Nonlinear System Identification of a Multi-story Building with Geometrical Nonlinearity Using a Deterministic Output-Only-Data Approach

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Abstract. Nonlinear system identification based on output-only data is challenging since the stochastic approaches require the structure to be excited by random input with a uniform Gaussian distribution. This paper applies a deterministic output-only approach to parameter estimation of a linear multi-story specimen with an amplitude-dependent geometrical nonlinearity. The approach is independent of the input type, value, and number but requires the excitation to be applied away from the nonlinearity. The vibration responses to high-amplitude excitations are taken into a subspace-based identification algorithm that simultaneously yields both nonlinear and underlying linear parameters. The process is verified by comparing the underlying linear parameters with the linear modal parameters of the structure under low-amplitude excitation. The results indicate a superior accuracy of the estimated parameters in the simulation and an acceptable confidence range for the experimental test.

Keywords: Nonlinear System Identification, Output-Only Measurements, Geometrical Nonlinearity, Subspace Identification, Multi-Story Building.

1 Introduction

Nonlinear system identification is generally an input-output data-driven process since the nonlinear systems' input (load) and output (response) are not proportional. However, many real-world nonlinear structures are subjected to unmeasurable environmental or operational loads. Statistical approaches such as Kalman Filter and stochastic subspace identification (SSI) algorithms can be implemented using the vibration response of nonlinear structures excited by white gaussian input. In such a case, a linearization in the domain with small steps, parameter updating, or additional terms for the basis function can be adopted for nonlinear parameter estimation [1]–[3]. Recently other approaches have also been introduced based on a mass-change scheme [4] and an input location-change scheme [1].

In this paper, the deterministic subspace identification algorithm recently used for the multi-degree of freedom (MDOF) system [1], [5], is applied to an experimental multi-story building with geometrical nonlinearity. It was demonstrated when an MDOF is excited, the linear and nonlinear elements attached to DOFs away from the external force can be identified at a time using the response of the whole DOFs. Hence, to identify elements attached to the DOFs which directly experience external force, one more test by applying force to a different DOF is required. In other words, the necessary condition for using this approach is the excitation and nonlinearity not to be at the same DOF. The objective of the present study is to extend the applicability range of the approach to experimental identification of structures with amplitude-dependent nonlinearity type based on vibration response only. Compared to the recent work, the present one is dedicated to simplifying the implementation of the algorithm for the users through the available system identification toolbox in MATLAB and using real vibration measurements.

In the first step of the nonlinear identification process, the velocity and displacement data are calculated from the first and second integration of the acceleration records. The state-space phase diagrams and restoring force-state maps are commonly used to detect the existence of nonlinearity and its type [6], [7]. As the best fit for the detected nonlinearity, then the basis function is defined and utilized as the input in the data-driven subspace algorithm [8]. The idea is using the fact that the oblique projection of the response data onto the input data (i.e., in their Hankel matrix arrangements) removes the input contribution, which is the same nonlinear contribution here, from the response data. The remaining space corresponds to the underlying linear system response. If there is no external force at the location of the measured response, or the force has stochastic nature, the remaining space corresponds to the underlying linear modal response. However, the quality of the identified parameters depends on the user-defined basis function to capture the nonlinearity, matrix decomposition operation (i.e., Hankel matrix rows representing the over-determined model order and columns representing the signal length), and noise contamination in the acceleration records. The integration is carried out within detrending and filtering schemes to reduce the process error. In the following sections, the time-domain identification steps are listed, and the experimental testing results are presented.

