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Abstract

This paper proposes a new precursor for monitoring coal-rock dynamic disasters based on a fiber bundle model (FBM), which has been validated in the study of material fracture and critical phenomena. First, the FBM was simulated using the Monte Carlo method to analyze the variations of force and energy. The derivative of energy was identified as a precursor characteristic for model failure. The Acoustic Emission (AE) features of coal-rock under uniaxial compression were also analyzed, and a constitutive model for coal-rock damage evolution under uniaxial compression was established using AE ringing count. Furthermore, the derivative of energy was calculated using the constitutive model to verify the simulation results and propose a new precursor indicator for coal-rock collapse. The research results provide useful guidance for preventing coal mine dynamic disasters.

Practical Applications

This study presents a novel methodology for the prediction of engineering geological hazards, focusing specifically on monitoring and preventing rockburst disasters in coal mines. The crucial precursor characteristics of coal and rock damage are unveiled by research findings, offering valuable insights for the accurate forecasting of potential disaster risks. This approach holds substantial promise not only within the coal mining sector but also across various engineering domains, including geotechnical engineering and other fields necessitating meticulous risk assessment. Implementation of this methodology empowers practitioners to refine disaster prediction, proactively ensuring the sustainability and safety of engineering ventures. It is recommended that project teams consider the integration of this innovative indicator alongside widely-adopted microseismic monitoring techniques, thus mitigating the limitations of existing geophysical monitoring methods. This innovative approach is poised to have a profound and transformative impact on the enhancement of geological hazard monitoring and engineering risk management, providing invaluable support for upcoming engineering projects.

Auther keywords: Fiber bundle model; Damage variable; Precursor; Acoustic emission; Coal burst

Introduction

In many regions of the world and especially in China coal is a primary energy source. However, as mining depths increase, coal and rock dynamic disasters such as coal burst and gas outburst have become more severe, posing a serious threat to the safety of underground personnel (Dou et al., 2014; Fan et al., 2020; Zhou et al., 2022). The occurrence of coal burst is a complex process that is characterized by suddenness and uncertainty and is often difficult to predict accurately. Therefore, accurate prediction of coal burst is crucial for coal mine safety.

Many studies have indicated that during the loading and damage of rock-like materials, a portion of the energy is released in the form of acoustic emission (AE), electromagnetic radiation, charge induction, and other forms (Aggelis, 2011; Baddari et al., 2011; Carpinteri et al., 2016; Ding et al., 2023; Li et al., 2016). Therefore, these non-destructive monitoring techniques are widely used in various fields, such as earthquake monitoring, structural damage monitoring, and coal and rock dynamic disaster monitoring (Behnia et al., 2014; Carpinteri et al., 2007; Donner et al., 2015; Lacidogna, et al., 2011; Lou et al., 2019). Specifically, the intermittent generation of AE time series reflects the increasing instability of the system, and the analysis of AE characteristic parameters can be used as a precursor to catastrophic events (Biswas et al., 2015; Iturrioz et al., 2013; Tanzi et al., 2023). During coal mine monitoring, it has been discovered that traditional non-destructive testing techniques have certain limitations. This is the result of the combined action of dynamic and static loads in deep coal mine rockburst disasters. (Dou et al., 2018; He et al., 2019). Acoustic emission can effectively monitor ultrasonic elastic waves caused by mining and blasting disturbances, providing early warning for coal and rock dynamic disasters, such as coal burst. However, for coal and rock dynamic disasters that occur under high static loads, the signals are relatively stable and weak before the occurrence of collapse, making it difficult to detect obvious precursor signals in advance.

