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(Article begins on next page)

Dakar Mosque gridshell: exploring the benefits of the improved multibody rope approach through postbuckling analysis

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Abstract

This paper presents a comparative study of a gridshell roof structure designed for the Islamic Cultural Center in Dakar, Senegal. The structure, conceived by Fragomeli & Partners, emulates the shape of a dune to seamlessly integrate with the surrounding environment. To define an optimal structural shape while preserving the architectural intent, two form-finding methodologies were employed. The first method, the Multibody Rope Approach (MRA), enables the calculation of the funicular structural shape in response to the imposed load configuration. The second method, Improved MRA (i-MRA), introduces slight geometric variations to the funicular shape to reduce the diversity of structural elements required for construction. This work investigates the impact of the geometric distortions introduced by i-MRA on structural stability. Linear buckling modal analysis has been conducted on the two generated geometries, taking into account various loading conditions and structural element slenderness. Through incremental load Geometrical Non-linear with Imperfection Analyses (GNIAs) the equilibrium paths are examined to analyze the postbuckling behaviour of the gridshells. In the final step, the elastoplas-

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tic constitutive law is incorporated to assess also the impact of material non-linearity. The models are examined through incremental load Geometrical and Material Non-linear Analyses (GMNIAs). The results indicate that the i-MRA geometry exhibits a more even stress distribution and better qualitative postbuckling behaviour.

Keywords: Gridshell, Multi-body Rope Approach, Postbuckling, Snap through instability, Nonlinear bifurcation

1. Introduction

Architecture and structural engineering are closely intertwined, with architecture influencing and being influenced by structural design. Over time, these disciplines have evolved hand in hand, responding to the ever-changing needs and aspirations of society. The dynamic relationship between architecture and structure has played a pivotal role in shaping a wide range of structural typologies and forms [1]. This holds in particular for the spatial structures in which the shape is strictly interconnected with the stress distribution. In this context, form-finding methodologies were introduced during the 1960s and 70s to establish the structural geometry as a function of the load field. These techniques have found extensive application, especially in the design of shells and gridshells [2]. Concerning structural stability, gridshells can be considered as single-layer reticulated shells. Architectural audacity and complexity require safety analyses for structural integrity beyond the standard procedures [3]. In this regard, the IASS (International Association for Spatial Structures) published comprehensive guidelines to buckling load evaluation of metal structures [4]. To incorporate all the non-univocal aspects behind buckling collapse, different analyses and approaches are indicated to assess the final design critical load. The elastic buckling load Q_{EB}

20 is usually evaluated through a Geometrically Nonlinear with Imperfection
 21 Analysis (GNIA) or Geometrically and Material Nonlinear with Imperfec-
 22 tion Analysis (GMNIA). The loading patterns, and the related displacement
 23 coordinate δ_i for which to evaluate the equilibrium path, are selected through
 24 the the linear eigenvalue extraction for the n design combination load factors
 25 Λ_n . The generic Euler load $Q_{CRi} = \Lambda_i q_i$, being q_i the total applied load in
 26 the $i - th$ combination, and the maximum modal displacement are then used
 27 as a set parameters for the GNIA. A linear static analysis of the structure
 28 loaded by Q_{CR} provides the critical displacement δ_{CR} , measured at the max-
 29 imum node displacement as a generalized coordinate. In a load-displacement
 30 graph, the straight line from the origin to the point (Q_{CR}, δ_{CR}) represents
 31 an upper-bound for all the possible equilibrium paths, as shown in Figure 1
 32 (recent authors refer to as energy barriers [5], [6], [7]). A geometrically non-
 33 linear analysis without imperfections would follow a lower stiffness path (blue
 34 dotted lines in Figure 1). This is due to the negative term of the geometric
 35 stiffness matrix, leading to the bifurcation point (marked by the blue triangle
 36 in Figure 1) associated with the global or local buckling branch. The bifur-
 37 cated branches represent the only equilibrium space of the system, therefore
 38 the resulting equilibrium path must lay within those if the control parameters
 39 of the analysis are increased. A valley in the equilibrium path indicates the
 40 presence of a snap-through instability. In this scenario, the system reaches
 41 to the *next* stable position (undergoing significant displacements) on the pos-
 42 itive slope portion of the valley at the same load level of the limit point. The
 43 presence of the imperfections (continuous blue line in Figure 1) further re-
 44 duces the system stiffness, since they are usually introduced from a geometric

45 deviation proportional to a critical eigenshape (or a combination of them).
 46 Q_{EBi} is therefore the limit point of the computed i equilibrium path using
 47 Λ_i as a load control analysis factor and a selected imperfections pattern. As
 48 illustrated in Figure 1b), Q_{EB} may result higher than Q_{CR} as a result of a
 49 hardening overall postbuckling branch. In such cases, the postbuckling be-
 50 havior is classified as benign [8], since to an increase of the external load,
 51 the stable nature of the bifurcation branch does not change (as it occurs for
 52 the Euler beam). The structure collapse would then take place in the form
 53 of a material failure or in correspondence with a local buckling secondary
 54 branch (dark grey curves in Figure 1). The interaction between local and
 55 global instabilities is a largely studied topic and of the central importance of
 56 reticulated structures [9], [10], [11], [12], [13]. These local buckling branches
 57 have always a negative slope portion in the global generalized coordinates
 58 plane, due to effect of geometric nonlinearity, as evidenced by Gioncu [14],
 59 [15], [16]. Thompson theorems ([17], [18]), [19] allow the composition of the
 60 final equilibrium path starting from discrete branching analysis. Because the
 61 resultant from a positive and negative slope is a negative one, the overall bi-
 62 furcated paths in Figure 1 are always unstable (dotted blue line). The effect
 63 of the imperfections can be measured through the erosion in the load curve
 64 between the limit point and the branch intersection cusps (more than one
 65 local buckling branch normally exists). Intuitively, sharper are these cusps
 66 deeper is the load erosion, meaning that the structure is more sensitive to
 67 geometrical imperfections. Moreover, to a vertical erosion corresponds an
 68 increase in the path curvature in proximity of the limit point, and therefore,
 69 a greater pre-buckling flexibility. Finally, a cusp bifurcation may occur as

70 the first equilibrium path bifurcation as in Figure 1a). In this case, the post-
71 buckling is identified as catastrophic, or ultra-catastrophic when both the
72 general coordinates reverse [8], [20]. In term of collapse mechanism, the limit
73 point translates into the well known snap-through buckling phenomenon [21],
74 [22], [23], [24]. Then, both the equilibrium paths in Figure 1 share such col-
75 lapse mechanism, but with a profound difference in how it is triggered. A
76 good indicator is the ratio $\frac{Q_{EB}}{Q_{CR}}$: when it is lower than 1, the corresponding
77 equilibrium path exhibits both a higher imperfection sensitivity and a higher
78 flexibility than the structures that have $\frac{Q_{EB}}{Q_{CR}} > 1$. In shell design, significant
79 knockdown factors must be introduced to keep the structure at load levels
80 safely below Q_{CR} [25], [26]. Conversely, when $\frac{Q_{EB}}{Q_{CR}} > 1$, there is still room
81 for postbuckling behaviour improvement through local buckling prevention
82 strategies that retard the overall collapse [27], [28]. The occurrence of local-
83 ized yielding increases the displacement response of the structure, therefore
84 the elasto-plastic buckling load Q_{EP} , evaluated through a GMNIA, can be
85 notably lower than Q_{EBi} . This is particularly relevant for flexible structures
86 and with the presence of stress spikes. A good indicator for this vulnerability
87 is the yielding load Q_Y : the level of external load at which the first yielding
88 stress is reached in any of the gridshell elements (observed by Kato in [29]).
89 Formal differences in the equilibrium path are given by three main factors,
90 that drive the overall stability behaviour: geometry, imperfections and con-
91 nections stiffness [30]. Concerning the former, comparisons between classical
92 spatial configurations have evidenced that geometries with high membrane
93 regime of internal forces (i.e. domes) exhibit smaller flexibility towards ver-
94 tical loads but increased imperfection sensitivity [31]. At the same time,

95 they also display a formal behavior transition when the shallowness ratio
96 is varied, contrary to bending driven geometries like barrel vaults. Such
97 change resembles the fundamental observations for continuous shell buckling
98 by Von Karman [32], and more recently summarized in [33], and for flat
99 arch stability [34],[35]. Anyway, reticulated systems suffer local instability
100 problems more than their continuous counterparts, such as connection fail-
101 ure or member (local) buckling, that can lead to progressive collapse of the
102 whole compartment [36], [37], [13], [38]. Freeform geometries have instead a
103 less predictable behavior, because the force fluxes have not a straightforward
104 path to the bearings [39]. Mesh pattern plays a significant role in direction-
105 ing such fluxes, and stability of freeform gridshells can be altered by it within
106 the same median surface [40]. Slenderness of the members is also a part of
107 the geometrical arrangement of a gridshell, and its effect on the equilibrium
108 path is both nonlinear and non monotonic when varied [41],[42]. However,
109 section members have often architectural constraints in shape and size, and
110 in general chosen towards a dead load minimization. This three level set
111 of geometrical features (median surface, mesh, slenderness members) concur
112 hierarchically in generating the tangential stiffness matrix along the equilib-
113 rium path [43], [44].

114 In this paper we tried to use qualitative studies of the equilibrium path shape
115 as a powerful tool for the conceptual design of a gridshell, testing two differ-
116 ent geometries generated through a Multibody Rope Approach (MRA) [45]
117 and its improved version (iMRA) [46]. The MRA is a form-finding method-
118 ology designed to define structural geometry [47] by dynamically solving a
119 structural model consisting of falling masses connected by inextensible ropes

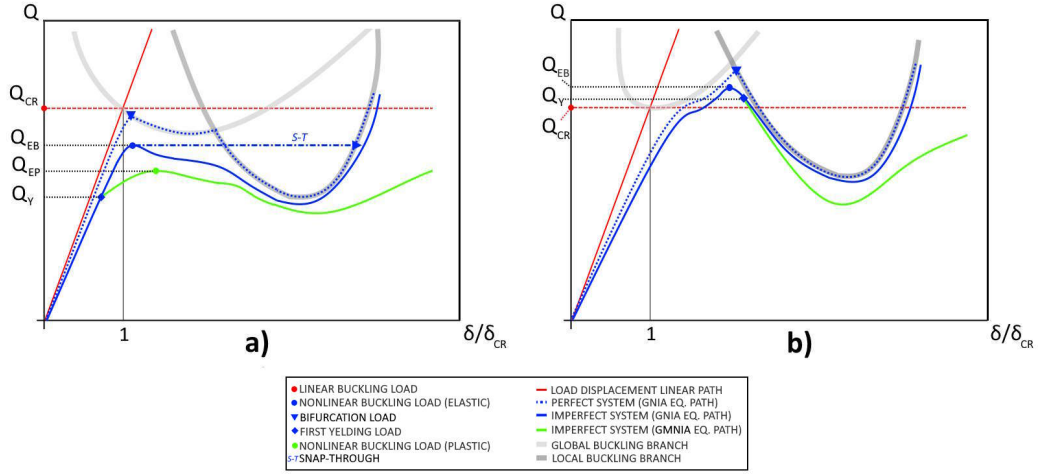


Figure 1: a) Equilibrium path of a reticulated shell with ultra-catastrophic post-buckling behavior. b) Benign (stiffening) postbuckling behavior.

120 [48, 49]. The i-MRA represents the enhanced version of the MRA [50]. The
 121 method is specifically designed for the form-finding and the optimization of
 122 the construction process of free-form gridshells [46]. A detailed study is con-
 123 ducted on the postbuckling behaviour of the geometries produced by these
 124 approaches [51]. In particular, we tested the influence of member slender-
 125 ness through GNIA and GMNIA and several imperfection magnitudes for
 126 separate load patterns. We instead kept a unique initial tessellation and
 127 hypothesized a perfect rigid quadrangular connection.

128 2. Form-finding techniques

129 The Multibody Rope Approach (MRA) is a specialized method designed
 130 specifically for determining the funicular shape of free-form gridshells. In
 131 this form-finding method, the shape of the structure is derived by modelling
 132 a multi-body physical system. This system consists of nodal masses inter-

133 connected by slack ropes, each with a length equal to l_{rope} . The structural
 134 shape can be calculated by solving the dynamic model of the system 1 and
 135 applying D'Alembert's principle to determine the equilibrium of the system.

136 The solution can be obtained as:

$$\vec{u}(t) = C_1 e^{-2\omega_n \zeta} + C_2 + \frac{C_3}{2\omega_n \zeta} t \quad (1)$$

137 Where C_1 , C_2 and C_3 are coefficients that can be calculated the system
 138 initial conditions.

$$C_1 = -\frac{2\omega_n \zeta \dot{\vec{u}}(t-\Delta t) - C_3}{(2\omega_n \zeta)^2} \quad (2)$$

$$C_2 = -\frac{(2\omega_n \zeta)^2 \vec{u}(t-\Delta t) + 2\omega_n \zeta \dot{\vec{u}}(t-\Delta t) - C_3}{(2\omega_n \zeta)^2} \quad (3)$$

$$C_3 = \vec{p}_i + \sum_{j=1}^{n_i} \left\{ k \cdot \vec{F}_{rope,ji} \right\} \quad (4)$$

139 In Equation 1, $\vec{u}(t)$ represents the position of the nodal masses, where
 140 t signifies the temporal instant at which the equation is computed. Addi-
 141 tionally, $\omega_n = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{c}{2\omega_n m}$ correspond to the natural frequency and
 142 critical damping of the system, respectively. These two quantities can be
 143 readily obtained by knowing the stiffness of the ropes k , the mass of the
 144 nodes m , and the damping coefficient of the system c .

