How students view the difficulty of mathematical tasks: factors that influence their perceptions

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Camilla Spagnolo[1] and Marta Saccoletto[2]

[1]Free University of Bozen/Bolzano, Italy, Camilla.Spagnolo@unibz.it

[2]University of Turin, Italy

This paper presents the first results of a qualitative study aimed at characterizing perceived difficulty. The purpose is to identify factors that could possibly influence (grades 9 and 10) students' perceived difficulty in any task. While the literature extensively discusses factors that influence task difficulty objectively, this study focuses on how students perceive it, by means of questionnaires. In this work, through the analysis of specific questions, we highlight some differences between students' perceived difficulty related to specific tasks after their resolution (firstly) and in a general sense (secondly).

Keywords: Mathematics education, qualitative study, difficulty perception, affect.

Introduction

The issue of difficulty in mathematics is a broad and fundamental topic in research on Mathematics Education. It involves many aspects that have been extensively discussed in literature, such as mathematical content (Radmehr & Drake, 2017) or text comprehension (Spagnolo et al., 2021). Several studies have highlighted some of the reasons behind students' difficulty in mathematics related to solving mathematics tasks. For example, in their research, Bolondi and colleagues (2018) explore the impact of different text variations on students' performance in mathematical tasks, suggesting that the wording of the task text can affect the difficulty of the task. Their findings indicate that task formulation is not universally better or worse for everyone, but can influence students' performance. The complexity of mathematical tasks can be also linked to the nature of the question, such as its numerical magnitude complexity (e.g. Thevenot & Oakhill, 2005; De Corte et al., 1988). To improve children's performance, it has been suggested that rewording the problem text can be effective, particularly among younger children, and the difficulties associated with rewording do not necessarily depend on the length of the resulting text (Vicente et al., 2007). The role of affective factors is also considered when investigating and interpreting students' behaviours and difficulties on mathematical problem-solving (Zan et al., 2006; McLeod, 1989). This study considers students' perspectives about the difficulty of a mathematical task. We want to examine what, according to them, could affect the difficulty of a mathematical task both in relation to specific tasks after solving them and in general. Little research focuses on this, and, in the field of Mathematics Education, no definition of "perceived difficulty" is being stated. *Difficulties* and *perceived difficulties* after solving task are two different (but closely related) aspects: task peculiarities might influence the student's idea of the task, and they might help to set up his/her *perceived difficulties* (Saccoletto & Spagnolo, 2022, p. 60). What we are interested in here is how students feel about the task, such as whether it is easy or difficult, and what leads them to make that evaluation. To this purpose, a first study discussed extensively in an expanded version of the paper presented at ICME-14 has been conducted (Saccoletto & Spagnolo, 2022), in which we qualitatively investigated factors that influence students' *perceived difficulty* about two specific mathematical tasks, which we recall in the next paragraphs,

after they solved it. In this work, we enlarge the first study, including what students think that *generally* influences, from their point of view, the difficulty of a mathematical task. Aspects that emerged from both analyses will be hence compared, to move toward a deeper and more inclusive investigation of what aspects characterise students' perceptions of difficulty of a mathematical task.

Theoretical and context background

We believe that when a student is confronted with a mathematical task, their perception of the task may be influenced not only by task-related factors, as shown in previous studies (Bolondi et al., 2018), but also by affective factors such as beliefs, emotions, and attitudes (McLeod, 1992; Zan et al., 2006). Among the works that address the need to develop theoretical frameworks on affect, we refer particularly to the study of Di Martino and Zan (2010) on attitude. Their study analysed 1600 essays written by Italian students from different grades, describing their experiences with mathematics, and we found some similarities with our own study in the way we analysed our students' answers. Their study identifies three categories of affective factors related to mathematics. The first category, called Emotional Dimensions, includes the emotional disposition of students towards mathematics, as well as their explicit emotional experiences related to the subject. This dimension can be positive or negative. The second category, Perceived Competence, is characterized by expressions such as "I succeed/fail in mathematics", "I understand/don't understand mathematics", and "I get good/bad marks in mathematics" (Di Martino & Zan, 2010 p. 38). This dimension can be characterized as high or low. The third category, Vision of Mathematics, is related to students' beliefs about what they need to do to be successful in mathematics, for example in terms of memory and recall. They observed that some students view mathematics as a set of rules to be memorized. This model was useful for better interpreting our results, as will emerge from the Discussion.

The answers provided by the students also suggest that metacognitive factors play an important role in problem-solving. Metacognition involves the decision-making process that problem solvers go through when choosing from different cognitive strategies to find solutions. These decisions are influenced by personal beliefs and values about learning and problem-solving, which are important in the acquisition and retrieval of knowledge (Radmehr & Drake, 2017). In particular, metacognitive experience is defined as "what the person is aware of and what she or he feels when coming across a task and processing the information related to it" (Efklides, 2008, p.279). Metacognitive experiences also include judgement of learning, estimation about effort and time that is needed and spent on the task, as well as estimating the correctness of the solution. Metacognitive experiences affect the decisions which students make in learning situations regarding effort allocation, time investment or strategy use (Efklides, 2008).