The statistical analysis of fracture and damage in heterogeneous materials has drawn wide attention from statistical physicists, and has also led to the study of phase transitions and critical phenomena in statistical physics related to the fracture behavior of materials (Alava et al., 2006; Pradhan et al., 2008). Recently, there have been some reports discussing the critical phenomena of material collapse (Batool et al., 2022; Dębski et al., 2021; Diksha & Biswas, 2022; Pradhan et al., 2019). Pradhan et al.(2019) used a fiber bundle model to analyze the relationship between force, elastic energy, and displacement during the tensile failure process of materials. According to this theory, the slope of elastic energy and displacement reaches its maximum value before the catastrophic failure of the material, which can be used as a precursor point for material catastrophic failure. Dębski et al. (2021) used the discrete element method to simulate the failure process of the fiber bundle model and confirmed the relationship between energy release and the maximum value of the derivative of elastic energy. Diksha & Biswas (2022) applied the fiber bundle model to analyze disordered solids and conducted a systematic analysis of avalanche size and energy release time series generated by the fiber bundle model using supervised machine learning. Roy (2023) numerically studied the failure process of disordered systems under external forces using the fiber bundle model. Diksha et al. (2023) simulated the process of fiber bundle fracture and believed that a measure of how unequal avalanche sizes are is potentially a crucial indicator of imminent failure. The obtained results can predict failure time. These research results not only expand the application of the fiber bundle model, but also provide new ideas for predicting and controlling coal rock dynamic disasters.

Despite extensive scholarly research, there is limited exploration regarding the application of the fiber bundle model to coal and rock materials, particularly concerning the existence of a maximum energy derivative phenomenon preceding catastrophic failure. Further experimental verification is needed. Previous studies have primarily utilized the Weibull distribution in the fiber bundle model, focusing predominantly on the shape parameter k, while the significance of the scale parameter λ has been overlooked. This paper addresses this research gap through a comprehensive investigation involving theoretical analysis, simulation, and experimental validation. In this work, the Monte Carlo method was employed to establish a fiber bundle model where elastic elements share the load, simulating the process of material fracture. Laboratory uniaxial compression tests on coal are performed, and a constitutive model of coal rock damage is developed based on AE ringing count. Validation of simulated results is conducted against experimental data.

Fiber Bundle Model

Establishment of Fiber Bundle Model

The fiber bundle model (FBM) (Peirce FT, 1926) is a simulation tool with a simple principle and the ability to reflect profound evolution processes, as shown in Fig. 1. The fibers are parallel to each other and fixed between parallel loading plates. A load parallel to the fibers can be applied to both ends of the fiber bundle by the loading plates until the stress threshold of the fiber bundle and, therefore, the rupture is reached. When a certain stress threshold is exceeded, some fibers will collapse, and the load they originally carried will be shared by the remaining unbroken fibers.

[Fig. 1 about here]

Assuming that the model consists of N parallel fiber bundles jointly bearing an external force F applied to the system, the relationship between the stress σ and the strain ε of each fiber is given by:

$$\sigma = \mu\varepsilon, \tag{1}$$

Where μ is the elastic constant. If a portion of fibers breaks, the load is distributed evenly among the remaining fibers. This type of loading model is called the equal-load-sharing (ELS) scheme (Daniels, 1945). Since ELS has a mathematical analytical form, it is helpful to study material failure problems in conjunction with the model. Therefore, the subsequent model only considers the distribution form of ELS. The strength of fibers is usually determined by the threshold x they can withstand. It is assumed that the strength of fibers follows a Weibull distribution, where the cumulative distribution function $P(x)$ is given by (Zheng et al., 2019):

$$P(x) = 1 - \exp\left(-\left(\frac{x}{\lambda}\right)^k\right), \tag{2}$$

its corresponding probability density function is:

$$p(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\exp\left(-\left(\frac{x}{\lambda}\right)^k\right), \tag{3}$$

Where k is shape parameter and λ is scale parameter. The parameter k determines the shape of the Weibull distribution. For a specific material, it can describe the brittle or plastic characteristics. The parameter λ determines the location and scale of the Weibull distribution, which can affect the strength and failure properties. Different materials have different physical properties and lifetime behaviors, which can lead to different shape parameters and scale parameters.

