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CONTROLLING COMPLETE SYNCHRONIZATION IN A COMPLEX NETWORK OF FITZHUGH-NAGUMO SYSTEMS

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Abstract: The control of network-coupled nonlinear dynamical systems is an active area of research in the nonlinear science community. Coupled oscillator networks represent a particularly important family of nonlinear systems. In this paper, we study the control of network-coupled limit cycle oscillators, particularly we present the complete synchronization dynamics of coupled FitzHugh-Nagumo neurons. In order to synchronize all state variables in the system, we regard the nonlinear feedback system as a MIMO system. Then, an exact decoupling feedback linearization algorithm with a state feedback with a Kalman Filter strategy are introduced respectively to design the synchronization controllers for the coupled neurons.

Keywords: Synchronization, FitzHugh-Nagumo Network, Complex Networks, Nonlinear Control

1. INTRODUCTION

Network-coupled dynamical systems can be found both in nature and science, and as a result the control of such systems has been the focus of a great deal of research from the nonlinear dynamics and complex networks communities (Strogatz, 2001). A particularly important family of network-coupled non-linear dynamical systems that plays an important role in modeling phenomena ranging from synchronization of power grids to cardiac excitation are networks of coupled oscillators. Control of coupled oscillator networks has also garnered signficant attention recently (Skardal and Arenas, 2015).

Since the discovery of chaos synchronization (Pecora and Carroll, 1990) the feasibility of synchronizing chaotic systems has been widely investigated. Over the last decade, many new types of synchronization have appeared (Pastor *et al.*, 2012). In this paper, the complete synchronization of networks of identical neurons—FitzHugh–Nagumo (FHN) neurons electrically coupled with gap junction in external electrical stimulation is the main focus.

On the other hand, with the development of control theory, various modern control methods, as described in Su and Xiaofan (2013); Skardal and Arenas (2016); Gao *et al.* (2014) have been successfully applied to synchronization of complex networks in recent years. In some of those studies, the control goal is to achieve a fixed point of the network or frequency synchronization. But in this work, the complete synchronization of the network is considered. Firstly, using exact decoupling feedback linearization algorithm for the MIMO nonlinear system. Then, considering the synchronization problem as a tracking problem and as the Gaussian white noise is usually used to simulate the ionic channel noise of neurons (Che *et al.*, 2009), we propose a state space feedback with a Kalman filter to estimate the state space variable which is not accessible.

In this work, we consider a complex network consisting of N identical linearly an diffusively coupled nodes of n-dimensional dynamical system described by

$$\dot{\mathbf{x}}_i(t) = f(\mathbf{x}_i(t)) + \sum_{j=1;j\neq i}^{N} c_{ij}\, a_{ij}\, \Gamma\left(\mathbf{x}_i(t) - \mathbf{x}_j(t)\right) \tag{1}$$

with $i = \{1, \ldots, N\}$ where $\mathbf{x}_i = [x_{i\,1}, \ldots, x_{i\,n}]^T$ is the state vector of the ith node; $t \in [0, +\infty)$ is continuous time; $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a continuous map; $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ is the *Adjancency matrix* of the network; $C = (c_{ij}) \in \mathbb{R}^{n\times n}$ is the *Coupling* or *Weight matrix* of the network; and $\Gamma = (\tau_{ij}) \in \mathbb{R}^{n\times n}$ is a matrix linking coupled variables, and if some pairs (i, j), $1 \leq i,\, j \leq n$, with $\tau_{ij} \neq 0$, then it means two coupled nodes are linked through their ith and jth state variables, respectively. For the sake of clarity, through this paper we will use $x_i(t) \equiv x_i$, taking for granted the temporal dependence of x_i.

The problem of *Complete Synchronization* (Pastor *et al.*, 2012) consists in the synchronization of the states of every node of a network called *Slaves Nodes* or just *Slaves*, to the dynamics of a single node called *Master Node* or simply *Master*. This is also known as *consensus with a leader* (Su and Xiaofan, 2013).

