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Leveraging 2:1 Parametric Resonance in a Notional Wave Energy Harvester

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Abstract. In ocean engineering applications, parametric resonance is normally detrimental for the stability of large structures, so the vast majority of effort in the literature is towards preventing and reducing it; similarly, parametric excitation is usually undesired for wave energy extraction too, since it often reduces the conversion ability. Conversely, this paper investigates the possibility to purposely introduce a 2:1 parametric resonance into a pitching wave energy harvester in order to inherently increase the energy absorption capabilities. Such a change in perspective is enabled by the use of a computationally efficient nonlinear hydrodynamic model (nonlinear Froude-Krylov force), that is able to articulate such a parametric instability at an early-development stage in a design-oriented simulation framework. The introduced 2:1 instability is found to be promising, since a significant amplification is obtained in the 2:1 region, where the oscillation amplitude is similar or even higher than in the 1:1 region.

Keywords: energy harvester, wave energy converter, nonlinear Froude-Krylov force, nonlienar hydrodynamics, paramteric resonance

1 Introduction

The recent global energy crisis brings upfront a renovated sense of urgency for sustainability and energy self-sufficiency; this is woven over the background of climate crisis, so innovative measures should contribute to an overall decarbonisation planning strategy [18]. Within this context, renewable energy harvesters are being developed extensively to contribute to the emancipation from fossil fuels [11]; one of the major form of still untapped renewable energy source is the ocean, whose waves can be used to produce clean energy, both at utility and small scales.

Wave energy harvesters (WEHs) are devices that respond to the external wave excitation force, typically with an oscillation motion, whose energy is converted via a power take-off (PTO) system. Although already technically viable, WEHs are required to improve their performance to become economically competitive with other more mature renewable energy sources. Efforts to this objective include holistic [21] or specific [22] optimization at a design stage , as well as control strategies including impedance-matching [6], linear time-invariant energy-maximisation control [3], or time-varying system parameters [13]. However, such approaches are normally based on linear models, which are proven to become unrepresentative when large motions occur [20]; to circumvent such an issue, data-driven approaches are also investigated [10].

Although representative nonlinear models exist, they usually are too computationally demanding to be used in early stages of design or control. Another consequence of such a computational limitation is the usual way nonlinearity or instability are handled; in fact, floating objects may be prone to dynamic instability, either in the yaw degree of freedom (DoF) [12, 19], or in the pitch and roll DoFs [15, 17]. Normally, such instability is discovered only after the preliminary design and effort is invested towards after-corrections or live-limitation [5]. However, having a representative and fast numerical model may enable to incorporate such instabilities already at the early stages of design. With this perspective, a few studies have proposed to embed parametric resonance into WEHs [1, 26], making it an enabling rather than a detrimental factor.

This paper proposes a novel floating WEH that is purposely designed to be prone to parametric resonance; a 2:1 resonance condition is defined in a heaving-pitching device, assuming that the energy extraction will be performed in the rotation DoF. Although 2:1 resonance is often studied in vibration energy harvesters, it has never been considered as a performance enhancer in wave energy converters, to the best of the author's knowledge; this is due to the inherent difficulty of describing parametric resonance in a design-oriented way. In this paper, parametric resonance is articulated and studied by means of a computationally efficient nonlinear Froude-Krylov (NLFK) force model [14]. The claim, herein demonstrated, is that parametric resonance can expand the operational bandwidth of the WEH. The remainder of the paper is organized as follows: Sect. 2 presents the mathematical model, Sect. 3 shows some results, while concluding remarks are finally drawn in Sect. 4.

2 Mathematical model

A generic heaving and pitching floater can be described via the following linear (LFK) and uncoupled equation of motion about the center of gravity, written in the frequency domain for compactness [9]:

$$\left[-\omega^2\left(\mathbf{M}+\mathbf{A}(\omega)\right)+j\omega\left(\mathbf{B}_v+\mathbf{B}(\omega)+\mathbf{B}_{PTO}\right)+\left(\mathbf{K}_h+\mathbf{K}_m+\mathbf{K}_{PTO}\right)\right]\boldsymbol{\xi}_2 = \\ =\mathbf{F}_d+\mathbf{F}_{FK_d}, \tag{1}$$