145 In the equation 4, the vector p_i represents the applied external force on
 146 a generic node i , while the forces transmitted by the ropes connected to the
 147 nodes are denoted by $\vec{F}_{rope,ji}$. It is important to note that the reaction of

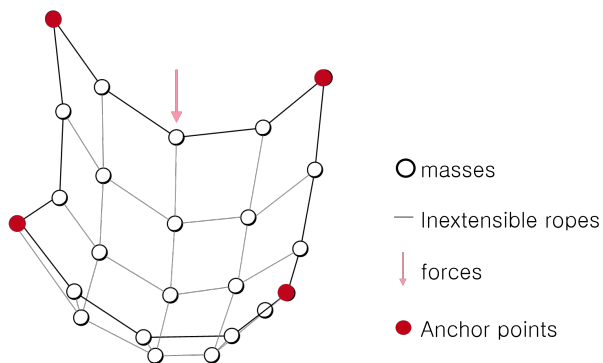


Figure 2: Dynamic model of falling bodies employed in the MRA and in the i-MRA.

148 the ropes can only be exerted when they are under tension. Thus, the forces
 149 $\vec{F}_{rope, ji}$ can be derived as follows:

$$\begin{cases} F_{rope} = 0 & \text{if } l < l_{rope} \\ F_{rope} = k(l - l_{rope}) & \text{if } l \geq l_{rope} \end{cases} \quad (5)$$

150 Starting with a basic mesh, the final solution is calculated through a step-
 151 by-step process. The conditions at each step rely on the solution from the
 152 previous one.

153 In the final solution, structural components can be classified into two sets
 154 based on their length. Elements shorter than l_{rope} are denoted as *loose ele-*
 155 *ments* and those with a length matching l_{rope} are labelled as *target elements*.

156 The i-MRA method stands as an advancement over the MRA, seeking
 157 not only to optimize geometry but also to improve the automation of the
 158 construction process by minimizing the variety of structural element typolo-
 159 gies [46]. In particular, two key techniques are employed to achieve this
 160 optimization. The first involves grouping the structural elements and assign-

161 ing different slack coefficients to each group. In this case, starting from the
 162 solution derived by the MRA, a new equilibrium configuration is calculated
 163 using the subsequent formulation for the forces transmitted by the ropes,
 164 denoted as $\vec{F}_{rope,ji}$.

$$\left\{ \begin{array}{ll} F_{rope} = 0 & \text{if } l < l_{rope,2} \\ F_{rope} = k(l - l_{rope,2}) & \text{if } l_{rope,2} < l \leq \gamma(l_{rope,1} - l_{rope,2}) + l_{rope,2} \\ F_{rope} = 0 & \text{if } \gamma(l_{rope,1} - l_{rope,2}) + l_{rope,2} < l < l_{rope,1} \\ F_{rope} = k(l - l_{rope,1}) & \text{if } l \geq l_{rope,1} \end{array} \right. \quad (6)$$

165 In Equation 6, the terms $l_{rope,1}$, $l_{rope,2}$, ..., $l_{rope,n}$ represent the lengths of
 166 the ropes belonging to various groups.

167 Through an iterative process, a funicular configuration is determined
 168 where the majority of the ropes, representing the structural elements, are ten-
 169 sioned. then, the final model undergoes slight geometric variations through
 170 the application of a repulsive force field on the nodal masses. This causes the
 171 nodes to move away from each other and also tensions the remaining slack
 172 ropes. The application of this method involves incorporating into the ex-
 173 ternal forces p_i an additional contribution q_i , which represents the repulsive
 174 forces as defined in Equation 7.

$$q_i = -k_{rep}(l_{rope} - l_{ij}) \quad (7)$$

175 The combination of these techniques enables the grouping of structural
 176 elements by length and, consequently, minimizes the number of beam ty-
 177 pologies required for the completion of the gridshell.



Figure 3: Islamic cultural center in Dakar, Senegal.

178 **3. Case Study: The Dakar Mosque**

179 The case study outlined in this paper focuses on the roof of the Islamic
180 Cultural Center in Dakar, Senegal. The roof of the mosque, depicted in
181 Figure 3, is designed by the architectural firm Fragomeli+Partners [52] in
182 collaboration with Wafai Architecture. The distinctive roof design draws
183 inspiration from the desert dunes, harmonizing with the surrounding envi-
184 ronment.

185 *3.1. Geometry Outline*

186 The design of the Dakar mosque roof consists of a steel gridshell span-
187 ning meters 63 meters by 56 meters. The shape is defined with the aim of
188 creating a free span, resting solely along its perimeter, eliminating the need
189 for intermediate pillars, as depicted in Figure 4.

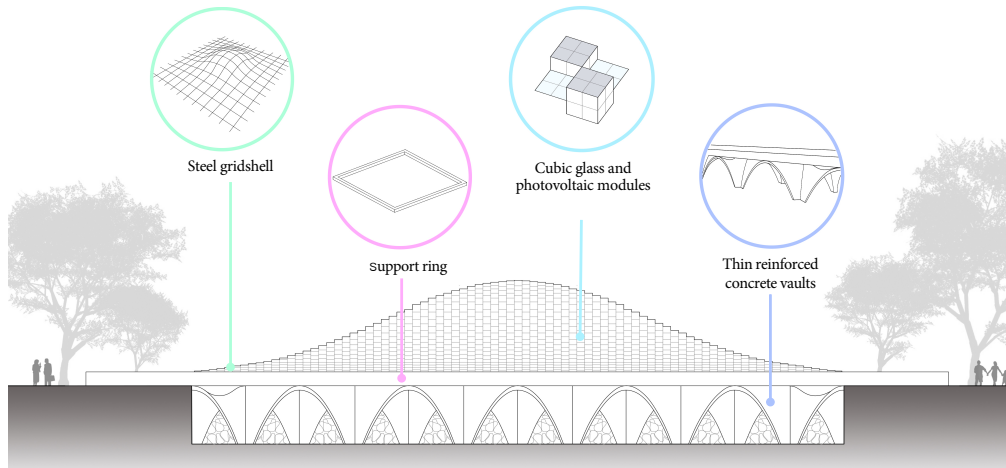


Figure 4: The concept behind the structural design of the Dakar Mosque.

190 Such geometry is primarily influenced by architectural design, introducing
 191 specific constraints, including a distinctive curvature that distinguishes it
 192 from conventional vaulted structures.

193 These constraints have significant implications for both the quantitative
 194 and qualitative aspects of structural behaviour characteristics. In general,
 195 the form-finding process involves applying the loads that act on the structure
 196 permanently to the numerical model. This allows for defining a structural
 197 form that is a funicular shape with respect to the loads that predominantly
 198 act on the structure during its service life. In the specific case of the roof
 199 of the Dakar mosque, to meet the architectural requirements, a load field
 200 different from the one actually experienced by the structure was required
 201 (as depicted in Figure 5). This deviation of the force field from the real
 202 one allows for achieving the curvature inversion regions that characterize the
 203 shape of the desert dune requested by the architects. At the same time, this
 204 change defines a structural form that is no longer funicular with respect to

205 the actual permanent loads. Therefore, the structural form with curvature
206 change is less-than-optimal compared to the funicular form.

207 In this paper, two distinct form-finding methodologies were employed to
208 establish the shape of the roof. The form-finding methodologies are used to
209 define optimal geometries, notwithstanding the above mentioned constraints.

210 In particular, the application of the Multibody Rope Approach (MRA)
211 [48] and the improved Multibody Rope Approach (i-MRA) [46] enables the
212 derivation of a geometry that achieves a structurally feasible configuration
213 while preserving the initial architectural design.

214 A parametric model of the structure was defined to derive the dynamic
215 hanging net model subject to the load system depicted in Figure 5. By
216 applying the MRA and i-MRA techniques to solve the equilibrium of the
217 system, two configurations were obtained, both aligned with the vision of
218 the architectural designer.

219 The Dakar mosque case presents a clear distinction in the geometries re-
220 sulting from the utilization of the MRA and i-MRA form-finding methods.
221 The MRA technique results in a configuration with a structural hierarchy
222 where the main bearing structure is the central arch (Figure 6a). The struc-
223 tural elements required to construct the MRA shape are distinguished by 19

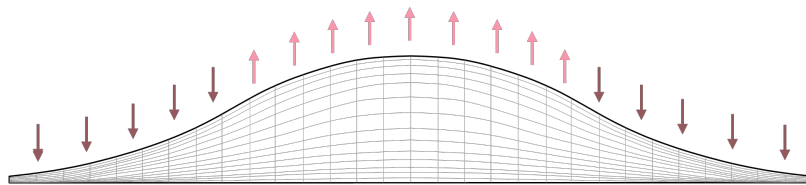


Figure 5: Force field applied to define architectural design.

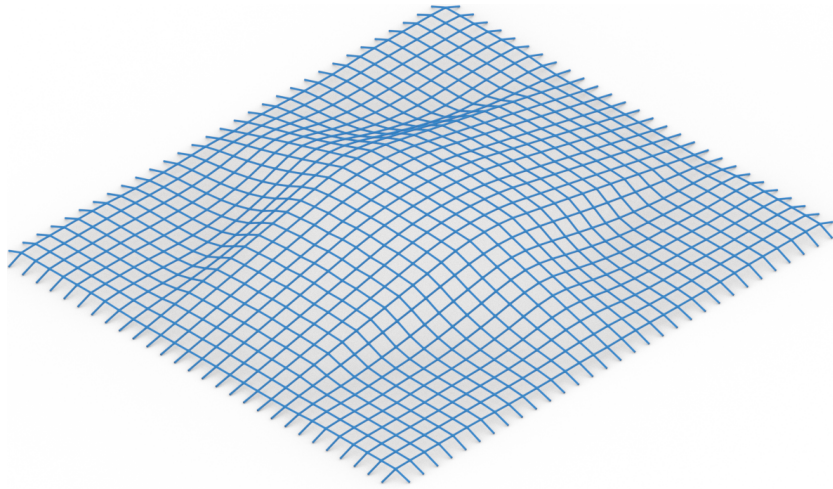
224 different lengths.

225 In comparison, the i-MRA approach creates a smoother and regular ge-
226 ometry (Figure 6b). In this configuration, there are no self-evident structural
227 hierarchies.

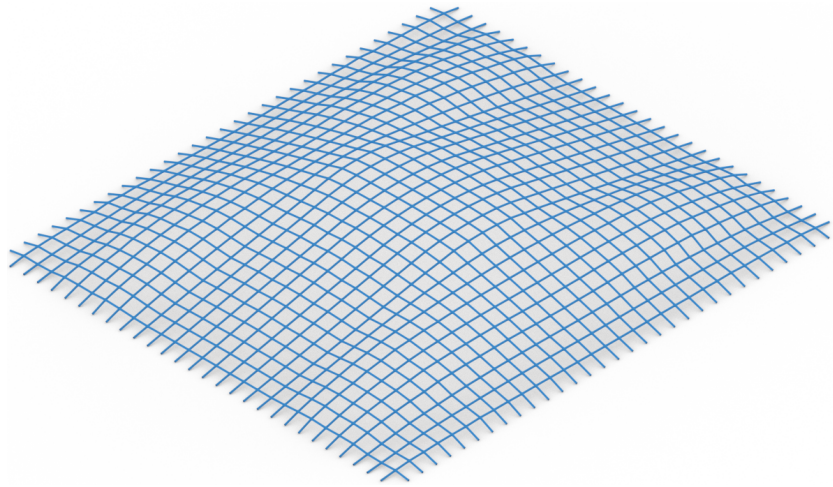
228 The i-MRA method, being designed to maximize the tensioned ropes in
229 the hanging net model, promotes a greater degree of uniformity in the final
230 geometry. Thus, a larger number of elements are engaged in supporting the
231 applied loads. Consequently, only 8 element typologies define the i-MRA
232 geometry.

233 The distribution of various types of structural elements for the two dis-
234 tinct structural configurations is illustrated in Figure 7 through histograms.
235 Specifically, the MRA method involves establishing a target length, which,
236 in this case, was set at 2.00m. Implementing this method results in ob-
237 taining 954 elements, constituting 53% of the total elements, with a length
238 precisely matching the set target. Conversely, the remaining 47% of struc-
239 tural elements exhibit lengths different from the target. These elements are
240 characterized by 18 distinct lengths, ranging from a minimum of 1.78m to a
241 maximum of 2.02m.

242 In the case of using i-MRA, it is possible to identify various target lengths.
243 This allows grouping structural elements into a smaller number of types. In
244 the specific case of the examined gridshell, four different target lengths were
245 set at 1.85m, 1.90m, 1.95m, and 2.00m. In this scenario, it can be observed
246 that 99% of structural elements have a length precisely matching one of
247 these targets. This enabled the grouping of structural elements, reducing the
248 number of different types from 19 to just 8.



(a) MRA



(b) i-MRA

Figure 6: Geometries obtained with the form-finding techniques.