2 Deterministic Nonlinear System Identification

Let u and y be the input (i.e., external force) and output (i.e., displacement) vectors of a q-DOF system with p nonlinearities, then the equation of motion can generally be stated as

$$\begin{bmatrix} \dot{y}(t) \\ \ddot{y}(t) \end{bmatrix} = A_c \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix} + B_c u(t) + N_c f(y(t), \dot{y}(t)) \tag{1}$$

where $\dot{y}$ and $\ddot{y}$ are the first and second derivatives of y, f is the nonlinear function, and the continuous-time state-space parameters have the following arrangement

$$\begin{aligned}
&A_c[1:2:2q;1:2:2q]=[0]_{q\times q}, A_c[1:2:2q;2:2:2q]=[\mathrm{I}]_{q\times q}\\
&A_c[2:2:2q;1:2:2q]=-[M^{-1}K]_{q\times q}, A_c[2:2:2q;2:2:2q]=[-M^{-1}C_d]_{q\times q}\\
&B_c[1:2:2q;1:m]=[0]_{q\times m}, B_c[2:2:2q;1:m]=[M^{-1}\Gamma]_{q\times m}\\
&N_c[1:2:2q;1:p]=[0]_{q\times m}, N_c[2:2:2q;1:p]=[M^{-1}\eta]_{q\times p}
\end{aligned} \tag{2}$$

where m is the number of applied inputs, respectively, M, K, C_d, and I, are the mass, stiffness, damping, and identity matrices, respectively. Also, the components of Γ are one at DOFs under external inputs and zero elsewhere, and η contains the coefficient of nonlinearities attached to different DOFs. Assuming the applied input u is unknown but its location (i.e., excited DOFs) is known, the idea is to choose a basis function appropriate to f and then apply it to a deterministic (input-output) subspace identification algorithm instead of u. The process will lead to estimating A_c and N_c at DOFs with no external load or $\Gamma=0$. If the applied load and the nonlinearity are not at the same location, N_c can be estimated by only one test, otherwise, one more test is required by applying load at another DOF. Hence, the representation and arrangement in Eqs (1) - (2) aid to include or exclude the identified components concerning the excited DOF.

The conditions for implementing the oblique-projection-based nonlinear subspace algorithm have been presented in [1], [5]. Once the left, singular and right matrices are calculated from SVD (P)= L . S . R, and P is the oblique projection of the future output Hankel matrix $Y_{r+1:2r}$ onto the past output-input Hankel block $\begin{bmatrix} Y_{1:r} \\ F_{1:r} \end{bmatrix}$ along the future input Hankel matrix $F_{r+1:2r}$, the observability matrix can be determined as

$$O=L\left[1:lr;\ 1:2q\right](S\left[1:2q;1:2q\right])^{1/2} \tag{3}$$

Then A and N, the discrete-time counterparts of A_c and N_c, can be achieved by solving $X_f=AX_p+NF_p$, where $X_p=P^{\dagger}O$ is the past state, and $X_f=P[1:lr-l]^{\dagger}O\left[1:lr-l;1:n\right]$, future states, and '†' denotes the pseudo-inverse. In another fashion, one can directly obtain $A=\left(O\left[l+1:lr;1:n\right]\right)^{\dagger}O\left[1:lr-l;1:n\right]$, and solve $L[1:lr;2q+1:lr]Y_fU_f^{\dagger}=L[1:lr;2q+1:lr]N$, to determine N.

In this paper to ease the implementation and comprehension of the algorithm, the N4SID command in MATLAB is called and performed to estimate the time-continuous deterministic state-space parameters. As the system output, the displacement data are integrated from the acceleration measurements. Such a linear deterministic process that normally estimates the state-space parameters A_c and B_c, now yields A_c and N_c, since instead of load measurements u, the nonlinear basis function f is used as the input. The flowchart of the simplified algorithm is presented in Figure 1.