At a certain moment, n fibers are assumed to be broken. The damage variable d is defined as:

$$d = \frac{n}{N}, \tag{4}$$

As N approaches infinity, the Δ at each step becomes very small, so the loading process can be considered quasi-static. On the other hand, for larger N values, the damage factor d can be treated as continuous, so it can be approximated by $P(x)$.

Considering the strain ε of each fiber bundle as representative of the strain in the process of coal compression, the constitutive relationship between the stress and strain of coal rock, considering the damage variable, can be expressed as:

$$\sigma(\varepsilon) = E\varepsilon\big(1 - P(\varepsilon)\big), \tag{5}$$

Where σ is the stress and *E* is the elastic modulus.

Strain is considered as the control parameter, and construct the energy E^e as:

$$E^e(\varepsilon) = \frac{E}{2}\varepsilon^2\big(1 - P(\varepsilon)\big). \tag{6}$$

Warning Sign of Collapse

From the above formula, the maximum values of functions (5) and (6) regarding stress and strain energy are sought. By setting the derivative of stress and energy, the following is obtained:

$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = E\big(1 - \varepsilon_c p(\varepsilon_c) - P(\varepsilon_c)\big) = 0, \tag{7}$$

$$\frac{dE^e(\varepsilon)}{d\varepsilon} = \frac{E}{2}[2\varepsilon_m(1 - P(\varepsilon_m) - {\varepsilon_m}^2 p(\varepsilon_m)] = 0, \tag{8}$$

From which the following values are obtained:

$$\varepsilon_c = \lambda k^{-\frac{1}{k}}, \tag{9}$$

$$\varepsilon_m = \lambda\left(\frac{2}{k}\right)^{\frac{1}{k}} = \varepsilon_c 2^{\frac{1}{k}} > \varepsilon_c, \tag{10}$$

In this context, ε_c is defined as the strain when the stress reaches its maximum value, also known as critical strain. On the other hand, ε_m represents the strain when the energy reaches its maximum value.

From the above analysis, it can be concluded that energy has a maximum value, but it always appears after the critical strain ε_c. when plotting $dE^e/d\varepsilon$, which is the derivative curve of energy with respect to strain, it is always possible to observe a maximum value. By calculating the second derivative of energy with respect to the strain and setting the derivative function equal to zero, the following is obtained:

$$\frac{d^2E(\varepsilon)}{d\varepsilon^2} = \frac{E}{2}\left[2\left(1 - P\left(\varepsilon_p\right)\right) - 4\varepsilon_p p\left(\varepsilon_p\right) - {\varepsilon_p}^2 p'\left(\varepsilon_p\right)\right] = 0, \tag{11}$$

By substituting Eq. (2) and (3) into Eq. (11), the following is obtained:

$$\frac{E}{2}\exp\left(-\left(\frac{\varepsilon_p}{\lambda}\right)^k\right)\left(k^2\left(\left(\frac{\varepsilon_p}{\lambda}\right)^k - 1\right)\left(\frac{\varepsilon_p}{\lambda}\right)^k - 3k\left(\frac{\varepsilon_p}{\lambda}\right)^k + 2\right) = 0, \tag{12}$$

From which the following value is obtained:

$$\begin{aligned} \varepsilon_p &= \lambda 2^{-\frac{1}{k}} \sqrt[k]{\frac{k^2 - \sqrt{k^2 + 6k + 1}k + 3k}{k^2}}, \\ &= \varepsilon_c k^{-\frac{1}{k}} 2^{-\frac{1}{k}} \sqrt[k]{k^2 - \sqrt{k^2 + 6k + 1}k + 3k} < \varepsilon_c. \end{aligned} \tag{13}$$

In Eq. (13) the term $k^{-\frac{1}{k}}$ is less than or equal to 1, and the term $2^{-\frac{1}{k}}$ is less than 1. Meanwhile, It is observe that $\sqrt[k]{k^2 - \sqrt{k^2 + 6k + 1}k + 3k}$ is always less than 1, irrespective of the value of k. Therefore, it follows that ε_p is always smaller than ε_c. Here, ε_p is defined as the corresponding strain when the $dE^e/d\varepsilon$ reaches its maximum value. The above analysis shows that the strain ε_p is always found before ε_c. As far as coal rock is concerned, when the sample reaches the ε_c in the compression failure process, most of the samples will be destroyed in a short period of time, showing strong brittle characteristics. Therefore, the strain ε_p can be used as a precursor of coal rock collapse. Compared with previous studies (Dębski et al., 2021; Pradhan et al., 2019), this paper considers the shape parameter k and scale parameter λ of Weibull distribution at the same time, so that the coal-rock loading process can be simulated with greater accuracy.

