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On a Time Domain Calderón Preconditioned CFIE Discretized with Convolution Quadratures

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***Abstract*—Several standard integral equations in the time domain suffer from at least one of the following limitations: 1) conditioning breakdowns, 2) internal resonances, or 3) DC-instabilities. The standard time domain combined field integral equation (CFIE) being no exception, is plagued by the large time step and dense discretization breakdowns. Calderón preconditioning strategies are commonly proposed to address these issues in the frequency domain but, to this day, they were not available for convolution quadrature CFIEs. This work will fill this gap proposing a new Calderón approach leading to a well-conditioned and resonant-free convolution quadrature discretized equation.**

I. Introduction

Integral formulations are widely used for the simulation of electromagnetic scattering from a perfectly electrically conducting object. Among them, the time domain combined field integral equation (CFIE) is particularly attractive being free from spurious internal resonances. Its effectiveness notwithstanding however, this equations is plagued by severe ill-conditioning for increasingly refined meshes and/or when large time steps are used. In the frequency domain, Calderón preconditioning approaches are used to cure some conditioning issues, but they are known to require special care not to introduce spurious resonances, something well mastered in the frequency domain, but not available for convolution quadrature time domain schemes. This work fills this gap by proposing the first available Calderón CFIE formulation for convolution quadrature methods [1]. One of the challenges to be solved was a suitable counterpart of the frequency domain localization to suppress Calderón resonances. In our time domain context we have achieved this by introducing a lossy term in the Laplace parameter and obtaining the resulting electric operator with a suitable choice of discretization strategies. Theoretical developments are complemented by numerical experiments showing the practical relevance of the new formulation.

II. Background and notations

This work addresses the problem of the time-domain scattering by a perfectly electrically conducting object in free space. The current $\mathbf{j}_\Gamma$ induced by the incident electromagnetic fields $(\mathbf{e}^{\text{inc}}, \mathbf{h}^{\text{inc}})(\mathbf{r}, t)$ at the surface of the scatterer Γ can be evaluated by solving the combined field integral equation (CFIE), the linear combination of the electric field integral equation and the magnetic field integral equation,

$$\mathcal{C}\mathbf{j}_\Gamma(\mathbf{r},t) = \frac{\alpha}{\eta_0}\hat{\mathbf{n}}\times\mathbf{e}^{\text{inc}}(\mathbf{r},t) + (1-\alpha)\,\hat{\mathbf{n}}\times\hat{\mathbf{n}}\times\mathbf{h}^{\text{inc}}(\mathbf{r},t)\,, \quad (1)$$

$$\mathcal{C} = -\alpha\mathcal{T} + (1-\alpha)\,\hat{\mathbf{n}}\times\mathcal{M}, \quad (2)$$

where $\mathcal{T}$ is the electric field integral operator (EFIO), $\mathcal{M}$ is the magnetic field integral operator (MFIO), η_0 is the characteristic impedance of the background, $\hat{\mathbf{n}}$ is the outpointing normal of Γ and $\alpha \in [0;1]$ [2].

In this work, the Rao-Wilton-Glisson (RWG) basis functions $(\mathbf{f}_n^{\text{rwg}})_{N_s}$ discretized the current whereas rotated RWG $(\hat{\mathbf{n}}\times\mathbf{f}_n^{\text{rwg}})_{N_s}$ test the EFIO and rotated Buffa-Christiansen (BC) ones $(\hat{\mathbf{n}}\times\mathbf{f}_n^{\text{bc}})_{N_s}$ test the MFIO. By denoting $\mathcal{L}$ the Laplace transform and δ the time Dirac delta, the Laplace semi-discretized operators are defined as [2]

$$[\widetilde{\mathbf{T}}(s)]_{m,n} = \int_\Gamma \hat{\mathbf{n}}\times\mathbf{f}_m^{\text{rwg}}\mathcal{L}\left(\mathcal{T}\left(\mathbf{f}_n^{\text{rwg}}\delta\right)\right)(s)d\Gamma, \quad (3)$$

$$[\widetilde{\mathbf{M}}(s)]_{m,n} = \int_\Gamma \hat{\mathbf{n}}\times\mathbf{f}_m^{\text{bc}}\mathcal{L}\left(\mathcal{M}\left(\mathbf{f}_n^{\text{rwg}}\delta\right)\right)(s)d\Gamma. \quad (4)$$