Research aim

This study broadly aims to delineate some of the aspects that characterize perceived difficulty regardless of the task. In this paper we focus particularly on the differences that have emerged between students' difficulty perception after solving specific tasks and difficulty perception in a more general sense.

RQ1. Investigating students' perceived difficulties without referring to a specific mathematical task, which factors that influence students' perceived difficulty of mathematical tasks arise?

RQ2. What are the differences between the factors influencing students' perceived difficulty with mathematical tasks after solving them (previous study) and those that arose in response to RQ1?

For this purpose, we did a preliminary qualitative investigation focusing on two questionnaires administered to students in grades 9 and 10. Both questionnaires investigated students' perceived difficulty. Firstly, students were administered a questionnaire with questions related to 2 mathematical tasks, which would explicit their perceived difficulty in relation to these two tasks. Specifically, we asked them to indicate a level of difficulty (from 1 to 10) and to express any difficulties they encountered while solving the two chosen tasks (Saccoletto & Spagnolo, 2022). We then constructed categories. The method we used to construct them is inductive: the categories of analysis were constructed by reasoning from the specific to the whole and focusing on the particular rather than the general. We based our conclusions on the database of protocols (students' responses consisted of rich descriptive data). We then administered the second questionnaire to the same students, with the general objective of having students make explicit the elements or characteristics that make a mathematical task difficult. The results were again categorized and we examined whether the categories constructed on the specific cases worked in the general case.

Methods

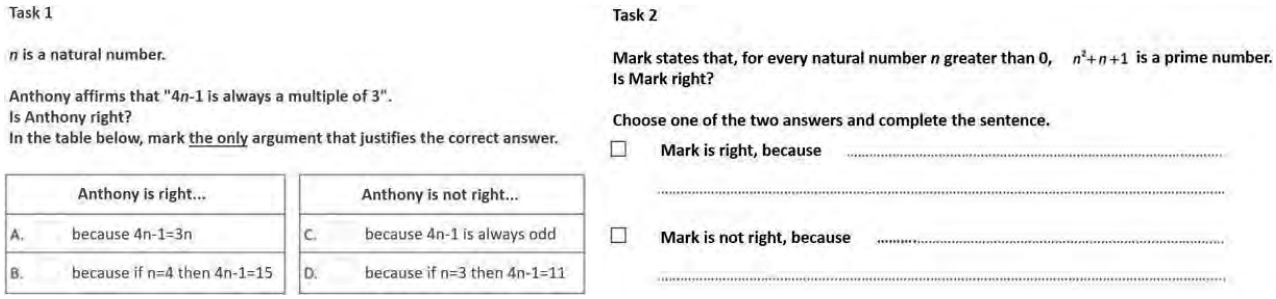
The study is carried out in two qualitative phases. Both phases included a first part of protocol collection (by protocols we refer to the students' answers given to a questionnaire) and a subsequent phase of analysis. The second phase was carried out with the specific aim of highlighting some differences linking perceived difficulty in relation to mathematics tasks after their resolution and perceived difficulty of students in a general sense. The discussion of the results will focus on this second phase.

Sample and questionnaire

The experiment involved 148 students: two grade 9 classes and two grade 10 classes (from a Humanistic school curriculum called "Scienze Umane" in Italy) and three grade 9 classes (from a Scientific school curriculum (called "Istituto tecnico" in Italy). Both questionnaires were administered during regular school activities. Students filled out the questionnaire through Google forms (using classroom computers), and a researcher was present during the administration. The sample chosen for the experimentation on specific tasks (represented in Figure 1) consisted of students in grade 9 (who were attending the last part of the school year) and grade 10 (who were attending the first part of the school year) because the tasks chosen were designed for those school levels. The experimentation carried out in general took place in the last part of the school year and therefore students of grade 9 were involved in order to make the results of the questionnaires (of both experimentations) comparable.

The first questionnaire referred to two specific mathematics tasks (represented in Figure 1). We asked the students to solve the tasks and, for each one, to respond to specific questions related to the perceived difficulties. The second questionnaire aimed to investigate non-cognitive factors. In particular, we investigated attitudes and beliefs such as negative or positive attitudes towards mathematics to bring out explicitly aspects related to perceived difficulty in general. The difference between questionnaires is that the first questionnaire investigates students' perceived difficulty

regarding two specific mathematical tasks, while the second questionnaire is not related to specific tasks but investigates perceived difficulty in general.



Task 1

n is a natural number.