Monte Carlo method

In this section, the fracture process of the FBM is simulated to verify the conclusions obtained above. To simulate the process of fiber bundle fracture, Monte Carlo method (James, 1980; Kroese et al., 2014) is applied. Monte Carlo simulations use random samples to estimate the probability distribution and expected values of a system or process. They are useful when dealing with complex systems or those with inherent randomness or uncertainty. Since the fracture of fiber bundles can be modeled using the Weibull distribution, Monte Carlo simulations can be utilized to reproduce the Weibull distribution of fiber bundles and model the process of fiber fracture. Since the Monte Carlo method mainly uses random numbers to solve the calculation problems of the FBM, the simulation process of the FBM can be represented by mathematical variables. The simulation steps can be described in mathematical language as follows:

(1) Generate a random matrix A of size $m \times n$ from a Weibull distribution.

(2) Initialize a non-zero matrix B of size $m \times n$ to store load increments, with all values set to an equal constant.

(3) Determine a loading rate v.

(4) Compute the number of non-zero elements a in array B, locate the positions of these elements within B, increase their values by $c=v/a$, and update the resulting array as B.

(5) Compare the elements of arrays A and B. If an element in array B is greater than the corresponding element in array A, set both elements in arrays A and B to 0 at same position, resulting in new arrays A and B. Otherwise, arrays A and B remain unchanged.

(6) Repeat steps 4 and 5 until all elements in array A are 0. End the simulation.

In the simulation process described above, the number of elements in matrix A represents the number of fibers in the bundle, and each element is assigned a Weibull distribution to represent the strength of the fiber. The values in matrix B simulate the changing load values. By comparing the values in matrix B with the corresponding positions in matrix A, when there exists an element in B that is greater than that in A, the element in A at the corresponding position is assigned to zero, indicating that a fiber has broken. Steps 4 and 5 are repeated, simulating the entire process of fiber bundle fracture. The simulation ends when all elements in matrix A become zero, indicating that all fibers have fractured.

Simulation Results

Setting m=500, n=100, and the fiber quantity N=5×10^4. Having a sufficient number of fibers can avoid abnormal simulation results. The loading rate is set to V=500, which depends on the number of fibers and the simulation effect. Too high rate can cause the fiber bundle to be completely destroyed in a few cycles, while too low rate means that the fibers will break one by one, which is not consistent with the actual situation. In the Monte Carlo method described above, the first step is to determine the free matrix A that follows the Weibull distribution. From Equation (2) and Eq. (3), it can be seen that the parameters that affect the Weibull distribution function are the scale parameter λ and the shape parameter k. The maximum likelihood estimation method is used to estimate the parameters of the Weibull distribution, with detailed solution process available in (Cohen, 1965). Let us assume that x_1, x_2, ..., x_n are the strengths of the samples measured, and based on the Weibull probability density function given in Eq. (3), the likelihood function can be expressed as (Murshudov et al., 1997):

$$L(x_i, \lambda, k) = \prod_{i=1}^{n} \frac{k}{\lambda} (\frac{x_i}{\lambda})^{k-1} exp\,(-\left(\frac{x_i}{\lambda}\right)^k). \quad (14)$$

Based on the actual sample test results, the uniaxial compressive strength of 8 coal samples were calculated in this paper. The values of the uniaxial compressive strengths are respectively 12.15, 12.35, 9.7, 8.67, 7.5, 6.25, 10.25, and 11.27 MPa. The calculated values of the scale parameter and shape parameter are k=5.75 and λ=10.58, respectively. In particular, when the shape parameter is 1 and 2, the Weibull distribution corresponds to the exponential distribution and Rayleigh distribution, respectively, whose simulation under these two conditions has been discussed by the authors.

