Kinematic modelling of swerve-drive-based mobile robots

Proceedings of the 7th Jc-IFToMM International Symposium (2024)
The 30th Jc-IFToMM Symposium on Theory of Machines and Mechanisms
22-23 June 2024

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Abstract
In the field of service robotics, wheeled mobile robots have a central role in precision agriculture, logistics, healthcare, inspection and maintenance and cleaning. This paper proposes a kinematic model for swerve-drive robots having two or more locomotion units. The kinematics of swerve-drive systems have already been addressed for specific robots, but a general approach is still missing.

Keywords: Kinematic Modelling; Omnidirectional Mobile Robots, Robotics, Swerve-Drive Systems

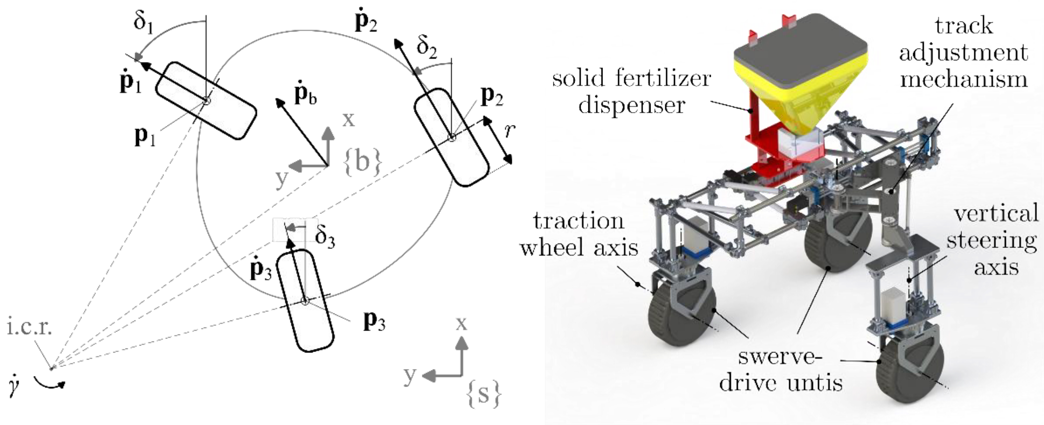
1. Introduction

In the last decades, service robotics has gained a lot of attention from both academia and companies. According to the 2023 Report from the International Federation of Robotics (IFR) [1], around 158 thousand service robots were sold in 2022 for professional use, a 48% increment from 2021. The main application sectors are precision agriculture, logistics, healthcare, inspection and maintenance, cleaning, and defense. Most of these devices are mobile robots able to navigate in different environments and interact with their surroundings. In the field of land-based mobile robots, wheeled mobile robots (WMRs) have a central role in robot locomotion architecture because of their simplicity in design, modelling, construction, and programming. Regarding their locomotion system, WMRs can be classified into the following categories: omnidirectional robots with no steering wheels (e.g., robots based on mecanum and omni-wheels), robots with no steering wheels but either one or several fixed wheels with a common axle (i.e., robots with a differential-drive locomotion system), and robots with no fixed wheels, but at least two independent steering wheels (i.e., swerve-drive based robots). Both in remote-controlled mode and autonomous mode, kinematic models of these locomotion systems are essential to control the robot motions. For this reason, this paper proposes a general kinematic model that can be used to model swerve-drive-based mobile robots with n swerve-drive units ($n \geq 2$). In the literature, the kinematics of swerve-drive systems have already been addressed for specific cases like for platforms with two [2], three [3], and four [4] locomotion units, but a general approach is still missing. In the remainder of this document, the swerve drive systems are called with the acronyms $nSWD$ where n is the number of locomotion units.

2. **Kinematic model of a generic swerve-drive system**

Let's consider a generic swerve-drive system composed of n swerve-drive units, as the one shown in Fig. 1 (**a**). Each swerve-drive unit i is composed of a motorwheel mounted on a vertical steering fork, that is actuated by means of a steering motor. Thus, each swerve-drive unit (also referred to as locomotion unit in this text) has two degrees of actuation: the wheel rotation θ_i around its own axis and the vertical steering angle δ_i. The wheel axis and the steering axis of each swerve-drive unit are incident, i.e. the wheel-ground contact point lies on the steering axis.