where $\boldsymbol{\xi}_2$ is the $2{\times}1$ state vector, composed of heave (z) and pitch (θ), $\mathbf{M}$ the diagonal inertia matrix, $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ the diagonal frequency-dependent added mass and radiation damping, $\mathbf{B}_v$ is the uncoupled linear viscous damping, $\mathbf{K}_m$ is the uncoupled linear mooring stiffness, $\mathbf{K}_h$ the diagonal linear hydrostatic stiffness, $\mathbf{F}_d$ and $\mathbf{F}_{FK_d}$ are the diffraction and linear dynamic FK forces, and $\mathbf{B}_{PTO}$ and $\mathbf{K}_{PTO}$ the diagonal PTO damping and stiffness. It is assumed that only the pitching motion is used to extract energy; however, the choice of the PTO coefficients is not trivial and is dependent on the model (either linear or nonlinear). Therefore, both for simplicity and to enable a meaningful first comparison of the inherent system nonlinear behavior, such PTO parameters are set to zero; then, the pitching angle, naturally a measurement of energy absorption by the hull, is also used as a proxy for power extraction ability.

The linear hydrodynamics is computed using a linear Boundary Element Method (BEM) software, such as Nemoh [2] or WAMIT [24]. The characterizing quantities of interest are the radiation added mass, radiation damping, diffraction and dynamic Froude-Krylov forces, and hydrostatic stiffness. The computation is performed in frequency domain and considering the mean wetted surface, defined by the floater at rest and the still water level.

The NLFK version of (1) alternatively computes $\mathbf{F}_{FK_d} - \mathbf{K}_h \boldsymbol{\xi}_2$ in a nonlinear way, i.e. considering the actual instantaneous wetted surface, defined as the displaced floater with the free surface at each time step, as shown in Fig. 1.

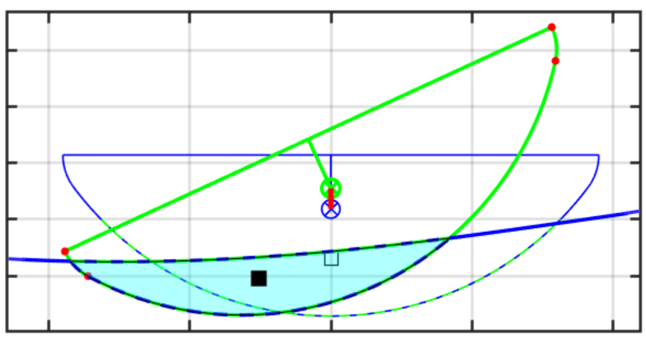


Fig. 1: Snapshot of a displaced hull, shown in green thick solid line, with instantaneous free surface elevation, shown in blue thick solid line, enclosing the instantaneous submerged volume (shaded area). The crossed-circle marker shows the position of the centre of gravity, while the square shows the centre of buoyancy (empty: LFK; full: NLFK). The rest position (LFK) is shown in thin line. Adapted from [14].

Froude-Krylov generalized forces ($\mathbf{F}_{FK}$), divided into translational forces ($\mathbf{f}_{FK}$) and torques ($\boldsymbol{\tau}_{FK}$), integrate the undisturbed pressure field (p_u) on the instantaneous wetted surfaces (S_w) as follows:

$$\mathbf{f}_{FK}(t) = \mathbf{f}_g + \mathbf{f}_p = \mathbf{f}_g + \iint\limits_{S_w(t)} p_u(x,y,z,t)\ \mathbf{n}\ dS, \tag{2a}$$

$$\boldsymbol{\tau}_{FK}(t) = \boldsymbol{\tau}_g + \boldsymbol{\tau}_p = (\mathbf{r}_g - \mathbf{r}_R) \times \mathbf{f}_g + \iint\limits_{S_w(t)} p_u(x,y,z,t)\ (\mathbf{r} - \mathbf{r}_R) \times \mathbf{n}\ dS, \tag{2b}$$

where $\mathbf{f}_g$ is the gravity force, $\boldsymbol{\tau}_g$ its contribution to the torque, $\mathbf{n}$ is the unity vector normal to the surface, $\mathbf{r}$ is the generic position vector, $\mathbf{r}_R = (x_R, y_R, z_R)'$ is the reference point around which the torque is computed, and likewise $\mathbf{r}_g$ is the position vector of the centre of gravity.