Based on the analysis in Section 2.1, The force F, energy E^e, and energy derivative $dE^e/d\varepsilon$ of the FBM during the simulated fracture process can be easily obtained, along with their relationship curve, as shown in Fig. 2.

[Fig. 2 about here]

In Fig. 2, the horizontal axis represents the number of cycles in the simulation process, indicating the number of iterations. The vertical axis shows the normalized dimensionless energy and force values. It can be seen from Fig. 2 that the model can be considered stable before the force reaches its maximum value, and it becomes unstable after the force reaches its maximum value, which is consistent with the actual situation. The strain ε_m in the figure represents the strain at the maximum energy, which appears in the unstable phase, while the strain ε_p represents the strain at the maximum derivative of energy, which appears before the critical strain ε_c. The simulation results are consistent with the theoretical analysis. As the model enters the unstable phase after the force reaches its maximum value, the fibers subsequently accelerate fracture, so it is believed that the ε_p can be used as a precursor to model failure, and also as a precursor indicator for coal collapse.

Test Equipment and Test Procedure

Sample preparation

A study was conducted on four coal samples from Xinjiang province, China. To ensure consistency, all samples were prepared in accordance with the suggested shape and size by the International Society for Rock Mechanics (ISRM), which have a cylindrical with a diameter of 50 mm and a height of 100 mm. Both ends of the sample had a flatness error of less than 0.02 mm. Table 1 presents the basic parameters of the coal samples tested.

[Table 1 about here]

Test equipment

In this study, the experimental system consisted of a loading system, an acoustic emission (AE) system, as shown in Fig. 3. An MTS C45.104 testing machine was used in the loading system, with a maximum load capacity of 300 kN. The AE instrument utilized a PCI-Express8 multi-channel AE system manufactured by American Physical Acoustics Company, along with nano 30 miniature sensors. The sensor has a frequency range of 125~750 kHz, and it is equipped with a 2/4/6 voltage preamplifier that allows for a selectable range of 20, 40, and 60 dB. The AE threshold was set to 40 dB to minimize the impact of ambient noise, and the sampling rate was set to 1 MHz. Prior to the start of the experiment, tight contact between the specimen and the upper and lower loading heads of the testing machine was ensured. Additionally, a pre-load of 50 N was applied to the specimen to mitigate potential errors caused by its unevenness. All samples underwent uniaxial compression tests at a displacement-controlled rate of 1 mm/min. The stress-strain curves are presented in Fig. 4.

[Fig. 3 about here]

Experimental Results

Strength and deformation characteristics

After conducting uniaxial compression tests, the strength and elastic modulus of coal specimens can be calculated, as shown in Table 1. Figure 4 presents the stress-strain curve of coal specimens under uniaxial compression conditions. It can be observed that the stress curves of samples C-1 and C-2 drop sharply after reaching compressive strength, indicating an instantaneous failure characteristic. In the laboratory, it has been observed that when a sample experiences macroscopic failure, coal fragments can be ejected from the face, and even a phenomenon called "coal burst" can occur, where the coal specimen completely explodes. In contrast, the curves of C-3 and C-4 did not show a sharp drop after reaching the compressive strength of the coal specimen.

Instead, the stress decreased to a certain value, then increased, and then decreased again until failure occurred, exhibiting a gradual failure characteristic. This is because after the coal sample reaches its peak strength, localized damage occurs, the stress adjusts, and a relatively stable structure is formed, and the remaining part still has a certain load-bearing capacity.

