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# A constitutive framework for the chemo-mechanical behaviour of unsaturated non-expansive clays

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**Abstract.** Both osmotic and matric suction changes have a significant influence on the mechanical behaviour of clays. Despite the different types of interactions at the microstructural level, both suctions have a relevant effect on the fabric of non-expansive clays. Starting from experimental observations at the laboratory scale, it is possible to identify some common features characterizing the mechanical response of non-expansive clays to salinity and degree of saturation changes. This paper presents an elastoplastic framework to reproduce the behaviour of unsaturated clayey soils upon changes in the salt concentration of the pore fluid. In particular, it presents a strategy to include osmotic suction induced by pore fluid salinity in BBM-like models [1]. The model was implemented in the Thebes code and it was calibrated on experimental data performed on Boom clay [2] and remoulded loess [3].

## 1 Introduction

The chemo-hydro-mechanical response of clays is relevant to many geotechnical applications, including slope stability, earth construction and pollution containment systems. Changes in both the degree of saturation and the chemical composition of the pore fluid may have significant effects and lead to volume change and variation of clay permeability, retention properties, compressibility and shear strength. The combined effect of the degree of saturation and pore fluid chemistry should thus be considered for a rational design, especially in the presence of severe safety requirements. In particular, unsaturated clays with salts dissolved in the pore fluid are subjected to both matric suction and osmotic suction, which is related to the molar concentration of the dissolved salt in the pore fluid. Both matric and osmotic suction changes affect the evolution of the clay microstructure, which has been proven to be a fundamental aspect to consider for a reliable prediction of unsaturated clay behaviour. As evidenced by ESEM (Environmental Scanning Electron Microscopy) and MIP (Mercury Intrusion Porosimetry) tests, compacted clay microstructure consists of clay clusters or aggregates. After compaction, this microfabric evolves with water content. In particular, increasing matric suction reduces aggregate size, which in turn may induce an increase in macro-porosity. Many studies also investigated the effect of pore water salinity on the microstructure of saturated clays, concluding that an increase in osmotic suction may also induce an increase in macro-porosity [4]. In expansive clays, this is triggered by the increase in the attractive forces between the faces of clay platelets in the aggregates,

whereas for non-expansive clays a relevant role is played by electrical interactions between the faces and edges of clay particles.

Modelling approaches empirically accounting for microstructure changes have been proven to work well for expansive clays, while less attention has been paid to the hydro-chemo-mechanical behaviour of non-expansive clays. Although in geo-environmental applications both matric and osmotic suction may change simultaneously, experiments on non-expansive clays have been mostly carried out considering separately either osmotic or matric suction changes. The coupled effect of matric and osmotic suction on an illitic clay (Boom clay) has been experimentally investigated, both microscopically and macroscopically, by [2]. In the context of low-activity geomaterials, the joint effect of partial saturation and pore fluid saline concentration has been also investigated for loess [3-5].

Starting from the experimental evidence collected in [6] and [7], this contribution proposes a constitutive framework capable of reproducing the mechanical effects of simultaneous matric and osmotic suction changes. The approach relies on the similarity between the phenomenological effects of matric and osmotic suction on the material mechanical behaviour, which in turn depends on fabric evolution [8]. The paper employs the well-established Barcelona Basic Model (BBM) proposed by [1] to reproduce the macroscopic behaviour of the material due to partial saturation. To replicate the macroscopic behaviour of non-expansive saturated clays subjected to changes in pore fluid salinity, it uses the model presented by [7]. This work aim is to provide a simple strategy to reproduce the chemo-mechanical behaviour of non-expansive unsaturated clays by joining

the BBM and the model proposed by [7], according to the approach presented in [8]. The model, implemented in the Thebes code [9-10], is used to simulate experiments on compacted Boom clay and a remoulded loess, subjected to different stress paths at varying matric and osmotic suctions. The suggested procedure is sufficiently general and can be applied to most constitutive models developed in the framework of elastoplasticity with generalized hardening.