Anthony affirms that "4*n*-1 is always a multiple of 3".
Is Anthony right?
In the table below, mark the only argument that justifies the correct answer.

Anthony is right...		Anthony is not right...	
A.	because 4n-1=3n	C.	because 4n-1 is always odd
B.	because if n=4 then 4n-1=15	D.	because if n=3 then 4n-1=11

Task 2

Mark states that, for every natural number *n* greater than 0, n^2+n+1 is a prime number.
Is Mark right?

Choose one of the two answers and complete the sentence.

☐ Mark is right, because .....

.....

☐ Mark is not right, because .....

.....

Figure 1: Task 1 administered to Grade 08 Italian students by INVALSI in 2017 and task 2 administered to Grade 10 Italian students by INVALSI in 2014, www.gestinv.it

For the first questionnaire we choose two mathematics INVALSI tasks, since they are statistically validated. In the Italian context, we have the possibility to track some students' difficulties over time thanks to INVALSI tests (tests with the purpose of measuring students' levels of competence in relation to the Italian curricular Guidelines) which were administered since in 2008 in grades 2, 5, 8, 10 and 13 from the National Institute for the Evaluation of the Educational System. Therefore, this quantitative validation, provide a useful tool to implement our qualitative analysis. We paid attention to argumentative questions relating to the Numbers area. With the help of the teachers of the classes involved in the experimentation, we selected tasks whose content had already been dealt with. This decision made it possible to exclude that the perception of difficulty was influenced by the fact that the students did not know the topic. The two chosen items involved mathematical similarities and differences. From one hand, task 1 was a multiple-choice task that required recognition of a correct argumentation, while task 2 was an open-ended task that required to produce an argumentation. On the other hand, for both items, the content was related to literal calculation and both tasks could be solved using the same strategy: proving the falsity of a statement through a counterexample. The questions for tasks 1 and 2 are the same and are intended to inquire students' ideas and to link them - in a strictly qualitative way - with students' attitudes, beliefs or peculiar INVALSI items elements. For example, we asked students to evaluate from 1 to 10 the difficulty of task 1, and we asked them to explain why they assigned a specific level of difficulty to Task 1 and Task 2.

Analysis method

The study used an inductive approach to analyse the data, starting from students' answers and working towards a broader understanding of the overall patterns and trends. There are significant regularities in data collection and data analysis procedures that have been useful to construct categories of analysis. Since few qualitative studies about students' perceived difficulty in performing a mathematical task have been carried out (as we clarified in the introduction), we used constructive grounded theory (Charmaz, 1994) as a research method.

Qualitative analysis of results

Analysis of students' answers to the first questionnaire (Saccoletto & Spagnolo, 2022) highlighted several aspects related to perceived difficulty connected to a specific task. For example, many

students explicitly refer to the time that was necessary to solve the tasks, as in the answer "Because I was able to solve it quickly", or to the procedure used to solve the problem, as in the response "Because it does not require complicated calculations". In addition, we found explicit references to emotions, for example, "I get anxious even if there is no grade", or to the fact that they were not sure about the given answer, in fact some students state "I am not sure about the answer". Furthermore, some students refer to previous experiences with similar questions, as in the case of the answer "because it's not the first time I've been asked questions like this". The answers were read several times and the categories were designed and modified gradually.

As a result of this first analysis, we outlined many categories related to the main aspects mentioned by the students in their answers. We then considered these categories and re-read answers to unify, compare and try to address the main aspects these categories referred to. Four macro categories emerged from this analysis: *Resolution strategy*, *Capability and experience*, *Emotions*, and *Task formulation*. In addition, a fifth category groups all those responses where the answer is missing or where students stated "it's just hard" and no reason was given. Some answers were assigned to more than one category.

In "Resolution strategy" (chosen by the 37.8% of the sample) we grouped together those answers in which students explicitly refer to the kind of strategy or process that, in students' view, was needed to solve the problem. The attention is on what students need to do to get the solution.

The second category, "Capabilities and experience" is the most widespread (39% of the sample) and concerns answers that refer to students' perceived capabilities or competence and previous experience that affect their perceived difficulty of the task. This category includes students' answers linked with their personal experiences and familiarity with this kind of task, reinforcing the idea that a problem is easier if it is similar to something already known. Moreover, this category contains answers that refer to what students are (not) able to do or what they (not) know. The focus (core) of these answers is students' self perception, in general, or in referring to these tasks. The category also emphases students who are (not) sure about what they know or did. Finally, this category concerns answers in which students refer to the fact they did (not) solve the task easily or smoothly. These responses usually refer, more or less explicitly, to some obstacles students encountered in tackling the problems, or to the time students invested in solving the task. Generally, the task is perceived easier if students were engaged for a short time or if they reported that they had an insight and that they solved the problem on the first try.

