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RF Beamforming and Subcarrier Allocation using Beam Squint in mmWave Systems

Nancy Varshney and Swades De

***Abstract*—In a multi-user wideband millimeter wave (mmWave) communication system, the existing works optimize the hybrid precoder by assuming a priori subcarrier allocation. However, the beam squint effect in mmWave affects the gain over the subcarriers differently for different beam steer directions. Thus, the design of radio frequency (RF) precoder and subcarrier allocation are intertwined. Based on this observation, in this letter we propose a sub-array hybrid precoder design wherein we jointly estimate the RF precoder and subcarrier allocation and then design the baseband precoder. Our numerical results demonstrate the benefits of the proposed strategy compared to the existing approach wherein subcarrier allocation precedes the RF and baseband precoder design.**

***Keywords:* Beam squint, millimeter wave (mmWave), RF precoder, sub-array hybrid precoder, subcarrier allocation**

I. INTRODUCTION

Recently hybrid beamforming at millimeter waves (mmWaves) has gained a lot of attention. Hybrid beamforming allows spatial channel reuse with a small number of radio frequency (RF) chains. However, in wideband mmWave channel, the deployment of a large number of antenna elements to form narrow spatial beams leads to beam squint effect. Different antennas receive time-delayed copies of the same signal that give rise to beam squint in frequency domain wideband channels, i.e., different subcarriers (SCs) of the orthogonal frequency division multiple (OFDM) symbol see different angle-of-arrival (AoA) for the same path. There are two broad hybrid precoder architectures: fully-connected and partially-connected (or sub-array) hybrid precoder. Though fully connected architecture offers improved performance, its control overhead and complexity resulting from channel estimation and precoder design are high [1]. Therefore, this paper focuses on a sub-array hybrid beamformer design over a wideband mmWave using OFDM in a multi-user scenario.

Several works considered mmWave channel employing OFDM in designing hybrid beamforming precoder, but they did not consider beam squinting. In [2], RF and baseband (BB) precoders were designed for wideband channels using OFDM without factoring beam squint effect, i.e., it considered the array response to be frequency independent. Furthermore, most of the hybrid precoder design optimization works focus on optimizing the BB precoder to reduce inter-user interference and number of RF chains. The authors in [3] studied the effect of beam squint on channel estimation with OFDM blocks and used zero-forcing BB precoder to mitigate beam squinting. Very little consideration has been given to selecting an RF precoder that accounts for beam squint. It is not possible to have a frequency-dependent steering weight vector. Hence, the RF precoder is the same over the whole bandwidth. However, because mmWave communication relies heavily on exact beam alignment, beam squint causes significant performance loss.

In literature, RF precoder is either designed using instantaneous channel state information (CSI) or channel's second-order characteristics [4], [5]. In [4] the authors designed dynamic hybrid beamformers for wideband mmWave channel with beam squint, assuming a priori knowledge of SC allocation to the users, whereas RF precoder was designed using instantaneous channel information. Similarly, in [6], [7], hybrid precoder designs for a multi-user system were proposed in which RF precoder was designed based on the channel covariance matrix, assuming prior SC allocation. We denote this as the SC-RF-BB approach. This approach assumes that the users' CSI is estimated in the beam training phase, and based on this CSI, the gNodeB (gNB) performs SC allocation. Consequently, using this effective CSI over all SCs, the RF and BB precoders are designed for the data transmission phase.

However, the SC-RF-BB approach is not optimal because of the following reasons. Suppose one sub-array of a hybrid precoder generates a beam of width φ steered at angle θ covering multiple users. Due to beam squint, the maximum array gain at different SCs occurs at different angles. For instance, at center frequency maximum gain is at θ while at n^{th} SC it occurs at $\theta + \Delta\theta_n$, as shown in Fig. 1. In this case, allocating the SC to a user with AoA closer to $\theta + \Delta\theta_n$ is more justified as it will deliver a higher gain. Thus, if the steering angle changes, then the SC allocation scheme will also vary. Accordingly, the selection of RF weight vector will affect SC allocation. Additionally, the beamwidths during beam training and data transmission phases are not the same. As evident from Fig.1, the beam squint effect varies with beamwidth. In the beam training phase, a beam of width φ^t estimates the users' CSI over different SCs. When $\varphi^t \neq \varphi$ estimating a prior SC allocation policy before designing the RF precoder for data transmission phase will vary the effective channel at each SC, affecting performance. Further, to achieve some threshold quality-of-service (QoS) and fairness in multi-user scenarios, especially when the number of users is higher than the number of RF chains, the SC-RF-BB approach cannot take advantage of channel variations over SCs to meet the desired QoS level. Again, this is because the RF precoder is designed based on second-order channel statistics, AoA, and channel correlation.

