Composed Graphic Scales in the European Military Treatises and Manuals from the 17th to the 19th Centuries

Composed graphic scales in the European military treatises and manuals from the 17th to the 19th centuries

Marco Giorgio Bevilacqua[1][0000-0002-6063-0333] and Roberta Spallone[2][0000-0003-3430-1402]

[1] University of Pisa, Largo Lucio Lazzarino, 1, 56122, Pisa, Italy
marco.giorgio.bevilacqua@unipi.it
[2] Politecnico di Torino, Viale Pier Andrea Mattioli, 39, 10125 Turin, Italy
roberta.spallone@polito.it

Abstract. This paper describes the results of an extensive research study on European military architecture treatises since the 17th century, which was aimed at identifying types, innovative aspects, and areas of application of composed graphic scales. Such scales, used in astronomical, cartographic, and fortification fields, take on the significance of a precision tool for project drawing and tracing on terrain in the executive field. Starting from the study by Amelio Fara on the graphic scale proposed by the military architect Raimondo Montecuccoli in the mid-17th century, the graphical analysis, carried out following the descriptions of the treatise writers, made it possible to hypothesize the steps of scale tracing, starting from a simple graphical scale, and to verify the measurement examples proposed by some authors. The methods of construction and use of such scales reverberated in the manuals used in military and engineering schools in the following century.

Keywords: Composed graphic scales, Military architecture treatises, Graphic analysis.

1 Introduction

Amelio Fara's 2010 essay describing Raimondo Montecuccoli's manuscript of military architecture, named by Fara himself "Trattato A" (1643), emphasizes the composed scale Montecuccoli drew to define a submultiple of the unit of measurement. In Fara's words, "this is an innovative element with respect to the previous cartographic and architectural culture" that highlights a significant contribution of Montecuccoli to the progress of architectural drawing [1].

Starting with this assertion, the authors of the present paper undertook extensive research on European military treatises published since the beginning of the 17th century, with an aim of identifying different types, innovative aspects, and areas of application of composed graphic scales, also defined as *a trasversali* (i.e., with oblique lines) or transversal scales and sometimes called Tychonic scales, since they were attributed to the Danish astronomer Tycho Brahe (1546–1601). These scales, used in astronomy, cartography, and military architecture, take on the significance of a precision tool for

project drawing and tracing on the terrain in the executive field, as has been noted by the authors of this paper. The graphical analyses of these scales, carried out by deconstructing them following the treatise writers' descriptions, made it possible to hypothesize the steps of scale tracing, starting from a simple graphical scale, and to verify the examples of measurement proposed by some authors. For each graphic scale analyzed, the conversion of the ancient unit of measurement to meters refers to Angelo Martini's *Manuale di Metrologia* (1883) [2].

The graphic scale by Montecuccoli represents a paradigm for those drawn later. The scale of 150 *verghe olandesi* (i.e., ancient Dutch rods, more properly Rhineland rods; 1 rod equals 3.7674 m) was first subdivided into 15 parts to obtain segments of 10 rods; then, drawing 10 parallels and subdividing the last horizontal section into 10 parts allowed for the possibility of reading on the scale itself a submultiple, the *piede* (foot), in the decimal system of the rod. Note that it is possible to read the rod units on the upper and lower main horizontal sections, as well as the feet on the intermediate horizontals at the intersection with the transversals (Fig. 1).

