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ABSTRACT

Gridshell structures require an intricate design activity that shall comply with several design goals of diverse nature. This design phase can be approached with different methods and strategies and usually requires multiple competencies from different scientific fields. In this context, a common benchmark, called FreeGrid, is proposed to the scientific and practitioners' communities in order to test and compare different approaches to the design and optimization of steel gridshells on the bases of ad-hoc defined performance metrics. FreeGrid sets three design baseline problems: a barrel vault, a parabolic dome, and a hyperbolic paraboloid, having their spring line partially not constrained (free-edge) and subjected to uniform and piecewise uniform load conditions. Participants are called to modify the baseline gridshell(s), observing a limited number of design constraints (related to geometry, external constraints and material), in order to improve their structural, buildability, and sustainability performances through the maximization of a bulk quantitative performance metric. Specifically, the structural performance metric accounts for both ultimate and serviceability behavior, through the calculation of the critical Load Factor and maximum vertical displacement; the buildability performance metric includes the evaluation of face planarity, uniformity of structural joints and members; the sustainability performance metric is based on the structure embodied carbon. This paper describes the baseline gridshells setups, the proposed performance metrics and the recommended method for performance assessment. The complete data of the baseline structures are made available according to an Open Data policy, together with postprocessing utilities intended to align the procedure to obtain the performance metrics.

Terminology

Entity	Geometry	Structure / construction	Computational model
0D	vertex	joint	node
1D	edge / line / arc	member	element
2D single entity	face	panel	—
2D overall entity	mesh	grid	discretization

1. Introduction

Gridshells are lightweight structures that cover large spans, characterized by their elegance and capacity to create dramatic and iconic spaces. Gridshells are spatial structures, whose geometry is generally defined by approximating a reference continuous surface through a discrete mesh of linear structural members [1,2,3]. They are extremely efficient structures, being their shape optimized to carry loads mainly through axial forces, thus allowing to save structural material with respect to conventional structures in bending. They are suitable for creating sustainable design since they are characterized by lower embodied energy with respect to conventional structures. They are highly versatile, having potential to be used for newly built structures, for adaptive reuse of historical buildings or as temporary structures. Due

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Nomenclature

A	generatrix arc length.
b	edge length.
B	generatrix span length.
C	member cross-section.
CP	Concentrated Plasticity.
d_j	distance between the j-th vertex and the best fitting plane π.
DBG	Design Baseline Gridshell.
DP	Distributed Plasticity.
DSG	Design Solution Gridshell.
f	gridshell rise / index for f-th 2D entity (face, panel).
f_y	yield stress / steel grade.
F	total number of 2D entities (face, panel).
FEM	Finite Element Method.
g	self-weight per unit length.
GMNA	Geometrically and Materially Non-linear Analysis.
h	gridshell maximum height.
i	index for i-th 1D entity (edge/arc/line, member, element).
j	index for j-th 0D entity (vertex, joint, node).
J	joint type.
K_0	tangent stiffness.
K_l	stiffness limit value.
$\tilde{l}$	coefficient of variation of member lengths.
L	length of the spring line.
L^*	length of the free edge.
LC	Load Condition.
$\widehat{LF}$	critical Load Factor.
M	total number of 1D entities (edge/arc/line, member, element).
n_f	number of incident vertices.
N	total number of 0D entities (vertex, joint, node).
p_f	face perimeter.
P	bulk performance metric.
P_b	buildability performance metric.
P_s	structural performance metric.
P_{su}	sustainability performance metric.
q	load per unit area.
Q	point load.
s	projection on the horizontal plane of the tributary area of a structural joint.
S	horizontal projection of the gridshell surface.
$\breve{S}$	gridshell surface.
v	valence of 0D entities (vertex, joint, node).
W	gridshell embodied carbon.
x,y,z	reference axes.
α	environmental impact correction coefficient.
$\widehat{\delta}_z$	maximum vertical displacement.
$\widehat{\delta}_{z,l}$	vertical displacement limit value.
ε_y	yield strain.
$\gamma_s, \gamma_b, \gamma_{su}$	weighting factors of partial performance metrics.
σ	stress.
$\overline{\Delta}$	face out-of-planarity metric.
Δ_f	f-th face out-of-planarity metric.
0	subscript referring to the Design Baseline Gridshell quantities.

to these advantages, a significant number of fascinating gridshells have been built all over the world, covering a wide variety of forms (overall shape and tessellation), materials (e.g., steel [1], timber [4], bamboo [5], composite [6]) and structural types (e.g. single [7] or double [9] layer, bending-active [8] or pre-formed [7]). Despite this great potential, gridshell design is so far limited to few extremely specialized practitioners because of the higher level of complexity in the overall design and fabrication process with respect to conventional structures.