While generic NLFK solvers for arbitrary complex floaters must rely on a meshed representation of $S_w(t)$, which becomes the computational bottleneck [16, 23, 25], a faster analytical representation is available for axisymmetric and prismatic floaters [14], which is herein implemented.

3 Results

The geometry of the floater is shown in Fig. 1. The rotational moment of inertia of the floater is chosen such that the natural frequency in the heaving DoF (ω_3) is twice the natural frequency in the pitching DoF (ω_5). Although real sea states are short-crested and multidirectional [4], long-crested waves are herein considered to clearly investigate the device attitude for different wave incoming frequency and highlight the parametric resonance response. A fine grid of wave periods (T_w) and wave heights (H_w) is simulated in both the linear (LFK) and nonlinear (NLFK) models. Simulations are run for long enough to reach steady state; the amplitude of the response, either in heave (z) or pitch (θ) is computed as half the distance from peak to trough of the last two wave periods, in order to consider the expected period doubling of the nonlinear response. Overall results are shown in Fig. 2, normalized with respect to the draft of the floater (D) and the pitching natural period (T_5).

According to the linear model, normally implemented in the field for design and control, the response is obviously proportional to the wave height. The heaving response reaches a peak at $T_w = T_3 = \frac{1}{2}T_w$, and remains about constant for higher periods; the pitching response sharply increases at $T_w = T_5$, well above physically meaningful values, due to the linearity assumption and the absence of viscous dissipation mechanisms: the white region in Fig. 2 represents capsizing of the hull, namely when the pitching angle is above the arbitrary threshold of $50°$. Note that the introduction of additional viscous damping may have averted the numerical capsizing; however, no artificial corrective mechanisms have been added in the LFK model in order to highlight that the NLFK model is inherently preventing unrealistic results, even without viscous damping. This is an evidence of how the NLFK model is more consistent to the physics of the wave-structure interaction.

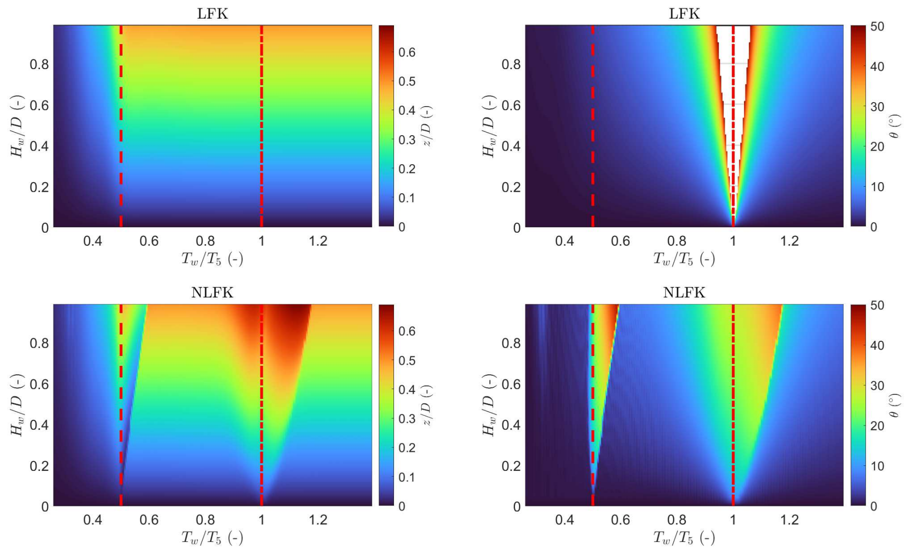
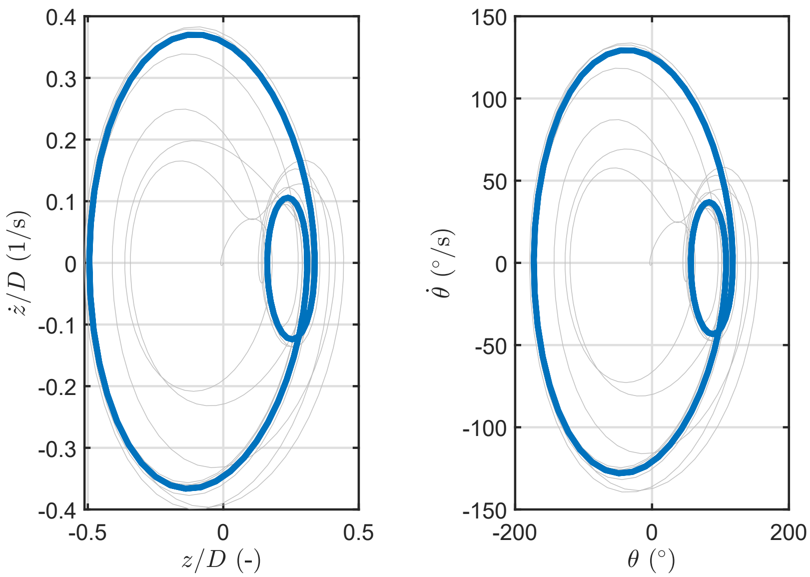