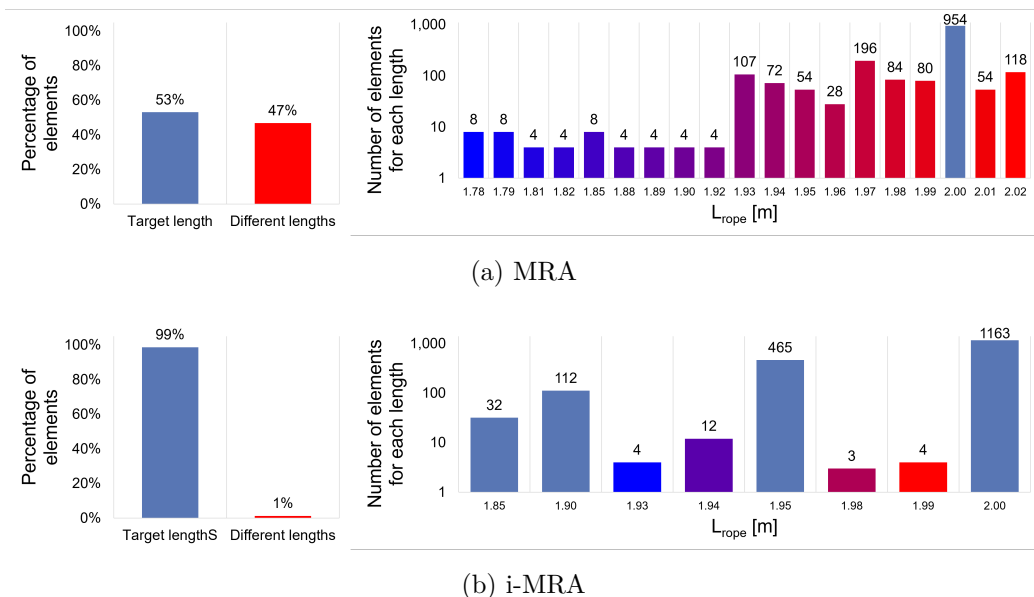


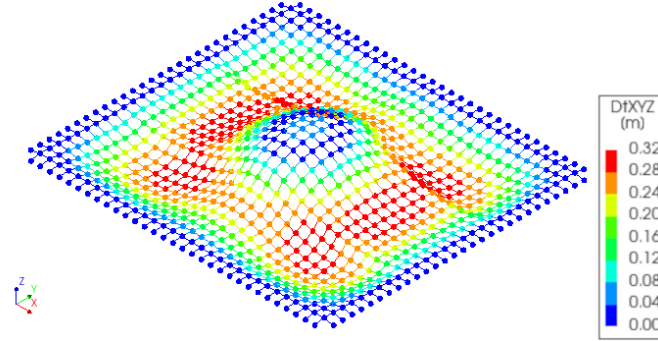
Figure 7: Structural element typologies distribution in the two structural configurations.

249 *3.2. Preliminary analyses*

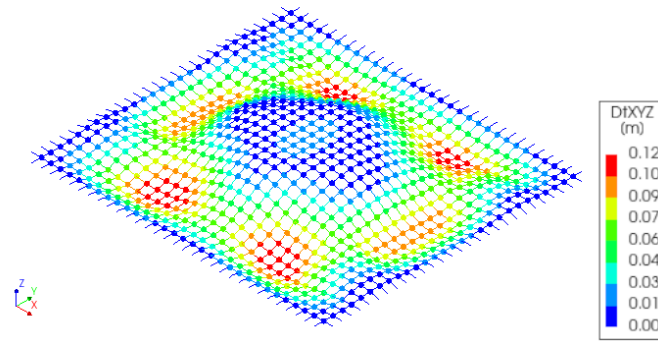
250 We employed a finite element model of the gridshell by using quadratic
 251 three-node beam elements to investigate some fundamental structural prop-
 252 erties. S355 steel circular hollow cross-section (CHS 19.1x8.0 mm) has been
 253 used and the base perimeter has been modelled as a series of full constraints.

254 The structural elements constituting both structures are fully constrained
 255 at their extremities. Choosing this type of constraint in a quadrangular mesh
 256 gridshell is essential for establishing a hyperstatic structural scheme. Indeed,
 257 opting for doubly hinged structural elements would result in a statically
 258 undetermined structural scheme.

259 In order to compare the two different shapes and evaluate the effects of
 260 geometric variations introduced by the i-MRA method, static and dynamic
 261 analyses have been conducted using the commercial finite element software



(a) MRA



(b) i-MRA

Figure 8: Displacement due to self-weight.

262 Diana(R) (Dianafea bv, The Netherlands) [53]. Superstructure weight (solar
 263 glass panels) has been modelled as concentrated nodal loads, in addition to
 264 elements self-weight, considered as distributed.

265 In Figure 8 the displacement profiles obtained for the two geometries
 266 have been reported. for both cases, the maximum displacements occur at
 267 the curvature inversion regions of the roof. These areas represent struc-
 268 tural weaknesses in terms of the response to vertical loading. Moreover, the

269 displacements are minimal near the supports and the central section of the
270 structure, aligning with the expected behaviour. This emphasizes the rigidity
271 of the central dome-like section compared to the curvature inversion zones.
272 The behaviour validates the well-established effectiveness of vaulted struc-
273 tures in supporting vertical loads. Indeed, the larger displacements occur in
274 areas where the roof deviates further from the dome-like geometry.

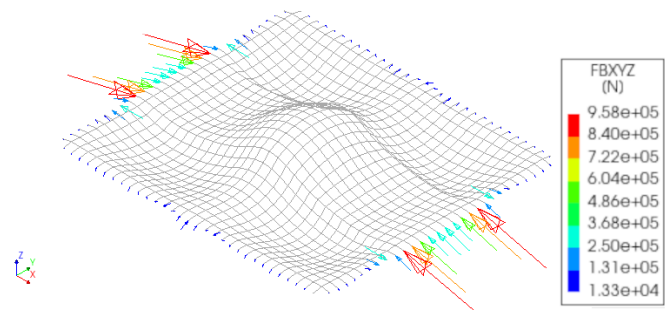
275 It is worth noting that the MRA shape exhibits a maximum displacement
276 that is more than twice the value observed in the improved i-MRA case.
277 However, even in the most extreme scenario, the maximum displacement
278 remains well below 1/200th of the main roof span. Thus, the deformations
279 observed in both cases are within acceptable limits and do not compromise
280 the overall stability and functionality of the roof structure.

281 Figure 9 illustrates a comparative analysis of the reaction forces acting
282 on the rigid supports.

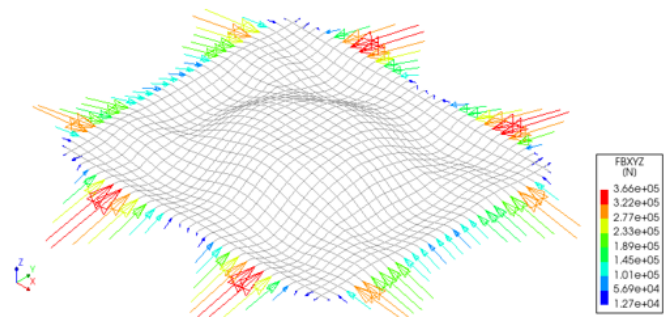
283 A clear pattern emerges in the MRA-derived geometry. It is evident
284 the presence of an arching behaviour along the main span direction, within
285 a narrow central zone of the roof. Significantly lower reaction forces are
286 observed on the supports not directly influenced by the central arch. This
287 hierarchy in the reaction distribution implies that the central arch is mainly
288 responsible for supporting the vertical loads.

289 In contrast, the support reactions are more equally distributed and bal-
290 anced in the i-MRA shape.

291 Arching mechanisms are present in both horizontal directions, indicating
292 bidirectional structural behaviour. The result is a more efficient load-sharing
293 capacity of the second geometry.

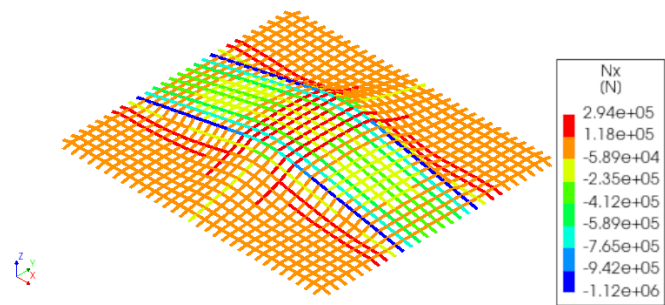


(a) MRA

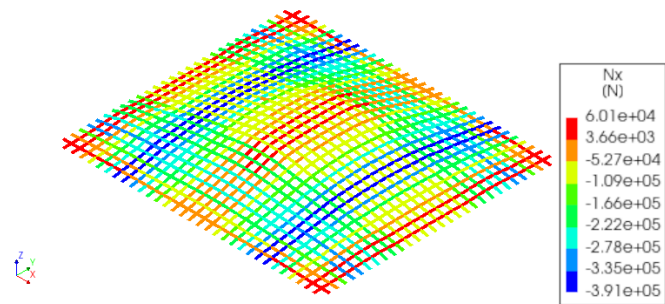


(b) i-MRA

Figure 9: Bearing reactions due to self-weight.



(a) MRA



(b) i-MRA

Figure 10: Axial force in the structural elements due to self-weight.

294 The axial force and bending moment generated by dead loads are depicted
295 in Figure 10 and 11, respectively. An obvious difference between the two
296 designs emerges considering the internal force patterns.

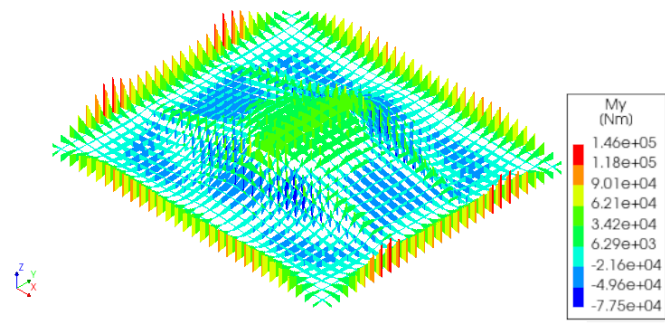
297 In the MRA structure, higher compressive forces are concentrated in the
298 central arch. In contrast, the lateral elements are either lightly loaded or
299 subjected to tension.

300 The colour-scaled graph effectively visualizes these distinct structural hi-
301 erarchies. Indeed, the compressed central arch appears clearly distinguished
302 from the secondary elements.

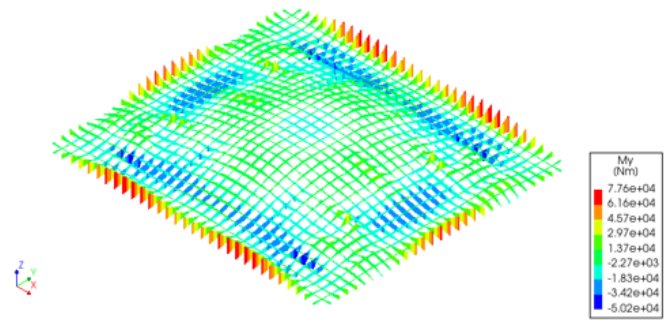
303 By opposition, the i-MRA geometry demonstrates a more balanced load
304 distribution. Compressed arches are present in both directions. This results
305 in a significant reduction in the extent of tensile regions compared to the
306 MRA case. Moreover, in the MRA geometry, the maximum compressive
307 axial forces are three times higher than those observed in the i-MRA case.
308 Similarly, the maximum tensile axial forces are almost five times greater.

309 Figure 11 shows that the structural elements experience bending moments
310 in both the studied geometries.

311 As expected, the magnitude of bending moments within the gridshell
312 is relatively small. Nevertheless, it is worth noting that the i-MRA con-
313 figuration demonstrates improved performance in this regard. Specifically,
314 the maximum bending moment observed in the MRA case is approximately
315 twice the corresponding bending moment in the i-MRA geometry. This dis-
316 crepancy indicates that the i-MRA approach achieves a more balanced dis-
317 tribution of bending moments, leading to a more efficient and optimized
318 configuration.



(a) MRA



(b) i-MRA

Figure 11: Bending moment in the structural elements due to self-weight.

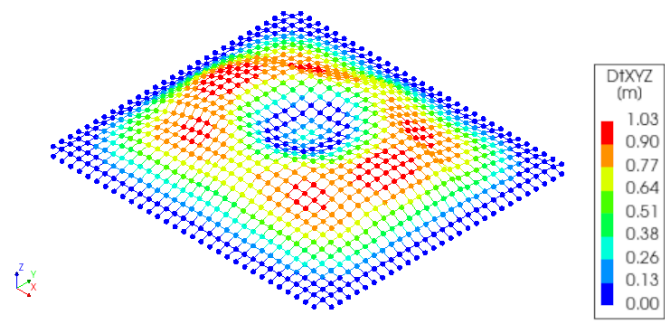
319 Considering the static analyses, it can be concluded that the i-MRA con-
320 figuration leads to a more balanced distribution of forces. Thus, the i-MRA
321 method demonstrates its ability to reduce structural hierarchies and enhance
322 the overall performance of the structure.

323 Finally, the comparison between the two geometries in terms of modal
324 frequencies and modal shapes is reported. The free vibration analysis of the
325 gridshells shows that they have similar eigenvalues (frequencies), as reported
326 in Table 1. However, the first eigenshapes reported in Figure 12 demonstrate
327 two different configurations. In particular, the eigenshape associated with
328 the MRA geometry exhibits higher deformation in the curvature inversion
329 region. In contrast, the i-MRA configuration reveals four distinct structural
330 zones where deformations are symmetrically distributed around the central
331 axes.

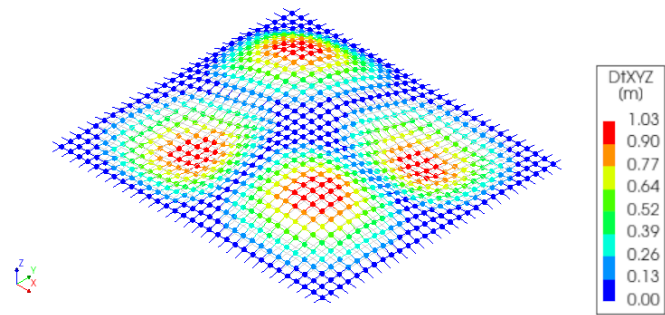
332 Furthermore, the i-MRA shape demonstrates a generalized frequency shift
333 of approximately -0.1 Hz compared to the MRA configuration. Thus, the
334 geometric variations introduced by the i-MRA method induce a slight mod-
335 ification in the dynamic response, in favour of a higher overall stiffness.