Defining node 1 as *Master* $x_1 \triangleq x_M$ and with no loss of generality, the complex network can be written as

$$\begin{aligned}
\dot{\mathbf{x}}_M &= f(\mathbf{x}_M) + \sum_{j=2}^{N} c_{Mj}\, a_{Mj}\, \Gamma\left(\mathbf{x}_M - \mathbf{x}_j\right) \\
\dot{\mathbf{x}}_i &= f(\mathbf{x}_i) + c_{iM}\, a_{iM}\, \Gamma\left(\mathbf{x}_i - \mathbf{x}_M\right) \\
&\quad + \sum_{j=2;j\neq i}^{N} c_{ij}\, a_{ij}\, \Gamma\left(\mathbf{x}_i - \mathbf{x}_j\right) + \mathbf{u}_i
\end{aligned} \tag{2}$$

with $i = \{2, \ldots, N\}$, so that the *Master-Slave* synchronization problem now can be reduced to design a control input $\mathbf{u}_i$, $i = \{2, \ldots, N\}$ to achieve

$$\lim_{t\to\infty} \|\mathbf{x}_i - \mathbf{x}_M\| = \lim_{t\to\infty} \|\mathbf{e}_i\| \longrightarrow 0 \tag{3}$$

for all i, where $\mathbf{x}_M$ is the state of the leader.

To accomplish the goal stated in (3), one must analyze the dynamical system associated to the tracking error between the *Master* and every *Slave* in the network, defined as $\mathbf{e}_i = \mathbf{x}_i - \mathbf{x}_M$ for all $i = \{2, \ldots, N\}$. This generates the following dynamical system

$$\dot{\mathbf{e}}_i = \dot{\mathbf{x}}_i - \dot{\mathbf{x}}_M \tag{4}$$

As we mentioned, we work with a network of FHN neurons. The FitzHugh–Nagumo model is a two-dimensional simplified form of the Hodgkin–Huxley model and in several cases represents efficiently the membrane's voltage dynamics in neurons (Ermentrout and Terman, 2010). The model captures all the qualitative properties of the Hodgkin–Huxley dynamics, in the sense that it results in the same bifurcation diagram (loci of fixed points with respect to model's parameters) (Rigatos, 2013). The FHN model comprises two differential equations:

$$\begin{aligned}
\dot{x}_{i\,1} &= -x_{i\,2} - x_{i\,1}(x_{i\,1} - 1)(x_{i\,1} - \lambda) + v_i \\
\dot{x}_{i\,2} &= \epsilon(x_{i\,1} - \delta\, x_{i\,2}) \\
y_i &= x_{i\,1}
\end{aligned} \tag{5}$$

where $x_{i\,1}$ represents the membrane voltage (measurable output of the system y_i); $x_{i\,2}$ is a slow recovery variable; and v_i represents an external control input. For specific values of its parameters $(\lambda,\, \epsilon,\, \delta)$ (5) it exhibits sustained oscillations in its state variables and in such a case limit cycles will also appear in the associated phase diagram. The detailed stability analysis of models of biological neurons is given in (Ringkvist and Zhou, 2005).

2. SYNCHRONIZATION OF FHN NETWORK

Each unit (node) of the network behaves like a FHN system as in (5). For simplicity, it is written in a more general form as

$$\begin{aligned}
\dot{x}_{i\,1} &= f_1(\mathbf{x}_i) + v_i \\
\dot{x}_{i\,2} &= f_2(\mathbf{x}_i)
\end{aligned} \tag{6}$$

We analize a complex network of N nodes that behaves as in (6), linearly coupled by the first state variable $x_{i\,1}$ $\left(\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ in } (1)\right)$, where the final goal is the one established in (3).