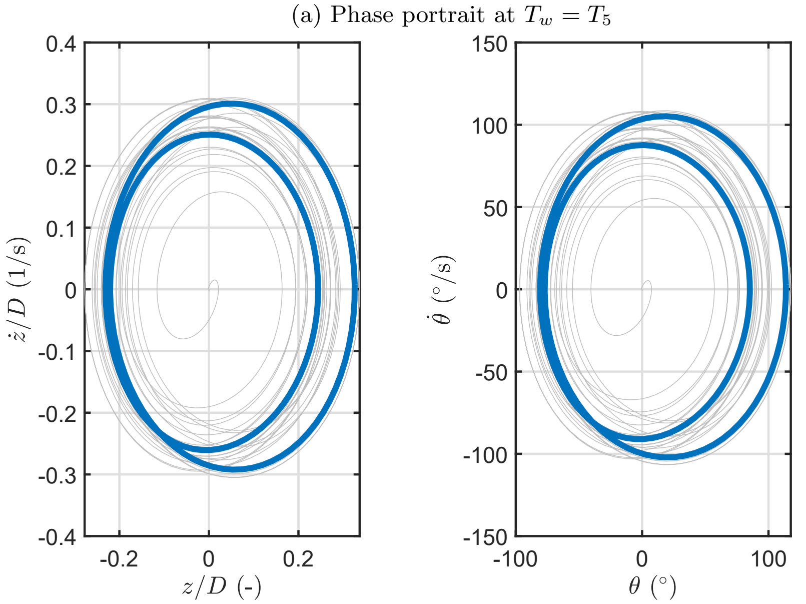
Fig. 2: Amplitude of response for heave (left) and pitch (right), according to the linear (top) and nonlienar (bottom) models. The dash-dot and dashed red lines highlight one or half of the pitching natural, respectively. The white region indicates capsizing, i.e. when the pitching angle is above the arbitrary threshold of 50°.

Figure 2 clearly shows the region of parametric resonance: due to the 2:1 resonance, there is a sharp increase of the pitching motion around $T_w = \frac{1}{2}T_5$; the amplitude also presents a bending towards larger periods as the wave height increases, highlighting the nonlinearity of the response. Note that this bending behaviour contributes to enlarge the operational bandwidth of the response, covering the region between $T_w = \frac{1}{2}T_5$ and $T_w = T_5$. In the same 2:1 resonance region, it is possible to appreciate a nonlinear coupling between the heaving and pitching DoFs, since there is a decrease of heaving motion as the pitching angle increases; this is usually detrimental for WEH that exploit heaving for the extraction [15], while is beneficial for WEH working on the pitching DoF. Conversely, in the 1:1 parametric resonance region, the coupling between heave and pitch causes a significant increase in the heave motion, which drains energy from the pitching DoF.

To further investigate the build-up of the nonlinear response, Figs. 3a and 3b show the phase portraits of the time-series response at the 1:1 and 1:2 resonance conditions, respectively. Figure 3a presents a period doubling, where the amplitude of motion is similar in the two consecutive cycles. An asymmetric response can be noted, where the mean is shifted to positive values for both heave and pitch. Period doubling is also found in Fig. 3b, but the two amplitudes are significantly different; in addition, the asymmetric response is towards negative



(a) Phase portrait at $T_w = T_5$



(b) Phase portrait at $T_w = \frac{1}{2}T_5$

Fig. 3: Phase portraits. The full time series, hence comprising of the transient, is shown in think grey line, whereas the latest two periods of oscillation are highlighted in think blue line.