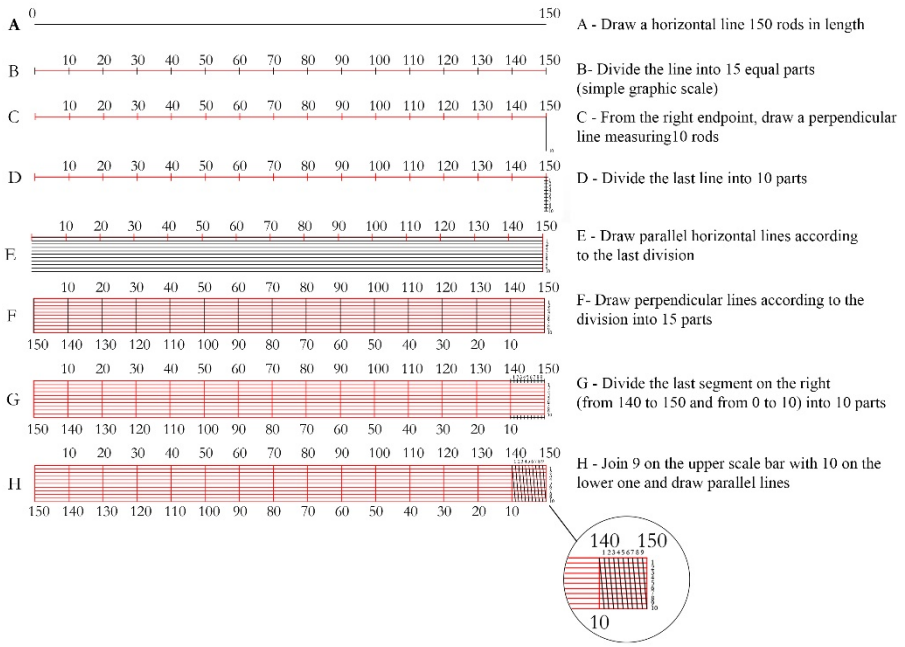

Fig. 1. Construction of the composed graphic scale by Raimondo Montecuccoli (Fara 2010, p. 41, fig. 11). Graphic analysis by R. Spallone.

2 Composed graphic scales: Synoptical overview

The use of composed graphic scales seems to have consolidated in European fortification culture only in the mid-17th century; in fact, no references are found in the main

treatises on military architecture of the previous century, such as in those by Lanteri (1557), Maggi and Castriotto (1st ed. 1564, 1583–84), De' Marchi (posthumous ed. 1599; 1st ed. 1554), Lorini (1st ed. 1596), and Lanteri-Zanco-Lupicini (1601), to name just a few.

From the 1730s onward, the use of such scales is noted in Floriani's treatise (1654, 1st ed. 1630) [3], proposed in *pertiche* (perches) of 10 *piedi di Ferrara* (ancient Ferrarese feet) each (1 perch equals 4.0385 m) in the drawings of fortifications in Figure 14A of the last chapter of Book One, and in that of Sardi (1639) [4] to express the scale of representation of various figures in *piedi geometrici* (geometric feet, more properly ancient Venetian feet) of 10 ounces of 10 minutes (1 foot, one-fifth of *passo geometrico*—geometric piece—, equals 0.3437 m) in the figures *Quarta* and *Vigesima Quarta* of Book Two, and in *verghe olandesi* (ancient Dutch rods of 10 feet each) in some of the "Dutch" figures of Book Three (Figs. *Prima, Undecima, Duodecima, Terza Decima, Quintadecima*). Although the use of these scales broadens in various forms and types over time, only a few treatises devote space to describing their construction and use. Those by Dögen, Bitanvieu, Guarini, and Milliet Dechales are certainly among these; their proposed analyses of the scales allow first a typological and formal classification, as described in the following paragraphs.

2.1 Composed graphic scales with alternating transversals

Among the treatises consulted, *L'architecture militaire moderne ou fortification...* by Matthias Dögen (1648 1st French edition, original edition 1647) [5] is the first in which explicit reference is made to the composed graphic scales, specifically, how to construct and use them (Book One, XVIII, pp. 203–204). The scale is represented in vertical, with the segment AB—divided into 10 equal parts—expressing the length of the reference measure in Rhineland rods in the scale of the drawing and the letters EFGHIKLMN highlighting the points of division (Fig. 2).