Defining node 1 as *Master* $x_1 \triangleq x_M$ as in (2) the complex network now can be written as

$$\begin{aligned}
\dot{x}_{M\,1} &= f_1(\mathbf{x}_M) + \sum_{j=2}^{N} c_{Mj}\, a_{Mj}\, (x_{M\,1} - x_{j\,1}) + v_M \\
\dot{x}_{M\,2} &= f_2(\mathbf{x}_M) \\
\dot{x}_{i\,1} &= f_1(\mathbf{x}_i) + c_{iM}\, a_{iM}\, (x_{i\,1} - x_{M\,1}) \\
&\quad + \sum_{j=2;j\neq i}^{N} c_{ij}\, a_{ij}\, (x_{i\,1} - x_{j\,1}) + v_i \\
\dot{x}_{i\,2} &= f_2(\mathbf{x}_i) \quad i = \{2, \ldots, N\}
\end{aligned} \tag{7}$$

Moreover, $f_1(\mathbf{x}_i)$ in (6) can be written as the sum of a linear and a nonlinear scalar field in the following form

$$\begin{aligned}
f_1(\mathbf{x}_i) &= -x_{i\,2} - x_{i\,1}(x_{i\,1} - 1)(x_{i\,1} - \lambda) \\
&= \underbrace{\left[-\lambda\, x_{i\,1} - x_{i\,2}\right]}_{f_1(\mathbf{x}_i)_l \text{ (linear)}} + \underbrace{\left[-x_{i\,1}^3 + x_{i\,1}^2\,(1+\lambda)\right]}_{f_1(x_{i\,1})_{nl} \text{ (nonlinear)}}
\end{aligned} \tag{8}$$

As $f_2(\mathbf{x}_i)$ is linear, the aforementioned operation is unnecessary. The final expression of the complex network is given by

$$
\begin{aligned}
\dot{x}_{M\,1} &= f_1(\mathbf{x}_M)_l + f_1(x_{M\,1})_{nl} \\
&\quad + \sum_{j=2}^{N} c_{Mj}\, a_{Mj}\,(x_{M\,1} - x_{j\,1}) + v_M \\
\dot{x}_{M\,2} &= f_2(\mathbf{x}_M) \\
\dot{x}_{i\,1} &= f_1(\mathbf{x}_i)_l + f_1(x_{i\,1})_{nl} + c_{iM}\, a_{iM}\, \Gamma\,(x_{i\,1} - x_{M\,1}) \\
&\quad + \sum_{j=2;j\neq i}^{N} c_{ij}\, a_{ij}\,(x_{i\,1} - x_{j\,1}) + v_i \\
\dot{x}_{i\,2} &= f_2(\mathbf{x}_i) \qquad i = \{2,\ldots,N\}
\end{aligned}
\tag{9}
$$

2.1 *Error dynamical system*

To achieve the goal stated in (3), one must analyze the dynamical systems associated to the tracking error between the *Master* and every *Slave* in the network.

$$
\begin{cases}
\dot{e}_{i\,1} &= \dot{x}_{i\,1} - \dot{x}_{M\,1} \\
\dot{e}_{i\,2} &= \dot{x}_{i\,2} - \dot{x}_{M\,2} \qquad i = \{2,\ldots,N\}
\end{cases}
\tag{10}
$$

which expanding yields

$$
\begin{aligned}
\dot{e}_{i\,1} &= f_1(\mathbf{x}_i)_l + f_1(x_{i\,1})_{nl} + c_{iM}\, a_{iM}\,(x_{i\,1} - x_{M\,1}) \\
&\quad + \sum_{j=2;j\neq i}^{N} c_{ij}\, a_{ij}\,(x_{i\,1} - x_{j\,1}) + v_i - f_1(\mathbf{x}_M)_l \\
&\quad - f_1(x_{M\,1})_{nl} - \sum_{j=2}^{N} c_{Mj}\, a_{Mj}\,(x_{M\,1} - x_{j\,1}) - v_M \\
\dot{e}_{i\,2} &= f_2(\mathbf{x}_i) - f_2(\mathbf{x}_M)
\end{aligned}
\tag{11}
$$