## 2 Constitutive framework

Matric suction is related to capillary forces, acting on solid particles in granular materials and on clusters/aggregates of clay particles. Both soil deposition and compaction may induce an ‘open fabric’ [1], which generally refers to the existence of macrovoids, i.e. voids between the clusters/aggregates of particles. If an unsaturated material is subjected to mechanical loading at constant suction, plastic strains may develop. From a constitutive point of view, the higher the matric suction, the larger the size of the elastic domain, and as a consequence, the position of the Normal Compression Line (NCL) in the compression plane moves to a higher void ratio as the matric suction increases. Therefore, if an unsaturated material with an open fabric is subjected to wetting at constant stress, a reduction in matric suction (i.e. a reduction in capillary forces) makes the current void ratio non-compatible with the applied stress and thus compressive plastic strains develop (wetting induced collapse). Mathematically, the possibility of having plastic strains related to both mechanical loading and suction decrease has been modelled by the Loading-Collapse (LC) curve, as proposed by [1] in the BBM model and then recognised as a fundamental feature in any constitutive model for unsaturated soils. However, it is acknowledged that also pore water salinity influences the fabric of non-expansive clays. The extent of this influence depends on the soil formation process and the electrochemical environment. When deposition of non-expansive clays occurs in salty water, the material tends to have a flocculated (open) fabric, whereas a face-to-face aggregated fabric is formed in freshwater. This has been proven to work also for compacted clays, at least at sufficiently low-stress levels. Recently, [7-11] proposed a link between the fabric of non-expansive clayey materials and their phenomenological behaviour upon chemo-mechanical actions, evidencing the role of the open fabric imparted upon deposition on the following compression behaviour. Upon specimen preparation, a larger osmotic suction conveys the material to a more open fabric. Consistently, the position of the NCL moves to higher void ratios and its slope changes with increasing pore fluid salinity. When an open fabric is created by interaction with a saline fluid and then salinity is decreased (i.e. osmotic suction decreases), compressive irreversible strains are anticipated, in the same way as wetting induces collapse for unsaturated soils. Experimental evidence (see e.g. [6]) proves that a reduction in osmotic suction at a given confining stress may cause irrecoverable volumetric

compression, i.e. desalinization-induced collapse, especially if the material is initially normally consolidated. Vice versa, if the material is strongly overconsolidated (i.e. the fabric is closed), desalinization induces swelling, consistently with the increase of repulsive forces between particles caused by the increase in thickness of the double layer, which is an elastic process. The analogy with matric suction also works for mechanical loading at constant osmotic suction, as its increase induces an increase of the elastic domain [6, 11, 12]. Figures 1 and 2 present two schematics of the effects due to both matric and osmotic suction reduction in clay with an open and a closed fabric.

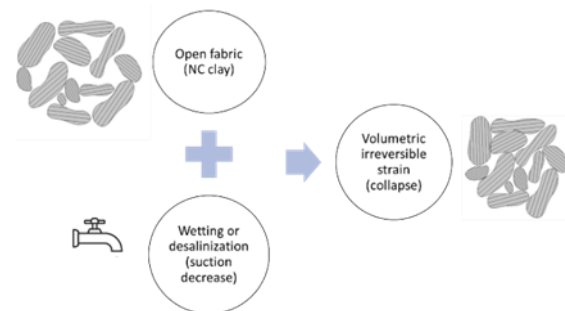


Fig. 1. Schematic of suction reduction effects on clay with open fabric.

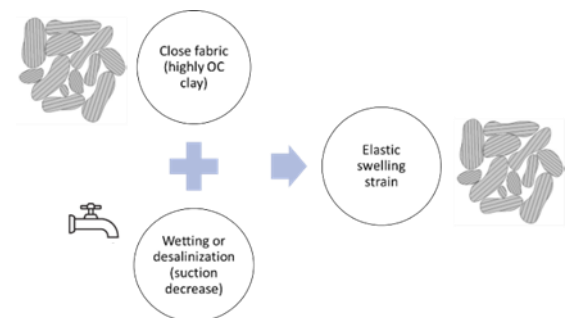


Fig. 2. Schematic of suction reduction effects on clay with closed fabric.