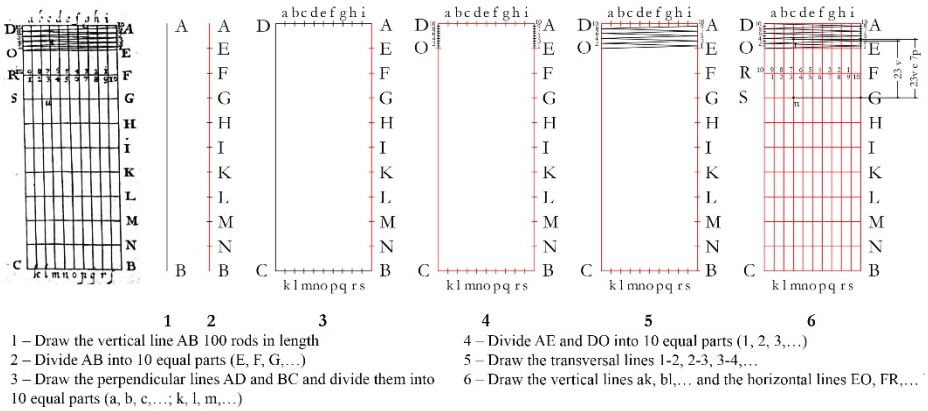

Fig. 2. Construction of the composed scale by Matthias Dögen (1648, Tav. K, Fig. LXXXV). Graphic analysis by M. G. Bevilacqua.

The two equal segments AD and BC, drawn orthogonal to line AB, are also divided into 10 equal parts: *abcdefghi* on AD and *klmnopqrs* on BC represent the division points. AE, the 10th part of line AB, like the corresponding DO on line DC is further divided into 10 parts—1234...10.

Transversal connecting lines are therefore drawn by alternately joining the points of the DO line with those of AE, returning a graphic pattern of multiple Vs arranged vertically. Nine lines parallel to AB and DC, placed at a constant distance from each other, complete the scheme. Constructed in this way, the scale allows not only the reading of the main units—the rods—but also of the first subunits—the feet—as Dögen demonstrates in the hypothesis that segment AB represents 100 rods, and the measure of 23 rods and 7 feet can be taken with a compass. The transverse lines, in fact, applying the criteria of similarity between the triangles thus drawn, identify on the upper 10ths of the intermediate vertical parallels the 10 progressively increasing portions of the submultiples, moving from left to right and vice versa.

Therefore, returning to the case described by Dögen, if a length of 20 rods is to be measured with the compass, in the scale of the drawing, the needles will be pointed at the points G and E, respectively; if of 23 rods at points G and 3 of the segment AE; if of 23 rods and 7 feet at the points *u*—corresponding to G on *cm*, the 7th parallel from AE—and *t*, the intersection of *cm* with the transversal from point 3 of AE to point 4 of DO.

Although the reference to rods is explicit, it is no accident that the author marks the scale with letters to highlight its replicability for any unit of measure and for any reference length.

If compared with that proposed by Montecuccoli, with transversals that are parallel to each other and in which the measurement of submultiples is made by moving from the fundamental line on the intermediate parallels always in the same direction, Dögen's scale requires shifts from left to right and vice versa depending on whether the starting dimension is even or odd. It is also worth pointing out that Dögen, along with other treatise writers of the time, including Montecuccoli and Sardi, prefers, for convenience of use and ease of calculation, to subdivide the duodecimal system of the rod into 10 subunits, thus proposing a resulting longer measure of the foot (Dögen, 1648, pp. 28, 29).

Silvère de Bitainvieu (the pseudonym of Jean Du Breuil) dedicates ample space to composed graphic scales in *Pratique V* of *Traitè II de L'Art universel des fortifications*... of 1674 (Fig. 3). The composed scales, here drawn horizontally, are proposed in configurations of different lengths: 20, 80, 100, and 160 *toises* of 6 *pieds du Roi* each (1 *toise* equals 1.9490 m). The pattern of multiple inverted V-shaped transversals, already found in Dögen, is extended here to the entire length of the scales and is functional, in the first instance, to the measurement of the single *toise* but not of the submultiple, the foot. The smaller one of 100 *toises* (PQ in fig. 3) is particularly interesting for the present study, since it demonstrates the possibility of measuring one single *toise,* which is otherwise unreadable on simple linear scales.