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To this end, in this paper we propose a novel approach wherein we jointly estimate the RF precoder and SC allocation policy leveraging beam squinting effect to maximize the net throughout. After that, the BB precoder is designed for minimizing inter-beam interference. We denote this approach as the RF-SC-BB approach. To cater to multiple users, the majority of the research works to date considered serving single user over the wideband mmWave channel per RF unit while time multiplexing the co-located users in the group [4], [7], [8]. Therefore, the problem of SC allocation does not arise. However, when the user population is higher than the total number of RF chains, it was shown in [9] that it is optimal to group and serve multiple users over OFDM per beam. In such a scenario, as we will demonstrate in this paper, our proposed RF-SC-BB approach outperforms the existing SC-RF-BB approach. *Notations:* In the paper notations $|\mathbf{a}|$ and $|\mathcal{A}|$ represent absolute value of vector $\mathbf{a}$ and size of set $\mathcal{A}$, respectively. $\mathbf{a}^*$ and $\mathbf{a}^H$, respectively, represent optimal value and conjugate-transpose of vector $\mathbf{a}$. $\bigcup$ represents union.

II. System Model

We consider a multiuser scenario of M users distributed in a 2-dimensional space using Cox process with angular spread σ_{spread}. The gNB is equipped with sub-array hybrid precoder having N_{RF} RF chains connected to a uniform linear array consisting of N_t antenna elements. Each sub-array is connected to a non-overlapping set of N_t/N_{RF} antenna elements generating one independent beam of width φ. The users are grouped into N_{RF} groups by k-means clustering. Users in a group are served using OFDMA by a single RF chain connected to a sub-array. We assume that each user has a single antenna. Let $\mathcal{K}_j$ denote the set of users located in coverage area of beam $j = \{1, \dots, N_{RF}\}$. The mmWave channel is divided into N_c SCs.

A. Channel model

At mmWaves, either a line-of-sight (LoS) link or non-LoS (NLoS) link exists between user and gNB. The probability of k^{th} user located at distance d_k from gNB having a LoS link is $\Pr(d_k) = \min\{\bar{d}_1/d_k, 1\}(1 - e^{-d_k/\bar{d}_2}\} + e^{-d_k/\bar{d}_2}$, where $\bar{d}_1 = 18$ and $\bar{d}_2 = 36$ [10]. Then, the link path losses are

$$\begin{aligned} \mathrm{PL}_{LoS}(d_k) &= a_1 + 10m_1\log_{10}(d_k) + \mathcal{N}(0,\sigma_1^2)\ \text{[dB]} \\ \mathrm{PL}_{NLoS}(d_k) &= a_2 + 10m_2\log_{10}(d_k) + \mathcal{N}(0,\sigma_2^2)\ \text{[dB]} \end{aligned} \tag{1}$$

Here, a_1 and a_2 are frequency dependent constants, m_1 and m_2 are path loss exponents, and σ_1^2 and σ_2^2 are shadowing variances respectively for LoS and NLoS links. The path loss in linear scale is expressed as

$$\mathrm{PL}_k = \begin{cases} \Pr(d_k)10^{\mathrm{PL}_{LoS}(d_k)/10}, & \text{LoS} \\ (1-\Pr(d_k))10^{\mathrm{PL}_{NLoS}(d_k)/10}, & \text{NLoS}. \end{cases} \tag{2}$$

The user-gNB channel has $L_k \ll N_t$ multipath components (MPCs). Therefore, the channel between the k^{th} user and the gNB over the n^{th} SC at frequency f_n is represented as

$$\mathbf{h}_k[n] = \sqrt{\frac{\mathrm{PL}_k}{L}}\sum_{l=1}^{L}\alpha_{k,l}e^{-j2\pi\tau_{k,l}f_n}\mathbf{r_T}(f_n,\phi_{k,l})^H \in \mathbb{C}^{1\times N_t} \tag{3}$$