### 2.1 Matric suction effect: Barcelona Basic Model

The classical BBM, developed in the framework of hardening elastoplasticity, uses net stress  $\sigma'' = \sigma - u_a$  and matric suction  $s = u_a - u_w$  as independent variables, where  $\sigma$ ,  $u_w$  and  $u_a$  are the total stress, pore water pressure and pore air pressure, respectively. The elastic changes of specific volume  $v$  are related to the variations of  $p'$  and  $s$  as follows:

$$dv^e = -\kappa \frac{dp''}{p''} - \kappa_s \frac{ds}{s + p_{atm}} \quad (1)$$

where  $\kappa$  is the slope of the elastic swelling line,  $\kappa_s$  is the compressibility coefficient for matric suction changes and  $p_{atm}$  is the atmospheric pressure. As the water content influences the soil fabric, BBM defines the Normal Compression Lines (NCL) for different values of suction in the  $v - \ln(p'')$  plane as:

$$v = N_s - \lambda_s \ln \frac{p''}{p_r} \quad (2)$$

where  $N_s$  is the specific volume for a reference mean stress  $p_r$  and  $\lambda_s$  is the slope of the Normal Compression Line. Both  $N_s$  and  $\lambda_s$  are functions of suction (see Fig. 5). BBM assumes the loading-collapse (LC) yield curve in the  $s - p''$  plane as:

$$p_0 = p_r \cdot \left( \frac{p_0^*}{p_r} \right)^{\frac{\lambda_0 - \kappa}{\lambda_s - \kappa}} \quad (3)$$

where  $p_0$  is the pre-consolidation pressure at matric suction  $s$ ,  $p_0^*$  is the isotropic pre-consolidation pressure at full saturation ( $s=0$ ) and  $\lambda_0$  is the slope of the normal compression line in saturated conditions. Eq. (3) introduces the dependence of the size of the yield surface on suction. In particular, the larger the matric suction, the larger the size of the yield surface.

## 2.2 Osmotic suction effect: the Musso-Scelsi Della Vecchia model

Musso et al. [7] proposed a constitutive model (MSD model in the following) for saturated low to medium-activity clays, based on the phenomenological and experimental evidence regarding the role of fabric in saturated conditions as induced by different salt concentrations. Formulated in the framework of elastoplasticity with generalized hardening, the chemo-mechanical model has been successfully used to reproduce the effects of changes in the applied osmotic suction and stress. The MSD model uses the Terzaghi effective stress  $\sigma'$  and the osmotic suction  $\pi$ , as the environmental variable. Osmotic suction has been set to depend on the chemical activity of the components in solution and, at low concentrations, can be described by the van't Hoff equation [13], as  $\pi = icRT$ , where  $i$  is the number of dissolved species (e.g. 2 for NaCl and 3 for CaCl<sub>2</sub>),  $c$  is the molar concentration of the electrolyte (mol/m<sup>3</sup>),  $R$  is the universal gas constant (8.3144 J mol<sup>-1</sup>K<sup>-1</sup>) and  $T$  is the absolute temperature (K). In this model, the elastic changes of specific volume are related to variations of  $p'$  and  $\pi$  as:

$$dv^e = -\kappa \frac{dp'}{p'} - \kappa_\pi \frac{d\pi}{\pi + \pi_{ref}} \quad (4)$$

where  $\kappa_\pi$  is the elastic compressibility for changes in osmotic suction and  $\pi_{ref}$  is a reference osmotic suction. As the pore fluid chemistry influences the soil fabric, the normal compression line in the semilogarithmic plane  $v - \ln(p')$  depends on the saline concentration:

$$v = N_c - \lambda_c \ln \frac{p'}{p'_{rc}} \quad (5)$$

where the intercept  $N_c$  and slope of the Normal Compression Line  $\lambda_c$  are both functions of the osmotic suction, while  $p'_{rc}$  is a reference pressure (here taken as 1 kPa). The original expression of the yield curve in the  $\pi - p'$  plane can be found in [7], but assuming that the dependence of the volumetric virgin compressibility  $\lambda$

on the pore fluid salinity for non-expansive clays is quite limited (i.e.  $\lambda_c = \lambda_0$ ) and that  $N_0 = N_c$ , the evolution of the size of the yield surface with osmotic suction for an isotropic stress state can be simplified as

$$p'_c = p_0^* \cdot \left( \frac{\pi + \pi_{ref}}{\pi_{ref}} \right)^{\frac{\kappa_\pi}{\lambda_0 - \kappa}} \quad (6)$$

where  $p'_c$  is the preconsolidation pressure corresponding to osmotic suction  $\pi$ . Like in the BBM, plastic strains may develop due to either a reduction of the osmotic suction or a stress change.