According to the coal specimen crack propagation process, the stress-strain curve can be divided into four stages: (1) compaction stage: at the beginning of loading, the stress is small, and the coal specimen has a faster axial strain rate due to the presence of many primary cracks. (2) Linear elastic stage: after the compaction stage, the stress of the coal sample begins to steadily increase and secondary cracks appear, at which point the stress-strain curve is approximately a sloping straight line. (3) Crack propagation stage: the stress reaches its yield limit, cracks begin to accelerate and form multiple crack clusters. (4) Failure stage: the coal sample reaches its maximum load-bearing capacity, the cracks merge and penetrate, and instantaneous or gradual failure characteristics begin to appear

[Fig. 4 about here]

AE Behaviors

The characteristics of AE signals are closely related to the deformation and failure process of coal specimens under uniaxial compression and can reflect the evolution of damage during loading. Figure 5 shows the relationship between stress-time and AE ring-down counts for specimens C-1, C-2, C-3, and C-4 during uniaxial compression. It can be observed that during the compaction stage, the AE ring-down counts are very low and can be ignored. This is because the initial cracks inside the coal specimens are closed and compacted. Only a small amount of low-energy AE signals are generated from some rough surfaces, mixed with some noise. As the coal specimens develop tiny cracks during the mid-loading stage, the AE signals gradually increase. During the crack propagation stage and failure stage, the density of AE signals increases as the cracks accelerate and propagate, leading to a rapid increase in AE ringing counts.

From Fig. 5, it can be observed that the AE ringing counts become extremely active when the stress reaches its peak value, and the maximum value of AE ringing counts is achieved at the stress peak. This is because the specimen immediately generates a significant stress drop when reaching the stress peak, indicating that macroscopic cracks have occurred and released significant energy. Additionally, the ringing counts increase significantly with every stress drop, as shown in Fig. 5(c) and Fig. 5(d). The above analysis demonstrates that there is a good correlation between AE ringing counts and coal damage under uniaxial compression.

[Fig. 5 about here]

Damage evolution model of coal based on AE characteristics.

Heiple et al. (1981) conducted long-term research on material damage and fracture processes using AE technique and found that the AE ring-down counts are one of the features that can better describe the changes in material damage among multiple parameters of AE. This is because it is proportional to the strain energy released by particle dislocations and movement, fracture, and crack propagation in the material. Therefore, this paper uses the AE ringing count and the cumulative AE ringing count to establish a coal damage evolution model. The damage variable of the material was originally proposed by Kachanov (1958). According to the definition of the damage variable, assuming that the cumulative AE ringing count when all N fibers in the material are broken is C, the average AE ringing count C_0 when each fiber is damaged is calculated as:

$$C_0 = \frac{C}{N}, \tag{15}$$

When n fibers are broken, the cumulative AE ringing count at this moment is C_n, which can be expressed as:

$$C_n = C_0 n = C\frac{n}{N}, \tag{16}$$

Therefore, the damage variable based on ringing counts can be defined as:

$$d = \frac{C_n}{C}, \tag{17}$$

There are many microcracks and voids randomly distributed inside the coal. According to the statistical damage theory, it is assumed that the failure probability of the coal microstructure follows the Weibull distribution. The microstructure size includes enough voids and cracks and can also be considered as small enough to adopt the concept of continuum mechanics (J. Zhou & Chen, 2013). The probability density function of the Weibull distribution is shown in Eq. (3).

The damage variable d defined above has a value range of 0 to 1, which represents the cumulative degree of microscopic damage in the material. Here, 0 represents an undamaged material, while 1 represents a completely damaged material. However, after the test is stopped, the specimen still has a certain bearing capacity, but the calculated value of the d is 1, which does not match the actual situation. To eliminate this influence, a critical damage can be introduced to modify the damage variable d based on the effect of load on AE. When the test stops, the damage variable d is controlled by the critical damage, which is more in line with the actual situation. Therefore, the modified damage variable can be expressed as:

$$d = d_0\frac{C_n}{C} = d_0\left(1 - \exp\left(-\frac{\varepsilon}{\lambda}\right)^k\right), \tag{18}$$

Where d_0 is the critical damage, which is multiplied by the damage variable d to obtain a modified d that can reflect the residual strength of the coal, making the obtained model closer to the actual situation. The critical damage d_0 reflects the damage condition of the specimen after loading. To simplify calculations, d_0 is taken as:

$$d_0 = 1 - \frac{\sigma_p}{\sigma_r}, \tag{19}$$

Where σ_p represents peak strength and σ_r represents residual strength.