In this case, we propose to separate in the first state of (11) the coupling terms of the *Master* with the ith *Slave*. Reorganizing, the aforementioned equation remains

$$
\begin{aligned}
\dot{e}_{i\,1} =& f_1(\mathbf{x}_i)_l - f_1(\mathbf{x}_M)_l + c_{iM}\, a_{iM}\,(x_{i\,1} - x_{M\,1}) \\
&+ c_{Mi}\, a_{Mi}\,(x_{i\,1} - x_{M\,1}) + f_1(x_{i\,1})_{nl} \\
&- f_1(x_{M\,1})_{nl} + \sum_{j=2;j\neq i}^{N} c_{ij}\, a_{ij}\,(x_{i\,1} - x_{j\,1}) \\
&- \sum_{j=2;j\neq i}^{N} c_{Mj}\, a_{Mj}\,(x_{M\,1} - x_{j\,1}) + v_i - v_M
\end{aligned}
\tag{12}
$$

With this proposal, the error dynamics can be written as

$$
\begin{aligned}
\dot{e}_{i\,1} &= f_1(\mathbf{e}_i)_l + c_{iM}\, a_{iM}\, e_{i\,1} + c_{Mi}\, a_{Mi}\, e_{i\,1} + g_i(\mathbf{y}) \\
&\quad + v_i - v_M \\
\dot{e}_{i\,2} &= f_2(\mathbf{e}_i) \qquad i = \{2,\ldots,N\}
\end{aligned}
\tag{13}
$$

where

$$
\begin{aligned}
g_i(\mathbf{y}) &= f_1(x_{i\,1})_{nl} - f_1(x_{M\,1})_{nl} \\
&\quad + \sum_{j=2;j\neq i}^{N} c_{ij}\, a_{ij}\,(x_{i\,1} - x_{j\,1}) \\
&\quad - \sum_{j=2;j\neq i}^{N} c_{Mj}\, a_{Mj}\,(x_{M\,1} - x_{j\,1})
\end{aligned}
\tag{14}
$$

For simplicity we consider $v_M = 0$ avoiding the control action in the *Master* node so thus the network achieves the tracking of the dynamics of this node. Finally, the error linear dynamics can be expressed using matrix notation as it follows

$$
\begin{aligned}
\dot{\mathbf{e}}_i &= \begin{bmatrix} -\lambda + c_{iM}\, a_{iM} + c_{Mi}\, a_{Mi} & -1 \\ \epsilon & -\delta\,\epsilon \end{bmatrix} \mathbf{e}_i \\
&\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} g_i(\mathbf{y}) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i = A_{\mathbf{e}_i}\, \mathbf{e}_i + \begin{bmatrix} 1 \\ 0 \end{bmatrix} g_i(\mathbf{y}) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i
\end{aligned}
\tag{15}
$$

2.2 *Control design*

We propose the control scheme showed in Fig. 1

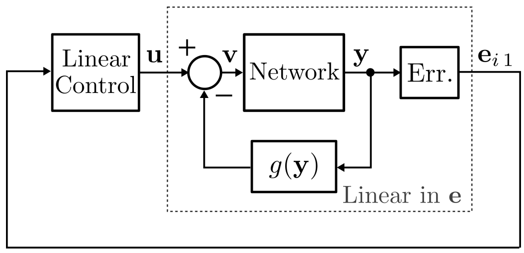

Fig. 1. Control scheme proposed

Then, choosing

$$
v_i = -g_i(\mathbf{y}) + u_i \tag{16}
$$

the resultant system is linear in terms of $\mathbf{e}_i$, that is

$$
\dot{\mathbf{e}}_i = A_{\mathbf{e}_i}\mathbf{e}_i + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_i \tag{17}
$$

Now the goal established in (3) can be achieved by an apropiate design of u_i such that the origin of (17) is asymptotically stable.