Looking at the current state of the art on gridshell design, some general considerations are outlined in the following:

1. The research on gridshell design is polarized between two communities [10,11]: the structural design community, made of consultants and practicing engineers, and the structural mechanics community, made of academics. The first one is mainly interested on the overall design process, which is usually defined on a case-by-case basis, according to the specific design constraints. The second one mainly addresses modelling and analysis from a more general perspective, with a specific focus on the factors that influence buckling phenomena. This polarization also emerges from the extensive literature mapping in [12], which reviewed 327 papers about gridshells, selected from different databases according to selected topics and categorized according to different criteria: about 46% of the reviewed papers deal with analysis, while 40% deal with design and innovation.
2. If compared with the design of conventional structures, gridshell design presents a higher level of complexity, due to the need to satisfy several design goals, which require quite different competences. For this reason, gridshell design should necessarily be tackled according to a multidisciplinary approach, which involves not only architects, structural engineers and builders, but also other experts, e.g., in mathematics and computer graphics. Furthermore, design goals are deeply co-related, so that an improvement obtained for a single design aspect may decrease another. Currently, there is no method to holistically evaluate the performance of a gridshell and its improvements.
3. Several approaches are available to tackle the design of form-resistant structures, at both the design conceptual phase and the design development phase [13]. They can be divided into the following categories: *heuristic approach*, such as trial-and-error [14], design from precedent [15,16], bio-inspired design [17], or typology-inspired design [18]; *experimental approach* [19], through in-scale or full-scale physical models; *computational approach*, e.g., optimization [20,21], computer-based graphic statics [22], dynamic relaxation method [23,24], Artificial Intelligence [25,26]; *geometry-driven approach* [27,28,29].

On the basis of the three issues outlined above, a benchmark on design and optimization of gridshells, called FreeGrid, is proposed by the Authors of the present paper as members of its Steering Committee with the following aims, relevant to both the scientific and design practice fields:

i. bridging the academic structural mechanics community and the designers to secure a two-way transfer of knowledge;
ii. gathering multiple knowledge and input from all the fields of competence involved in gridshell design around a common realistic design problem;
iii. testing the readiness of different approaches to the conceptual design of form-resistant structures, selecting the most suitable ones to design practice and setting their best practices;
iv. setting multiple performance metrics for the assessment of the gridshell performances in a holistic perspective that can help the

designer to provide justification and reasoning for its design choice.

Overall, the achievement of the four aims above is expected to result in spreading the gridshell structural type in the current design practice and in widening its application.

To reach these objectives, FreeGrid proposes three design case studies, called Design Baseline Gridshells (DBGs): a barrel vault, a parabolic dome and a hyperbolic paraboloid. DBGs are steel single-layer gridshells, discretized with a quadrangular mesh and characterized by free-edges. This latter feature is dictated by the observation that horizontal spring line and/or perfect infinitely rigid constraints seldom occur in built gridshells. Usually and more and more frequently, gridshells include edges free of constraints eventually bounded by stiffening members [30]. Moreover, free-edge gridshells represent a challenge in the design practice due to the fact that free edges contribute to alter the membrane behavior. Participants to the benchmark are called to propose Design Solution Gridshells (DSGs) able to improve the overall performances of DBGs.

FreeGrid benchmark has been conceived to address both reproducibility and fair comparability [31] of studies and related results. To this aim, FreeGrid sets requirements to participants intended to secure the full description of (a) the geometrical and mechanical features of the design solutions, (b) the methods adopted for design/-optimization/performance assessment, (c) the obtained results. Moreover, FreeGrid precisely and analytically adopts objective, purely quantitative performance metrics to evaluate the best design solution.