The system can be modelled as a rigid body in the plane of motion, that is considered parallel to the perfectly flat and smooth ground. If out of the plane rotations and translations are neglected, the



(**a**) (**b**)

Fig. 1 (**a**) Representation of a generic swerve drive system with $n = 3$ swerve-drive units; (**b**) Render of Agrimaro robot designed for precision agriculture in greenhouses that adopts three swerve-drive units.

body is free to move on the XY plane of the $\{s\}$ reference frame (rf). The pose of the body with respect to (wrt) the fixed-space frame $\{s\}$ is defined by the position vector ${}^{s}\mathbf{p}_b = [x, y, 0]^T$ and the vector of Euler angles (body-*XYZ*) $[0,0,\gamma]$ equivalent to the orientation matrix ${}^{s}\mathbf{R}_b = Rot(z,\gamma)$ that expresses the orientation of frame $\{b\}$ wrt frame $\{s\}$.

The space velocity twist of the system is defined as $\mathbf{V}_b^* = [\omega, \dot{p}_b]^T = [0,0,\dot{\gamma},\dot{x},\dot{y},0]^T$, or in its three-dimensional form $\mathbf{V}_b = [\dot{\gamma},\dot{x},\dot{y}]^T$. The velocity twist and its time derivative can be expressed wrt frame $\{s\}$ or frame $\{b\}$, e.g. ${}^{s}\mathbf{V}_b = {}^{s}[\dot{\gamma},\dot{x},\dot{y}]^T$ and ${}^{b}\mathbf{V}_b = {}^{b}[\dot{\gamma},\dot{x},\dot{y}]^T$. Both ${}^{s}\mathbf{V}_b$ and ${}^{b}\mathbf{V}_b$ refer to the velocity twist of the body relative to the space frame $\{s\}$, but their components are expressed in two different reference frames. In this model, the goal is to derive the relationship between the space velocity twist ${}^{b}\mathbf{V}_b$ expressed relative to the body frame $\{b\}$ and the actuation vector $\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T, \ldots, \mathbf{q}_n^T]^T$, where $\mathbf{q}_i = [\theta_i, \delta_i]^T$.

The position of the center of the generic locomotion unit i wrt the body frame $\{b\}$ is described by the two-dimensional vector $\mathbf{p}_i = [x_i, y_i]^T$. This point is considered the attachment point of the swerve-drive unit to the system chassis. Under the assumption of rigid body, the velocity of the center points of the locomotion unit i can be evaluated as:

$$\begin{bmatrix} \dot{\mathbf{p}}_i \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{p}} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} \times \begin{bmatrix} \mathbf{p}_i \\ 0 \end{bmatrix} \;\Leftrightarrow\; \dot{\boldsymbol{p}}_i = \begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \boldsymbol{V}_b = \boldsymbol{h}_i \boldsymbol{V}_b \tag{1}$$

At the same time, under the assumption of pure-rolling constraint, the velocity of the center points of the locomotion unit i can be evaluated as:

$$\dot{\mathbf{p}}_i = r \begin{bmatrix} c_{\delta_i} \\ s_{\delta_i} \end{bmatrix} \dot{\theta}_i = \boldsymbol{s}_i \dot{\theta}_i \tag{2}$$

where $c_{\delta_i} = \cos(\delta_i)$ and $s_{\delta_i} = \sin(\delta_i)$ and r is the wheel radius. Given the Eq. (1) and Eq. (2), the following relationship can be written:

$$h_i \mathbf{V}_b = \boldsymbol{s}_i \dot{\theta}_i \tag{3}$$

If all locomotion units are considered, the following equation can be written:

$$\begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \\ \vdots \\ \boldsymbol{h}_n \end{bmatrix} \mathbf{V}_b = \begin{bmatrix} \boldsymbol{s}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{s}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{s}_n \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} \quad \Leftrightarrow \quad H\, \mathbf{V}_b = S\, \dot{\boldsymbol{\theta}} = S(\boldsymbol{\delta})\, \dot{\boldsymbol{\theta}} \tag{4}$$