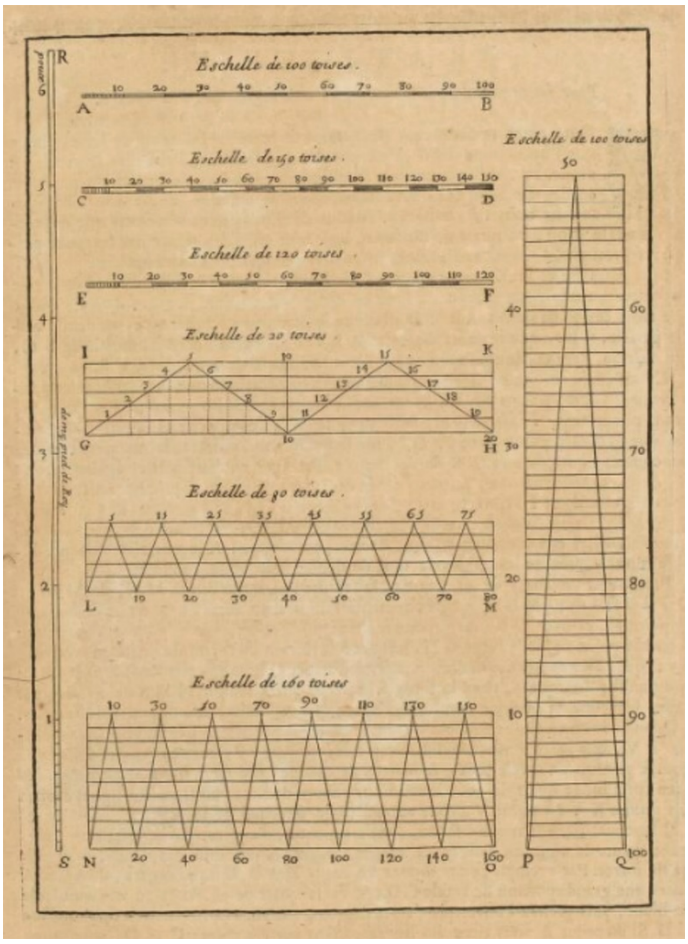

Fig. 3. Composed scales by Silvère de Bitainvieu (1674, p. 65).

2.2 Composed graphic scales with parallel transversals

The use of these scales provides "those who do not wish to make use of arithmetic" precision measurements even for drawings of wide fortifications represented in reduced scales on paper, as Guarino Guarini himself points out in his *Trattato di fortificatione* (1676, p. 43) [7]. The scale that Guarini presents, arranged vertically, is divided into 10 parts, each corresponding to one *pertica* or *trabucco* of 6 *piedi liprandi* each (1 *trabucco* equals 3.086420 m). The transversals, here arranged in parallel as in Montecuccoli, are extended to the entire length of the scale and make it possible to take measurements of feet along the additional 6 lines parallel to the fundamental AD, which in this way—and graphically—is divided into 60-minute parts regardless of its length on the sheet of paper (Fig. 4).