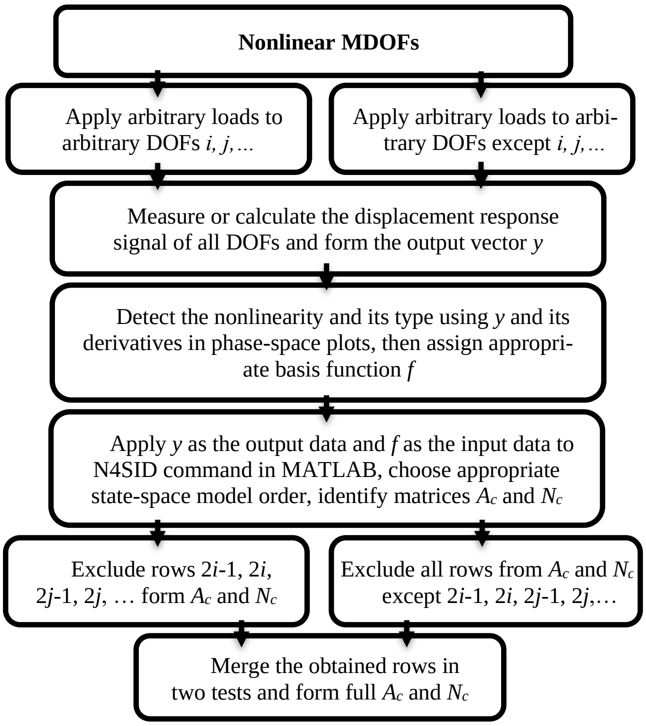

Figure 1. Flowchart of the double-test deterministic nonlinear system identification algorithm

The 'n4sid' command can be set and performed as

$$Sys \;=\; \mathbf{n4sid}(Data, Order, 'Ts', 0, 'form', 'canonical', Option) \tag{4}$$

where *Data* is set the input-output data through the 'iddata' command as

$$Data \;=\; \mathbf{iddata}(y, f, dt) \tag{5}$$

and *Option* accounts for the subspace weighting system that can be chosen canonical variate analysis (CVA) via the 'n4sidOptions' command

$$Option \;=\mathbf{n4sidOptions}('N4weight', 'CVA') \tag{6}$$

In Eq. (4),'*Order*' refers to the state-space model order that needs to be chosen by the user either a priori if the DOFs are known (i.e., two times of DOFs *Order* = 2*q*), or during the identification process by observing nonzero singular values (e.g, *Order* = 1:4*q*) or using a stabilization diagram. The next item '*Ts*' refers to the discrete/continues-time model switch and since the physical parameters such as stiffness and damping need to be estimated, the value of '*Ts*' is chosen 0 to perform the continues-time identification. The item '*form*' determines the arrangement of the estimated components, that following Eq. (2), it must be chosen '*canonical*'. The identification results can be extracted as $A_c = Sys$.A and $N_c = Sys$.B. and in discrete MDOFs with noisy output measurements can be estimated accurately [1], [5].

3 Experimental testing

The structure under study has frequently been studied in [4], [9] for linear and nonlinear parameter estimation. Figure 2 displays the test setup that consists of a lab-scale multi-story building with a thin elastic string at the last floor as the nonlinearity source when the vibration amplitude is high.

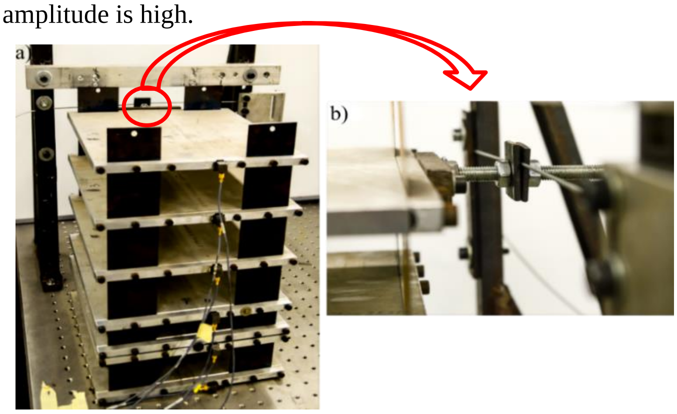

Figure 2. a) Multi-story building test setup, b) thin wire attached to the last floor triggering geometrical nonlinearity

Within the new tests, the specimen has been excited at four elevations with different load levels and simply using hammering tests. The acceleration responses have been recorded on four floors. The specimen can be deemed as an underlying linear structure with a geometric nonlinearity localized at the last floor and it is assumed not to be activated under low-level excitations. Hence, the response signals measured in four tests under high-level impulse trains are taken into the nonlinear identification process introduced in Figure 1 for estimating both nonlinear and underlying linear parameters. Also, the free decay response signals measured in four other tests under low-level impulse are utilized in the stochastic subspace identification (SSI) process and the identified linear modal parameters are compared with those achieved from the underlying linear system in nonlinear tests

