Bifurcation Analysis of a Nonlinear Vehicle Model on Banked Road

Lecture Notes in Mechanical Engineering

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Bifurcation Analysis of a Nonlinear Vehicle Model on Banked Road

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Abstract. Towards the transition to automated driving, lateral stability of the vehicle represents a key requirement to guarantee the safety of passengers and vulnerable road users, especially during emergency operating conditions where nonlinearities arise. The present paper aims at investigating the effect of road banking angle on vehicle plane motion stability. To perform this analysis, a pure lateral nonlinear double track model is numerically derived for an oversteering vehicle. Lateral load transfer and its distribution among the axles are included for exploiting the tyre saturation region. The stability analysis is conducted by searching for the vehicle steady-state conditions and deriving the linearised equations around the equilibrium points. Moreover, the phase-plane plot is adopted to draw the states trajectories and to identify potential unstable regions. Finally, the bifurcation analysis as function of the road banking angle is investigated to highlight possible change of the phase portrait topology. The results show that a saddle-node bifurcation may occur when the vehicle is negotiating a certain level of bank road angle, affecting the vehicle yaw stability region.

Keywords: Stability analysis · nonlinear vehicle dynamics · bifurcation · phase-plane analysis

1 Introduction

Road safety represents a primarily concern when dealing with the standard and automated driving. Reducing dangerous situations for passengers and vulnerable road users is strictly related to understanding the vehicle limits and its stability, especially when the vehicle is engaged in an emergency manoeuvre. The analysis of vehicle stability under nonlinear operating conditions is a well-established topic: many authors have investigated the limit conditions due to different inputs (such as the vehicle speed, the steering wheel angle, the road friction) or vehicle parameters (such as the influence of the position of the centre of mass or the front/rear roll stiffness distribution) [1–3]. As shown in these works, the main cause of vehicle instability lies in the saturation of the rear axle cornering characteristics, which is a typical behaviour of oversteering vehicles. This condition leads to the presence of a couple of unstable equilibrium points, which can be clearly highlighted on a phase-plane plot. Most previous articles focus

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their analysis on the vehicle alone, adopting the single- or double-track models, while the authors of [4, 5] extend the analysis including driver dynamics in the set of equations of motion governing vehicle dynamics. In particular, the authors demonstrated that, for an understeering vehicle, the coupling of vehicle and driver dynamics can result in a stable and unstable limit cycle above a critical speed. The attempt to extend the classic two-states phase-plane approach is performed by the authors of [6] where they proposed an enhanced three-state model which accounts for the longitudinal vehicle motion and the effect of the tyre slip combination on the system stability. The phase-plane plot is an effective tool, commonly used to investigates nonlinear systems, which allows possible unstable regions to be described through the phase-portrait. This approach makes it possible to determine the stability boundary of the vehicle, which can be proficiently used to set constrains for control purposes, as done by the authors of [7] where phase-plane analysis is used to define the boundary of stability, in term of sideslip angle and sideslip velocity. Moreover, different phase-plane types have been used throughout the literature, as well-explained by the survey on lateral stability criterion presented in [8], where a classification of the different phase-plane types is presented and qualitatively evaluated in terms of measurability, relevance, sensibility, and controllability.

Nevertheless, the main literature studies are limited to bifurcation analysis on flat and horizontal road surfaces, whereas the effect of the banking of the road on stability is not deeply investigated. Thus, this paper aims at exploring the effects of the road banking angle in the nonlinear operative conditions, searching for possible stability limits and bifurcations through a classic phase-plane analysis on the yaw rate and sideslip angle plane. The paper is divided as follows. In Sect. 2 the vehicle model and the main hypothesis are presented. Moreover, the methodology to determine the presence of fixed points is discussed. Section 3 depicts the phase-portrait starting from the a grid of initial conditions to the vehicle subject to different levels of bank angle. Finally, some conclusion of bifurcation analysis are derived in Sect. 4.

2 Model and Test Workflow

2.1 Vehicle Model

In this paper, a nonlinear vehicle model negotiating banked turns is under investigation. The model is a double-track vehicle (see Fig. 1) which is characterised by two degrees of freedom (lateral and yaw motion). The following assumption are made:

- The vehicle body is assumed to be rigidly connected to the axles, thus roll and pitch motion are neglected.
- Vehicle velocity V is considered constant and the effect of tyre longitudinal forces on vehicle dynamics is neglected.
- Lateral load transfers, i.e. the variation of the vertical tyre forces while cornering, are modelled considering a constant distribution factor $k_{F/R}$ between the front and rear axle is assumed equal to 40:60.
- The wheels are always in contact with the ground; thus the rollover dynamics is neglected.

