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Data-driven Approximation of Linear Switched Systems

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This contribution addresses the problem of learning dynamical Linear Switched System (LSS) models from input/output observations [1]. The method is illustrated on systems switching between two modes, according to the value of an exogenous signal $p(t) : \mathbb{R}_+ \to \{0, 1\}$. The LSS G maps its input $u(t)$ to the output $y(t)$, i.e. $y(t) = G[u(t), p(t)]$, see [1] for a rigorous definition.

First, we observe that G can be viewed as a family of linear time-varying (LTV) systems, each corresponding to a fixed switching trajectory. In particular, restricting $p(t)$ to the set of square-wave signals $p_{\omega_0}(t)$ of frequency ω_0, the collection of periodic LTV systems G_{ω_0} (indexed by ω_0) is defined by $y_{\omega_0}(t) = G_{\omega_0}[u(t)] \triangleq G[u(t), p_{\omega_0}(t)]$. According to Zadeh's theory [3], G_{ω_0} is represented by an ω_0-periodic transfer function $H_{\omega_0}(\mathrm{j}\omega, t)$, that admits a complex Fourier expansion with coefficients $H^{(n)}_{\omega_0}(\mathrm{j}\omega)$, $n \in \mathbb{Z}$. Isolating $n = 1$, we define the bivariate function $F(\mathrm{j}\omega, \mathrm{j}\omega_0) \triangleq H^{(1)}_{\omega_0}(\mathrm{j}\omega)$, that is a purely I/O representation, measurable by experiment or simulation in periodic steady-state conditions. Hence, the training dataset is a collection of evaluations $F(\mathrm{j}\omega^{(k)}, \mathrm{j}\omega_0^{(h)})$.

With this premise, we look for a model $\tilde{G}$ whose associated bi-variate function $\tilde{F}(\mathrm{j}\omega, \mathrm{j}\omega_0)$ approximates F in a least-squares sense. To this aim, we adopt a Wiener-like model structure [2] for $\tilde{G}$, whose output is $\tilde{y}(t) = \tilde{G}[u(t), p(t)] = \sum_{i=1}^{\bar{\imath}} \phi_i[p](t) \cdot \psi_i[u](t)$, where $\phi_i[\cdot]$ and $\psi_i[\cdot]$ are LTI systems. Its $\tilde{F}$-function can be written in pole-residue form as

$$\tilde{F}(\mathrm{j}\omega, \mathrm{j}\omega_0) = (\mathrm{j}\pi)^{-1} \sum_{i,j} r_{ij} (\mathrm{j}\omega - \alpha_i)^{-1} (\mathrm{j}\omega_0 - \beta_j)^{-1}. \tag{1}$$

We highlight that in the selected model structure, the components $\phi_i[p](t), \psi_i[u](t)$ are the outputs of scalar LTI systems. Model fitting, i.e. optimization of poles α_i, β_j and residues r_{ij}, can be performed using a suitable adaptation of a multivariate rational fitting algorithm. In our experiments, we used the Vector Fitting (VF) algorithm in two steps. First, we view ω_0 as a parameter and run VF to find a set of basis poles α_i to approximate the frequency dependence w.r.t. ω for all sampled values $\omega_0^{(h)}$ collectively. Then, a second run of VF, with fixed α_i, gives poles β_j and residues r_{ij}. Finally, $\tilde{G}$ results from assigning $\phi_i(s) = (s - \alpha_i)^{-1}$, $\psi_i(s) = \sum_j r_{ij}(s - \beta_j)^{-1}$. The input/output stability of the proposed model structure is guaranteed by enforcing strictly negative real part of the estimated poles α_i, β_i using standard techniques. The proposed approach is demonstrated using several benchmark examples of practical interest, including a Buck voltage regulator commonly used to stabilize the microprocessor power supply in electronic systems.

References

[1] I. V. Gosea, M. Petreczky, and A. C. Antoulas. Data-driven model order reduction of linear switched systems in the loewner framework. *SIAM Journal on Scientific Computing*, 40(2):B572–B610, 2018.

[2] P. Van den Hof, R. Toth, and P. Heuberger. *Model structures for identification of linear parameter-varying (LPV) models*, pages 15–34. MTA, 2009.

[3] L. Zadeh. Frequency analysis of variable networks. *Proceedings of the IRE*, 38(3):291–299, 1950.