Mode	MRA	i-MRA	Mode	MRA	i-MRA
1	1.03E+00	9.09E-01	11	2.11E+00	1.85E+00
2	1.07E+00	1.06E+00	12	2.21E+00	2.03E+00
3	1.08E+00	1.14E+00	13	2.25E+00	2.25E+00
4	1.18E+00	1.27E+00	14	2.52E+00	2.26E+00
5	1.81E+00	1.52E+00	15	2.75E+00	2.28E+00
6	1.83E+00	1.53E+00	16	2.82E+00	2.45E+00
7	1.88E+00	1.64E+00	17	2.89E+00	2.73E+00
8	1.92E+00	1.72E+00	18	2.96E+00	2.74E+00
9	2.05E+00	1.81E+00	19	3.04E+00	2.84E+00
10	2.11E+00	1.83E+00	20	3.06E+00	2.85E+00

Table 1: Eigen-frequencies obtained by the dynamic modal analysis.



(a) MRA



(b) i-MRA

Figure 12: Mode 1 eigen-shapes obtained with the dynamic modal analysis.

336 4. Equilibrium Paths

337 Moving ahead to a deeper structural analysis, the behaviour of the two
338 configurations of the roof with respect to stability is now investigated. The
339 finite element program LUSAS [54] has been used and the structures have
340 been modelled by adopting finite elements with a formulation based on the
341 modified Timoshenko hypothesis for thick beams to the continuum theory.
342 In the adopted formulation, the deformation due to shear is considered and
343 the cross-sections remain plane and undistorted under deformation but do
344 not remain normal to the beam axis. In the model, each structural element is
345 divided into 4 different finite elements adopting a 4 nodal mesh. Analogous
346 static schemes and material characteristics to previously described analyses
347 have been employed.

348 In order to study the effects of the elements slenderness on the instability
349 phenomena, three different cross-sections were investigated. The slenderness
350 ratio $\lambda = \sqrt{\frac{AL^2}{J}}$ is defined as a function of the cross-section area A , structural
351 elements length L and bending inertia J . In the case studies, a standardized
352 length was employed for the definition of the slenderness ratio. This adopted
353 length is the predominant dimension among the elements in both structures.
354 Specifically, this length aligns with the target length used in the MRA, es-
355 tablished at $L = 2.00m$. The dimensions of the tubular cross-section have
356 been chosen to investigate three possible slenderness ratios with λ values of
357 50, 100, and 150.

358 Three different magnitudes of imperfection have been evaluated. The
359 purpose is to assess the effects of geometric variations introduced during the
360 construction phase on the instability phenomena.

361 In particular, the geometry is deviated from the initial one considering
362 the shape of the first buckling mode of the structure. The buckled geometry
363 is scaled to investigate three possible maximum deviations. Operating this
364 way, the obtained results provide an index about the imperfection sensitivity
365 of the initial geometry, being the eigenshapes intrinsically connected to the
366 geometric stiffness matrix K_G . The first scenario represents the maximum
367 accepted tolerance during the construction phase, equal to 1 cm. In the
368 remaining scenarios, the maximum deviations have been defined as a function
369 of the structure free span. The reason is to consider potential flaws due to
370 the complexity of realizing such an advanced structure. The, the maximum
371 deviations are set equal to $L/1000 = 6cm$ and $L/500 = 12.5cm$.

372 Finally, two load patterns are investigated. In the first loading condition,
373 vertical loads are uniformly distributed as concentrated forces acting on all
374 the structural nodes. This configuration represents the most common loading
375 condition during the structure service life. Actually, the structure is always
376 subject to self-weight and permanent.

377 The second scenario focuses on a situation where just half of the structure
378 is subjected to vertical loads. This condition is relevant as it represents
379 an asymmetrical load scenario that can significantly impact the structural
380 stability through force flows imbalance.

381 *4.1. Linear Buckling*

382 In this section, the results of the linear eigenvalue buckling analysis of
383 the two structural geometries under two different loading conditions are re-
384 ported. The finite element models have been realized as described in Section
385 4 by using the software LUSAS [54]. Buckling state is reached through the

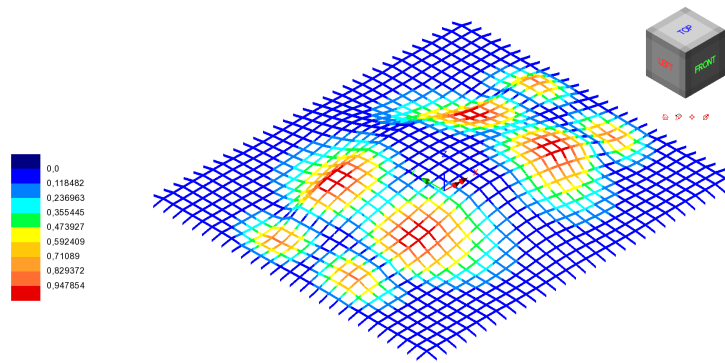
386 extraction of the eigenvalue λ from the problem $\det(K - \lambda K_G) = 0$, where K
387 and K_G are the global and geometric stiffness matrices respectively. Lanczos
388 solution method has been used and the error norm resulted in lower than
389 $1E - 11$ for all the computed modes.

390 The first buckling mode of the two geometries is illustrated in Figure 13
391 and 14, respectively for the symmetrically and the asymmetrically distributed
392 loads. The eigenshapes illustrated are the linear buckling deformations with
393 the lowest energy. Consequently, they are utilized as a reference in investi-
394 gating the impact of the imperfections on structural behaviour.

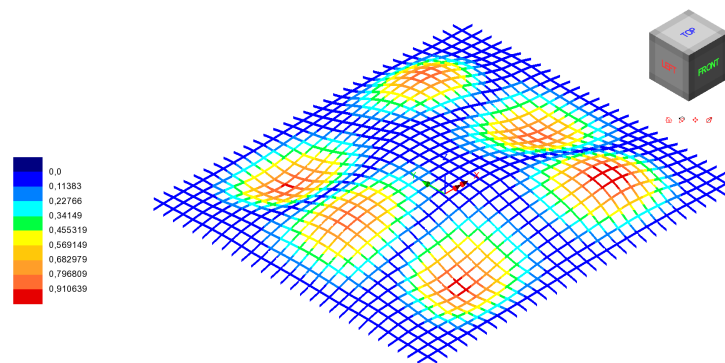
395 Finally, Table 2 presents the linear buckling eigenvalues for both struc-
396 tural configurations, considering three different slenderness ratios. The eigen-
397 values represent the critical load multipliers that lead to structural instabil-
398 ity. As a result, the table reports the relative difference in load multiplier
399 between the studied geometries.

400 The differences, denoted as Δ , are computed as percentages. This cal-
401 culation is performed according to Equation 8, where ω_{MRA} represents the
402 buckling load multipliers associated with the MRA structure, and ω_{i-MRA}
403 corresponds to those related to the i-MRA structure.

$$\Delta[\%] = 100 \left(1 - \frac{\omega_{MRA}}{\omega_{i-MRA}} \right) \quad (8)$$

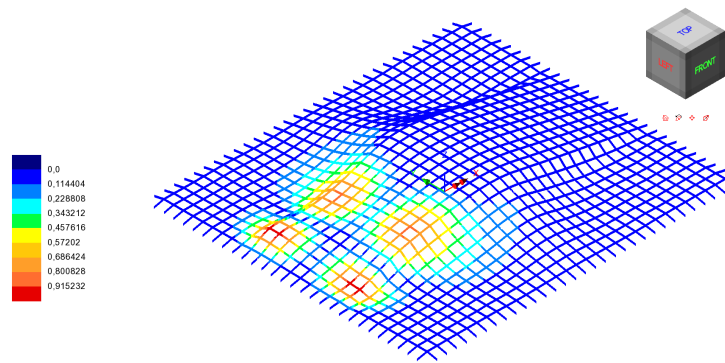


(a) MRA

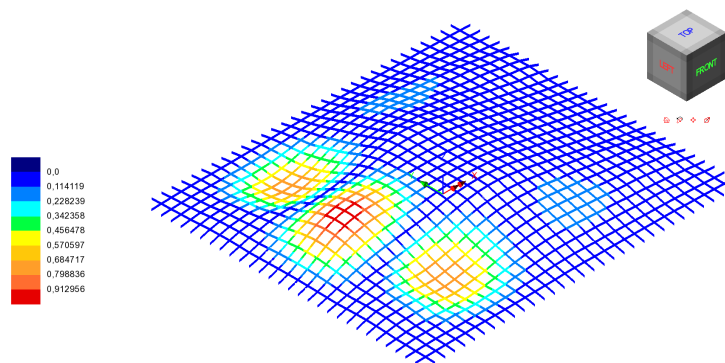


(b) i-MRAd

Figure 13: Mode 1 eigen-shapes obtained with linear buckling analysis on the symmetrical loaded structures.



(a) MRA



(b) i-MRA

Figure 14: Mode 1 eigen-shapes obtained with linear buckling analysis on the asymmetrical loaded structures.

λ	Mode	MRA		i-MRA		Difference %	
		Sym	Asym	Sym	Asym	Sym	Asym
50	1	1935.7	3041.6	1204.1	1412.8	37.8%	53.6%
	2	1973.3	3454.8	1305.2	1502.8	33.9%	56.5%
	3	2194.0	3795.0	1351.1	1664.2	38.4%	56.1%
100	1	94.8	156.0	72.7	91.1	23.3%	41.6%
	2	97.8	174.3	77.3	94.4	21.0%	45.8%
	3	104.5	192.4	78.3	100.2	25.1%	47.9%
150	1	17.4	27.6	14.0	18.6	19.5%	32.6%
	2	18	30.6	14.2	18.9	21.1%	38.2%
	3	18.9	33.1	14.4	19.8	23.8%	40.2%

Table 2: Dakar Mosque linear buckling eigenvalues (load multipliers) considering symmetric and asymmetric load condition.

404 4.2. GNIAs

405 To investigate the post-buckling behaviour of the structure, an incremen-
406 tal load Geometrically Nonlinear Analysis with Imperfections (GNIA) has
407 been conducted. The co-rotational formulation [55] has been employed to
408 account for geometrical non-linearity. Body strains have been derived at
409 each step from the rigid body motion, considering both the element and the
410 force vector rotation. Step by step, therefore the geometric stiffness matrix
411 has been computed taking into account the elements rotations and deforma-
412 tions, and consequently the equilibrium paths have been outlined.

413 Imperfections have been modeled using the first linear buckling mode
414 associated with each loading condition. The eigenshapes have been scaled to
415 have a maximum deviation from the original geometry as defined in 4.

416 The GNIA has been performed to determine the loading path considering
417 both the symmetrical and the asymmetrical loading patterns. The analyses
418 have been then repeated in order to asses the effects of geometrical imper-
419 fections and changes in the structural element slenderness.

420 The initial study was conducted by varying the slenderness of the struc-
421 tural components while maintaining constant the imperfection. The displacement-
422 load paths of the gridshells with a maximum geometric imperfection of 1 cm
423 are presented in Figure 15. In the graphs, the sum of the structural element
424 reactions F_z to the applied loads Q is shown as a function of the displacement
425 of the central structural node δ . Results are reported for both geometries,
426 considering two distinct loading conditions and three different element slen-
427 derness λ . Furthermore, to emphasize the relationship between the linear
428 buckling and the nonlinear load path, the linear buckling load is depicted as

429 a dashed horizontal line.

430 In Table 3, the results of the analyses are reported in terms of Euler load
431 Q_{CR} , elastic buckling load Q_{EB} and Yielding load Q_y . These loads represent
432 critical situations in terms of structural integrity. For this reason in the Table
433 is also reported the design load Q_d defined as $\min\{Q_{CR}, Q_{EB}, Q_y\}$.

434 In the table, a comparison is made between the two structures, consid-
435 ering the relative difference for each critical loading situation. Specifically,
436 the differences, denoted as Δ_Q , are reported as percentages and calculated
437 as per Equation 9, where Q represents Q_{CR} , Q_{EB} , Q_y , or Q_d , depending on
438 the specific situation under consideration.

$$\Delta_Q[\%] = 100 \left(1 - \frac{Q_{MRA}}{Q_{i-MRA}} \right) \quad (9)$$

439 It is worth noting that the numbers in the table represent vertical loads
440 that are deemed positive when acting in the direction of gravity acceleration.

441 It should be highlighted that the yielding load Q_y is the lower critical load
442 for the MRA case, considering all the possible slenderness values. Thus, the
443 stress distribution considering the minimum between the Euler load Q_{CR} and
444 the elastic buckling load Q_{EB} is computed. The stress distribution related
445 to the structures with a slenderness $\lambda = 100$ is reported in Figures 16 and
446 17.

447 Moreover, the displacements to this loading condition with a slenderness
448 $\lambda = 100$ is reported in Figures 18 and 19. The displacement configuration is
449 key to understand the structural behaviour in the critical loading conditions.