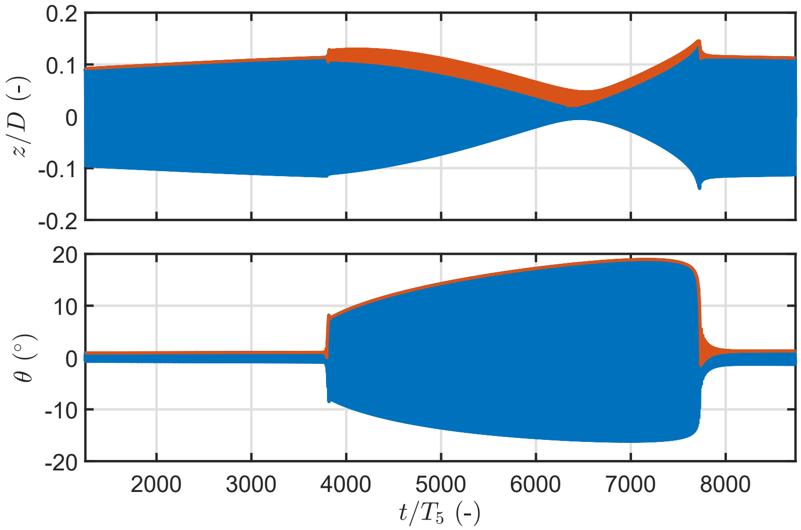


Fig. 4: Down-chirp time series response, with frequency (ω) sweep from 2.15 ω_5 to 1.85 ω.

values for the largest amplitude, whereas the smallest oscillation is at the peak, i.e. at positive values.

Finally, a down-chirp signal is simulated to highlight the regions of instability and the interaction between the two DoFs. The frequency (ω) sweep shown is from 2.15 ω_5 to 1.85 ω, and about 10^4 wave periods have been simulated to ensure a smooth transition. Figure 4 shows the time series response, highlighting the envelope of the peaks, which are also plotted in Fig. 5 against the related instantaneous frequency.

From Fig. 4 it can be noted that the pitch response is always essentially zero, apart from the abrupt rise caused by parametric resonance; once it kicks in, a substantial fall of the heave response is obtained, demonstrating an internal transfer of energy between the two degrees of freedom. The frequency range where parametric resonance appears is evident form Fig. 5: the 2:1 area is properly included within the pitching response region, but its extension is asymmetrically oriented towards lower frequencies; similarly, the peak of the response is found at $\omega \approx 1.91\omega_5$, hence lower than $2\omega_5$.

4 Conclusions

This paper proposes to define at the design-stage a 2:1 resonance in a wave energy harvester in order to expand the operational bandwidth of the system, hence the ability to extract energy in a wider range of sea states. The response amplitude is demonstrated to be indeed increased in the targeted area of interest, i.e. at a wave period half of the pitch natural period. Such a result is promising, since it

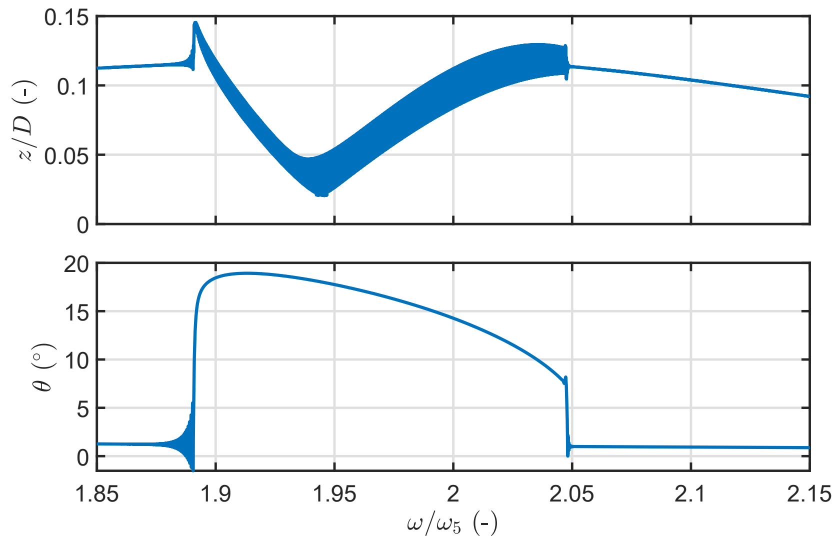


Fig. 5: Amplitude of the down-chirp response against instantaneous frequency.

confirms the potentiality of the approach; however, further steps are required to evaluate the practicality of the implementation. In particular, a power take-off system will be introduced and optimized, to quantify the improvement in power extraction, rather than power absorption (pithing angle). Finally, by framing the NLFK model into a control-oriented structure via model order reduction [7], it will be investigated the possibility to formulate an optimal control problem [8] to lead the system into parametric resonance condition and further enlarge the operational bandwidth.