4 Results and discussion

4.1 Linear modal analysis

The modal parameters of the multi-story specimen are estimated using low-level responses in the SSI algorithm. Figure 3 depicts the stabilization of the identified natural frequencies and damping ratios associated with the first acceleration data set. The stabilization diagrams for the displacement data obtained using the Kalman filter integration scheme with a cut-off frequency of 3 to 200 Hz, follow almost the same trend and values. Table 1 represents the linear modal parameters estimated based on displacement corresponding to four low-level tests. The mean and CV (coefficient of variation that is standard deviation to mean) values in the last two rows indicate a limited uncertainty bound for the natural frequencies whereas the damping ratios have a wider uncertainty bound. Figure 4 demonstrates the difference between the power spectral density (PSD) of the displacement data associated with two typical linear and nonlinear tests. The main harmonic (peak) shifts are observable in the nonlinear response of all floors, while for the fifth floor the multi-harmonics are also significant due to the nonlinear element attached to this floor.

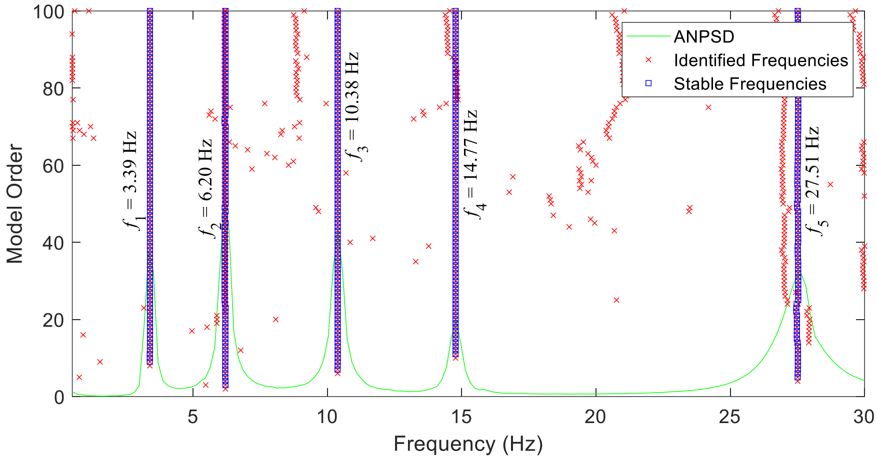

Figure 3. Stabilization diagram of identified natural frequencies within a linear test

Table 1. Natural frequency and damping ratio estimated using four low amplitude data sets

Data Set	Natural Frequency (Hz)					Damping Ratio (%)				
1	3.41	6.20	10.40	14.70	27.41	0.37	0.29	0.42	1.12	0.86
2	3.41	6.20	10.40	14.28	27.46	0.24	0.30	0.40	1.48	1.08
3	3.41	6.19	10.37	14.76	27.70	0.26	0.32	0.37	0.52	0.84
4	3.41	6.20	10.38	14.77	27.88	0.55	0.34	0.32	0.49	0.84
Mean	3.41	6.20	10.39	14.63	27.61	0.36	0.31	0.38	0.90	0.91
CV($\times 10^{-4}$)	0.00	16	19	157	797	3889	645	11316	5333	1209