where $\boldsymbol{\delta} = [\delta_1 \quad \delta_2 \quad \cdots \quad \delta_n]^T$. Eq. (4) represents the relationship between the space velocity twist $\mathbf{V}_b$ and the actuation variables vector $\mathbf{q}$. The matrix H depends only on the location of the swerve-drive units wrt the $\{b\}$ frame, while the matrix $S = S(\boldsymbol{\delta})$ is state-dependent. In fact, for swerve-drive systems, it is impossible to derive a linear transformation that maps the actuation vector $\mathbf{q}$ into the velocity twist of the system $\mathbf{V}_b$. The matrix H is a $2n \times 3$ matrix, while the S matrix is a $2n \times n$ matrix.

The system has three degrees of freedom and $2n$ actuation variables. Let's imagine that each locomotion unit has a unique position vector, then for $n \geq 2$ the system has $2n - 3$ actuation redundancy. Therefore, an arbitrary choice of the actuation variables could result in skidding of the wheels. The actuation variables must be chosen on a three-dimensional surface in the $2n$ dimensional actuation space. For this reason, a set of $2n - 3$ constraints must be fullfilled to avoid skidding of the wheels. This set of kinematic constraints are defined by Eq. (5).

$$\begin{gathered} \forall\, i \neq j, \quad \dot{\mathbf{p}}_i^T(\mathbf{p}_i - \mathbf{p}_j) = \dot{\mathbf{p}}_j^T(\mathbf{p}_i - \mathbf{p}_j) \\ \forall\, i \neq j, \quad \left((x_i - x_j)c_{\delta_i} + (y_i - y_j)s_{\delta_i}\right) \omega_i = \left((x_i - x_j)c_{\delta_j} + (y_i - y_j)s_{\delta_j}\right) \omega_j \end{gathered} \tag{5}$$

In other words, the velocity components of the center of two swerve-drive units on the direction that connects the two centers of the two swerve-drive units must be equal, i.e. the assumption of rigid body must be fulfilled.

From Eq. (4), the forward Kinematic equation can be written as:

$$\mathbf{V}_b = H^{\dagger}\, S(\boldsymbol{\delta})\, \dot{\boldsymbol{\theta}} \tag{6}$$

where $H^{\dagger}$ is the pseudo-inverse of the H matrix. Therefore, given the actuation variables $\boldsymbol{\delta}$ and $\dot{\boldsymbol{\theta}}$ it is possible to compute the velocity twist of the system. It is important to underline that the actuation variables $\boldsymbol{\delta}$ and $\dot{\boldsymbol{\theta}}$ must be chosen according to the kinematic constraint (5).

In the same way, the relation between the velocity twist $\mathbf{V}_b$ and the wheels speeds $\dot{\boldsymbol{\theta}}$, can be written as:

$$\dot{\boldsymbol{\theta}} = S^{\dagger}(\boldsymbol{\delta})\, H\, \mathbf{V}_b \tag{7}$$

Nevertheless, Eq. (7) does not represent a useful tool to compute the inverse kinematics of the system, since the matrix S is state dependent. A practical way to solve the inverse kinematics of the system is to consider each locomotion unit independently. Given the desired velocity twist, the velocity $\dot{\mathbf{p}}_i$ of point $\mathbf{p}_i$ can be evaluated through Eq. (3) (i.e., $\dot{\mathbf{p}}_i = [\dot{x}_i, \dot{y}_i] = \mathbf{h}_i\mathbf{V}_b$). The norm of the velocity vector can be computed as $|\dot{\mathbf{p}}_i| = \sqrt{\dot{\mathbf{p}}^T_i\dot{\mathbf{p}}_i}$ and, therefore, the actuation variables associated to the locomotion unit i can be evaluated as:

$$\begin{cases} \delta_i = \operatorname{atan2}(\dot{y}_\mathrm{i}/|\dot{\mathbf{p}}_\mathrm{i}|, \dot{x}_\mathrm{i}/|\dot{\mathbf{p}}_\mathrm{i}|) + k\pi \\ \dot{\theta}_i = (-1)^k\, |\dot{\mathbf{p}}_\mathrm{i}|/\mathrm{r} \end{cases} \quad \text{where } k \in \mathbb{N} \tag{8}$$