Writing the complete system (17) in a matrix form, we obtain

$$
\dot{\mathbf{e}} = \mathrm{diag}(A_{\mathbf{e}_2},\, \ldots\, , A_{\mathbf{e}_N})\,\mathbf{e} + \left(\mathbb{I}_{N-1} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \mathbf{u} \tag{18}
$$

where $\mathbf{e} = [\mathbf{e}_2, \ldots, \mathbf{e}_N]^T \in \mathbb{R}^{2(N-1)\times 1}$; $\mathbf{u} = [u_2, \ldots, u_N]^T \in \mathbb{R}^{(N-1)\times 1}$; $\mathbb{I}_{N-1}$ is a $(N-1)$ identity matrix; and the operator $\otimes$ represents the usual *Kronecker* matrix product. The system in (18) is completely decoupled since the variables of every $\dot{\mathbf{e}}_i$ do not interact with other node error dynamics. This allows to design separately every control action u_i. As it is a linear system, any linear control can be propose. In particular we propose

$$u_i = -K_i \hat{\mathbf{e}}_i \tag{19}$$

where $\hat{\mathbf{e}}_i$ is an estimation of $\mathbf{e}_i$. Choosing K_i such as $\left(A_{\mathbf{e}_i} - \begin{bmatrix}1\\0\end{bmatrix} K_i\right)$ is Hurwitz and

$$\dot{\hat{\mathbf{e}}}_i = \left(A_{\mathbf{e}_i} - L_i \begin{bmatrix}1 & 0\end{bmatrix}\right) \hat{\mathbf{e}}_i + \begin{bmatrix}1\\0\end{bmatrix} u_i + \begin{bmatrix}1 & 0\end{bmatrix} y_{\mathbf{e}_i} \tag{20}$$

where $y_{\mathbf{e}_i}$ is the difference of membrane voltage between *Master* and ith *Slave*.

The Gaussian white noise is usually used to simulate the ionic channel noise of neurons, which is actually the most common disturbance in neuroscience (Che *et al.*, 2009). Then, the state-space description of each $\dot{\mathbf{e}}_i$ remains as follows

$$\begin{aligned} \dot{\mathbf{e}}_i &= A_{\mathbf{e}_i}\, \mathbf{e} + \begin{bmatrix}1\\0\end{bmatrix} u_i + G_i\, w_{\mathbf{e}_i} \\ y_{\mathbf{e}_i} &= \begin{bmatrix}1 & 0\end{bmatrix} \mathbf{e}_i + n_{\mathbf{e}_i} \end{aligned} \tag{21}$$

where $w_{\mathbf{e}_i}$ and $n_{\mathbf{e}_i}$ are two white, zero-mean, mutually uncorrelated noise signals; and the observer described in (20) can be implemented with a Linear Kalman Filter considering the Gaussian noise described before. The time-varying Kalman gain L_i can be computed as it follows (Bay, 1999)

$$\begin{aligned} \dot{P}_i &= -P_i C_{\mathbf{e}_i}^T W_i^{-1} C_{\mathbf{e}_i} P_i + G_i V_i G_i^T + A_{\mathbf{e}_i} P_i + P_i A_{\mathbf{e}_i}^T \\ L_i &= P_i C_{\mathbf{e}_i}^T W_i^{-1} \end{aligned} \tag{22}$$

where $V = E\{w_{\mathbf{e}_i} w_{\mathbf{e}_i}^T\}$, $W = E\{n_{\mathbf{e}_i} n_{\mathbf{e}_i}^T\}$ and $E\{\bullet\}$ represent the expected value of $\bullet$.

3. NUMERICAL EXAMPLE

The network configuration it's shown in Fig. 2. For simplicity, we consider $c_{ij} = c_{ji} = \sigma$.

