, "Experimental observation of optical bound states in the continuum," Phys. review letters **107**, 183901 (2011).
16. S. Weimann, Y. Xu, R. Keil, *et al.*, "Compact surface fano states embedded in the continuum of waveguide arrays," Phys. review letters **111**, 240403 (2013).
17. J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, "Molding the flow of light," Princet. Univ. Press. Princeton, NJ [ua] (2008).
18. D. R. Smith, J. B. Pendry, and M. C. Wiltshire, "Metamaterials and negative refractive index," Science **305**, 788–792 (2004).
19. A. Lagarkov and V. Kissel, "Near-perfect imaging in a focusing system based on a left-handed-material plate," Phys. Rev. Lett. **92**, 077401 (2004).
20. X. You, C. Fumeaux, and W. Withayachumnankul, "Tutorial on broadband transmissive metasurfaces for wavefront and polarization control of terahertz waves," J. Appl. Phys. **131**, 061101 (2022).
21. M. Robnik, "A simple separable hamiltonian having bound states in the continuum," J. Phys. A: Math. Gen. **19**, 3845 (1986).
22. M. I. Molina, A. E. Miroshnichenko, and Y. S. Kivshar, "Surface bound states in the continuum," Phys. review letters **108**, 070401 (2012).
23. A. Jain and C. Shastry, "Bound states in the continuum for separable nonlocal potentials," Phys. Rev. A **12**, 2237 (1975).
24. B. Zhen, C. W. Hsu, L. Lu, *et al.*, "Topological nature of optical bound states in the continuum," Phys. review letters **113**, 257401 (2014).
25. L. Pilozzi, M. Missori, and C. Conti, "Observation of terahertz transition from fano resonances to bound states in the continuum," Opt. letters **48**, 2381–2384 (2023).
26. A. Ospanova, M. Cojocari, P. Lamberti, *et al.*, "Broadband transparency of babinet complementary metamaterials," Appl. Phys. Lett. **122** (2023).
27. I. Allayarov, A. B. Evlyukhin, and A. Calà Lesina, "Multiresonant all-dielectric metasurfaces based on high-order multipole coupling in the visible," Opt. Express **32**, 5641–5658 (2024).
28. M. V. Cojocari, A. K. Ospanova, V. I. Chichkov, *et al.*, "Pseudo-anapole regime in terahertz metasurfaces," Phys. Rev. B **104**, 075408 (2021).
29. V. A. Fedotov, N. Papasimakis, E. Plum, *et al.*, "Spectral collapse in ensembles of metamolecules," Phys. review letters **104**, 223901 (2010).
30. A. Ospanova, M. Cojocari, and A. Basharin, "Modified multipoles in photonics," Phys. Rev. B **107**, 035156 (2023).
31. A. A. Basharin, E. Zanganeh, A. K. Ospanova, *et al.*, "Selective super-invisibility effect via compound anapole," Phys. Rev. B **107**, 155104 (2023).