## 2.3 Combining matric and osmotic suction effects

For soils where the pore liquid is pure water, the BBM links the saturated preconsolidation pressure  $p_0^*$  to the unsaturated yield mean net stress,  $p_0$ , through Eq. (3). Following the same rationale, the chemo-mechanical model introduces a relation between the preconsolidation pressure for the material saturated with distilled water,  $p_0^*$ , and the yield mean effective stress for material saturated with saline solutions,  $p'_c(\pi)$ , through Eq. (6). Both yield curves in the  $p' - s$  plane and in the  $p' - \pi$  plane plays the role of the Loading Collapse (LC) curve and have similar trends (see Fig. 3). The approach we propose to combine the effect due to the partial saturation with that caused by the salinity in the soils is to add them in terms of Loading Collapse curve. To model the joint effects of pore fluid salinity and partial saturation within a unique model, the osmotic suction is introduced in the BBM model as an equivalent matric suction. Introducing in the BBM changes in the matric suction  $s_\pi$  equivalent to the osmotic suction would cause similar effects to those caused by the osmotic suction changes in the MSD model.

According to the MSD model, when a saline pore fluid is present, the preconsolidation pressure increases from  $p_0^*$  to  $p'_c$ . The equivalent matric suction  $s_\pi$  is the matric suction that would be obtained from the LC curve of the BBM passing from the saturated pre-consolidation pressure,  $p_0^*$ , to  $p'_c$ , as depicted in Figure 3. Imposing that  $p_0 = p'_c$ , the expression of  $\lambda_s(s_\pi)$  can be obtained from Eq. (3):

$$\lambda_s = (\lambda_0 - \kappa) \cdot \frac{\ln \frac{p_0^*}{p_r}}{\ln \frac{p'_c}{p_r}} + \kappa \quad (7)$$

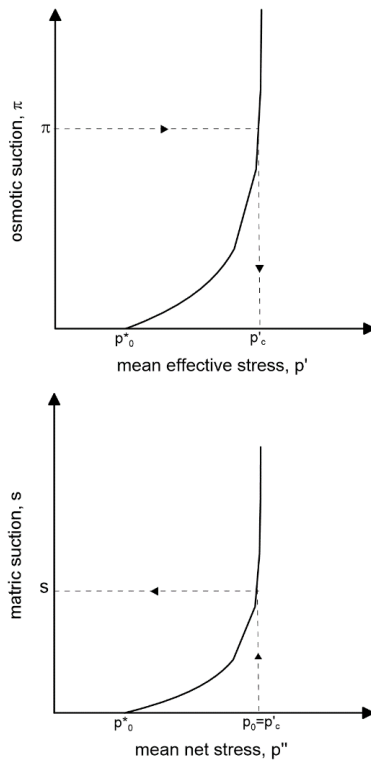
Equivalent matric suction (Eq. 8) is obtained by exploiting the link between the compressibility and matric suction in BBM, modified to reproduce the increase in compressibility for increasing osmotic suction (see, e.g. [14]):

$$s_\pi = -\frac{1}{\beta} \ln \left[ \frac{\lambda_s + \lambda_0(r-2)}{\lambda_0(r-1)} \right] \quad (8)$$

where  $r$  is a parameter already present in the original BBM formulation. When the saline pore fluid is present in an unsaturated soil of given matric suction  $s$ , the

partial saturation and salinity are taken as equivalent in terms of void ratio and fabric. The equivalence of the microstructural effects of the matric and osmotic suction leads to a common Loading Collapse curve for both effects. The BBM is enhanced by introducing an equivalent suction  $s_\psi$ , used in the same way as suction in the original BBM:

$$s_\psi = s + s_\pi \quad (9)$$



**Fig. 3.** Procedure to obtain the equivalent matric suction.

A yield surface for the first-ever chemical loading has been also introduced, to reproduce the plastic volumetric strain related to the change in soil structure upon the first salinization. Therefore, a yield function conceptually similar to the suction increase SI yield locus of the BBM has been introduced, as

$$\pi = \pi_0 \quad (10)$$

The maximum osmotic suction ever experienced by the soil  $\pi_0$  should also be converted to an equivalent matric suction  $s_{\pi_0}$  in order to be introduced to model. Once this yield locus is reached, the plastic volumetric strain increment is evaluated as:

$$d\varepsilon_{vs} = -\frac{\lambda_\pi}{v} \frac{d s_{\pi_0}}{(s_{\pi_0} + p_{atm})} \quad (11)$$

where  $\lambda_\pi$  is the stiffness parameter for changes in (osmotic) suction for virgin states of the soil.

### 3 Model predictions

#### 3.1 Compacted Boom clay

Mokni et al. [2] performed oedometer tests to investigate the hydro-mechanical response of compacted Boom clay (a kaolin-illite clay of low-medium activity) subjected to different matric and osmotic suction. Figures 4 and 5 show the experimental results and model prediction with respect to the distinct effects of matric and osmotic suction. Specimens had been obtained by mixing dry Boom clay powder with saline solutions (4M, 5.4M of NaNO<sub>3</sub>, corresponding to  $\pi=11000$  kPa and 20000 kPa, respectively). After compaction, specimens had been subjected to either  $s=500$  kPa or  $s=0$  kPa (saturated conditions) and finally loaded and unloaded at constant suction. Figures 4 and 5 show the effects of matric suction (Fig. 4) and osmotic suction (Fig. 5) on the compression behaviour of compacted Boom clay. It is evident that the similar fabric effects are also inducing a similar mechanical response in terms of the relative position of the normal compression lines and the preconsolidation pressure increase. Model predictions are satisfactory not only along constant suction compression paths but also in terms of salinization path ( $\pi=31$  MPa) at constant matric suction ( $s=500$  kPa), as shown in Figure 6. The chemo-mechanical path applied consists of a mechanical loading at constant matric suction up to the vertical stress of 50 kPa, followed by salinization at constant vertical stress and subsequent loading/unloading. In all the simulations,  $p_0^*$  has been set equal to 90 kPa,  $p_c=3000$  kPa and  $\pi_0=1$  kPa. Model parameters used for the simulations are summarized in Table 1.

It is worth noting that the model can be calibrated following the calibration procedure described in [1] for the BBM model, performing tests with distilled water, and in [7] for the MSD model, performing tests on the saturated material with changes in pore fluid chemistry. No dedicated tests accounting for both matric suction and salinity changes are needed.

#### 3.2 Remoulded loess

Experimental data [3] on a remoulded loess coming from the Gansu Province (China) have been also exploited to check the predictive capabilities of the proposed constitutive framework. The remoulded loess had been statically compacted in an oedometer to a void ratio of 0.53 at a gravimetric water content of 12.3%. Oedometer tests were then performed, by loading the specimens up to 25 kPa and then exposing them to a given value of total suction  $\psi_0$  (via relative humidity control). Due to the lack of information related to the as-prepared suction of the specimens, the drying stage has not been simulated and, according to the volume change data presented by [3], the change in the void ratio of this stage has been considered negligible. Later, samples had been soaked with different solutions (i.e. distilled water, NaCl solution, Na<sub>2</sub>SO<sub>4</sub> solution), corresponding to several values of initial osmotic suction  $\pi_{in}$  and finally subjected to a standard oedometer loading/unloading, up to a maximum vertical stress of 1600 kPa. The model has been used to reproduce material response in the  $e$ - $\log\sigma_v$  plane. Material parameters related to unsaturated