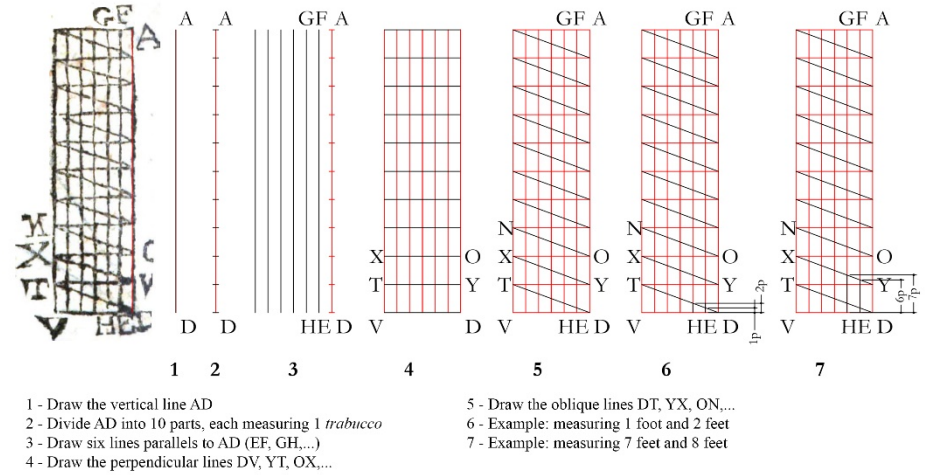

Fig. 4. Construction of the composed scale by Guarino Guarini (1676, Table I, Fig. 5). Graphic analysis by R. Spallone.

Claude-François Milliet Dechales, in *Proposition III* of the second book of *L'art de fortifier, de defendre, et d'attaquer les place...* (1677) [8], proposes a horizontal model similar to Guarini's with parallel transversals extended to the entire length of the reference line.

The scale of 100 *toises* allows the measurement of the single unit of *toise* (Fig. 5). By appropriately increasing the distance between the horizontal parallels and dividing the space into 6 parts with the drawing of 5 intermediate horizontals, the scale would allow for the reading of the foot; in which case, as the author himself specifies, "pour éviter la confusion" it would be advisable to use different colors, shading, dashes, or dots in drawing the intermediate lines.

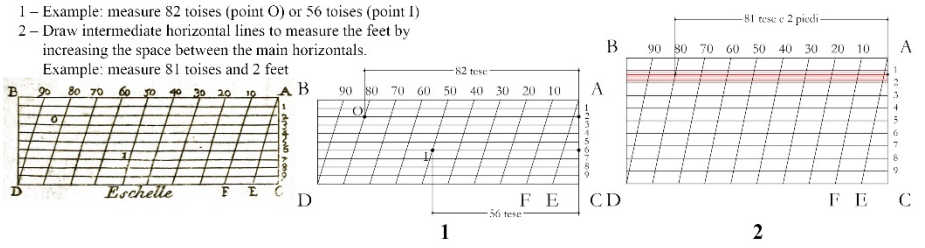

Fig. 5. Construction of the composed scale by Claude-François Milliet Dechales (1677, Planche 10, fig. 23, p. 99). Graphic analysis by M. G. Bevilacqua.

3 Composed scales in the manuals for educational purposes

Defining land measurement, cartography, and topography as scientific and applied methodologies for territory surveying and as disciplines of study at the academic institutions that preceded the founding of modern polytechnics involved, between the second part of the 18th century and the first part of the 19th century, the drafting of specific manuals whose authors belonged, in many cases, to the military ranks or the academic system. In such manuals, the discourse on composed graphic scales seems to reverberate in its didactic function the innovations achieved in military treatises since the previous century.

In-depth research on the transformation and new foundation of military and engineering schools in the Age of Enlightenment and, in particular, on the three disciplines mentioned above in their modern setting, will provide an accurate framing of the moment of landing in the didactic manuals of composed graphical scales and will most likely reveal new proposals and applications of them.

In this research phase, the volumes of Louis Charles Dupain de Montesson, Alessandro Righini di San Giorgio, and Giovanni Curioni may be considered as exempla.

The scales analyzed are classifiable as composed graphic scales with parallel transversals and do not present innovations from the point of view of graphic construction or reading methods or, again, with respect to the possibility of precisely reading units and submultiples. However, they turn out to be interesting and original with respect to the applications that link the composite scales proposed in the manuals to local realities, in terms of the use of units, and to measurement practices.