450 Finally, the displacement-load paths have been extracted for increasing
451 magnitude of imperfections. The first linear buckling eigen-shape has been

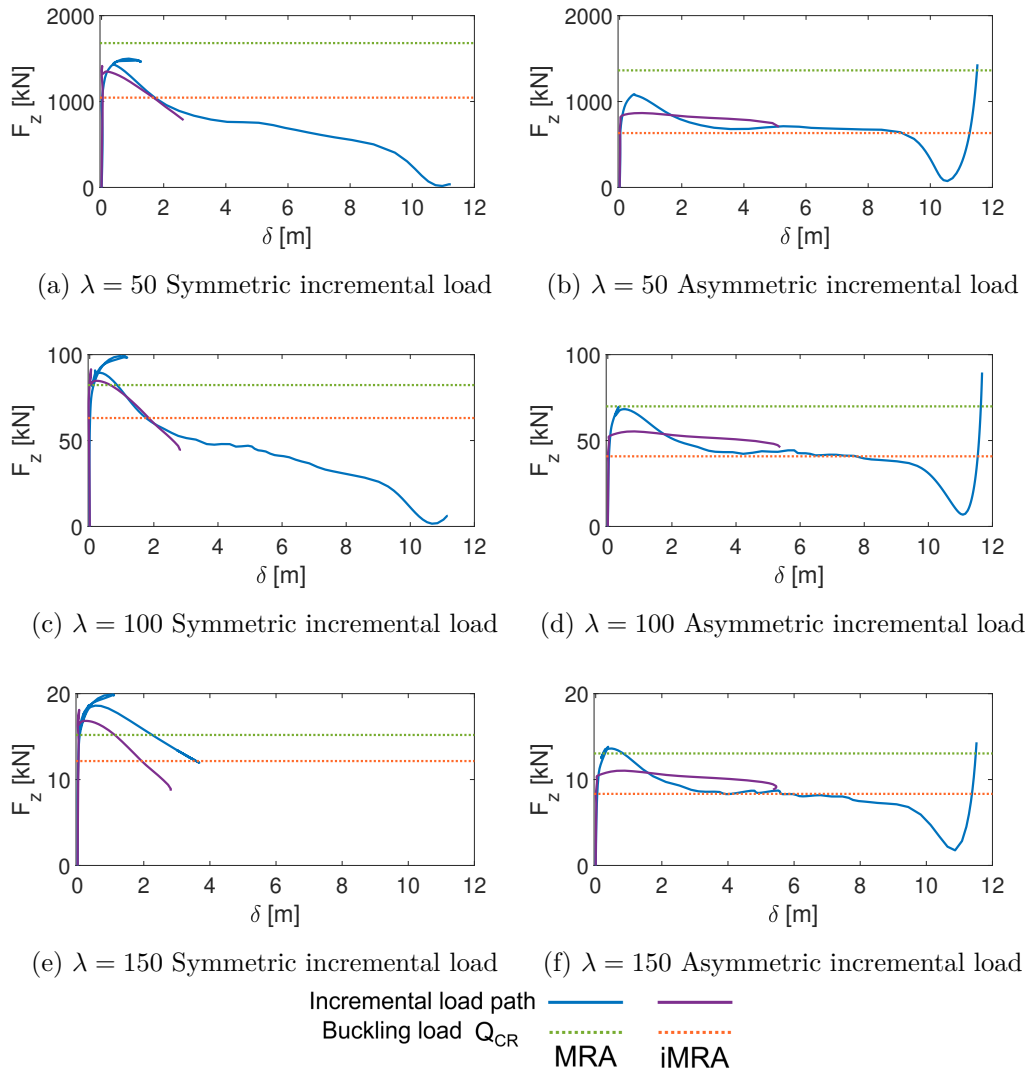
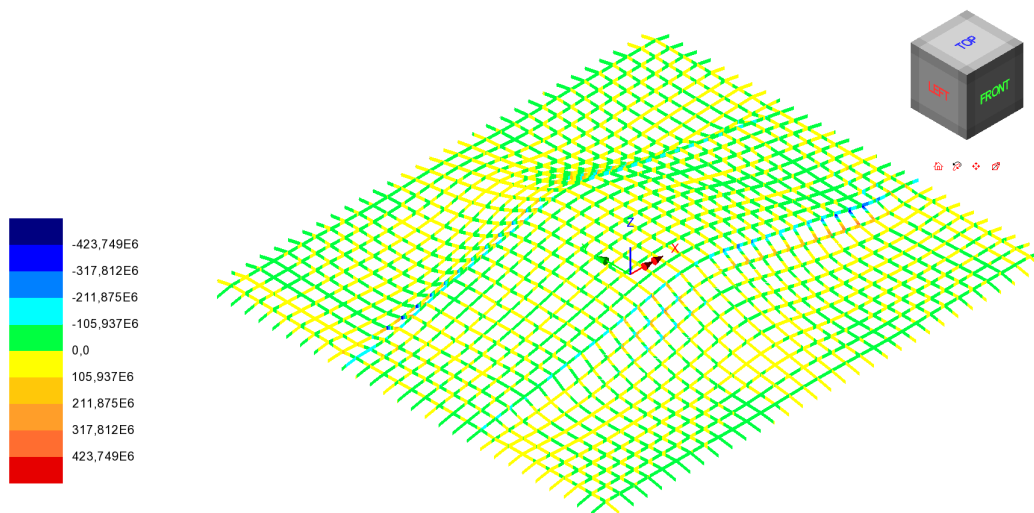


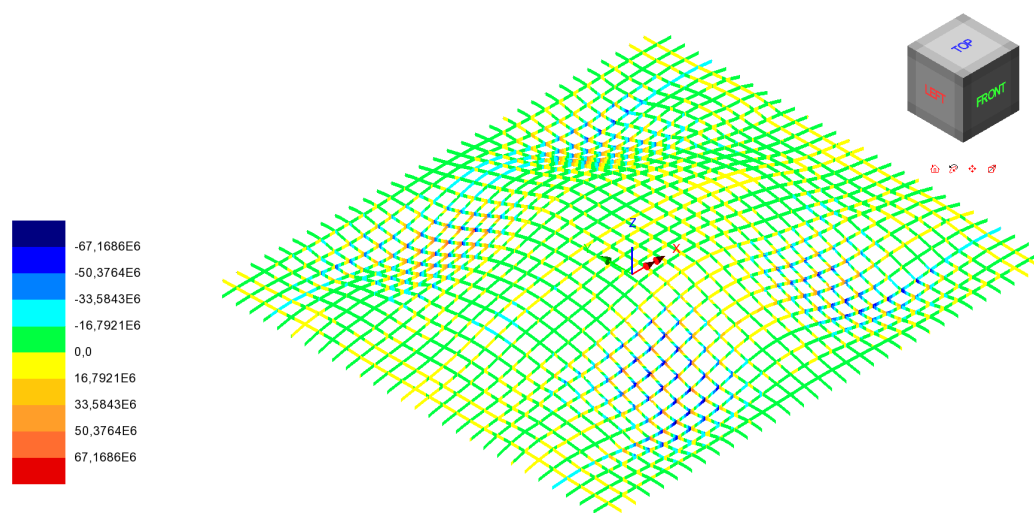
Figure 15: Displacement-load path of the central node of the Dakar mosque gridshell considering an incremental load geometric nonlinear analysis.

	λ	MRA		iMRA		Difference [%]	
		Sym	Asym	Symmc	Asym	Sym	Asym
Q_{CR}	50	1680.1	1362.6	1045.2	632.9	37.8%	53.6%
Q_{EB}		1428.2	1084.3	1349.0	866.0	5.5%	20.1%
Q_y		607.6	343.2	925.3	483.8	-52.3%	-41.0%
Q_{CR}	100	82.3	69.9	63.1	40.8	23.3%	41.6%
Q_{EB}		90.9	68.2	84.8	55.3	6.7%	18.9%
Q_y		75.0	43.2	83.2	51.3	-11.0%	-18.7%
Q_{CR}	150	15.2	13.0	12.2	8.3	20.0%	36.1%
Q_{EB}		18.6	13.6	16.8	11.0	9.3%	19.1%
Q_y		15.2	10.8	16.5	10.1	-8.5%	6.6%
Q_d	50	607.6	343.2	925.3	483.8	-52.3%	-41.0%
	100	75.0	43.2	63.1	40.8	15.9%	5.6%
	150	15.2	10.7968	12.2	8.3	20.1%	22.8%

Table 3: Critical instability loads in [N] considering the structural element slenderness $\lambda = [50, 100, 150]$ and both symmetrical and asymmetrical load patterns.

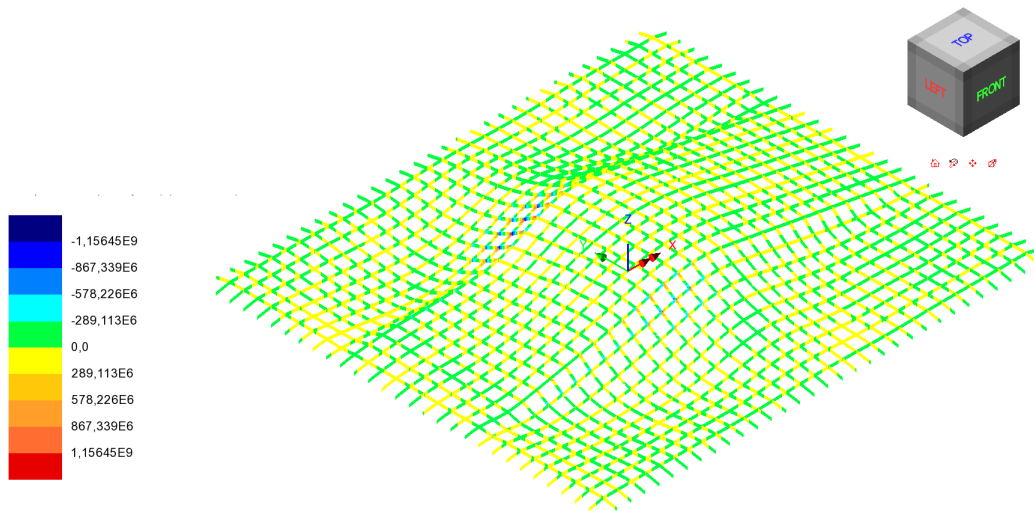


(a) MRA: $|\sigma_z| = 424MPa$

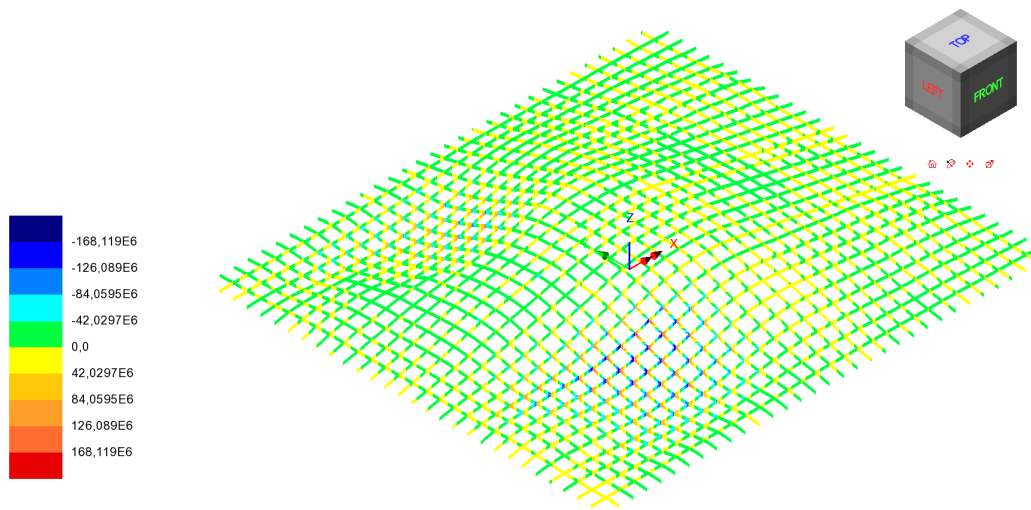


(b) i-MRA: $|\sigma_z| = 67MPa$

Figure 16: Stress distribution [MPa] related to the minimum critical load between Euler load Q_{CR} and the elastic buckling load Q_{EB} in the symmetrical load configuration.