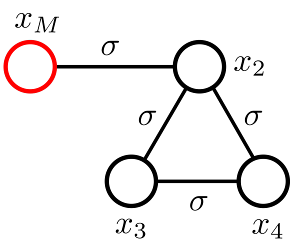

Fig. 2. Network structure

$$A = \begin{bmatrix} 0&1&0&0\\1&0&1&1\\0&1&0&1\\0&1&1&0 \end{bmatrix} \quad C = \begin{bmatrix} 0&\sigma&0&0\\ \sigma&0&\sigma&\sigma\\ 0&\sigma&0&\sigma\\ 0&\sigma&\sigma&0 \end{bmatrix} \quad \Gamma = \begin{bmatrix}1&0\\0&0\end{bmatrix} \tag{23}$$

In the following, we present the dynamics of each node using the notation proposed in (2).

$$\begin{aligned}
\dot{x}_{M\,1} &= f_1(\mathbf{x}_M) + \sigma\,(x_{M\,1} - x_{2\,1}) \\
\dot{x}_{M\,2} &= f_2(\mathbf{x}_M) \\
\dot{x}_{2\,1} &= f_1(\mathbf{x}_2) + \sigma\,(x_{2\,1} - x_{M\,1}) + \sigma\,(x_{2\,1} - x_{3\,1}) \\
&\quad + \sigma\,(x_{2\,1} - x_{4\,1}) + v_2 \\
\dot{x}_{2\,2} &= f_2(\mathbf{x}_2) \\
\dot{x}_{3\,1} &= f_1(\mathbf{x}_3) + \sigma\,(x_{3\,1} - x_{2\,1}) + \sigma\,(x_{3\,1} - x_{4\,1}) + v_3 \\
\dot{x}_{3\,2} &= f_2(\mathbf{x}_3) \\
\dot{x}_{4\,1} &= f_1(\mathbf{x}_4) + \sigma\,(x_{4\,1} - x_{2\,1}) + \sigma\,(x_{4\,1} - x_{3\,1}) + v_4 \\
\dot{x}_{4\,2} &= f_2(\mathbf{x}_4)
\end{aligned} \tag{24}$$

Thus, the dynamical systems of the tracking error between *Master* and every *Slave* as in (13) remains as follows

$$\begin{aligned}
e_{2\,1} &= x_{2\,1} - x_{M\,1} = f_1(\mathbf{e}_2) + 2\sigma\, e_{2\,1} + g_2(\mathbf{y}) + v_2 \\
e_{2\,2} &= x_{2\,2} - x_{M\,2} = f_2(\mathbf{e}_2) \\
e_{3\,1} &= x_{3\,1} - x_{M\,1} = f_1(\mathbf{e}_3) + g_3(\mathbf{y}) + v_3 \\
e_{3\,2} &= x_{3\,2} - x_{M\,2} = f_2(\mathbf{e}_3) \\
e_{4\,1} &= x_{4\,1} - x_{M\,1} = f_1(\mathbf{e}_4) + g_4(\mathbf{y}) + v_4 \\
e_{4\,2} &= x_{4\,2} - x_{M\,2} = f_2(\mathbf{e}_4)
\end{aligned} \tag{25}$$

where each $g_i(\mathbf{y})$ is given by

$$\begin{aligned}
g_2(\mathbf{y}) &= f_1(x_{2\,1})_{nl} - f_1(x_{M\,1})_{nl} + \sigma\,(x_{2\,1} - x_{3\,1}) \\
&\quad + \sigma\,(x_{2\,1} - x_{4\,1}) \\
g_3(\mathbf{y}) &= f_1(x_{3\,1})_{nl} - f_1(x_{M\,1})_{nl} + \sigma\,(x_{3\,1} - x_{2\,1}) \\
&\quad + \sigma\,(x_{3\,1} - x_{4\,1}) - \sigma\,(x_{M\,1} - x_{2\,1}) \\
g_4(\mathbf{y}) &= f_1(x_{4\,1})_{nl} - f_1(x_{M\,1})_{nl} + \sigma\,(x_{4\,1} - x_{2\,1}) \\
&\quad + \sigma\,(x_{4\,1} - x_{3\,1}) - \sigma\,(x_{M\,1} - x_{2\,1})
\end{aligned} \tag{26}$$