3.1 Dupain and the measurement of agricultural land

Dupain published the volume La science de l'arpenteur dans toute son étendue [9] in manuscript form in 1766. In the frontispiece, the author lists his titles of Captain of Infantry and Engineer-Geographer of the King's Encampments and Armies in the dedication to the Prince of Condé. This was Louis de Bourbon-Condé, governor of the Champagne region since 1751, a position that explains the interest in surveying land and, in particular, vineyards.

In the initial section, devoted to geometric scales, Dupain specifies the function of the scale to make known the dimensions of a figure and to form similar ones and references the *toise* (1.9490 m) as a unit of measurement. The *toise* contains 6 feet; one foot equals 12 inches, one inch equals 12 lines, and one line equals 12 points. The composed scale allows the smallest measurements to be taken accurately for drawings of high detail. This type of scale can be traced on a copper or wooden ruler, i.e., on a measuring instrument or a paper sheet. The obvious simplicity of the scale is complemented by the interesting reference to this dual possibility of use. A description of the graphic construction of the scale, which allows for reading *toises* and feet, follows, including some examples. This is a scale of 5 *toises*, the 6 parallels allowing the foot to be identified as a submultiple (1 *toise* equals 6 feet). The first *toise* is subdivided by a diagonal that allows the feet to be read (Fig. 6).

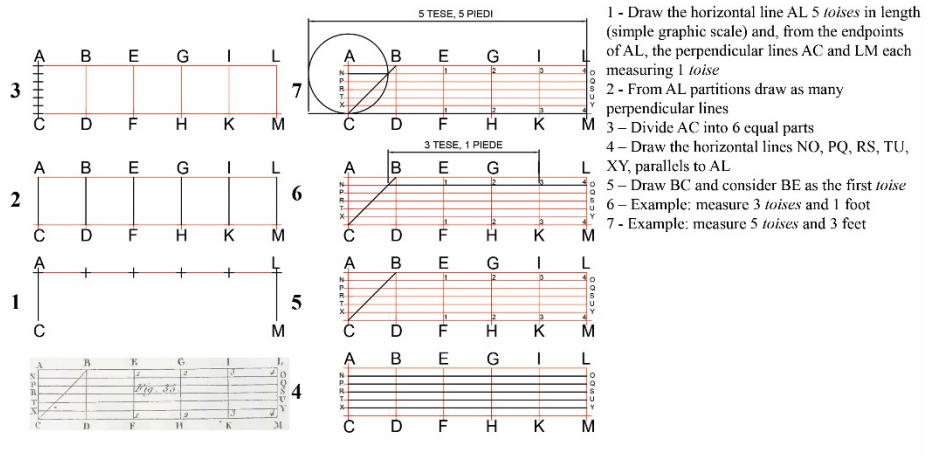

Fig. 6. Construction of the composed graphic scale in toises and feet by Louis Charles Dupain de Montesson (1766, pl. 2, fig. 35). Graphic analysis by R. Spallone.

3.2 Righini and Curioni: The military and engineering schools

The texts by Righini and Curioni, both published in Turin, represent, almost simultaneously, the interest in topography by the military schools established by the Ministry of War of the Kingdom of Sardinia and the emerging Schools of Application for Engineers, respectively.

Righini, a lieutenant-colonel in the Royal Corps of the General State, first published the *Corso completo di Topografia* for student officials in 1856 [10]. His scales, *a transversali*, in *trabucchi* or meters, testify to the transition in Piedmont between local units of measurement and those referring to the metric system (Fig. 7).

The composed scale in meters, designed for the 1:2000 reduction ratio, allows one to evaluate the meter's hundreds, tenths, and units. Like the Montecuccoli scale, one can read the two submultiples at the head of the scale where the subdivisions into tenths and units are present.