where $\alpha_{k,l}$, $\tau_{k,l}$, and $\phi_{k,l}$ are the small scale channel fading, delay, and angel-of-arrival (AoA) of l^{th} MPC of k^{th} user. The gNB array response vector $\mathbf{r_T}(f_n, \phi_{k,l})$ at an offset angle $\phi_{k,l}$ is expressed as

$$\mathbf{r_T}(f_n,\phi_{k,l}) = \frac{1}{\sqrt{N_t}}\left[1, e^{\frac{-j2\pi f_n d'\sin\phi_{k,l}}{\lambda_c f_c}}, \dots, e^{\frac{-j2\pi f_n d'(N_t-1)\sin\phi_{k,l}}{\lambda_c f_c}}\right]^T \tag{4}$$

where f_n/f_c is the beam squint factor at frequency f_n, d' is the inter element ULA spacing, and λ_c is the carrier wavelength. For simplicity, we assume perfect channel knowledge.

B. Precoder

The sub-array hybrid precoder in the data transmission phase is composed of RF and BB precoder. In each sub-array, the same RF beamforming weight vector is applied at all the SCs. The analog precoding matrix $\mathbf{A}_{RF}$ of the sub-array hybrid structure is of the form

$$\mathbf{A}_{RF} = \{\mathbf{a}_1, \dots, \mathbf{a}_{N_{RF}}\} \in \mathbb{C}^{N_t\times N_{RF}}. \tag{5}$$

Here $\mathbf{a}_j \in \mathbb{C}^{N_t\times 1}$ is the RF weight vector of j^{th} sub-array. In general the beam's half power beamwidth is $\varphi \approx 360/(\pi N_t/N_{RF})$ (degrees). The phase shifters have discrete phases that are controlled by b bits and decide the steering direction. Therefore, the possible steering directions (θ) are uniform over the 2π range from the set $\Theta = \{2\pi i/2^b | i = 0, \dots, 2^b - 1\}$. The resulting RF weight vector of the j^{th} sub-array steered at angle $\theta_j \in \Theta$ for all frequencies is

$$\mathbf{a}_j = \begin{cases} e^{-j\frac{2\pi}{\lambda_c}d'(m-1)\sin\theta_j} & \forall m \in [(j-1)\frac{N_t}{N_{RF}} + 1 : j\frac{N_t}{N_{RF}}], \\ 0 & \text{otherwise}. \end{cases} \tag{6}$$

Due to beam squint, each SC experiences different gains for the same RF precoder. Therefore, the effective gain experienced by a user at frequency f_n situated at an offset angle ϕ from a beam steered at an angle θ is [11]

$$\begin{aligned} G(\phi,\theta,f_n) &= \left|\mathbf{r_t}(f_n,\phi)^H\mathbf{a}_j(\theta)\right|^2 \\ &= \left|\frac{1}{\sqrt{N_t}}\sum_{m=1}^{N_t} e^{-j\frac{2\pi}{\lambda_c}d'(m-1)(\frac{f_n}{f_c}\sin\phi+\sin\theta)}\right|^2. \end{aligned} \tag{7}$$

As shown in Fig. 1, the beam gains fall off more sharply in narrower beams (higher N_t) and hence, are more affected by the beam squinting effect.

In a sub-array hybrid precoder each RF unit transmits one data stream. Thus, over n^{th} SC the j^{th} RF unit transmits

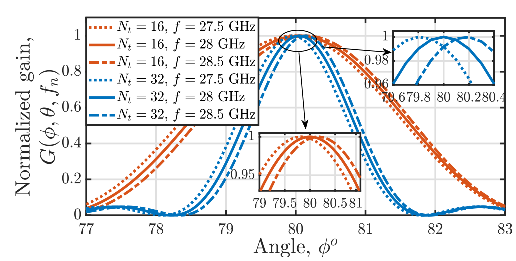

Figure 1: Illustration of beam squinting effect at 28 GHz with $N_{RF} = 1$ connected to N_t antennas and steered at 80^o.

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signal $x_j[n]$ only. At baseband, $x_j[n]$ is multiplied by BB beamforming weight $d_j[n]$ to reduce inter-beam interference resulting from the sidelobes of the other $N_{RF}-1$ concurrent beams over the n^{th} SC. Let $\mathbf{D}[n]$ denote the BB precoder at n^{th} SC, which is of the form

$$\mathbf{D}[n] = \text{diag}\{d_1[n], \dots, d_{N_{RF}}[n]\} \tag{8}$$

such that $\sum_{n=1}^{N_c} \text{Tr}\{\mathbf{D}[n]\mathbf{D}[n]^H\} \leq P_{max}$, where P_{max} is the maximum transmit power.

