Wiener-Hopf Solution of Diffraction by a PEC Wedge in Anisotropic Media

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In this work we present our recent work on novel Wiener-Hopf (WH) formulation for the analysis and the study of scattering of wedges [1] immersed in complex materials [2-3]. We start from an original study of the perfect electrically conducting (PEC) wedge immersed in uniaxial ($\varepsilon_x=\varepsilon_y\neq\varepsilon_z$, $\mu_x=\mu_y\neq\mu_z$) and biaxial (all different ε_i, μ_i i=x,y,z) media illuminated by plane waves, where the wedge has an aperture angle of $2\pi-2\gamma$. The uniaxial case has been studied by Felsen in [4], however generalization and exploitation of this case has not been further carried out in literature.

Our formulation allows the analysis of uniaxial case and biaxial case by developing an original theory in the framework of Wiener-Hopf technique, which allows closed form analytical solution in spectral domain. The theory takes inspiration from the one presented in [5] for stratified media with a generalization to angular regions [2-3]. Moreover, in anisotropic media different from uniaxial and biaxial media, a closed form solution is not available, however our theory allows to resort to Fredholm factorization technique that gets approximate solutions preserving physical meaning [6],[1]. From trasversalization of Maxwell's equations in oblique Cartesian coordinates, we obtain a matrix differential problem of first order in terms of the field components tangent at the boundaries. In spectral domain, by using Laplace transform, we apply the characteristic Green's function procedure to the system of differential equation yielding a spectral solution before imposing boundary conditions. The solutions represent functional equations that relate the spectral field components along two faces of the angular region. By imposing the boundary conditions the combination of functional equations of each angular region yields a set of GWHEs for the angular region problem ($-\gamma<\varphi<\gamma$). The imposition of PEC boundary conditions on the two faces of the wedge simplifies the problem (vanishing tangent electric field components), however the presence of biaxial medium yields two types of propagation constants. For this reason, the GWHEs present plus functions defined in a complex plane, and minus functions defined into two different complex planes (related to the two propagation constants). We note that the two complex planes of the minus functions are related to the complex plane of the plus functions. The proposed new and original extension of WH technique for the uniaxial and biaxial cases is based on a decoupling procedure among the equations due to the two propagation constants and on a suitable definition of spectral mappings to resort to two systems of Classical Wiener-Hopf Equations (CWHEs) in two new complex planes. Once completed this not easy procedure, simple closed form factorizations allow solutions in spectral domain. Further detail and numerical results will be presented at the conference. Moreover, the sets of derived WH equations are not limited to angular region problems, for example in [7] functional equations of different kind of regions are coupled together.

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