References

1. Abdelkhalik, O., Zou, S., Robinett, R., Bacelli, G., Wilson, D., Coe, R.: Control of Three Degrees-of-Freedom Wave Energy Converters Using Pseudo-Spectral Methods. Journal of Dynamic Systems, Measurement, and Control **140**(7), 074,501 (2018). DOI 10.1115/1.4038860
2. Babarit, A., Delhommeau, G.: Theoretical and numerical aspects of the open source BEM solver NEMOH. Proceedings of the 11th European Wave and Tidal Energy Conference. (September 2015), 1–12 (2015). DOI hal-01198800
3. Carapellese, F., Pasta, E., Paduano, B., Faedo, N., Mattiazzo, G.: Intuitive LTI energy-maximising control for multi-degree of freedom wave energy converters: The PeWEC case. Ocean Engineering **256**, 111,444 (2022). DOI 10.1016/J.OCEANENG.2022.111444
4. Cervelli, G., Parrinello, L., Moscoloni, C., Giorgi, G.: Comparison of the ERA5 Wave Forecasting Dataset Against Buoy Record. Instrumentation Mesure Métrologie **21**(3), 87–95 (2022). DOI 10.18280/i2m.210301

5. Davidson, J., Kalmár-Nagy, T.: A Real-Time Detection System for the Onset of Parametric Resonance in Wave Energy Converters. Journal of Marine Science and Engineering **8**(10), 819 (2020). DOI 10.3390/jmse8100819
6. Faedo, N., Carapellese, F., Pasta, E., Mattiazzo, G.: On the principle of impedance-matching for underactuated wave energy harvesting systems. Applied Ocean Research **118**, 102,958 (2022). DOI 10.1016/J.APOR.2021.102958
7. Faedo, N., Dores Piuma, F.J., Giorgi, G., Ringwood, J.V.: Nonlinear model reduction for wave energy systems: a moment-matching-based approach. Nonlinear Dynamics **102**(3), 1215–1237 (2020). DOI 10.1007/S11071-020-06028-0
8. Faedo, N., Giorgi, G., Ringwood, J.V., Mattiazzo, G.: Optimal control of wave energy systems considering nonlinear Froude–Krylov effects: control-oriented modelling and moment-based control. Nonlinear Dynamics **109**(3), 1777–1804 (2022). DOI 10.1007/s11071-022-07530-3
9. Fontana, M., Casalone, P., Sirigu, S.A., Giorgi, G., Bracco, G., Mattiazzo, G.: Viscous Damping Identification for a Wave Energy Converter Using CFD-URANS Simulations. Journal of Marine Science and Engineering 2020, Vol. 8, Page 355 **8**(5), 355 (2020). DOI 10.3390/JMSE8050355
10. Gioia, D.G., Pasta, E., Brandimarte, P., Mattiazzo, G.: Data-driven control of a Pendulum Wave Energy Converter: A Gaussian Process Regression approach. Ocean Engineering **253**, 111,191 (2022). DOI 10.1016/J.OCEANENG.2022.111191
11. Giorgi, G.: Parametric resonance exploitation due to periodic damping in an electromechanical energy havester. International Journal of Mechanics and Control **22**(01), 77–85 (2021)
12. Giorgi, G., Davidson, J., Habib, G., Bracco, G., Mattiazzo, G., Kalmár-Nagy, T.: Nonlinear Dynamic and Kinematic Model of a Spar-Buoy: Parametric Resonance and Yaw Numerical Instability. Journal of Marine Science and Engineering 2020, Vol. 8, Page 504 **8**(7), 504 (2020). DOI 10.3390/JMSE8070504
13. Giorgi, G., Faedo, N.: Performance enhancement of a vibration energy harvester via harmonic time-varying damping: A pseudospectral-based approach. Mechanical Systems and Signal Processing **165**, 108,331 (2022). DOI 10.1016/J.YMSSP.2021.108331
14. Giorgi, G., Sirigu, S., Bonfanti, M., Bracco, G., Mattiazzo, G.: Fast nonlinear Froude–Krylov force calculation for prismatic floating platforms: a wave energy conversion application case. Journal of Ocean Engineering and Marine Energy **7**(4), 439–457 (2021). DOI 10.1007/S40722-021-00212-Z
15. Gomes, R.P.F., Henriques, J.C., Gato, L.M., Falcao, A.F.d.O.: Experimental Tests of a 1:16th scale model of the spar-buoy owc in a large scale wave flume in regular waves. pp. 1–10 (2018)
16. Guerinel, M., Zurkinden, A.S., Alves, M., Sarmento, A.: Validation of a partially nonlinear time domain model using instantaneous froude-krylov and hydrostatic forces. In: Proceedings of the 10th European Wave and Tidal Energy Conference. Technical Committee of the European Wave and Tidal Energy Conference, Aalborg (2013)
17. Habib, G., Giorgi, G., Davidson, J.: Coexisting attractors in floating body dynamics undergoing parametric resonance. Acta Mechanica **233**(6), 2351–2367 (2022). DOI 10.1007/S00707-022-03225-3
18. Novo, R., Marocco, P., Giorgi, G., Lanzini, A., Santarelli, M., Mattiazzo, G.: Planning the decarbonisation of energy systems: The importance of applying time series clustering to long-term models. Energy Conversion and Management: X **15**(January), 100,274 (2022). DOI 10.1016/j.ecmx.2022.100274