C. Achievable rate

On n^{th} SC, the baseband of j^{th} RF unit transmits unit energy signal $x_j[n]$ to user $k \in \mathcal{K}_j$ where $j \in \{1, 2, \dots, N_{RF}\}$. Therefore, let $\mathbf{x}[n] = [x_1[n], x_2[n], \dots x_{N_{RF}}[n]]^T$ be the input signal to the BB precoder. Then, assuming k^{th} user is allocated on n^{th} SC, the received signal is $y_k[n] = \mathbf{h}_k[n]\mathbf{A}_{RF}\mathbf{D}[n]x[n] + m[n]$, where $m[n]$ is noise variable with distribution $\mathcal{N}(0, \sigma^2)$. Therefore, achievable data rate of j^{th} beam on n^{th} SC is

$$R_{j,k}[n] = \log_2\left(1 + \frac{s_{j,k}[n]\,|\mathbf{h}_k[n]\mathbf{a}_j d_j[n]|^2}{\sigma^2 + \sum_{i\neq j} s_{i,k}[n]|\mathbf{h}_k[n]\mathbf{a}_i d_i[n]|^2}\right) \tag{9}$$

where $s_{i,k}[n] \in 0, 1$ is the SC allocation variable. $s_{i,k}[n] = 1$ if user k allocated to SC n by i^{th} RF unit and 0, otherwise.

III. RF-SC-BB Design

A. Problem formulation

We first jointly design the SC allocation and RF precoder, and then design the BB precoder. To ensure fairness to LoS and NLoS users, we proportionally allocate the SCs to all the users in each beam using OFDMA, which is equivalent to maximizing sum log-rate. Thus, the optimization problem is

$$\begin{aligned} \mathcal{P}1: &\max_{\mathbf{s},\mathbf{D},\mathbf{A}_{RF}} \sum_{j=1}^{N_{RF}} \sum_{k\in\mathcal{K}_j} \ln\left(\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]\right) \\ \text{s.t.}\quad & \text{C1: } \sum_{k\in\mathcal{K}_j} s_{j,k}[n] \leq 1, \forall j, n;\ \text{C2: } s_{j,k}[n] \in \{0, 1\},\ \forall j, n, k \\ & \text{C3: } \sum_{n=1}^{N_c} \text{Tr}\{\mathbf{D}[n]\mathbf{D}[n]^H\} \leq P_{max};\ \text{C4: } \mathbf{a}_j \in \Theta. \end{aligned} \tag{10}$$

Constraints C1 and C2 ensure binary SC allocation with no sharing within a beam. Constraint C3 limits maximum power and constraint C4 limits the possible beam steering directions. $\mathbf{s}$ and $\mathbf{D}$ are set of all $s_{j,k}[n]$ and $\mathbf{D}[n]$, respectively. $\mathcal{P}1$ is a mixed integer non-convex optimization problem. Therefore, we decompose it to solve for $(\mathbf{s}, \mathbf{D}, \mathbf{A}_{RF})$. We first fix $\mathbf{D}$ and optimize for $\mathbf{s}$ and $\mathbf{A}_{RF}$ in the following section.

B. RF precoder design and SC allocation

Initially assuming $d_j[n] = P_{max}/(N_c N_{RF})\ \forall n, j$ and zero inter-beam interference, we optimize for $\mathbf{s}$ and $\mathbf{A}_{RF}$. Thus, $\mathcal{P}1$ reduces to

$$\begin{aligned} \mathcal{P}2: \quad &\max_{\mathbf{s},\mathbf{A}_{RF}} \sum_{j=1}^{N_{RF}} \sum_{k\in\mathcal{K}_j} \ln\left(\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]\right) \\ \text{s.t.} \quad & \text{C1, C2, C4.} \end{aligned} \tag{11}$$

We find the RF precoder independently for each sub-array, given $\mathbf{h}_k[n]\ \forall k, n$. Let for j^{th} RF unit $\Phi_j = \{\phi_{k,l} | k \in \mathcal{K}_j\}$ be the set of all possible AoAs from all the users in $k \in \mathcal{K}_j$. Thus, the set of possible steering direction of j^{th} beam is

$$\Theta_j = \{\theta_j\} = \{\underset{\theta\in\Theta}{\text{argmin}}|\theta - \min(\Phi_j)|, \underset{\theta\in\Theta}{\text{argmin}}|\theta - \max(\Phi_j)|\}. \tag{12}$$