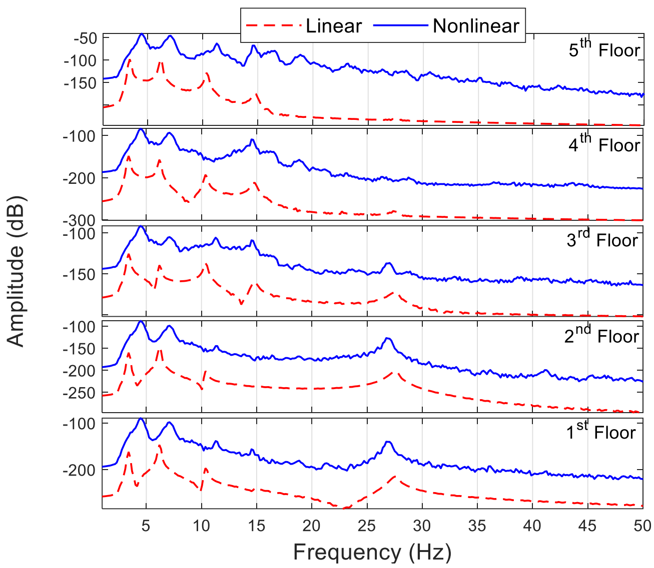

Figure 4. PSD of measurements at 5 floors in two linear and nonlinear tests

4.2 Nonlinear system identification

In this section, the nonlinear system identification procedure introduced by Figure 1 is implemented. The state-space phase diagrams are utilized to observe the nonlinearity and its type. In DOFs with no external force, the figure represents the nonlinear restoring force surface due to stiffness (acceleration-displacement) and damping (acceleration-velocity) elements. Figure 5 displays the state-space phase plot of the fifth floor in a typical nonlinear test and the corresponding fitted surface and curve indicating a cubic behavior. Hence polynomial type basis function is used to characterize and estimate the nonlinear dynamics of the specimen.

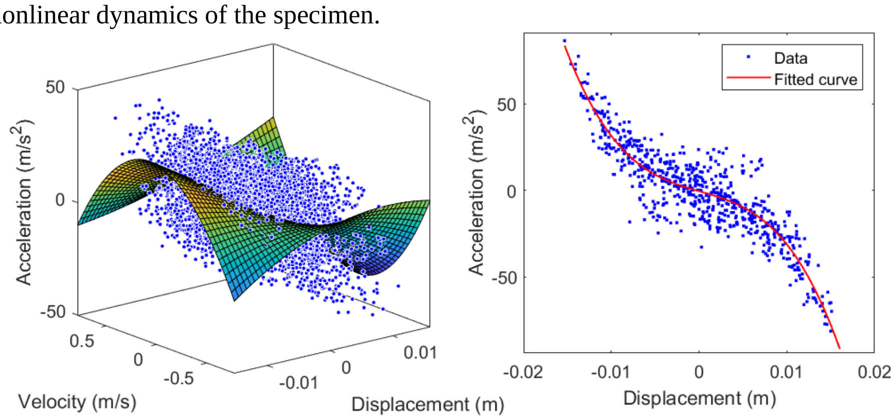

Figure 5. Nonlinear behavior in the last floor, a) restoring force surface b) zero-velocity section plot with cubic polynomial curve fitting

The 3rd order polynomial basis function $f = [y_5^2; y_5^3]$ is considered in terms of the fifth floor's displacement row-vector y_5 where the semicolon ';' separates two rows. Assuming Y and F are the Hankel matrix of $y = [y_1;\ldots;y_5]$ and f respectively, then performing the oblique projection operation on the past-future splits of the mentioned matrices, cancels the row-space of F from the response Y so that the resultant row space associates with the underlying linear system. In such a nonlinear state-space model identification process, both dynamical matrix A (i.e., leading to modal parameters of underlying linear system) and nonlinear parameter N (i.e., coefficients of f) are simultaneously determined. In the present study, the N4SID command in MATLAB is implemented according to Eqs. (4) - (6) using displacement response y as the output data, the basis function f as the input data and state-space model order 10 chosen from variable Hankel matrix rows 10 to100. The following figures represent the nonlinear identification results using displacement-only data measured under high-level excitation in four different tests. Figure 6 demonstrates the variation of the basis function coefficients estimated in different nonlinear tests. Figure 7 shows the underlying linear natural frequency estimates in four nonlinear tests compared to the mean of the natural frequency estimates previously obtained through the linear tests and presented in Table 1.