Hereinafter, the benchmark is fully described through the following sections: Section 2 describes in detail the DBGs setups; Section 3 is devoted to the definition of the performance metrics conceived to evaluate the fulfilment of selected design goals; Section 4 summarizes the design constraints; Section 5 discusses the proposed methods for structural analysis, while performances of DBGs are presented in Section 6; finally, Conclusions are outlined in Section 7.

2. Baseline gridshells

FreeGrid considers three types of gridshell (Fig. 1): a barrel vault, a parabolic dome, and a hyperbolic paraboloid, with simple, double gaussian positive, and double gaussian negative curvature, respectively. These 'Design Baseline Gridshells' (DBGs) have their spring line partially not constrained along a 'free-edge' [30]. Moreover, for comparison purposes, 'Background Gridshells' (BGs) are analyzed: the latter differ from DBGs only in the boundary conditions, being fully hinged along their boundaries. Although imperfections play a key role in gridshell design and optimization [32], for the sake of simplicity the FreeGrid DBGs are free from any kind of imperfections induced by constraints, load conditions, mechanical properties, or geometrical features.

2.1. Geometrical setups

The main geometrical features of the DBGs are shown in Fig. 1.

All the DBGs share the same parabolic generatrix, whose equation is given in Table 1, where the generatrix span $B = 30$ m is adopted as reference length, and $f = B/8$ is its rise. The horizontal plane $z = 0$ is referenced in the following as 'horizontal reference plane'. The generatrix arc length A is divided into 20 edges with uniform length b. The directrix is split with the same step b. Therefore, all DBGs are discrete translational surfaces, whose resulting mesh is homogeneous, and made by planar square faces, except for boundary ones that are obtained by intersection with the horizontal reference plane. The other main geometrical features of the DBGs are: the maximum height h above the horizontal reference plane; the lengths L and L^* of the spring line and free-edge referring to their continuous counterpart; the areas $\breve{S}$ and S of the DBG surface and of its horizontal projection.

The uniform, quadrangular, not-boundary-fitted meshes of the DBGs are intentionally set to leave room to their geometrical optimization by the Participants to the benchmark.

2.2. Structural setups

2.2.1. Structural members

All the structural members of the DBGs are made of steel S355 with a bilinear elastic-perfect plastic constitutive law, with Young's modulus E = 2.1e+ 5 MPa, Poisson's ratio $\nu = 0.3$, yield strength $f_y = 355$ MPa, and density $\rho = 7850$ kg/m^3.

Structural members have circular hollow cross-section and are not subjected to initial prestressing in all the DBGs. The structural members of a single DBG have the same cross section. The cross-section

Table 1
Geometrical features of the DBGs.

	Barrel vault	Parabolic dome	Hyperbolic paraboloid
Generatrix equation	$z = -\frac{x^2}{2B}+f, D = \left\{-\frac{B}{2} \le x \le \frac{B}{2}\right\}, A = \frac{B}{4}\sqrt{5}+B\cdot\ln\left(\frac{1+\sqrt{5}}{2}\right)$		
Directrix equation	$z = f,\ z = -\frac{y^2}{2B}+f,\ z = \frac{y^2}{2B}+f$ $D = \left\{-\frac{B}{2} \le y \le \frac{B}{2}, x = 0\right\},\ D = \left\{-\frac{B}{2} \le y \le \frac{B}{2}, x = 0\right\},\ D = \left\{-\frac{B}{2} \le y \le \frac{B}{2}, x = 0\right\}$		
f	$B/8$	$B/8$	$B/8$
h	$B/8$	$B/8$	$B/4$
L	A	$2/3\pi B$	$3/2B$
L^*	L	$L/2$	L
S	BL	$\pi B^2/4$	$B^2/2$
b	$A/20$	$A/20$	$A/20$