- The effect of road banking is modelled as a constant component of the weight force acting along the vehicle y-axis direction and applied to the centre of gravity. At the same time, the component of the weight force in the direction perpendicular to the road plane is reduced.
- The driver dynamics is not included in the set of equation of motion thus, the analysis is conducted at fixed steering angle.
- Tyre-contact forces are computed using the semi-empirical steady-state Pacejka Tyre Magic Formula [9]

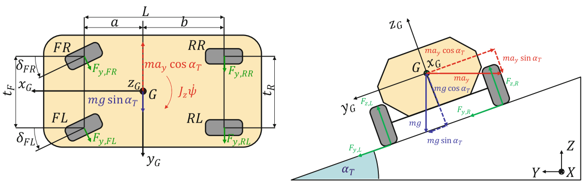

Fig. 1. Double-track nonlinear vehicle model scheme: on the left $x_G y_G$ plane, on the right YZ plane.

2.2 Equilibrium Points Search and Discussion

This section presents the method used to search for and discuss the presence of single or multiple steady-state conditions and the bifurcation analysis. From the theory of the nonlinear dynamics, a generic nonlinear system may exhibit more than one equilibrium point (or fixed point). In the specific case of vehicle plane motion (without considering the driver dynamics), the presence of more than one fixed point deals with oversteering vehicles [3].

Figure 2 synthetizes the adopted workflow to find and to discuss the model steady-state conditions. The procedure has been entirely developed in MATLAB/Simulink environment by exploiting the FSOLVE function, which has the objective of searching for the stationary solutions. A nonlinear vehicle Simulink model is set with the initial condition guess point $x_{0,start}$ and the input parameters (vehicle velocity V, steering wheel angle δ_{sw}, friction coefficient μ and bank angle α_T). The tyre lateral forces are gathered from the simulation results to a set of nonlinear equation of motion checking the presence of an equilibrium condition on lateral and yaw motion. If a solution $\boldsymbol{x}_0 = [\beta_0, r_0]$ is found, the equations of motion are linearised around that equilibrium point, and, to assess stability in its neighbourhood, the trace and the determinant of the Jacobian matrix are computed. This procedure allows to classify the nature of the fixed point and to determine its stability.

The bifurcation analysis on road banking angle is performed by repeating the workflow on the following set of banking angles: $[-45°, 45°]$.

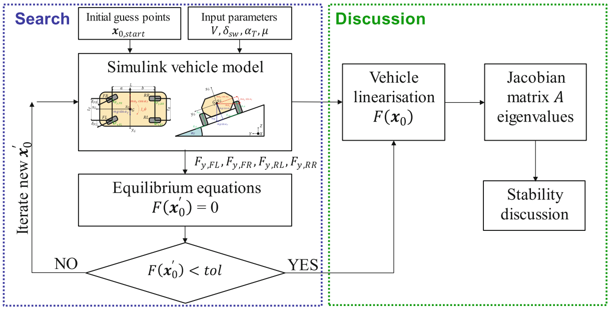

Fig. 2. Adopted workflow for the steady-state solutions and stability discussions.

3 Phase Plane Numerical Results

In this section the results from the bifurcation analysis and the numerical integration of the equation of motion of the model are presented. In this analysis an oversteering vehicle model is tested in all its operative working range, using the phase-plane ($\beta - r$) diagram. The states are represented by the vehicle sideslip angle β and the yaw rate r. The vehicle geometrical and inertial properties are listed in Table 1.

Table 1. Vehicle data

Parameter	*Value*
Mass [kg]	1600
Yaw moment of inertia [kgm^2]	2860
Wheelbase [m]	2.6
Front axle distance to CoG [m]	1.56
Rear axle distance to CoG [m]	1.04
Front and rear trackwidth [m]	1.54
Stiffness distribution factor $k_{F/R}$ [-]	40:60

The model is numerically integrated to draw the phase-portrait at a set velocity V, steering wheel angle δ_{sw} and banking angle α_T. The proposed analysis investigates the steady-state solution when the vehicle is running on a banked road at 100 km/h with steering wheel angle null. The vehicle model is integrated by setting a grid of initial conditions (β_0, r_0) with a span of 5°/s for the yaw rate and 5° for the sideslip angle respectively. In the following results the vertical load acting on each wheel is always higher than zero, thus rollover issues are not triggered.