As described in Section 4.2, due to the presence of inherent cracks and a relatively high porosity in coal, there is a compression stage during the compression process, resulting in fewer AE signals. Additionally, compared with the crack propagation and failure stages, the AE signals during the elastic stage are also minimal. If only the AE signals are used to construct the damage evolution model, it will lead to significant discrepancies between the model and the actual curve for both the compression and linear elastic stages. To mitigate this effect, the concept of compaction coefficient K (the ratio of the stress-strain derivative to the elastic modulus E) is introduced in this study (Gu et al., 2019). Since the derivative curve of stress-strain relationship approximates a logarithmic function (Liu et al., 2016), K can be described as:

$$K = \begin{cases} \log_n\left(a\frac{\varepsilon}{\varepsilon_s} + 1\right), 0 \le \varepsilon < \varepsilon_s \\ 1, \varepsilon \ge \varepsilon_s \end{cases}, \tag{20}$$

here, n is a constant obtained through experiments, ε_s is the yield strain. The coal damage model established in this paper can be expressed as follows:

$$\sigma = KE\varepsilon(1 - d). \tag{21}$$

Damage model validation

To verify the accuracy and effectiveness of the proposed damage model, this paper applies the model to experimental data of actual coal and compares the results. The parameters of the compression coefficient K were obtained by fitting the stress-strain curve of uniaxial compression, as shown in Table 2. Similarly, based on the relationship between time and strain, as well as time and AE ringing counts, the relationship between strain and cumulative AE ringing counts can be obtained and fitted using the Weibull distribution function. The fitting curve is shown in Fig. 6. From Fig. 6, it can be seen that the Weibull distribution function fits the cumulative ringing count very well, indicating that the curve follows the Weibull distribution. The damage variable parameters k and λ can be accurately obtained using the acoustic emission ringing count. The fitting parameters of each sample obtained by fitting using Eq. (18) are shown in Table 2, where R^2 represents the degree of curve fitting.

[Fig. 6 about here]

[Table 2 about here]

The theoretically calculated stress-strain curve obtained from Eq. (21) is in good agreement with the actual stress-strain curve, as shown in Fig. 7. This indicates that the coal damage constitutive model established in this paper is relatively reasonable. From Fig. 7, it can also be observed that the stress-strain curve described by Eq. (21) is generally slightly higher than the experimental values. This is due to the fitting error of the compression coefficient K. Compared to rocks such as granite and marble, coal has a relatively higher porosity. During uniaxial compression, the linear elastic stage of the curve is not significant, and the curve is concave upwards. This leads to some errors in the calculation of the elastic modulus E and affects the fitting of the compression coefficient K. Therefore, the obtained theoretical compression coefficient is relatively larger, resulting in a slightly higher strength than the experimental values.

In Fig. 7, the variation curve of the damage variable d with strain is also plotted. It can be observed that before the specimen enters the yield stage, the d is between 0 and 0.1. This indicates that only a small amount of AE signals are collected before the yield stage, and it also proves that the degree of damage to the specimen is small during this stage.

[Fig. 7 about here]

Early warning indicator

According to the established coal damage constitutive model, the energy can be calculated by the following equation:

$$E^e(\varepsilon) = K\frac{E}{2}\varepsilon^2\big(1 - P(\varepsilon)\big). \tag{22}$$

Based on the results of simulation analysis using the FBM presented in Section 2, it can be inferred that the maximum energy occurs after the critical strain ε_c, while the maximum energy derivative occurs before the ε_c. To further validate this conclusion through experiments, Figure 8 shows the stress-strain, energy-strain, and energy derivative-strain curves based on the coal damage constitutive model. It is evident that the energy derivatives of all four specimens reached the maximum value before the maximum stress was reached. The stress-strain curve shows that coal, as a material with high plastic deformation capacity, exhibits significant deformation during uniaxial compression but quickly collapses after reaching the peak stress, as evidenced by the occurrence of obvious coal burst phenomena in specimens C-1 and C-2, indicating strong impact tendency. Therefore, the maximum energy derivative can be used as a precursor indicator of impact ground pressure. For coal specimens with gradual damage characteristic, such as C-3 and C-4, the maximum energy derivative can also be used as a precursor indicator of the sample entering the unstable stage.