soil behaviour have been calibrated on the test involving soaking with distilled water, as shown in Figure 7, corresponding to a test with a total suction before soaking equal to 110.7 MPa. According to the very small deformation upon drying-wetting paths, the parameter  $\kappa_s$  has been set equal to 0.001 in the whole section. Figure 7 shows the experimental data and model predictions for a soaking test with distilled water. Figures 8 and 9 show experimental data corresponding to the unsaturated loess soaked with a 0.29 mol/l  $\text{Na}_2\text{SO}_4$  solution and a 0.17 mol/l  $\text{NaCl}$  solution (corresponding to osmotic suctions of  $\pi = 1.39$  Mpa and  $\pi = 0.74$  Mpa, respectively). The initial total suctions  $\psi_0$  were equal to 14.0 Mpa and 20.6 Mpa, respectively. In this case, the presence of compressive volumetric strain upon soaking is evident. From the modelling point of view, this volumetric strain is related to the first salinization path (eq. 11), rather than a collapse upon wetting. This evidence is consistent with the elastic reloading path evident in the compression curve after soaking, which would not be present after a wetting induce collapse path. Model parameters used for the simulations are again summarized in Table 1. As for the initial values of the internal variables, for the simulations in Figures 8 and 9,  $p_0^*$  were set equal to 50 kPa,  $p_r=3000$  kPa and  $\pi_0=5$  kPa. For the simulation in Figure 7, characterized by  $\psi_0=110$  Mpa,  $p_0^*$  was assumed equal to 170 kPa, consistent with the coupling between SI hardening and  $p_0^*$  evolution [1].

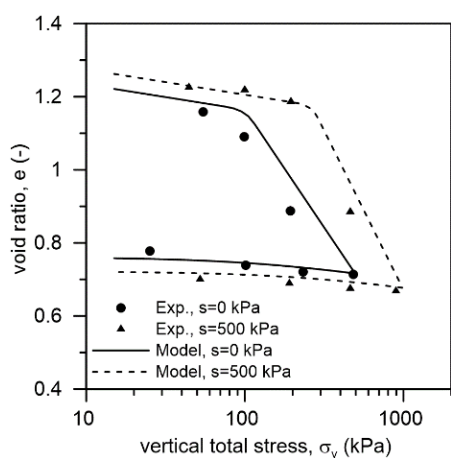


Fig. 4. Effect of matric suction on the compression behaviour of Boom clay (data from [2]).

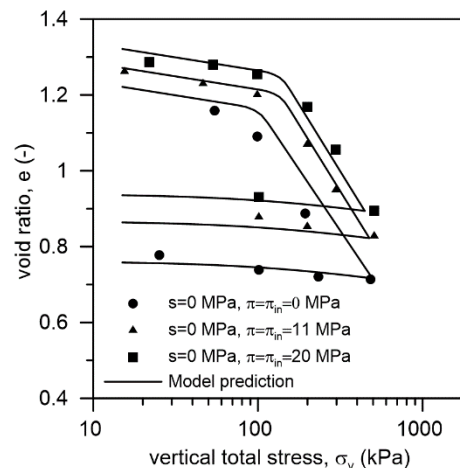


Fig. 5. Effect of osmotic suction on the compression behaviour of Boom clay (data from [2]).

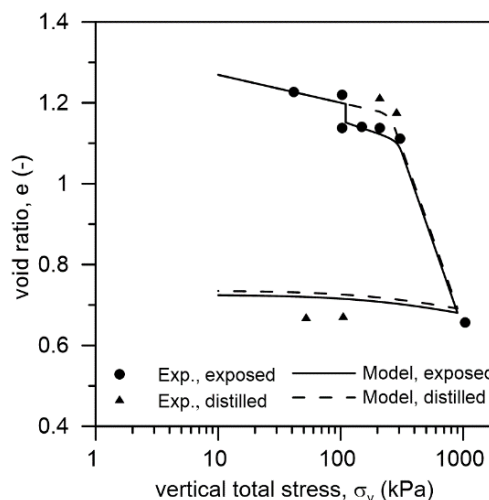


Fig. 6. Void ratio change along a chemo-mechanical loading path on compacted Boom clay (data from [2]).