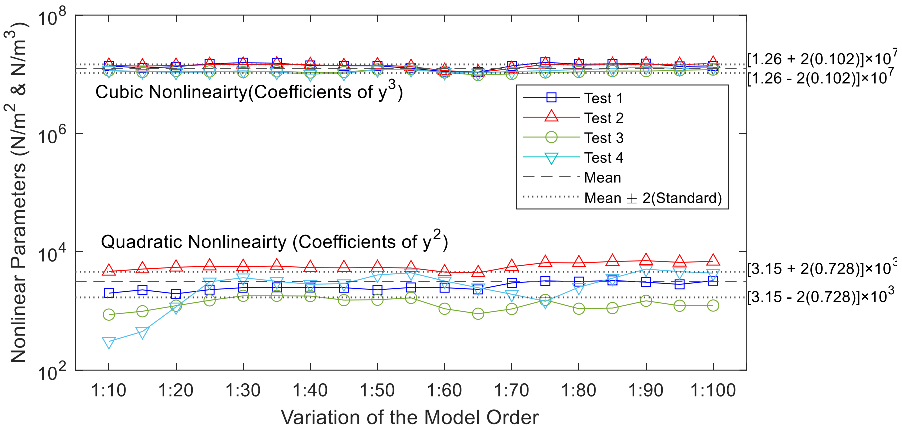

Figure 6. Nonlinear parameters estimated in different tests versus the model order variation

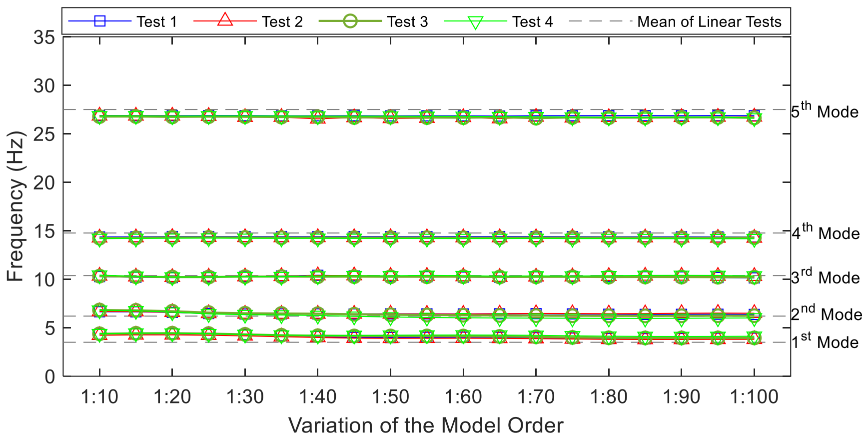

Figure 7. Underlying linear frequencies estimated in different tests vs the model order variation

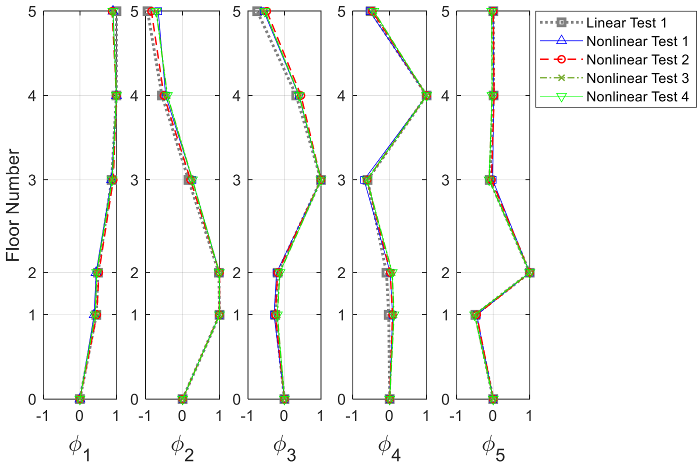

Figure 8. Underlying linear system mode shapes compared to mode shapes in linear Test 1