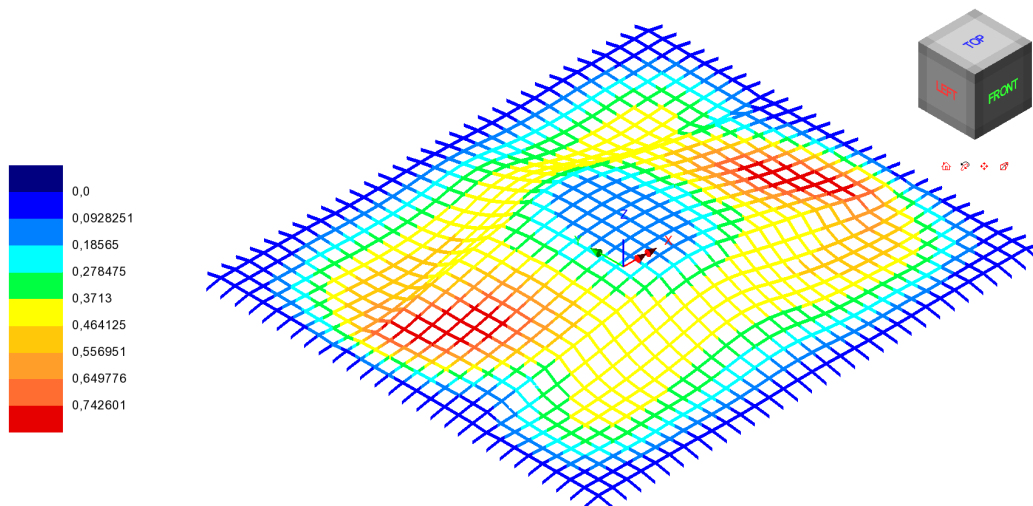


(a) MRA: $|\sigma_z| = 1157$ MPa

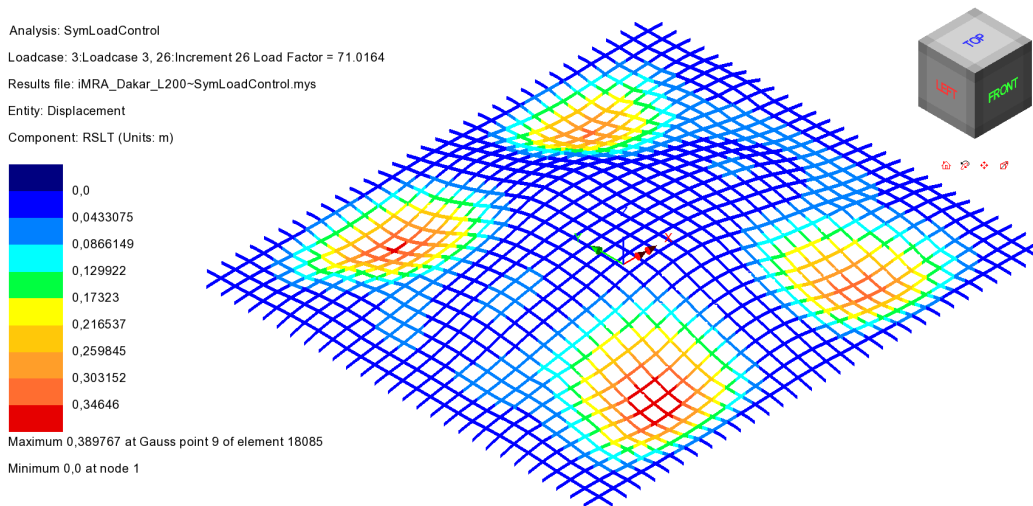


(b) i-MRA: $|\sigma_z| = 168$ MPa

Figure 17: Stress distribution [MPa] related to the minimum critical load between Euler load Q_{CR} and the elastic buckling load Q_{EB} in the asymmetrical load configuration.

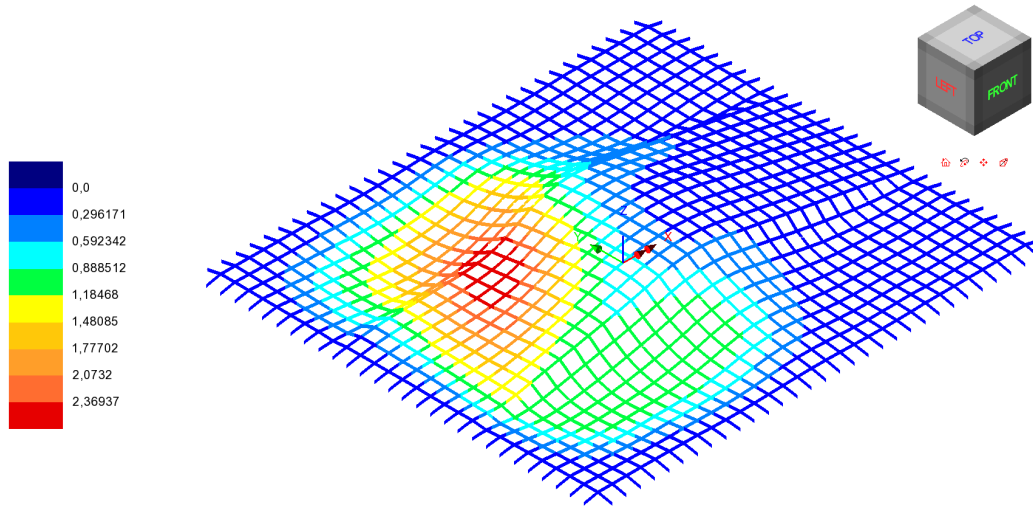


(a) MRA

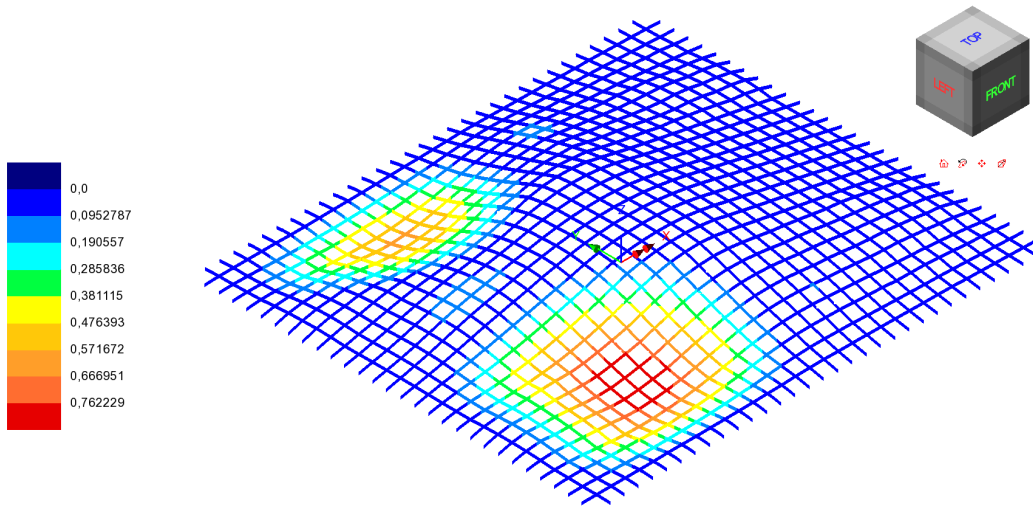


(b) i-MRA

Figure 18: Displacement distribution [m] related to the minimum critical load between Euler load Q_{CR} and the elastic buckling load Q_{EB} in the symmetrical load configuration.



(a) MRA



(b) i-MRA

Figure 19: Displacement distribution [m] related to the minimum critical load between Euler load Q_{CR} and the elastic buckling load Q_{EB} in the asymmetrical load configuration.

452 used to define the geometrical imperfections. The results for maximum de-
453 viations of 1cm , $L/1000 = 6\text{cm}$ and $L/500 = 12.5\text{cm}$ are shown in Figure
454 20. In this case, the slenderness of the structural elements has been kept
455 constant and equal to $\lambda = 100$.

456 It should be highlighted that the monotonously increasing branch shown
457 in Figure 20(a) represents an unstable equilibrium path. It is essential to
458 note that this equilibrium path can only be realized in a numerical solu-
459 tion. In practice, the behaviour of the structure will always adhere to the
460 stable equilibrium path. Consequently, during numerical simulations, after
461 attempting to follow the unstable path, the system will return to find equi-
462 librium configurations associated with the stable path.

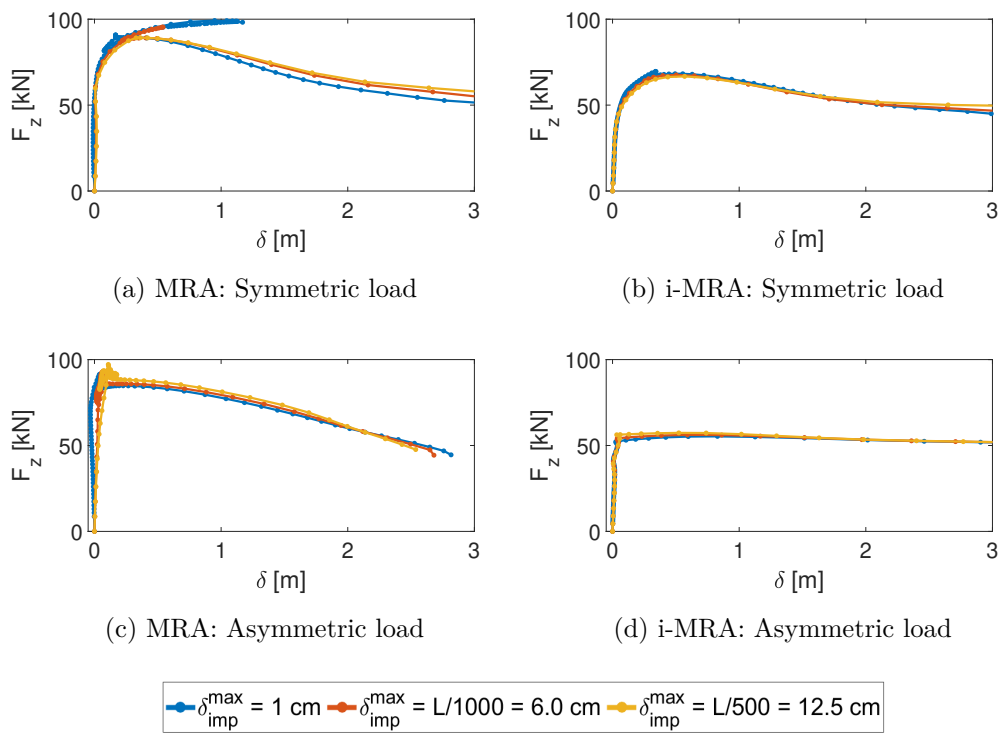


Figure 20: GNIA considering three magnitudes of imperfections on the Dakar mosque roof with $\lambda = 100$.

463 *4.3. GMNIAs*

464 In Section 4.2, the GNIA findings demonstrates that the yielding load Q_y
465 and instability loads (Q_{CR} and Q_{ED}) have the same order of magnitude. The
466 structural elements in some regions of the structure may reach the yielding
467 limit before the development of the instability phenomenon.

468 It is not possible to neglect the material non-linearity while investigating
469 the interaction between yielding and instability. Indeed, the yielding of some
470 structural components may alter the displacement field that is considered
471 during the GNIA.

472 The elasto-plastic steel S355 constitutive law is added into the FEM to
473 evaluate the effect of material non-linearity. An incremental load geomet-
474 rically and material nonlinear with imperfection analysis (GMNIA) is per-
475 formed on the structures with $\lambda = 100$. Initially, the structures with a
476 maximum deflection equal to $1cm$ are investigated. The analysis is repeated
477 for both the geometries and load configurations. In Figure 21, the GMNIA
478 results are compared to those obtained with the GNIA. A dashed red line
479 represents the F_z value at which the yielding limit is exceeded. It is there-
480 fore possible to investigate how the structural behavior varies after passing
481 yielding limit.

482 Finally, the analysis has been repeated for different magnitudes of imper-
483 fection to study the effects due to the combination of material non-linearity
484 and geometric deviations. The resulting load paths are reported in Figure
485 22.

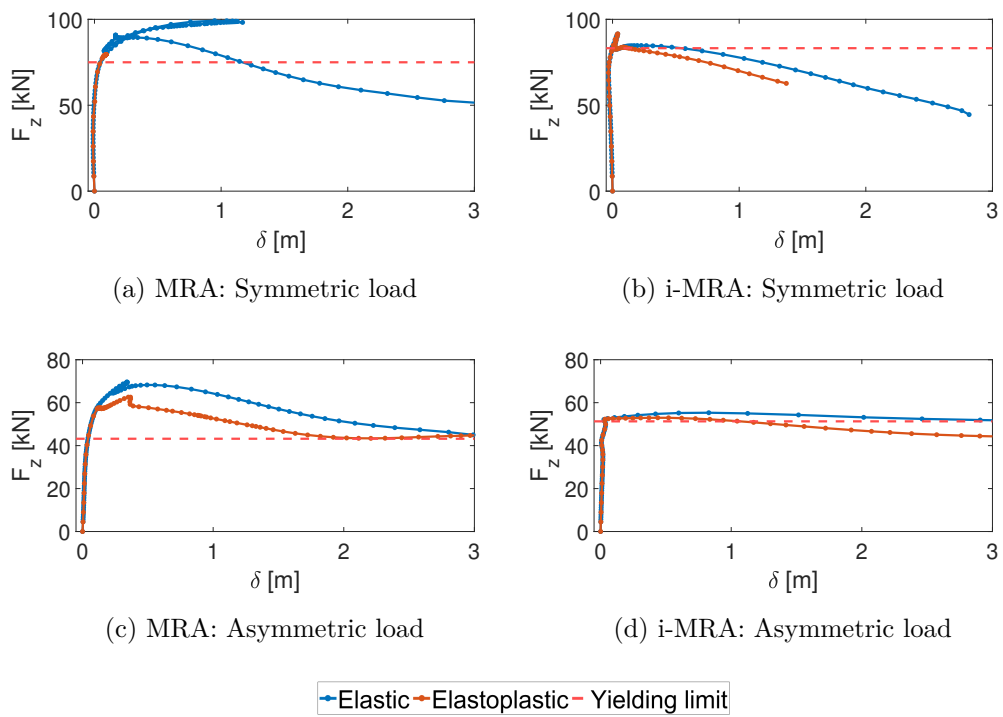


Figure 21: Comparison between GMNIAs with elastic and elasto-plastic material, $\lambda = 100$.