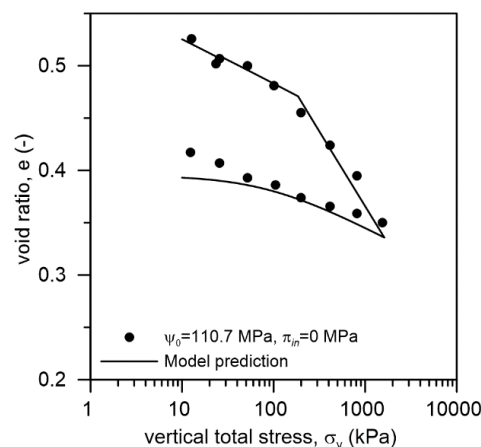


Fig. 7. Void ratio change along a chemo-mechanical loading path on remoulded loess, with  $\psi_0=110.7$  Mpa and  $\pi_{in}=0$  (data from [3]).

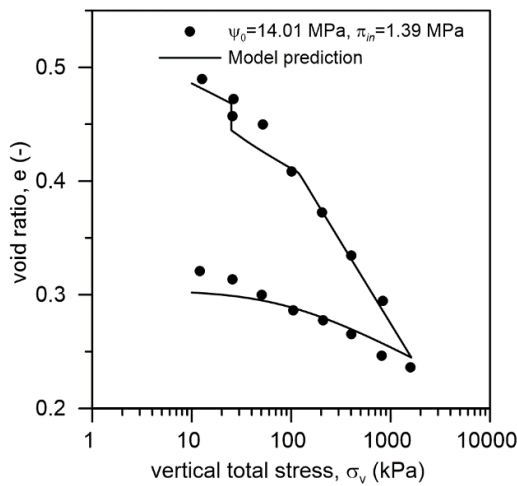


Fig. 8. Void ratio change along a chemo-mechanical loading path on remoulded loess, with  $\psi_0=14.1$  Mpa and  $\pi_{in}=1.39$  Mpa (data from [3]).

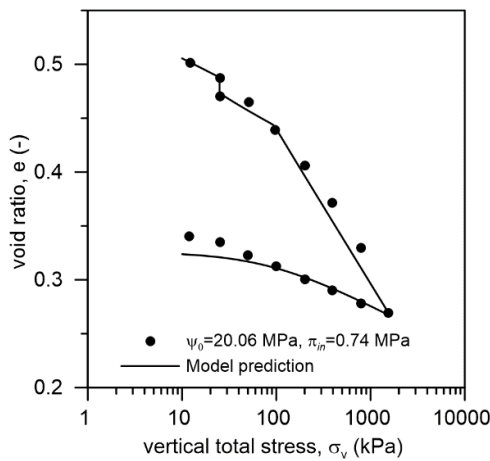


Fig. 9. Void ratio change along a chemo-mechanical loading path on remoulded loess, with  $\psi_0=20.6$  Mpa and  $\pi_{in}=0.74$  Mpa (data from [3]).

**Table 1.** Model parameters.

Material	Parameters
Boom clay	$\kappa=0.03$
	$\lambda_s=0.28$
	$\nu=0.33$
	$M=0.86$
	$\kappa_\pi=0.01$
	$r=0.65$
	$\beta=0.008 \text{ kPa}^{-1}$
Gansu loess	$\kappa=0.02$
	$\lambda_s=0.062$
	$\nu=0.40$
	$M=1$
	$\kappa_\pi=0.0001$
	$r=0.65$
	$\beta=0.00016 \text{ kPa}^{-1}$
	$\kappa_s=0.001$
	$\lambda_\pi=0.15$

## 4 Conclusions

Microstructural observations on low-medium activity clay materials show that an increase in both matric suction and pore fluid salt concentration may cause a transition from an open to a close fabric. This evidence allows introducing a constitutive framework for soils partially saturated with saline solutions, where both salinity and degree of saturation change simultaneously. In this paper, we proposed a simple elastoplastic model, joining together the well-known BBM with a recently published model, MSD, capable of accounting for the salinity of the pore fluid. The model introduces the effect of the osmotic suction in the BBM as an equivalent matric suction, i.e. a matric suction in the BBM which causes the same effects on void ratio as those caused by the osmotic suction in the MSD model. The model reproduces the stiffness changes of the soil induced by suction variations and the plastic chemical deformations, which develop when the clayey material is exposed to an increase in osmotic suction for the first time.

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