Thus, for $\delta_i \in [-\pi,\ \pi]$, there are two possible solutions to the inverse kinematic problem for each swerve-drive system. This is in accordance with the physics of the system: the velocity of the center of the wheel is the same if the wheel turns 180° around the vertical steering axis and it spins in the opposite direction.

3. **Application cases**

Let's consider three swerve-drive systems composed respectively of two, three and four swerve-drive units, as the one shown in Fig. 2.
For a 2SWD system like the one in Fig. 2 (**a**), Eq. (7) can be rewritten in the form:

$$\mathbf{V}_b = r\begin{bmatrix} 0 & 1/w & 0 & -1/w \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} c_{\delta_1} & 0 \\ s_{\delta_1} & 0 \\ 0 & c_{\delta_2} \\ 0 & s_{\delta_2} \end{bmatrix}\dot{\boldsymbol{\theta}} \tag{9}$$

For a 3SWD system like the one in Fig. 2 (**b**), Eq. (7) can be rewritten in the form:

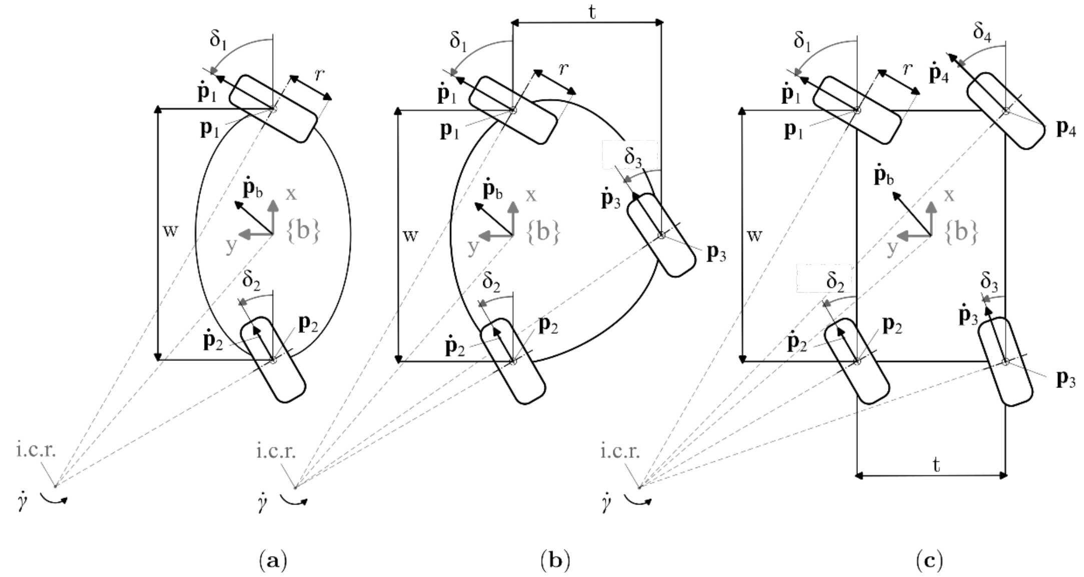


Fig. 2 Representation of three swerve-drive systems composed respectively of two (**a**), three (**b**) and four (**c**) swerve-drive units.

$$\mathbf{V}_b = r\begin{bmatrix} -t/(2\sigma_1) & 3w/(4\sigma_1) & -t/(2\sigma_1) & -3w/(4\sigma_1) & t/\sigma_1 & 0 \\ \sigma_2 & -\,tw/(4\sigma_1) & \sigma_2 & tw/(4\sigma_1) & w^2/(4\sigma_1) & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \end{bmatrix}\begin{bmatrix} c_{\delta_1} & 0 & 0 \\ s_{\delta_1} & 0 & 0 \\ 0 & c_{\delta_2} & 0 \\ 0 & s_{\delta_2} & 0 \\ 0 & 0 & c_{\delta_3} \\ 0 & 0 & s_{\delta_3} \end{bmatrix}\dot{\boldsymbol{\theta}} \tag{10}$$

where $\sigma_1 = t^2 + \frac{3}{4}w^2$ and $\sigma_2 = \frac{2t^2+w^2}{4t^2+3w^2}$.