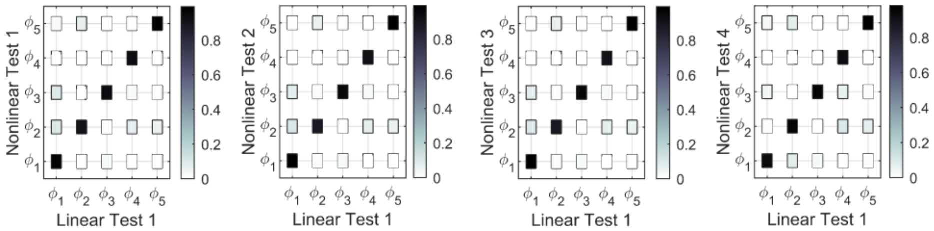

Figure 9. MAC diagrams between underlying linear system mode shapes and mode shapes

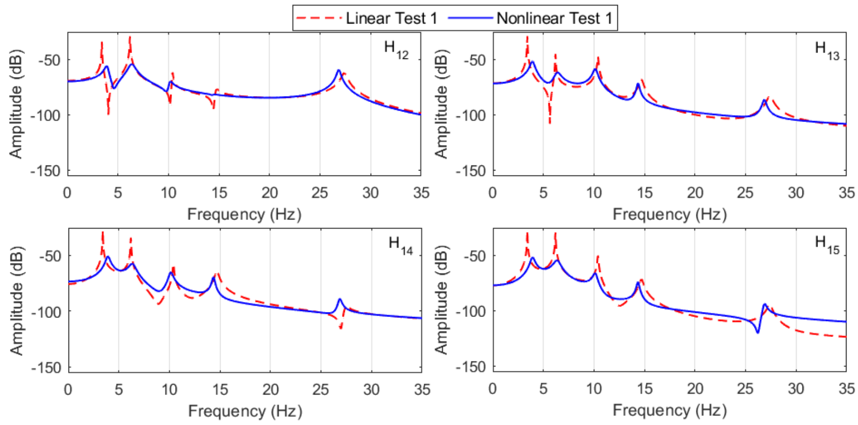

Figure 10. FRFs based on modal parameters obtained from linear and nonlinear tests

Figure 8 and Figure 9 depict the corresponding mode shapes and their MAC diagrams. Utilizing the identified modal parameters, frequency response functions (FRFs) are obtained according to Eq. (10) in [4]. However, the mode shape vectors are not mass normalized here, and since the modal mass scale factor is unknown, it is assumed to be one. Figure 10 displays FRFs based on modal parameters obtained through linear and nonlinear tests.

4.3 Discussion

Regarding Figure 6, almost all the estimates of the coefficient cubic term remain within the uncertainty bounds whereas the quadratic term coefficient estimated through Test 2 and 3 is not acceptable. According to Figure 7, the linear natural frequencies estimated based on the nonlinear test data sets indicate a deviation (max 14 %, in the first mode) from the linear test frequencies. The effect of high damping estimates is also observable within FRFs in Figure 10. The mode shapes in Figure 8 and their MAC values in Figure 9 indicate a minor discrepancy. It must be noticed that in each test, one floor (e.g., DOF) is excited whose corresponding linear and nonlinear parameters must be excluded. Since the nonlinearity is observed only at the fifth floor and in none of the present tests the excitation has not been applied to the fifth floor, the nonlinear parameter can correctly be estimated using the data of all tests. However, the underlying linear modal parameters are influenced because they are extracted here from a full-component dynamical system matrix of which rows/columns correspond to all floors, including the excited one. In other words, only the response data of floors with no external force must be taken into the identification process.

5 Conclusion

In this paper, the linear modal analysis and nonlinear system identification were carried out using vibration measurement of a multi-story building in laboratory conditions. The geometrical nonlinear behavior was observed on the fifth floor. Hence, the polynomial basis function was predicted and determined based on the displacement of the fifth floor. The basis function and displacement data were utilized as the input and output in a deterministic subspace identification algorithm to simultaneously estimate the nonlinear and underlying linear system parameters. The modal parameters of the underlying linear system were compared to those obtained from the linear test cases. The process error due to integration operation and inclusion of the dynamical matrix components of the excited floor in each test caused a bias in the natural frequency and damping ratio. Meanwhile, the mode shapes properly match in linear and nonlinear tests. On the other hand, the stability of the estimated nonlinear parameters verifies the capability of the simplified identification approach to be applied with MATLAB commands and experimental output-only measurements.

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