Next, for a $\theta_j \in \Theta_j$ we find $\mathbf{s}^*$ by relaxing $s_{j,k}[n]$ to be in interval $[0, 1]$. Then the optimization problem reduces to

$$\mathcal{P}3: \quad \max_{\mathbf{s}} \sum_{k\in\mathcal{K}_j} \ln\left(\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]\right);\ \text{s.t.} \quad \text{C1.} \tag{13}$$

Therefore, the Lagrangian of $\mathcal{P}3$ is

$$\mathcal{L}(\mathbf{s}, \boldsymbol{\lambda}) = \sum_{k=1}^{K} \ln\left(\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]\right) - \sum_{n=1}^{N_c} \lambda_n \left(\sum_{k=1}^{K} s_{j,k}[n] - 1\right) \tag{14}$$

where $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_{N_c}]$ are non-negative Lagrangian multipliers corresponding to C1. Applying Karush-Kuhn-Tucker (KKT) condition we have

$$\frac{\partial\mathcal{L}(\mathbf{s}, \boldsymbol{\lambda})}{\partial s_{j,k}[n]} = \frac{R_{j,k}[n]}{\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]} - \lambda_n \leq 0. \tag{15}$$

Subsequently, in (15) $s_{j,k}[n] = 1$ if the SC n is allocated to user $k \in \mathcal{K}_j$ then, $R_{j,k}[n] \neq 0$ and $R_{j,k}[n]/\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n] - \lambda_n > 0$. Otherwise, $R_{j,k}[n] = 0$ and $R_{j,k}[n]/\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n] - \lambda_n \leq 0$. This implies that SC n is allocated to user k by the following rule

$$k^* = \arg\max_{k\in\mathcal{K}_j} \frac{R_{j,k}[n]}{\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]}. \tag{16}$$

To satisfy the constraint C1, λ_n is set between $R_{j,k}[n]/\sum_{n=1}^{N_c} s_{j,k}[n]R_{j,k}[n]$ and one. Therefore, the optimal precoder design of j^{th} sub-array is obtained as

$$\theta_j^* = \arg\max_{\theta_j\in\Theta_j} \sum_{k\in\mathcal{K}_j} \ln\left(\sum_{n=1}^{N_c} s_{j,k}^*[n]R_{j,k}[n]\right). \tag{17}$$

Remark 1: While designing RF precoder, we aim to maximize throughput by using optimal RF precoder and thereafter select the corresponding optimal SC allocation policy. Further rate improvement is performed by optimizing BB precoder to reduce inter-user interference over each SC in the later stage.

C. BB precoder design

Let $u_{j,k^*}[n]$ be the receive BB beamforming weight of the optimal user k^* allocated over SC n on beam j. With the knowledge of $(\mathbf{A}_{RF}^*, \mathbf{s}^*)$, in this section we design the optimal BB precoders $\mathbf{D}^*$ at gNB and $\mathbf{u} = \{u_{j,k^*}[n]\}\ \forall k, j, n$ at user that maximizes total rate, i.e.,

$$\mathcal{P}4: \quad \max_{\mathbf{u},\mathbf{D}} \sum_{j=1}^{N_{RF}} \sum_{n=1}^{N_c} R_{j,k*}[n], \quad \text{s.t. C3.} \tag{18}$$

We solve $\mathcal{P}4$ by transforming weighted sum-rate maximization into weighted sum-rate mean square error (WMMSE) minimization problem using block coordinate descent optimization. Let $w_{j,k^*}[n]$ and $e_{j,k^*}[n]$, respectively, be the weight

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Algorithm 1 Baseband beamforming