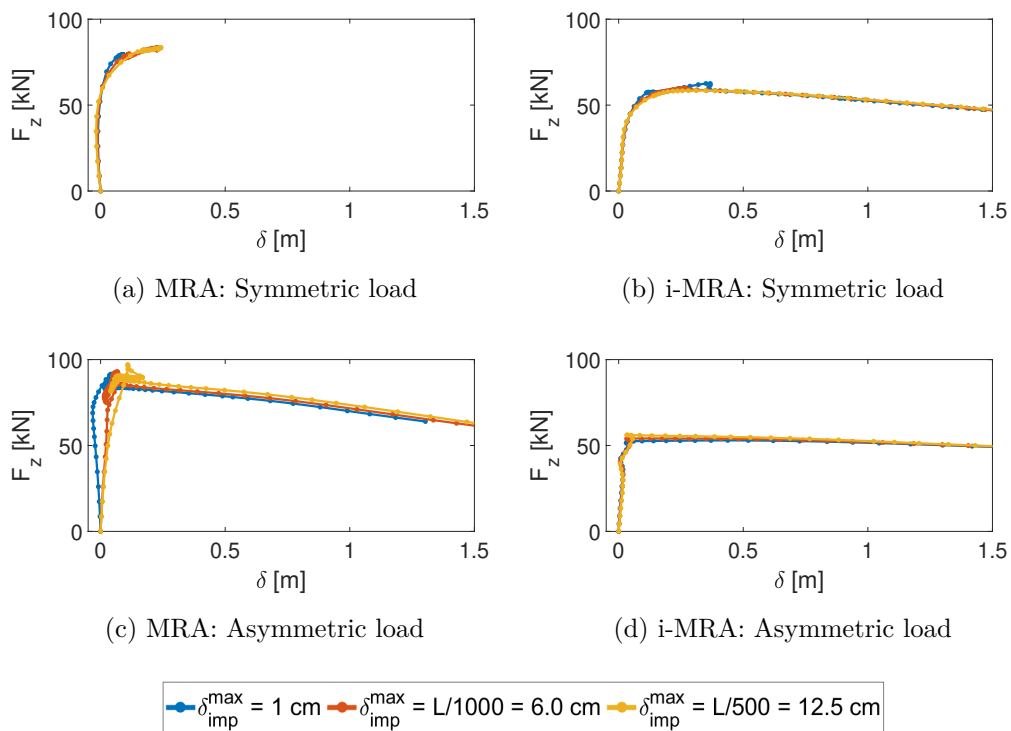


Figure 22: GMNIA considering elasto-plastic material and three magnitudes of imperfections on the Dakar mosque roof with $\lambda = 100$.

486 5. Discussion

487 The previous sections illustrate the results of the analyses carried out on
488 the roof of the Dakar mosque. These studies investigated the influence of
489 geometry, structural elements slenderness, material non-linearity, and imper-
490 fections on structural behaviour. The findings gathered from the analyzed
491 data are detailed in the following.

492 5.1. Effect of geometry: MRA vs. iMRA

493 The geometries generated using the two form-finding methodologies differ
494 significantly in terms of response with respect to the instability phenomena.

495 The linear buckling analysis produces eigenshapes that have distinct de-
496 formations patterns. As a result, the linear instability behaviour in the two
497 geometries is qualitatively different, as seen in Figure ?? and 14. The MRA
498 case shows typical linear buckling of lowered arches, with the main defor-
499 mations around the haunches of the central arch. This holds for both the
500 studied load patterns. Instead, the buckling deformation in the i-MRA shape
501 engages a greater number of structural members. Its behavior is similar to
502 that seen in flat shell stability, or form resistance structures in general.

503 The eigenvalues provided in Table 2 reveal that the MRA structure per-
504 forms better overall in terms of linear buckling resistance. Indeed, the critical
505 loads Q_{CR} are greater than those displayed by the i-MRA geometry for all
506 the investigated scenarios. As introduced, this result can be invalidated by
507 nonlinear analyses; when this happens, it generally implies for such cases the
508 presence of catastrophic postbuckling behavior. However, the results of lin-
509 ear buckling analyses are essential to carry out the GNIA. In details, they

510 have been used to define imperfection patterns, load patterns and factors.

511 In Section 4.2, the results of the GNIA for both the geometric configura-
512 tions have been reported. The analyses allows for a meaningful comparison
513 in terms of post-buckling behaviour. In the symmetrical load situation, the
514 behaviour of the two geometries is remarkably similar, as evident in Figure
515 15. Even for the critical load Q_{CR} is considerably different, the symmetrical
516 equilibrium load path are similar. The differences in terms of elastic buckling
517 load Q_{EB} are smaller than 10%, as reported in Table 3. The postbuckling
518 behaviour induced by the symmetric load pattern can classified as benign,
519 since $Q_{EB} > Q_{CR}$ with the exception of the MRA case with $\lambda = 150$. This
520 means that the equilibrium paths can be assimilated to those represented in
521 Figure 1b).

522 It is worth noting that the differences in postbuckling behaviour become
523 more pronounced when investigating the asymmetric loading case. In gen-
524 eral, the critical loads are smaller when the load is distributed asymmetri-
525 cally. Consequently, the asymmetrical loading condition results to be the
526 critical one. The equilibrium paths in Figure 15 describe a very different
527 exhibited behaviour by the two geometries. $Q_{EB} > Q_{CR}$ in the i-MRA struc-
528 ture indicates a benign postbuckling. In the MRA shape, however, Q_{EB} is
529 generally smaller than Q_{CR} . The structures exhibit catastrophic postbuck-
530 ling behaviour. As a result, the equilibrium paths can be associated to the
531 one represented in Figure 1a).

532 Figure 23 provides insights of the structure under asymmetrical loading
533 conditions with $\lambda = 100$. The graph illustrates the evolution of displacements
534 and stresses during the incremental load GNIA.

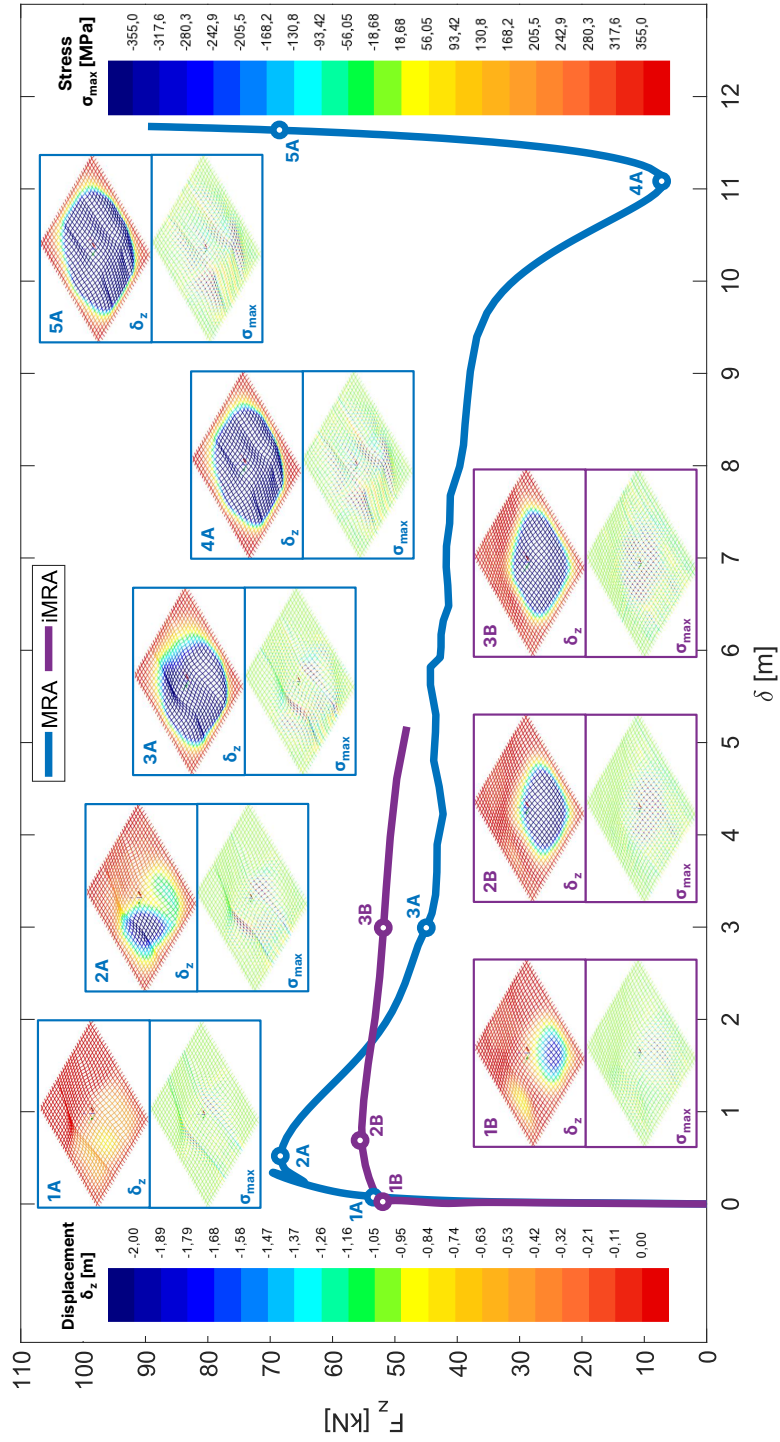


Figure 23: Postbuckling behaviour of the Dakar Mosque

535 In the initial phase, displacement increases approximately linearly as the
536 load increases in both of the studied structures. Snap-through occurs in
537 the MRA case with a sudden transition from peak configuration 2A to final
538 configuration 5A. The collapse is catastrophic with a complete overturning
539 of the roof.

540 Despite a reduced peak load Q_{EB} , the i-MRA configuration exhibits be-
541 nign post-buckling performance. In the post-peak phase, the structure ex-
542 hibits a plateau. The partial collapse experienced in the initial stages grad-
543 ually extends. There is a smooth transition between a partial and global
544 collapse.

545 The load-displacement graph in Figure 23 can be used to trace the col-
546 lapse mechanism. The stress graphs illustrate that there is a larger concentra-
547 tion of stresses in the MRA case. As a result, the source of this concentration
548 has been investigated, and the maximum axial force N_{max} and bending mo-
549 ment M_{max} have been computed for each load increment. Figures 24, 25, and
550 26 illustrate the values of N_{max} and M_{max} as the load increases. The Figures
551 refer to structures with slenderness λ 50, 100 and 150, respectively. In any
552 case, it is clear that the maximum axial force and bending moment are always
553 greater in the case of MRA than those on the i-MRA structure. This con-
554 centration implies that yielding will occur in specific structural nodes before
555 the critical buckling load is attained.

556 In order to investigate the interaction between yielding and instability
557 phenomena, the elasto-plastic constitutive law has been introduced in the
558 model. In Figure 21 the elastic model is compared with the GMNIAs results.

559 Implementation of the elasto-plastic constitutive law to the MRA model

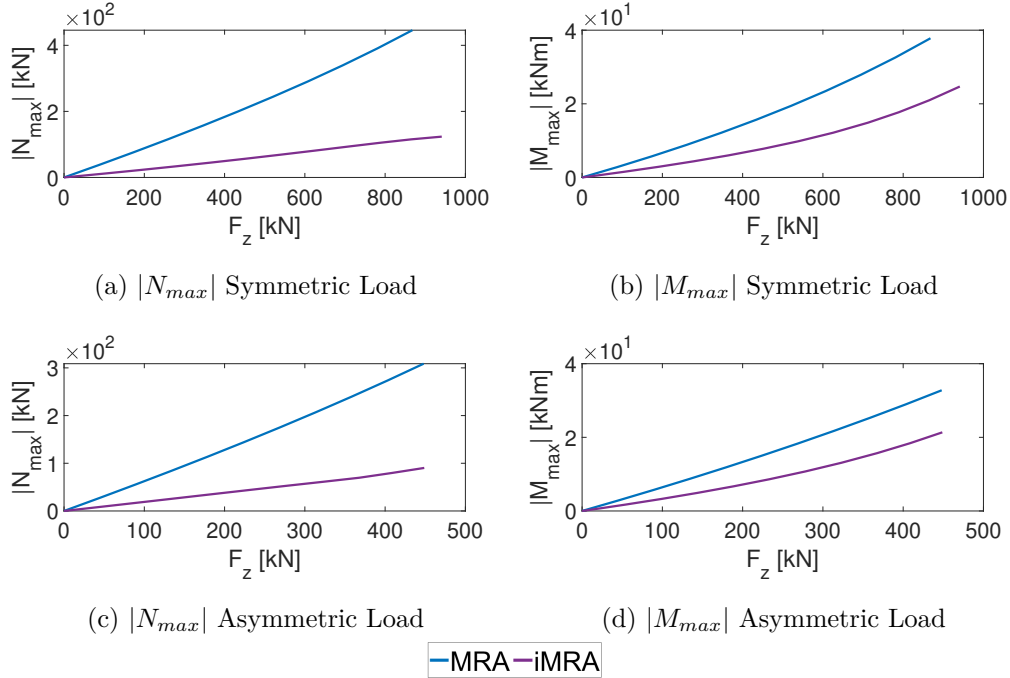


Figure 24: Maximum absolute axial force and bending moment in the structural elements with $\lambda = 50$ considering to increase the load to perform the GNIA.

560 reduces the linear buckling load, then results in $Q_{EB} < Q_{EP}$. In the case
 561 of i-MRA, yielding is observed in proximity to the elastic peak. As a re-
 562 sult, the elastic buckling load Q_{EB} is roughly equivalent to the elastic-plastic
 563 buckling load Q_{EP} . In either case, a reduction in stiffness is observed in the
 564 postbuckling tails.

565 5.2. Effect of member slenderness

566 Structural members' slenderness effects have been investigated by analysing
 567 structures with three different λ . In particular, Table 2 reports the eigenval-
 568 ues for the linear buckling analysis of structures characterised by λ equal to
 569 50, 100 and 150. The difference between the two structural configurations is

570 minimized when λ increases, as seen in the Table. Local instability phenom-
571 ena become more prevalent than global ones when slenderness rises. As a
572 result, the effects of the geometric differences between the two configurations
573 tend to be less significant.

574 Considering the GNIA, In Figure 15 the same qualitative behaviour is
575 observed for all the slenderness ratios. The major differences are related to
576 the yielding load Q_y . In Table 3, the design load results to be the yielding
577 one in the cases with thicker structural elements. For the i-MRA case, as the
578 slenderness increases the instability becomes the most critical phenomenon.