Finally, for a 4SWD system (Fig. 2 (**c**)), Eq. (7) can be rewritten in the form:

$$\mathbf{V}_b = r\begin{bmatrix} -t\sigma_3 & w\sigma_3 & -t\sigma_3 & -w\,\sigma_3 & t\,\sigma_3 & -w\,\sigma_3 & t\,\sigma_3 & w\,\sigma_3 \\ 1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/4 & 0 \\ 0 & 1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/4 \end{bmatrix}\begin{bmatrix} c_{\delta_1} & 0 & 0 & 0 \\ s_{\delta_1} & 0 & 0 & 0 \\ 0 & c_{\delta_2} & 0 & 0 \\ 0 & s_{\delta_2} & 0 & 0 \\ 0 & 0 & c_{\delta_3} & 0 \\ 0 & 0 & s_{\delta_3} & 0 \\ 0 & 0 & 0 & c_{\delta_4} \\ 0 & 0 & 0 & s_{\delta_4} \end{bmatrix}\dot{\boldsymbol{\theta}} \tag{11}$$

where $\sigma_3 = 1/\left(2(t^2 + w^2)\right)$.

To show the type of motions that can be achieved with these locomotion systems, the mobility of the three swerve-drive systems is illustrated using a sample trajectory. Fig. 3 shows the three systems while performing the sample trajectory, highlighted in purple in Fig. 3. During these simulations, the center of the $\{b\}$ frame (fixed to the moving swerve-drive system) follows a fifth order Bézier curve, while the orientation is computed to be directed towards a fixed point in the plane of motion

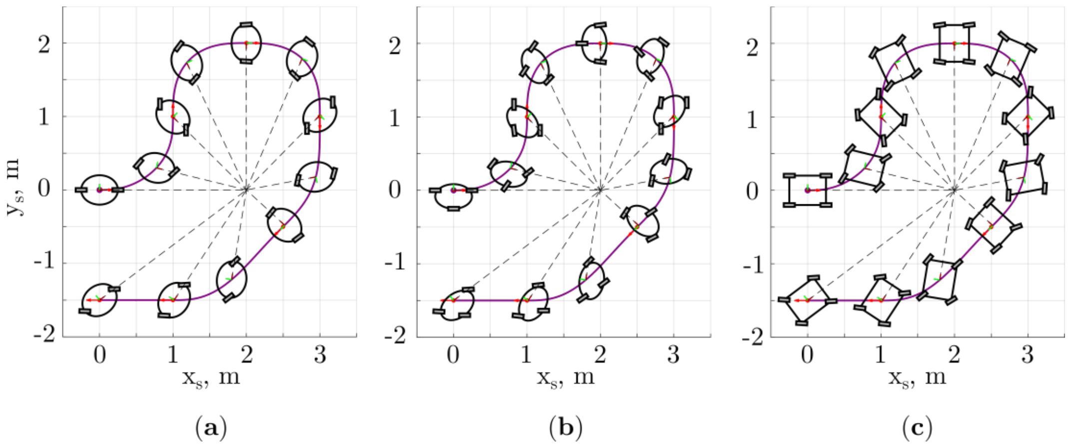


Fig. 3 Sample trajectory followed by (**a**) a 2SWD system, (**b**) a 3SWD system and (**c**) a 4SWD system.

(coordinates ${}^{s}[2,\ 0,\ 0]^{T}$). The pure kinematic simulation is performed in Matlab by providing at each iteration the position and orientation of the $\{b\}$ frame wrt a fixed $\{s\}$ frame. Then, the simulation computes the inverse kinematics of the system to evaluate the actuation variables needed to follow the desired motion.