Input: $N_{RF}, \mathbf{s}^*, \mathbf{A}^*_{RF}, \mathbf{h}_k[n]\ \forall k, j, n$
Output: $\mathbf{D}$ and $\mathbf{u}$
Initialize $\mathbf{D}[n] = \mathbf{I}_{N_{RF}} P_{max}/(N_c N_{RF})$
do
  Find $u_{j,k^*}[n]$ using (20) $\forall j, n$
  Find $w'_{j,k^*}[n] \leftarrow w_{j,k^*}[n]$ using (22) $\forall j, n$
  Find μ using (25)
  Find $d_j[n]$ using (24) $\forall j, n$
  Update $w_{j,k^*}[n]$ (22) $\forall j, n$
while $\left|\sum_j \log\det(\sum_n w'_{j,k^*}[n]) - \log\det(\sum_n w_{j,k^*}[n])\right| \leq \epsilon$

and mean-square estimation error of user k^* on n^{th} SC in j^{th} RF unit. Then, using WMMSE $\mathcal{P}4$ transforms to [12]

$$\mathcal{P}5: \min_{\mathbf{w},\mathbf{u},\mathbf{D}} \sum_{j=1}^{N_{RF}} \sum_{n=1}^{N_c} \left(w_{j,k^*}[n] e_{j,k^*}[n] - \log w_{j,k^*}[n]\right);\ \text{s.t. C3.} \tag{19}$$

Fixing $\mathbf{D}$ and minimizing weights w_{j,k^*} leads to general MMSE receiver u_{j,k^*}, which is given by (20) [12]

$$u_{j,k^*}[n] = \frac{\mathbf{h}_k[n]\mathbf{a}_j[n]d_j[n]}{\sigma^2 + \sum_{i\neq j} |\mathbf{h}_k[n]\mathbf{a}_i d_i n]|^2}. \tag{20}$$

Using this receiver, the corresponding mean square error is expressed as

$$\begin{aligned} e_{j,k^*}[n] = &\left|u^H_{j,k^*}[n]\mathbf{h}_{k^*}[n]\mathbf{a}^*_j d_j[n] - 1\right|^2 + \\ &\sum_{i\neq j} \left|u^H_{j,k^*}[n]\mathbf{h}_{k^*}[n]\mathbf{a}^*_i d_i[n]\right|^2 + \sigma^2_{j,k^*}[n] \left|u^H_{j,k^*}[n]\right|^2. \end{aligned} \tag{21}$$

Hence, the weight $w_{j,k^*}[n]$ is found as

$$w_{j,k^*}[n] = e_{j,k^*}[n]^{-1}. \tag{22}$$

Substituting these values in (18) we have the equivalence, as

$$\mathcal{P}6: \max_{\mathbf{D}} \sum_{j=1}^{N_{RF}} \sum_{n=1}^{N_c} \log_2\left(1 + \frac{\left|\mathbf{h}_{k^*}[n]\mathbf{a}^*_j d_j[n]\right|^2}{\sigma^2 + \sum_{i\neq j} |\mathbf{h}_{k^*}[n]\mathbf{a}^*_i d_i[n]|^2}\right); \text{s.t. C3} \tag{23}$$

where $\mathcal{P}6$ is convex in $d_j[n]\ \forall j, n$. Consequently, we solve it using KKT to obtain

$$d^*_j[n] = \frac{w_{j,k^*}[n] u^H_{j,k^*}[n]\mathbf{h}_{k^*}[n]\mathbf{a}^*_j}{\mu + |u^H_{j,k^*}[n]|^2 \sum_i |w_{i,k^*}[n]\mathbf{h}_{k^*}[n]\mathbf{a}^*_i|^2} \tag{24}$$

where μ is the Lagrangian multiplier corresponding to C3. Substituting value of $d^*_j[n]$ in C3, we have

$$\sum_{n=1}^{N_c} \sum_{j=1}^{N_{RF}} \frac{\left|w_{j,k^*}[n] u^H_{j,k^*}[n]\mathbf{h}_{k^*}[n]\mathbf{a}^*_j\right|^2}{\left(\mu + |u^H_{j,k^*}[n]|^2 \sum_i |w_{i,k^*}[n]\mathbf{h}_{k^*}[n]\mathbf{a}^*_i|^2\right)^2} \leq P_{max}. \tag{25}$$

Here, (25) is solved by numerical methods. Further, (25) is a decreasing function of μ, Therefore, if $\sum_{n=1}^{N_c} \text{Tr}\{\mathbf{D}_{\mu=0}[n]\mathbf{D}_{\mu=0}[n]\}^H \leq P_{max}$, then $\mathbf{D}^*[n] = \mathbf{D}_{\mu=0}[n]$; otherwise μ^* is found using bisection search. Algorithm 1 lists the steps to find BB precoder and Algorithm 2 describes the steps for complete joint SC allocation and hybrid precoder estimation.