The composed scale in trabucchi, on the other hand, has the transversals along its entire extent, analogous to that of Milliet Dechales and, equally allows for reading the unit and its submultiple, i.e., *trabucchi* and *piedi liprandi* (1 *piede liprando* = 0.514403m).

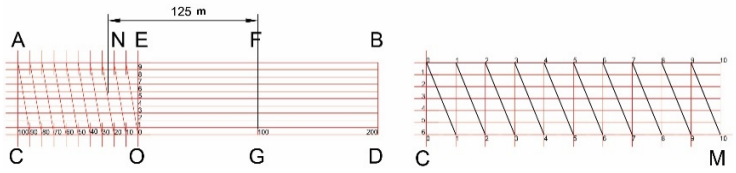

Fig. 7. The composed graphic scale in meters and *trabucchi* by Alessandro Righini di San Giorgio (1856, fig. 5 p.18 and fig. 6 p. 19). Graphic analysis by R. Spallone.

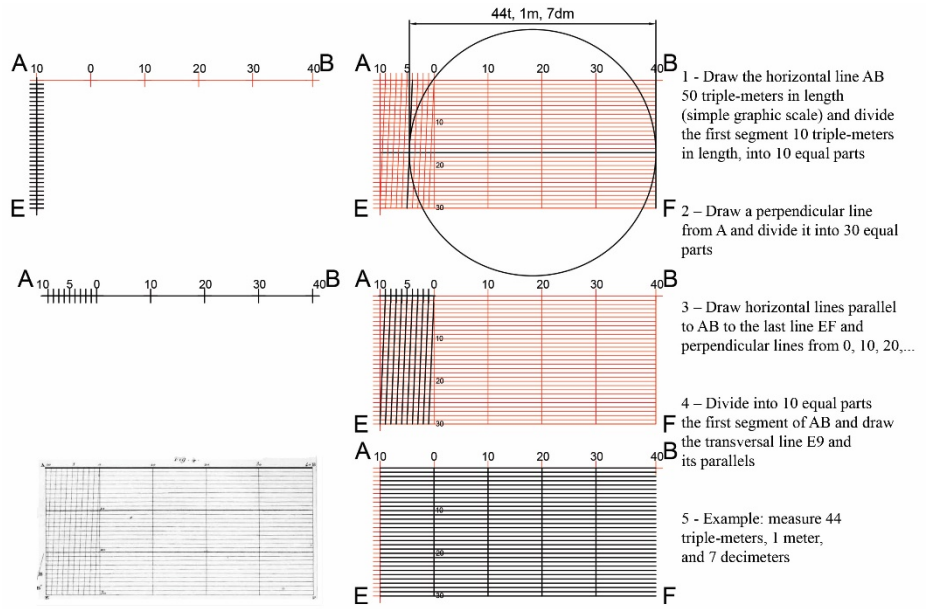

Fig. 8. Construction of the composed graphic scale in triple-meters, meters, and decimeters by Giovanni Curioni (1869, tav. 1 fig. 4). Graphic analysis by R. Spallone.

Curioni, engineer, architect, and professor at the *Scuola d'Applicazione per gl'Ingegneri*, published in 1869 the *Operazioni topografiche* [11], a text intended for professionals and students of engineering schools. The text articulately describes the Tychonic or *transversali* scales as distinct from the simple graphic scales.

The descriptions are helpful, as specified by the author, both in the design phase through drawing on paper and in the executive stage through tracing on the ground. The reasoning, also exposed through graphical schemes, describes the scales in meters, with the definition of two submultiples of the decameter (meter and decimeter), those in triple-meters (Fig. 8), which interestingly link the drawing with the surveying instrument, consisting of screwed wooden rods, also with two submultiples (m, dm), and those in *trabucchi*, the unit of measurement now in disuse in Piedmont, subdivided into 6 *piedi liprandi*. Again, the reading of submultiples is resolved in the scale head.