Figure 3 depicts the phase-portrait obtained from the numerical simulation with three different levels of banking (0°, 20°, 41°). The blue lines represent the states trajectories while the red circle, diamond and star are the three steady-states solutions obtained with the explained workflow. The circle is a stable focus, while diamond and star are two saddle-points. It is worth noting that by increasing the road banking angle, the unstable node approaches the stable one: after 40° of lateral slope a saddle-node bifurcation occurs: beyond this value no more stable region is available.

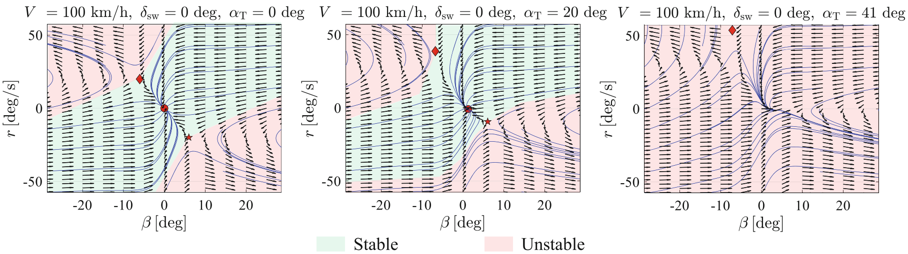

Fig. 3. Phase-portrait on $\beta - r$ plane of the nonlinear vehicle model running at 100 km/h with null steering wheel angle and three different level of banking angle (0°, 20°, 41°).

Figure 4 depicts the states trajectories obtained for a value of α_T equal to 20° in combination with a third axis represented by the Load Transfer Ratio (*LTR*), which is a quantity used to detect and predict the rollover risk [10]. *LTR* is defined as the relative vertical force on tyres between the right side ($F_{z,R}$) and left side ($F_{z,L}$) of a vehicle (Eq. 1):

$$LTR = \frac{F_{z,R} - F_{z,L}}{F_{z,L} + F_{z,R}} \tag{1}$$

Values lower than one indicates a normal force greater than zero for all the wheels. In this condition, tyres are always in contact with the ground.

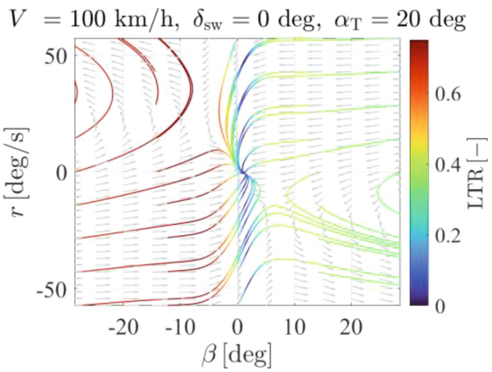

Fig. 4. Phase-plane using a third axis represented by the LTR values.

The numerical results show the presence of a clear bifurcation at a certain lateral slope value α_T.

It should be noted that the angle of banking that corresponds to the bifurcation is well above the common values encountered when a vehicle is travelling on normal roads.

However, looking at the distance between the stable equilibrium point (circle in Fig. 3) and the point closest to the stability boundary (star) in the phase plane graph, it can be seen that the progressive increase in transverse road inclination reduces the stability margin of the vehicle.

4 Conclusion

The conducted analysis of the vehicle stability in its nonlinear working points is crucial to define possible avoidable conditions, such as unstable steady state turning conditions. Albeit the bifurcation due to the steering angle and velocity is well established in the literature the analysis on lateral slope is not well investigated. To this aim, a nonlinear double track model has been developed and tested in different banked roads. The developed methodology revealed to be well suited to find the steady-state equilibrium conditions and, after the linearisation process, their stability discussion. A saddle-node bifurcation appears at certain level of lateral slope angle, since stable and unstable manifolds collides into a saddle-point. Further improvements will be devoted to the inclusion in the set of the equation the driver dynamics, roll dynamics and the wheel detachment dynamics.

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