The category "Emotions" refers to the fact that students explicitly consider their emotions in motivating the difficulty level they chose. This category is smaller than the previous ones, chosen only by the 2,9% of the sample. However, we decided to draw this category because these responses presented some peculiar aspects that could hardly be included in the remaining categories.

The fourth category represents considerations about the formulation of the task, in particular with respect to the text. Also this category is very small (4,3% of the sample), but as for the previous one, we think however it shows characteristics that are peculiar and that need to be considered. Finally, we note that these categories are not intended as exclusive, and some answers could be categorised referring to more than one category.

Starting with categories identified from a specific task, we categorized answers to general questions about the perceived difficulty. The protocols considered for analysis are answers to the questions:

- Do you agree with the statement "Mathematics is a difficult subject"? Why?
- Do you remember a mathematics question or task that particularly challenged you? Describe it briefly. If you want, you can also consider experiences from previous school grades.
- According to you, what particular factors or aspects can make a mathematics question difficult?

The analysis highlighted some differences. In particular, some responses led us to create a new category: Personal consideration. This category is understood as a student's personal reflection relative to his or her own success in mathematics. For example, three of the answers to the question "In your opinion, what particular elements or aspects can make a mathematical question difficult?" that we categorized as "Personal consideration" are:

- Student 1: It depends on how much you have studied, on how focused you are or on how much you apply yourself.
- Student 2: The most important thing is to pay attention in class and study if you don't do these things every question can be difficult.
- Student 3: Definitions, because for me it is easier to understand the concept of a formula than to learn that formula by heart.

Thus, we believe that the features that emerged from this qualitative study characterizing students' perceived difficulty in mathematics can be classified into the following 5 categories: *Resolution strategy, Capability and experience, Emotions, Task formulation* and *Personal consideration*.

Discussion and concluding remarks

We have shown in previous research that a student's perceived difficulty does not seem to be related to the ability to answer the question correctly (Saccoletto & Spagnolo, 2022). In this paper we highlight some key features that emerge from the analysis of two qualitative questionnaires. The categories that emerged from the analysis allow us to clarify some of the main aspects involved when a student expresses his or her perceived difficulty in relation to mathematical tasks and in a general sense.

There are obviously methodological differences between our investigation and the rich work by Di Martino and Zan (2010). For example, we chose to ask specific questions, and students' answers were securely shorter. Despite the differences, we think that the categories we identified in our study can be related to attitude construct, as defined by them. As far as the emotional dimension is concerned, the category emerged in analysing the answers to the first questionnaire. In addition, answers to the first and second questionnaire can be related to the vision about mathematics. For example, answering the first questionnaire, students explicitly describe methods that they think are necessary to solve the tasks. Moreover, the category "personal considerations" contains answers that express what students think is necessary to succeed in mathematics. Finally, in our case too, students may more or less

explicitly refer to their perceived competences in solving the chosen task and derive some ideas about their perceived knowledge and skills.

In particular, in assigning a level of perceived difficulty to a task, students seem to be influenced by factors more closely related to the task (such as elements of the text), factors related to their attitude or emotions, and metacognitive aspects (such as lack of ability to judge their own skills, knowledge, and abilities). In making explicit in a general sense aspects related to perceived difficulty, students seem to refer much more often to the text, personal preparation, specific content, or aspects related to the production of explanation or argumentation to support their results. Also completely missing are aspects related to emotions or time, but much more present are personal reflections on how to improve one's performance. We hence answer the research questions.

A1. Investigating students' perceived difficulties without referring to a specific mathematical task, the features that arose from this qualitative study characterizing students' perceived difficulty in mathematics can be classified into these categories: *Resolution strategy, Capability and experience, Task formulation* and *Personal consideration*.

A2. In our previous study (Saccoletto & Spagnolo, 2022), the category Emotions featured highly among the factors influencing students' perceived difficulty in mathematical tasks after solving them. Among those that emerged in response to RQ1, the Emotions category does not emerge, and in addition, some students' answers led us to create a new category: Personal Consideration (a category understood as a student's personal reflection relative to his or her own success in mathematics).

Categories we have defined starting from factors that influence students' perceived difficulty of mathematical tasks are essential to give a definition of perceived difficulty, but we think they should not be the only aspects that characterize it. Further study will help us move in one main direction: to give a *definition* of perceived difficulty. This preliminary study can be developed in several directions: we believe, for example, that it may be interesting to investigate perceived difficulty even before solving the task and relate it to perceived difficulty after solving it. Starting from the results of the qualitative phase, we would like to construct an adaptive questionnaire and to see if it is possible to organize the tasks (more than 2) in order of difficulty. Thus, we wonder whether it is possible to rank the tasks according to the perceived difficulty of the students. In addition, we are preparing a further study to examine the phenomenon from a quantitative perspective. Finally, we think it would be interesting to understand how students' perceived difficulty is related to teachers' perceived difficulty and whether it corresponds.

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