4 Conclusions

The use of composed graphic scales, which, as we have seen, became widespread in fortification drawings from the 1730s onward, is part of the broader panorama of tools—not only graphic tools—useful in solving complex problems, fostered by the development of scientific knowledge from the mid-16th century onward. Based on the principles of the similarity of triangles, such scales refine the precision of reading any gradation. The models analyzed, in fact, allow for measurements of greater precision

where the physical size of a simple graduated scale does not allow for legibility on paper and the consequent measurability of units and submultiples with compasses.

In 17th-century military culture, scale construction is therefore a preparatory operation to the elaboration of a project drawing or to the restitution of a survey drawing, providing a tool that allows conversion between drawing units and real units without recourse to arithmetic calculation. From the following century, methods of constructing and using such scales became established in the manuals used in military and polytechnic schools.

Acknowledgement

The research was realized in the framework of the project PRIN2022 INFORTREAT, Reconstructing the Early Modern bastioned front. INformation models for the fruition of constructive knowledge in FORtified architecture TREATises (16th-18th Century), CUP I53D23005420006, funded by the European Union – Next Generation EU, P.I.: M. G. Bevilacqua, University of Pisa; A.I.: R. Spallone, Politecnico di Torino.

Authors contribution

The authors confirm contribution to the paper as follows: Introduction: MGB and RS; Par. 2: MGB; Par. 3: RS; Conclusions: MGB and RS. All authors reviewed the results and approved the final version of the manuscript.

References

1. Fara, A.: L'architettura militare di Raimondo Montecuccoli. Da un inedito trattato agli Aforismi. Incidenze su Guarino Guarini. Critica d'arte 43-44, 33-72 (2010).
2. Martini, A.: Manuale di metrologia, ossia misure, pesi e monete in uso attualmente e anticamente presso tutti i popoli. Loescher, Torino (1883).
3. Floriani, P.P.: Difesa et offesa delle piazze di Pietro Paolo Floriani da Macerata. Opera non solo utile, e necessaria a' capitani, e governatori di fortezze, ma anco di sommo profitto a' studiosi dell'historie militari, così antiche, come moderne. (1 ed. 1630). Francesco Baba, Venezia (1654).
4. Sardi, P.: Corno dogale dell'architettura militare di Pietro Sardi Romano dedicato e consecrato al Serenissimo Principe di Venetia. I Giunti, Venezia (1639).
5. Dögen, M.: L'architecture militaire moderne ou fortification: confirmée par diverses histoires tant anciennes que nouvelles, & enrichie des figures des principales forteresses qui sont en l'Europe. (1 ed. 1647). Louys Elzevier, Amsterdam (1648).
6. Bitainvieu, S. de (Du Breuil, J.): L'Art universel des fortifications françoises, holandoises, espagnoles, italiennes et composées. Avec l'art d'attaquer les places fortifiées par les surprises & par la force, & aussi de défendre les places fortifiées contre les surprises & contre la force. Troisième edition reveuë, corrigée & augmentée. Jacques Du Breil, Paris (1674).
7. Guarini, G.: Trattato di fortificatione, che hora si usa in Fiandra, Francia, et Italia; composto in ossequio del sereniss. principe Lodovico Giulio cavagliere di Savoia. Appresso gl'heredi di Carlo Gianelli, Torino (1676).

8. Milliet De Chales, C.F.: L'art de fortifier, de défendre, et d'attaquer les places, suivant les méthodes françoises, hollandoises, italiennes & espagnoles. Le tout enrichy de figures en taille douce. Estienne Michallet, Paris (1677).
9. Dupain de Montesson, L.C.: La Science de l'Arpenteur. Goeury, Paris (1766).
10. Righini di San Giorgio, A.: Corso completo di Topografia. Tip. Scolastica di Sebastiano Franco e Figli e Comp., Torino (1856).
11. Curioni, G.: Operazioni topografiche. Tomo IV de L'arte di Fabbricare. Augusto Federico Negro, Torino (1869).