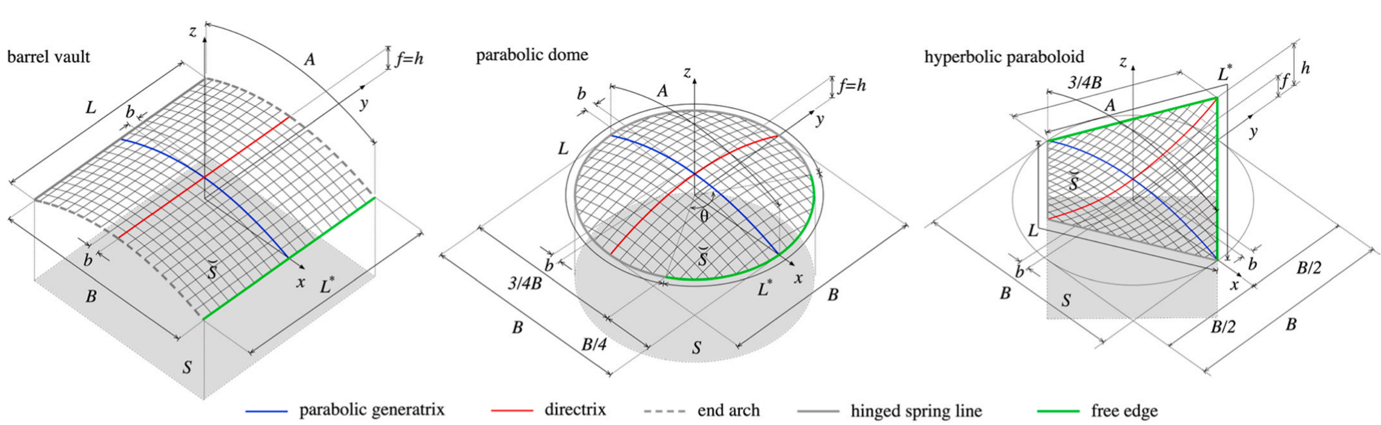

Fig. 1. FreeGrid DBGs: barrel vault, parabolic dome, hyperbolic paraboloid.

dimensions differ among the DBGs (Table 2) to ensure their comparable structural performances.

Steel type, cross section type and cross section dimensions are set intentionally uniform for all the structural members of each DGB in order to leave the way open to structural optimization by the Participants to the benchmark. Moreover, properties of the structural members are chosen according to structural criteria only, thus leaving room to improvements from the sustainability perspective.

2.2.2. Structural constraints

External constraints at the structural joints along the spring lines L are perfect hinges, except for the head arches of the barrel vault, along which only z- and x-wise joint displacements are constrained in order to avoid non-linear stiffening induced by the y-wise members. All the internal structural joints are rigid. The structural joints along the free-edge length L^* are not constrained.

2.2.3. Load Conditions

The design solutions shall be evaluated with reference to two Load Conditions (LC_k, $k = 1{:}2$). Both LCs are simplified and exploratory, intended to assess structural performances under ideal and controlled working conditions. Nevertheless, their moduli have the same order of magnitude of standardized design loads on gridshells. They are sketched in Fig. 2 in terms of piecewise uniform loads for the sake of clarity.

The first Load Condition (LC_1) cumulates the distributed self-weight of the structural members and the point loads $Q_{1,j} = (q_1 + q_2)s_j$ applied to all the structural joints, where s_j is the projection on the horizontal plane of the tributary area of the j-th structural joint, q_1 is the uniform distributed load that mimics the permanent weight-like load of a glass cladding, q_2 is the uniform distributed load that mimics the live snow-like load.

The second Load Condition (LC_2) mimics not only the effects of non-uniform vertical loads, but also the ones of wind-induced loads or other horizontal loads. LC_2 cumulates the distributed self-weight of the structural members, the point loads $Q_{2,1,j} = q_1 s_j$, and the point loads $Q_{2,2,j} = q_2 s_j$, where q_2 is applied on the surface with $x \geq 0$ on barrel vault and parabolic dome, and $y \geq 0$ on hyperbolic paraboloid.

Structural performances at Ultimate Limit States (ULS) are evaluated by setting $q_1 = 600\mathrm{N/m^2}$ and $q_2 = 1200\mathrm{N/m^2}$, while the ones at Serviceability Limit States (SLS) are assessed under $q_1 = 400\mathrm{N/m^2}$ and $q_2 = 800\mathrm{N/m^2}$.

3. Design goals and performance metrics

Participants to the benchmark are called to modify the above DBG(s) and conceive the Design Solution Gridshell(s) (DSG) in order to achieve seven selected Design Goals (DGs) in a genuine holistic perspective.