19. Orszaghova, J., Wolgamot, H., Draper, S., Taylor, P.H., Rafiee, A.: Onset and limiting amplitude of yaw instability of a submerged three-tethered buoy. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences (2020). DOI 10.1098/rspa.2019.0762
20. Ransley, E.J., Brown, S.A., Hann, M., Greaves, D.M., Windt, C., Ringwood, J., Davidson, J., Schmitt, P., Yan, S., Wang, J.X., Wang, J.H., Ma, Q., Xie, Z., Giorgi, G., Hughes, J., Williams, A., Masters, I., Lin, Z., Chen, H., Qian, L., Ma, Z., Chen, Q., Ding, H., Zang, J., van Rij, J., Yu, Y.H., Li, Z., Bouscasse, B., Ducrozet, G., Bingham, H.: Focused wave interactions with floating structures: a blind comparative study. Proceedings of the Institution of Civil Engineers - Engineering and Computational Mechanics **174**(1), 46–61 (2021). DOI 10.1680/jencm.20.00006
21. Rava, M., Dafnakis, P., Martini, V., Giorgi, G., Orlando, V., Mattiazzo, G., Bracco, G., Gulisano, A.: Low-Cost Heaving Single-Buoy Wave-Energy Point Absorber Optimization for Sardinia West Coast. Journal of Marine Science and Engineering **10**(3) (2022). DOI 10.3390/jmse10030397
22. Sirigu, S.A., Gallizio, F., Giorgi, G., Bonfanti, M., Bracco, G., Mattiazzo, G.: Numerical and Experimental Identification of the Aerodynamic Power Losses of the ISWEC. Journal of Marine Science and Engineering 2020, Vol. 8, Page 49 **8**(1), 49 (2020). DOI 10.3390/JMSE8010049
23. Tarrant, K.R., Meskell, C.: Investigation on parametrically excited motions of point absorbers in regular waves. Ocean Engineering **111**, 67–81 (2016). DOI 10.1016/j.oceaneng.2015.10.041
24. WAMIT, I.: WAMIT User Manual (2019). DOI 10.1017/CBO9781107415324.004
25. Wolgamot, H.A., Fitzgerald, C.J.: Nonlinear hydrodynamic and real fluid effects on wave energy converters. Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy **229**(7), 772–794 (2015). DOI 10.1177/0957650915570351
26. Zou, S.S., Abdelkhalik, O., Robinett, R., Korde, U., Bacelli, G., Wilson, D., Coe, R., Korde, U.: Model Predictive Control of parametric excited pitch-surge modes in wave energy converters. International Journal of Marine Energy **19**, 32–46 (2017). DOI 10.1016/j.ijome.2017.05.002