Algorithm 2 Joint SC allocation and hybrid beamforming

Input: $N_{RF}, M, \mathbf{h}_k[n]\ \forall k, j, n$ and user positions
Output: $\mathbf{s}^*, \mathbf{A}^*_{RF}, \mathbf{D}^*$, and $\mathbf{u}^*$
Group users into N_{RF} groups such that $\bigcup_{j=1}^{N_{RF}} \mathcal{K}_j = M$
for $j = 1$ to N_{RF} **do**
  Initialize $\mathbf{D}[n] = \mathbf{I}_{N_{RF}} P_{max}/(N_c N_{RF})$
  Assume zero inter-beam interference $\Rightarrow \mathbf{u} = \{1\}$
  Find Θ_j using (12)
  for $\theta_j \in \Theta_j$ **do**
    Find $s^*_{j,k}[n]$ and k^* using (16) $\forall n, j$
    Calculate $\sum_{k\in\mathcal{K}_j} \ln\left(\sum_{n=1}^{N_c} s^*_{j,k}[n] R_{j,k}[n]\right)$
  end for
  Find θ^*_j using (17)
end for
Find $\mathbf{D}^*$ and $\mathbf{u}^*$ using steps in Algorithm 1

IV. Complexity Analysis

In the proposed RF-SC-BB scheme, we use the same BB precoder as in SC-RF-BB approach. Therefore, for brevity, we do not compare the complexity of BB precoder for the two approaches. For fair comparison both the approaches are considered to allocate SCs to maximize the sum log-rate using (16) that requires $N_{RF}MN_c$ computations to determine $R_{j,k}[n]$ $\forall j, k, n$, and $|\mathcal{K}_j|$ comparisons to find k^*. In SC-RF-BB approach, SC allocation per beam is performed only once. Next, to find the covariance matrix of $\sum_{n=1}^{N_c} \mathbf{h}_{j,k^*}[n]/N_c$ in beam j, N_t^2 multiplications are required. Further, eigenvalue decomposition of covariance matrix to find RF precoder has worst-case complexity of $\mathcal{O}(N_t^3)$. Thus, the overall complexity of SC-RF-BB approach is $\mathcal{O}1 = \mathcal{O}(2N_{RF}|\Theta| + N_{RF}MN_c + N_c|\mathcal{K}_j| + N_t^2 + N_t^3)$. In contrast, in our proposed RF-SC-BB approach, for j^{th} beam the SC allocation is performed $|\Theta_j|$ times. Also, to compute Θ_j in (12) a maximum of $2|\Theta|$ searches are required. Therefore, the total computational complexity RF-SC-BB approach is $N_{RF}MN_c + \sum_{j=1}^{N_{RF}} |\Theta_j|(N_c|\mathcal{K}_j|)$. In the worst case this can be approximated as $\mathcal{O}(2N_{RF}|\Theta| + N_{RF}MN_c + N_{RF}|\Theta_j||\mathcal{K}_j|N_c) \ll \mathcal{O}1$.

V. Simulation and Results

For simulation setup, 3 set of user clusters are generated over a 60^0 angular area of radius 200 m using Cox process with $\sigma_{spread} = 4^o$ [13]. The mmWave system parameters are $f_c = 28$ GHz, rician channel with parameter 8 dB, $N_{RF} = 3$, $N_t = 96$ (32 per sub-array corresponding to beamwidth $\varphi \approx 3.58^o$), $N_c = 64$, $P_{max} = 1$, $b = 5$, $d' = \lambda/2$, $m_1 = 2, m_2 = 2.92, m_{5GHz} = 2, a_1 = 61.4, a_2 = 72, \sigma_1^2 = 33.64$, and $\sigma_2^2 = 75.69$ [14]. We also study the effect of beam squint at $f_c = 5$ GHz, with parameters $N_t^{5GHz} = 2$, rayleigh channel, path loss exponent $= 2$, and shadowing variance $= 44.36$.

Further, we compare the performance of the proposed RF-SC-BB approach with that of the SC-RF-BF approach. In SC-RF-BB approach, SC allocation in j^{th} beam is performed to maximize sum log-rate based on the effective CSI obtained from the beam training phase, i.e., $\mathbf{h}_k[n]\mathbf{a}^t_j$, where $\mathbf{a}^t_j$ is the RF precoder used during beam training phase by the j^{th} RF unit having beamwidth φ^t. It is notable that the beam training uses beam sweeping by partitioning the area into sectors equal to HPBW of the beam φ^t. Also, the beamwidth

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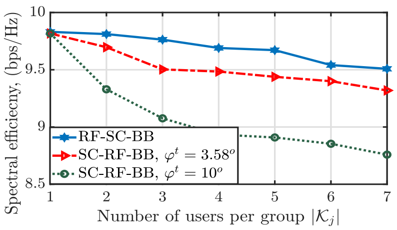

Figure 2: Spectral efficiency comparison of RF-SC-BB and SC-RF-BB [7] approaches at 28 GHz.