579 At the same time, yielding seems to be the main challenge for all the
580 slenderness ratios, if we consider the MRA geometry. This is a consequence
581 of the stress concentration observed in Figures 16 and 17.

582 5.3. Effect of material yielding

583 The GNIA results show that the yielding load Q_y is the most critical in
584 some of the investigated structural configurations. In particular, in the MRA
585 case, it is observed that the $Q_y < Q_{EB}$ for all the slenderness ratios. In order
586 to understand the effects related to material non-linearity, the elasto-plastic
587 constitutive law has been included in GMNIAs. The results of the GMNIAs
588 are shown in Figure 21, demonstrating that the yielding of the structural
589 nodes can alter the structural behaviour.

590 In the MRA case is evident that after the yielding the global structural
591 stiffness decreases. As a result, it is observed that $Q_{EP} < Q_{EB}$.

592 Considering the MRA geometry, the material non-linearity effects are
593 exploited only on the postbuckling load paths. The stiffness decreases only
594 after the peak and $Q_{EP} \approx Q_{EB}$.

595 The yielding of some structural nodes reduces the peak load in compar-
596 ison to the MRA situation. In contrast, there is no significant effect on the
597 behaviour of the i-MRA structure. Although in GNIA the peak load of the
598 MRA case was much higher, the concentration of stresses produced by such
599 geometry results in the yielding of structural nodes prior to the establish-
600 ment of buckling mechanisms. As a result of introducing an elasto-plastic
601 constitutive law, the disparity between the two peak loads relative to the two
602 analyzed geometries is greatly reduced due to local plastic flows.

603 *5.4. Imperfection Sensitivity*

604 The structure defined by $\lambda = 100$ was examined by taking into account
605 different levels of deviation from the basic geometry. The geometric imper-
606 fections were introduced by scaling the deformed relative to the first linear
607 buckling mode. The resulting geometry was scaled so that the maximum
608 distance between the deformed and undeformed configurations was 1 cm,
609 $L/1000 = 6.0cm$, and $L/500 = 12.5cm$.

610 A geometrically nonlinear analysis with imperfection was conducted on
611 the deformed configurations. The results of this analysis are shown in Figure
612 20.

613 The graphs indicate the presence of geometric deviations causes minor
614 changes to the equilibrium paths. Deviations from the initial geometry ap-
615 pear to increase the peak load. This phenomena has been widely documented
616 in the literature [56].

617 Finally, the analyses have been repeated after introducing material non-
618 linearity of the material; results are reported in Figure 22. Also in this case
619 a slight improvement in the peak load is observed as the geometric deviation

620 increases.

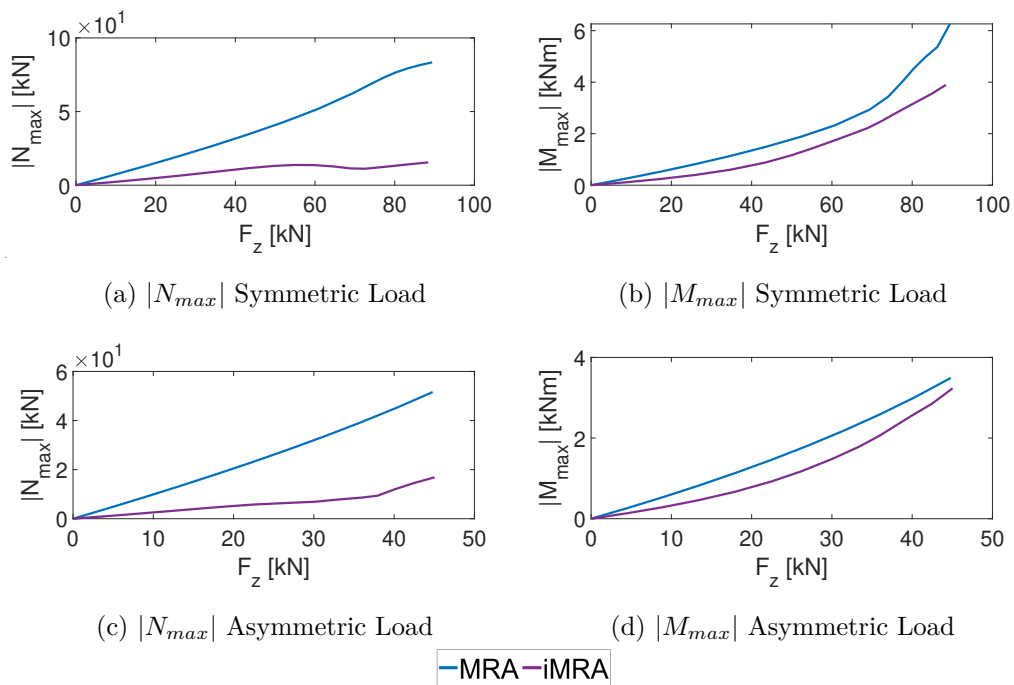


Figure 25: Maximum absolute axial force and bending moment in the structural elements with $\lambda = 100$ considering to increase the load to perform the GNIA.

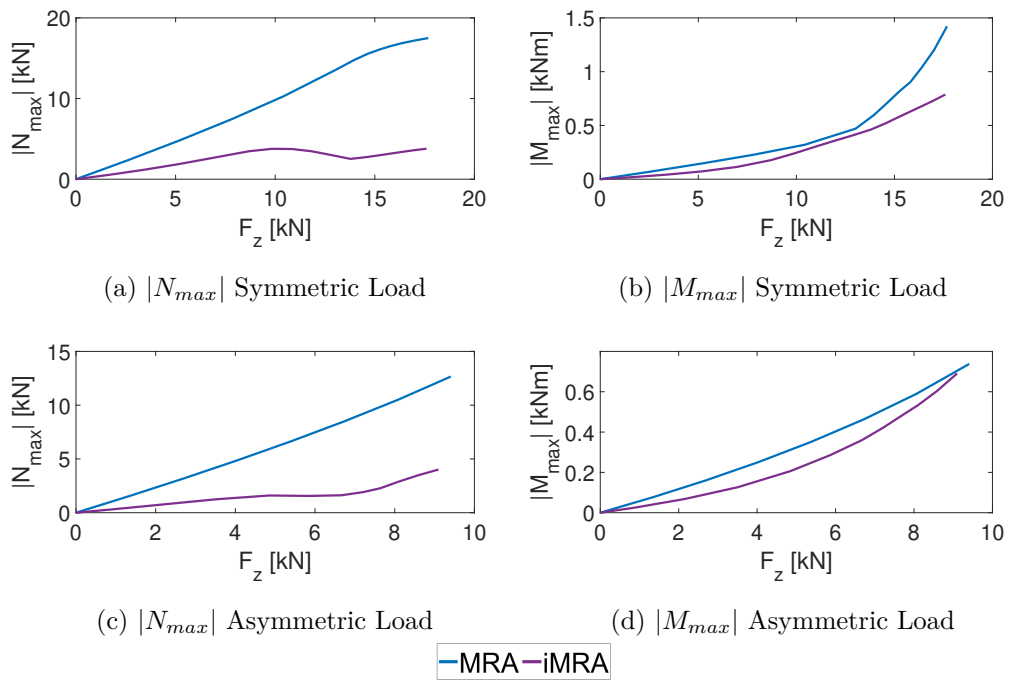


Figure 26: Maximum absolute axial force $|N_{max}|$ and bending moment $|M_{max}|$ in the structural elements with $\lambda = 150$ considering to increase the load to perform the GNIA.

621 **6. Conclusions**

622 In this paper, a comparative analysis of two form-finding methods with
623 respect to structural stability is provided. The case study is the gridshell
624 that constitutes the roof of the mosque in Dakar, Senegal. The Multibody
625 Rope Approach (MRA) and its enhanced variant, the i-MRA, were used
626 to generate two different structure geometries. A preliminary analysis was
627 performed to establish the differences between the two configurations in the
628 static and dynamic fields as a result of permanent loads. The two structural
629 geometries are then investigated through a linear eigenvalue buckling analysis
630 under two different loading conditions. Three distinct slenderness ratios λ of
631 the structural components are considered in the analyses.

632 The findings of these investigations are used to define the shape of the
633 geometric deviations from the starting geometry. The rescaled first linear
634 buckling mode define the geometry of structural imperfections. An incre-
635 mental load geometric nonlinear analysis with imperfection (GNIA) is then
636 performed under these conditions.

637 The outcomes define the equilibrium paths of the investigated structural
638 configurations.

639 Ultimately, the impacts of material nonlinearity are considered by intro-
640 ducing an elastoplastic constitutive law. Subsequently, the structures are
641 investigated by Geometrically incremental load and Material Nonlinear Im-
642 perfection Analyses (GMNIAs).

643 Different structural behaviours are exhibited by the two geometries. In
644 particular, The MRA case shows the typical behaviour of lowering arches.
645 Both static and instability analyses indicate this behaviour. This structure

646 shows a structural hierarchy where the central arch works as the main sup-
647 porting structure.

648 Conversely, the i-MRA case behaviour resembles a shell structure. In this
649 configuration, no obvious structural hierarchies are observed. The outcome
650 is a more even distribution of stress across the structure.

651 For the two investigated geometries, two distinct structural behaviours
652 are identified by the equilibrium paths resulting from the GNIA. The MRA
653 case is characterized by a higher elastic buckling load Q_{EB} . However, for
654 the MRA geometry $Q_{EB}/Q_{CR} < 1$, which is an indicator of catastrophic
655 postbuckling. On the contrary, in the i-MRA case, postbuckling can be
656 defined as benign being $Q_{EB}/Q_{CR} > 1$. Therefore, even if the peak load
657 Q_{EB} is lower, the i-MRA example exhibits a qualitatively better postbuckling
658 structural behaviour.

659 The ratios between the elastic buckling load Q_{EB} and the Euler critical
660 load Q_{CR} for the different structural configurations are displayed in Figure
661 27. It is evident that this ratio is always higher in the i-MRA scenario than
662 it is in the MRA case.

663 Finally, the GNMIA results demonstrated a reduction in the peak load
664 for the MRA case, obtaining $Q_{EB} > Q_{EP}$. This reduction is due to the stress
665 concentration induced by the MRA geometry. As a result, the yield strength
666 limit is exceeded before the buckling loads are reached.

667 This phenomenon is not observed in the i-MRA case. The lowering of
668 Q_{EB} resulting from the elastoplastic model introduction is evident in Figure
669 27 and is limited to the MRA scenario. As a result, the elasto-plastic buckling
670 loads Q_{EP} related to the two geometries are comparable.

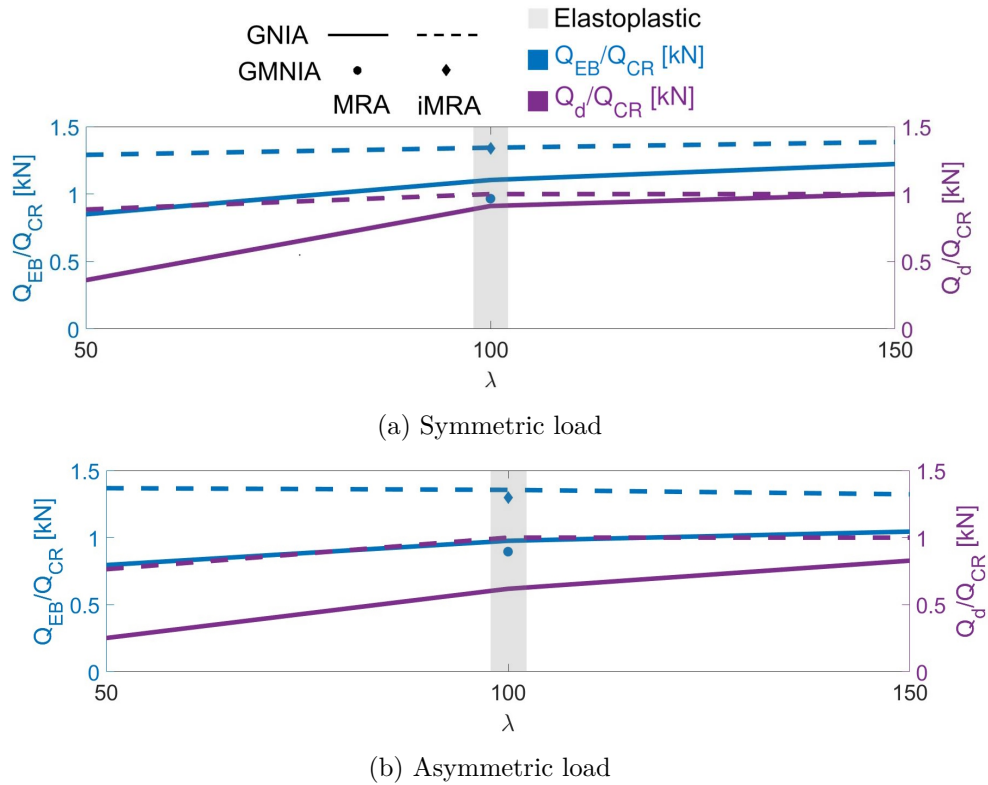


Figure 27: Ratios between the elastic buckling load Q_{EB} and the Euler critical load Q_{CR} compared with ratios between design load Q_d and the Euler critical load Q_{CR} , for the different structural configurations.

671 In conclusion, the i-MRA technique enables us to define a structural ge-
 672 ometry that is advantageous from a construction perspective. The i-MRA
 673 form exhibits superior stress distribution in relation to static loads. Further-
 674 more, a qualitatively better postbuckling behaviour is observed compared to
 675 the MRA case. Finally, if the non-linearity of the materials is considered,
 676 the peak loads are comparable in the two studied geometries.

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