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# Data-driven Approximation of Linear Switched Systems

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This contribution addresses the problem of learning dynamical Linear Switched System (LSS) models from input/output observations [1]. The method is illustrated on systems switching between two modes, according to the value of an exogenous signal  $p(t) : \mathbb{R}_+ \rightarrow \{0, 1\}$ . The LSS  $G$  maps its input  $u(t)$  to the output  $y(t)$ , i.e.  $y(t) = G[u(t), p(t)]$ , see [1] for a rigorous definition.

First, we observe that  $G$  can be viewed as a family of linear time-varying (LTV) systems, each corresponding to a fixed switching trajectory. In particular, restricting  $p(t)$  to the set of square-wave signals  $p_{\omega_0}(t)$  of frequency  $\omega_0$ , the collection of periodic LTV systems  $G_{\omega_0}$  (indexed by  $\omega_0$ ) is defined by  $y_{\omega_0}(t) = G_{\omega_0}[u(t)] \triangleq G[u(t), p_{\omega_0}(t)]$ . According to Zadeh's theory [3],  $G_{\omega_0}$  is represented by an  $\omega_0$ -periodic transfer function  $H_{\omega_0}(j\omega, t)$ , that admits a complex Fourier expansion with coefficients  $H_{\omega_0}^{(n)}(j\omega)$ ,  $n \in \mathbb{Z}$ . Isolating  $n = 1$ , we define the bivariate function  $F(j\omega, j\omega_0) \triangleq H_{\omega_0}^{(1)}(j\omega)$ , that is a purely I/O representation, measurable by experiment or simulation in periodic steady-state conditions. Hence, the training dataset is a collection of evaluations  $F(j\omega^{(k)}, j\omega_0^{(h)})$ .

With this premise, we look for a model  $\tilde{G}$  whose associated bi-variate function  $\tilde{F}(j\omega, j\omega_0)$  approximates  $F$  in a least-squares sense. To this aim, we adopt a Wiener-like model structure [2] for  $\tilde{G}$ , whose output is  $\tilde{y}(t) = \tilde{G}[u(t), p(t)] = \sum_{i=1}^{\tilde{i}} \phi_i[p](t) \cdot \psi_i[u](t)$ , where  $\phi_i[\cdot]$  and  $\psi_i[\cdot]$  are LTI systems. Its  $\tilde{F}$ -function can be written in pole-residue form as

$$\tilde{F}(j\omega, j\omega_0) = (j\pi)^{-1} \sum_{i,j} r_{ij} (j\omega - \alpha_i)^{-1} (j\omega_0 - \beta_j)^{-1}. \quad (1)$$

We highlight that in the selected model structure, the components  $\phi_i[p](t), \psi_i[u](t)$  are the outputs of scalar LTI systems. Model fitting, i.e. optimization of poles  $\alpha_i, \beta_j$  and residues  $r_{ij}$ , can be performed using a suitable adaptation of a multivariate rational fitting algorithm. In our experiments, we used the Vector Fitting (VF) algorithm in two steps. First, we view  $\omega_0$  as a parameter and run VF to find a set of basis poles  $\alpha_i$  to approximate the frequency dependence w.r.t.  $\omega$  for all sampled values  $\omega_0^{(h)}$  collectively. Then, a second run of VF, with fixed  $\alpha_i$ , gives poles  $\beta_j$  and residues  $r_{ij}$ . Finally,  $\tilde{G}$  results from assigning  $\phi_i(s) = (s - \alpha_i)^{-1}$ ,  $\psi_i(s) = \sum_j r_{ij} (s - \beta_j)^{-1}$ . The input/output stability of the proposed model structure is guaranteed by enforcing strictly negative real part of the estimated poles  $\alpha_i, \beta_i$  using standard techniques. The proposed approach is demonstrated using several benchmark examples of practical interest, including a Buck voltage regulator commonly used to stabilize the microprocessor power supply in electronic systems.

## References

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