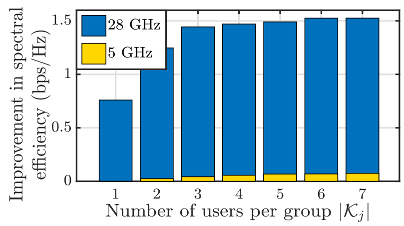

Figure 3: Gain in spectral efficiency using RF-SC-BB over RF-SC-BB with approach at 28 GHz and 5 GHz.

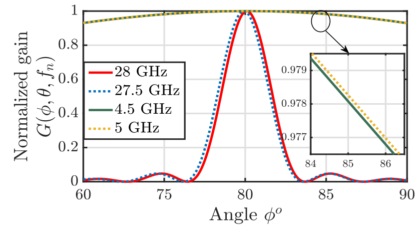

Figure 4: Illustration of beam squinting effects at 5GHz and 28GHz with $N_t^{5GHz} = 2$, $N_t^{28GHz} = 32$, and steering angle $= 80^o$.

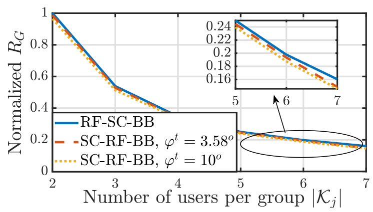

Figure 5: Comparison of normalized geometric mean rate of RF-SC-BB and SC-RF-BB [7] approaches at 28 GHz.

during the data transmission and the beam training is not necessarily the same, i.e, $\varphi \neq \varphi^t$. Therefore, for performance comparison, we consider two scenarios of training beamwidth $\varphi^t = \{3.58^o, 10^o\}$. In SC-RF-BB approach, the RF precoder is designed using eigenvalue decomposition of covariance of the average of CSI $\{\mathbf{h}_{k^*}[n]\}$ over all the SCs [6], [7].

Fig. 2 verifies that the RF-SC-BB approach achieves higher spectral efficiency over the SC-RF-BB approach. Further, it can be observed that in SC-RF-BB approach, spectral efficiency degrades at $\varphi^t = 10^o (\neq \varphi)$ because the SC allocation used in the training phase to design RF precoder leads to degraded output during data transmission phase that has $\varphi = 3.58^o$ leading to different array response and thus, different beam squint effect. The RF-SC-BB approach achieves improved spectral efficiency with sum throughput maximization as well. For brevity, we omit the analysis here.

Fig. 3 illustrates that the difference of spectral efficiency performance of the two schemes is more prominent at higher frequency, thus showing importance of the proposed scheme at mmWaves. Here, $N_{RF} = 1$, $\varphi^{28GHz} = 3.58^o$, and $\varphi^{5GHz} = 60^o$. The attenuation increases with frequency. Therefore, to maintain same power density at cell edge, the antenna array size required at 5 GHz is much smaller than the antenna array at 28 GHz frequency, resulting in broader beams and hence smaller beam squint, as shown in Fig. 4.

Further, we measure the user fairness by geometric mean rate given as $R_G = \left(\prod_{k=1}^{M} R_k\right)^{(1/M)}$, where R_k is the rate of k^{th} user. As shown in Fig. 5 the RF-SC-BB offers improved fairness over SC-RF-BB approach. This is again because the SCs are allocated proportionally depending on the effective channel gain $\mathbf{h}_k[n]\mathbf{a}_j^*$ after optimal RF precoder is estimated.

VI. Conclusion

In this letter, we proposed a joint subcarrier allocation and hybrid beamforming design for a sub-array hybrid beamformer in a multi-user wideband mmWave channel. We emphasized the interlink of RF and SC allocation policy. Furthermore, we demonstrated that, compared to the SC-RF-BB strategy, our suggested RF-SC-BB technique increases spectral efficiency and user fairness. We also showed that the performance difference is more prominent at mmWave than at sub-6 GHz.

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