Applying the control law defined in (16), each linear dynamical system $\dot{\mathbf{e}}_i$

$$\begin{aligned}
\dot{\mathbf{e}}_2 &= \begin{bmatrix} -\lambda + 2\sigma & -1 \\ \epsilon & -\delta\,\epsilon \end{bmatrix} \mathbf{e}_2 + \begin{bmatrix}1\\0\end{bmatrix} u_2 \\
\dot{\mathbf{e}}_3 &= \begin{bmatrix} -\lambda & -1 \\ \epsilon & -\delta\,\epsilon \end{bmatrix} \mathbf{e}_3 + \begin{bmatrix}1\\0\end{bmatrix} u_3 \\
\dot{\mathbf{e}}_4 &= \begin{bmatrix} -\lambda & -1 \\ \epsilon & -\delta\,\epsilon \end{bmatrix} \mathbf{e}_4 + \begin{bmatrix}1\\0\end{bmatrix} u_4
\end{aligned} \tag{27}$$

Notice that $A_{\mathbf{e}_3} = A_{\mathbf{e}_4}$ due to the lack of coupling with the *Master* node.

3.1 Results

In this section, numerical simulations are carried out to illustrate the proposed control scheme. The parameters of the model have been chosen so that (5) has a sustained oscillation (limit cycle) according to the analysis developed in (Ringkvist and Zhou, 2005). Explicitly: $\lambda = -0.5$, $\epsilon = 0.5$ and $\delta = 0.5$. On the other hand, the coupling strength $\sigma = 0.02$ has been chosen so that the system does not synchronize by itself according to (Jiang *et al.*, 2004).

We design every K_i gain as stated in (19) such that the eigenvalues of $\left(A_{\mathbf{e}_i} - \begin{bmatrix}1\\0\end{bmatrix} K_i\right)$ are $[-1, -1]$. On the other hand, to design the Kalman Filter as in (22) we assume that $G = [1\ -1]^T$ and the noise term $w_{\mathbf{e}_i}$ has zero mean and covariance $V = 0.5$ for every node in the network. Also, the measurement noise term is assumed to have zero mean and variance $W = 0.02$ as in (Yu *et al.*, 2012).

The evolution of the membrane voltage of every node ($x_{i\,1}$) in the network it's shown in Fig. 3. Also, in Fig. 4 the $x_{M\,2}$–$x_{i\,2}$ phase plane for every ith slave it's shown. The first seconds of simulation corresponds to the open loop network. At $t = 30$ the control developed it's applied. As it can be appreciated in both figures, after the controller is applied, the errors converge to zero and the complete synchronization is obtained.

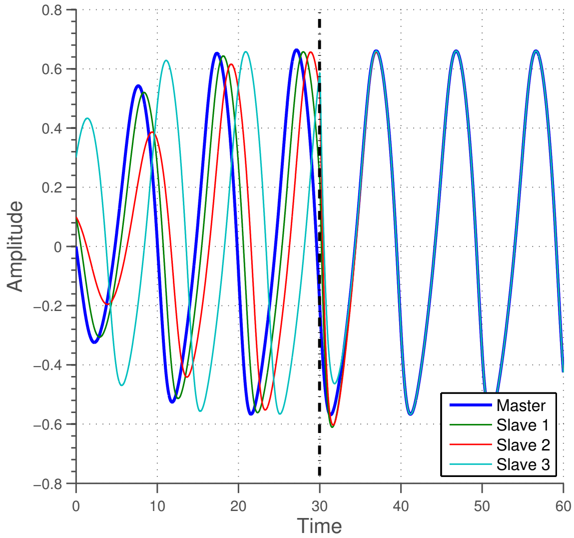

Fig. 3. Open and closed loop

Finally, to further evaluate the control strategy, we add Gaussian random noise with mean 0 and variance 0.02 as we stated before. Simulation with measurement noise it's shown in Fig. 5, where it can be appreciated the effect of the Kalman filter in the estimation of every $e_{i\,1}$.