The convolution quadrature method applied to (1) generates the following Marching-On-in-Time scheme

$$\mathbf{C}_0\mathbf{J}_n = (1-\alpha)\mathbf{N}\mathbf{G}_m^{-1}\mathbf{H}_n + \frac{\alpha}{\eta_0}\mathbf{E}_n - \sum_{q=0}^{n-1}\mathbf{C}_{n-q}\mathbf{J}_q, \quad (5)$$

where the gram matrix $\mathbf{N}$ and the mixed gram matrix $\mathbf{G}_m$ are inserted in the combined operator to match the discretization and to rotate the magnetic contributions, $(\mathbf{J}_n)$ is the array sequence expansion of the surface current on $(\mathbf{f}_n^{\text{rwg}})_{N_s}$ at any time step, $\mathbf{H}_n$ and $\mathbf{E}_n$ are the array sequences of $\hat{\mathbf{n}}\times\mathbf{h}^{\text{inc}}$ and $\hat{\mathbf{n}}\times\mathbf{e}^{\text{inc}}$ tested with their respective test basis functions and

$$\mathbf{C}_i = \mathcal{Z}^{-1}\Big(z \mapsto -\alpha\widetilde{\mathbf{T}}\left(\mathbf{s}_{\text{cq}}(z)\right) + (1-\alpha)\mathbf{N}\mathbf{G}_m^{-1}\widetilde{\mathbf{M}}\left(\mathbf{s}_{\text{cq}}(z)\right)\Big)_i\,, \quad (6)$$

where $\mathcal{Z}^{-1}$ is the inverse $\mathcal{Z}$-transform and $\mathbf{s}_{\text{cq}}(z)$ is fully determined by the implicit sheme used and the time step Δt.

III. Calderón strategies for the CFIE

The ill-conditioning of the time domain combined field integral equation is inherited from the electric field integral operator $\mathcal{T}$, which can be addressed by leveraging the Calderón identity

$$\mathcal{T}^2 = -\frac{\mathcal{I}}{4} + \mathcal{K}^2, \quad (7)$$

where the operator $\mathcal{K}$ is compact. The new operator $\mathcal{T}^2$ is therefore a second-kind integral operator similarly to the as the MFIO ($\mathcal{M} = \frac{\mathcal{I}}{2} - \mathcal{K}$). As a consequence, provided that an appropriate discretization scheme is used, both operator generate a well-conditioned MOT. A naive idea would be to consider their linear combination

$$-\alpha \mathcal{T}^2 + (1-\alpha)\mathcal{M}, \tag{8}$$

which is a second-kind integral operator too. However $\mathcal{T}^2$ shares some of the resonances of $\mathcal{M}$ and so does their linear combination. This is a well-known limitation that has been overcome in the frequency domain by complexifying the wave number of the operators used as preconditioner [3] . In this work we propose and investigate the convolution quadrature counterpart of such an approach where the Laplace parameter of the left electric operator $\mathrm{s}_{\mathrm{cq}}(z)$ is replaced by a parameter $\mathrm{s}_{\mathrm{m}}(z)$ corresponding to a lossy medium. The new parameter, however, should satisfy the following conditions

$$\forall z \in \mathscr{C}, \quad \mathrm{s}_{\mathrm{m}}(\overline{z}) = \overline{\mathrm{s}_{\mathrm{m}}(z)}, \tag{9}$$

$$\forall z \in \mathscr{C}, (\mathrm{s}_{\mathrm{m}}(z)\mathbf{x} = \lambda \mathbf{x}) \Rightarrow \lambda \in \mathbb{C}^+, \tag{10}$$

where $\mathscr{C}$ is the integration contour of the $\mathcal{Z}^{-1}$-transform. In this study, we choose

$$\mathrm{s}_{\mathrm{m}}(z) = \mathrm{s}_{\mathrm{cq}}(z)\sqrt{1+\frac{\sigma}{\mathrm{s}_{\mathrm{cq}}(z)}}, \tag{11}$$

where $\sigma \in \mathbb{R}^+$. The new regularized operator proposed is

$$-\alpha \mathcal{T}^{\sigma}\mathcal{T} + (1-\alpha)\mathcal{M}. \tag{12}$$

The space discretization of the lossy EFIO employs the BC basis functions and the rotated BC basis functions as source and test functions, respectively,

$$\begin{aligned}[\widetilde{\mathbb{T}}(s)]_{m,n} &= \int_\Gamma \hat{\mathbf{n}} \times \mathbf{f}_m^{\mathrm{bc}} \mathcal{L}\left(\mathcal{T}\left(\mathbf{f}_n^{\mathrm{bc}}\delta\right)\right)(s) d\Gamma, \\ \mathbb{T}_i^\sigma &= \mathcal{Z}^{-1}\left(z \mapsto \widetilde{\mathbb{T}}\left(\mathrm{s}_{\mathrm{m}}(z)\right)\right)_i,\end{aligned} \tag{13}$$

leading to the following MOT

$$\mathbf{Z}_0\mathbf{J}_i = \frac{\alpha}{\eta_0}\sum_{j=0}^{i}\mathbb{T}_j^\sigma \mathbf{G}_m^{-1}\mathbf{E}_{i-j} - (1-\alpha)\mathbf{H}_i - \sum_{j=1}^{i}\mathbf{Z}_j\mathbf{J}_{i-j}, \tag{14}$$

where

$$\mathbf{Z}_i = \mathcal{Z}^{-1}\left(-\frac{\alpha}{\eta_0}\widetilde{\mathbb{T}}(\mathrm{s}_{\mathrm{m}})\mathbf{G}_m^{-1}\widetilde{\mathbf{T}}(\mathrm{s}_{\mathrm{cq}}) + (1-\alpha)\widetilde{\mathbf{M}}(\mathrm{s}_{\mathrm{cq}})\right)_i. \tag{15}$$