[Fig. 8 about here]

Discussion and Conclusions

Although a simple FBM was adopted, in all the analysis results, whether it was the simulation of the FBM or the laboratory experiment, the precursor point before catastrophic failure of the material was consistently observed, that is, the maximum value of energy derivative was observed before the catastrophic collapse of the coal specimen. The coal damage constitutive model based on AE ringing count has also shown good agreement with the experimental curve. This seems to solve the problem of predicting coal specimen collapse well. However, some existing issues need to be further discussed to make the conclusions more reasonable.

First of all, there is a simulation issue. MATLAB was utilized to simulate the random fracture of the FBM, but it is difficult to simulate the real situation of coal failure. This is because using the Monte Carlo method to abstract FBM as an iterative process of random arrays ignores the structure and size of the coal sample, as detailed in Section 2.3. The advantage of doing so is that the model is simple and can intuitively evolve the fracture process, but it is also difficult to simulate the complex failure process of coal. Precursor indicator obtained through energy changes allow us to consider collapse from the point of view of coal specimen stress state, but this result is not always correct. For instantaneously damaged coal specimens, such as C-1 and C-2, the stress-strain curve shows that the failure occurs in a short time after reaching the peak stress, and the strain at the maximum value of the energy derivative can be regarded as the precursor indicator of catastrophic failure. But for coal specimens with the gradual failure characteristic, such as C-3 and C-4, the strain at the maximum energy derivative can only be considered that the specimen is about to enter the unstable stage. This also indicates that if there is a maximum value in the energy derivative due to damage, it cannot be immediately judged as the final catastrophic failure. On the other hand, if a small damage leads to the conclusion of catastrophic failure, this judgment is completely wrong.

In fact, this shows that the collapse of coal mass cannot be judged solely by the energy derivative. The development of AE technique can to some extent compensate for this deficiency. For example, the AE b-value (Carpinteri, et al., 2009). The magnitude of the b-value can indicate changes in micro-cracks of different scales in coal rock masses, reflecting the degree of damage to the coal rock mass. Many studies have reported this conclusion (Fritschen, 2010; Mondal & Roy, 2019; X. Zhou et al., 2023). Therefore, *b*-value and energy derivative can be used to judge whether coal rock mass will collapse, to better predict rock burst.

The FBM proposed in this paper has certain limitations. The equal-load-sharing model is employed, which means that the force acting on the FBM is evenly distributed among the unbroken fibers. This is primarily because this type of model can be solved analytically. However, microcracks in coal samples also significantly influence the development of local cracks, making the equal-load-sharing model inadequate. Therefore, further work is required to enhance the model and improve its ability to simulate real-world scenarios. Given the above comments, The research conclusions can be summarized as follows:

1) The FBM established by Monte Carlo method can quantitatively describe the process of fiber fracture, and the evolution of the FBM can correspond well with the AE ringing count of coal under uniaxial compression. The simulation results that there is a maximum value of the energy derivative before the catastrophic failure of the model.

2) A damage constitutive model of coal under uniaxial compression based on AE ringing count was established, laying a foundation for better understanding the evolution law of coal damage and revealing the intrinsic mechanism of coal damage.

3) Through analyzing the results of uniaxial compression tests on coal specimens, it is found that the energy derivative has a maximum value before the catastrophic failure of coal specimens, which can serve as a precursor for the collapse of coal specimens under uniaxial compression and provide theoretical guidance for the prevention and control of coal mine dynamic disasters.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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