4. CONCLUSION

In this paper, a control scheme for the complete synchronization of a network with a leader is developed. This strategy focus the synchronization as a tracking control problem. A simple MIMO exact decoupling feedback linearization technique based on the network structure is proposed, allowing to design a state feedback control for each *Master* - *Slave* tracking error dynamical system separately. Moreover, a Linear Kalman Filter is applied to estimate the unknown states and also improve the loop response in presence

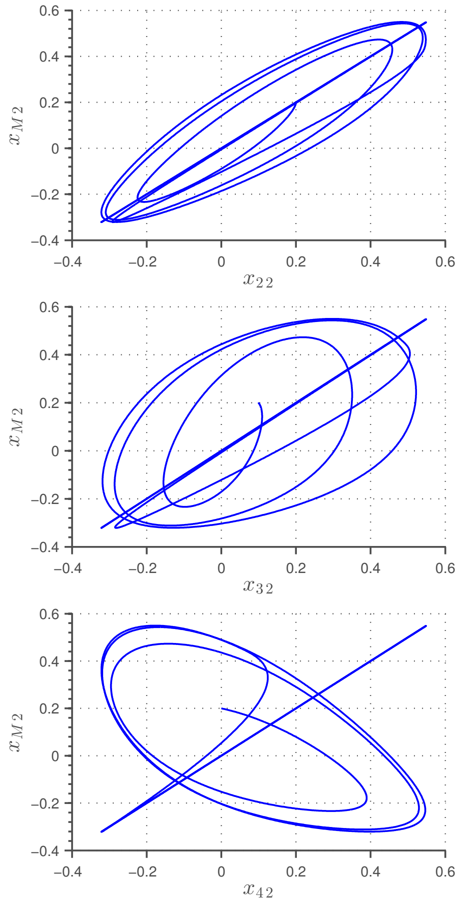

Fig. 4. Open and closed loop

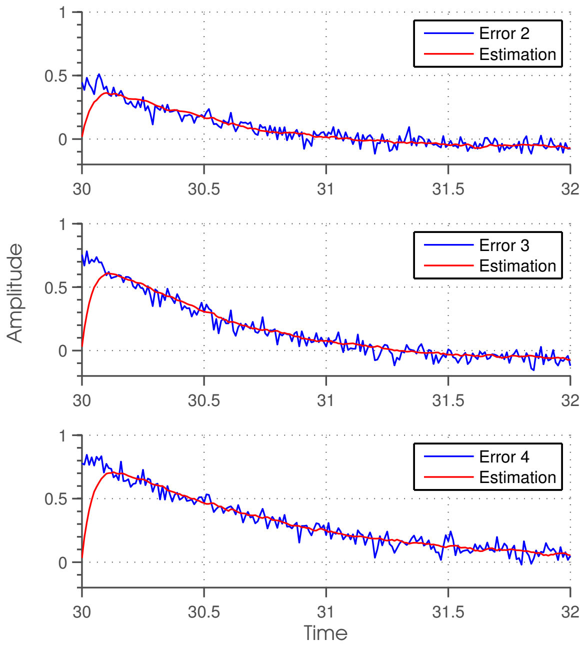

Fig. 5. Estimation of every $e_{i\,1}$ state

of measurement noise, which is a common issue in this kind of systems. Although we perform simulations to test the robustness of the control strategy developed in presence of parameter uncertainties, we propose as a future improvement to this work to add an adaptive strategy to handle this possible uncertainties in the model.

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