IV. Numerical Results

The newly proposed formulation (14) is tested on a sphere of radius $1\,\mathrm{m}$ illuminated by a Gaussian pulse plane wave

$$\mathbf{e}^{\mathrm{inc}}(\mathbf{r},t) = A_0 \exp\left(-\frac{\left(t - \frac{\hat{\mathbf{k}}\cdot\mathbf{r}}{c}\right)^2}{2\sigma_{\mathrm{bw}}^2}\right)\hat{\mathbf{p}}\,, \tag{16}$$

$$\mathbf{h}^{\mathrm{inc}}(\mathbf{r},t) = \frac{1}{\eta_0}\hat{\mathbf{k}} \times \mathbf{e}^{\mathrm{inc}}(\mathbf{r},t)\,, \tag{17}$$

where $\sigma_{\mathrm{bw}} = 6/(2\pi f_{\mathrm{bw}})$, $\hat{\mathbf{p}} = \hat{\mathbf{x}}$, $\hat{\mathbf{k}} = -\hat{\mathbf{z}}$, $A_0 = 1\,\mathrm{V\,m^{-1}}$ and $f_{\mathrm{bw}} = 250\,\mathrm{MHz}$ is the frequency bandwidth, which is chosen to be larger than the first resonant mode of the sphere $131\,\mathrm{MHz}$, this insuring that the resonance-free property of the scheme is verified. Since $\mathrm{s}_{\mathrm{cq}}(z)$ is inversely proportional to Δt, the lossy parameter σ in (11) is chosen equal to $0.1/\Delta t$.

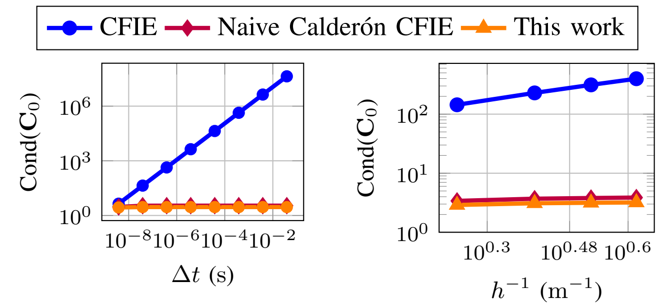

Fig. 1. Condition number with respects to the time step ($Ns = 270$).

Fig. 2. Condition number with respects to the mesh density (Δt=127 ns).

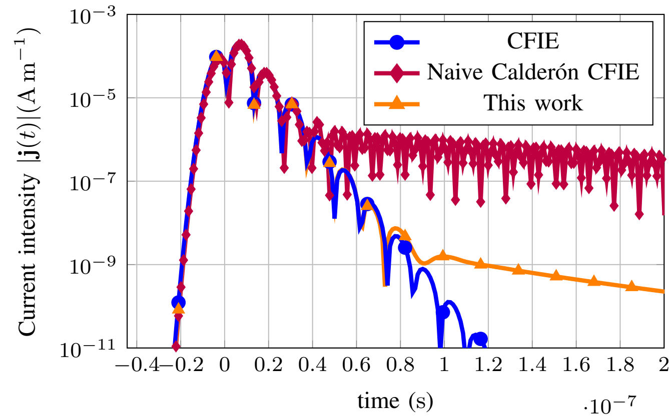

Fig. 3. Evolution in time of the current intensity at a specific point of the sphere with parameters $Ns = 750$ and $\Delta t = 1.15\,\mathrm{ns}$.

The conditioning studies over the sphere Fig. 1 and Fig. 2 show that the proposed formulation shares the same well-conditioning properties than that of a naive Calderón-preconditioned formulation (8), while the standard CFIE formulation (1) suffers from the dense discretization breakdown and the large time step breakdown. However, as expected, the naive Calderón CFIE (8) suffers from non-physical resonances illustrated in Fig. 3. This instability does not affect the CFIE and the proposed formulation. However, the proposed formulation (14) is affected by a semi-DC-instability, the amplitude of which is directly linked to the accuracy of the space discretization. Its analysis and resolution is object of current research and falls outside of the scope of this contribution.

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