To follow the sample trajectory, the systems must follow the velocity twist reference shown in Fig. 4 (**a**). As an example, Fig. 4 (**b**) and (**c**) show the computed actuation variables for a 4WD platform

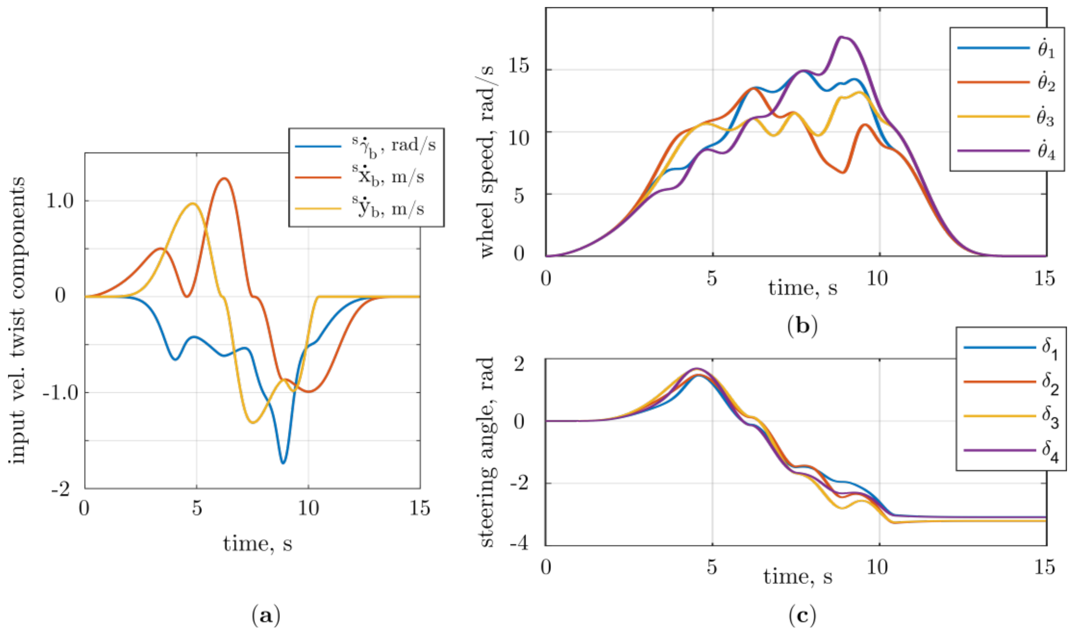


Fig. 4 (**a**) input velocity twist components, (**b**) required wheel speeds and (**c**) steering angles for a 4SWD system following the sample trajectory.

during the execution of the sample motion. In this example, the following geometric parameters have been adopted: $w = 0.5$ m, $t = 0.4$ m, and $r = 0.1$ m (Fig. 2). The actuation variables in Fig. 4 (**b**) and (**c**) have been evaluated using the inverse kinematics model proposed in the previous section.

It should be noted that, given a desired velocity twist of the robot, the inverse kinematics problem has four independent solutions. Usually, the steering angle velocity is limited. For this reason, a useful method to choose the best solution to the inverse kinematic problem is to select the solution that minimizes the steering angles change. Using this approach, given a continuous velocity twist reference, the needed actuation variables are continuous as represented by Fig. 4 (**b**) and (**c**) resulting in a smooth motion of the robot.

4. **Conclusion**

In conclusion, this paper proposes a general approach to derive the kinematics of swerve-drive robots having two or more locomotion units. Both in remote-controlled and autonomous mode, kinematic models of the platform locomotion systems are essential to control the robot motions. The result of this paper can be applied to different platforms to derive their kinematic models, as shown in Section 3 for 2SWD, 3SWD and 4SWD systems.

Acknowledgement

This work was supported by the National Center for the Development of New Technologies in Agriculture (Agritech). https://agritechcenter.it/ (accessed on 13 March 2024) and with the contribution of PIC3SeR Politecnico di Torino Interdepartmental Center for Service Robotics (https://pic4ser.polito.it/, accessed on 13 March 2024).

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