The whole performance assessment of each DSG develops in three conceptual steps (Fig. 3). First, each DG is expressed through a quantitative metric to be increased (↑) or decreased (↓) and made dimensionless with respect to the corresponding metric of the DBG (subscript 0). Second, DGs are grouped into three performance categories, covering the structural response (subscript s), the buildability (subscript b) and the sustainability (subscript su) of the DSG(s). Correspondingly, the single DG metrics are clustered into partial performance metrics $P_k(k = s, b, su)$. Finally, a bulk performance metric P is obtained as linear combination of the above partial ones.

Table 2
Cross-section dimensions of the structural members in the DBGs.

	Barrel vault	Parabolic dome	Hyperbolic paraboloid
Section type (type/diameter [mm]/thickness [mm])	O/139.7/14.2	O/101.6/10	O/101.6/10
Area [mm^2]	5596	2876	2876
Inertia [mm^4]	11157936	3052611	3052611

3.1. Structural goals and metrics

Gridshells are form-resistant structures, and the design and optimization of their shape traditionally mainly focus on their mechanical performances, e.g. [33,34,35]. As a result, they are as efficient as exposed to instability and deformability issues [36] at ULS and SLS, respectively.

Stability (ULS). The adopted metric to be increased is the critical Load Factor $\widehat{LF}$, with reference to both LC_1 and LC_2 defined in subsect. 2.2.3. $\widehat{LF}$ accounts for global, local or member elastic-plastic instability, and/or full plasticization of at least one cross section [37,38,39]. In particular $\widehat{LF} = \min\{LF_I, LF_P\}$ is defined as the minimum load multiplier between the LFs inducing (Fig. 4a):

- Global, local or member elastic-plastic instability, defined on the LF-displacement curve by the condition:

$$LF_I | dLF/d\widehat{\delta}_z = K_l, \tag{1}$$

where the limit value of the stiffness K is ideally null, and numerically set as $K_l = 0.02K_0$, being $K_0 = dLF/d\widehat{\delta}_z|_{\widehat{\delta}_z=0}$ the tangent stiffness.

Full plasticisation of at least one cross section, defined by the condition (Fig. 4b):

$$LF_P | \varepsilon^*_{\min} = \max_{i=1:M}\left[\frac{\min_z \varepsilon(z)}{\varepsilon_y}\right]_i = 1, \tag{2}$$

where $\varepsilon(z)$ is the absolute value of the strain in the generic fibre, ε_y is the strain corresponding to yield stress f_y, and M is the number of structural members. In Eq. (2) the minimum value of $\varepsilon(z)$ is selected by excluding a sufficiently small neighborhood of the neutral axis fibre, where the strain is null by definition.

The DSGs shall comply with $\widehat{LF} \geq 1$.

Deformability (SLS). The adopted quantitative metric to be decreased is the modulus of the maximum vertical displacement $|\widehat{\delta}_z|$ over the whole gridshell, under LC_1 and LC_2 load conditions. The DSGs shall comply with $|\widehat{\delta}_z| \geq \widehat{\delta}_{z,l} = B/200$.

The structural performance metric is averaged over the Load Conditions LC_k ($k = 1{:}2$) and is defined as

$$P_s = \frac{\sum_{k=1}^{2} \widehat{LF}_{k,0} \Big/ \frac{|\widehat{\delta}_{z,k}|}{|\widehat{\delta}_{z,k,0}|}}{2} \tag{3}$$

where the subscript “0” refers to the metrics of the DBGs.

Under the setup conditions specified in Sect. 2, the BGs fulfil the selected structural requirements at both ULS and SLS, while the DBGs do not.

3.2. Buildability goals and metrics

To the Authors’ best knowledge, buildability performances (also known as ‘fabrication-aware design’) are not thoroughly and unanimously defined in literature, in spite of their paramount design role and recent excellent proposals [40,41,42]. As such, the selected DGs, the related metrics and the resulting partial metric are intended to be an intentionally not all-encompassing, although rigorous, buildability performance model. In particular, the selected buildability DGs only refer to the geometry of 2D gridshell panels coincident with the faces, 1D line-like structural members coincident with edges, and 0D joints at vertices. Hence, in a truly conceptual design perspective, the adopted DGs do not include issues related to the panel-face offset and to the joint 3D manufacturing (e.g., kinks at joints, edge offset [41]).

Face out-of-planarity. Face planarity is the most widespread and

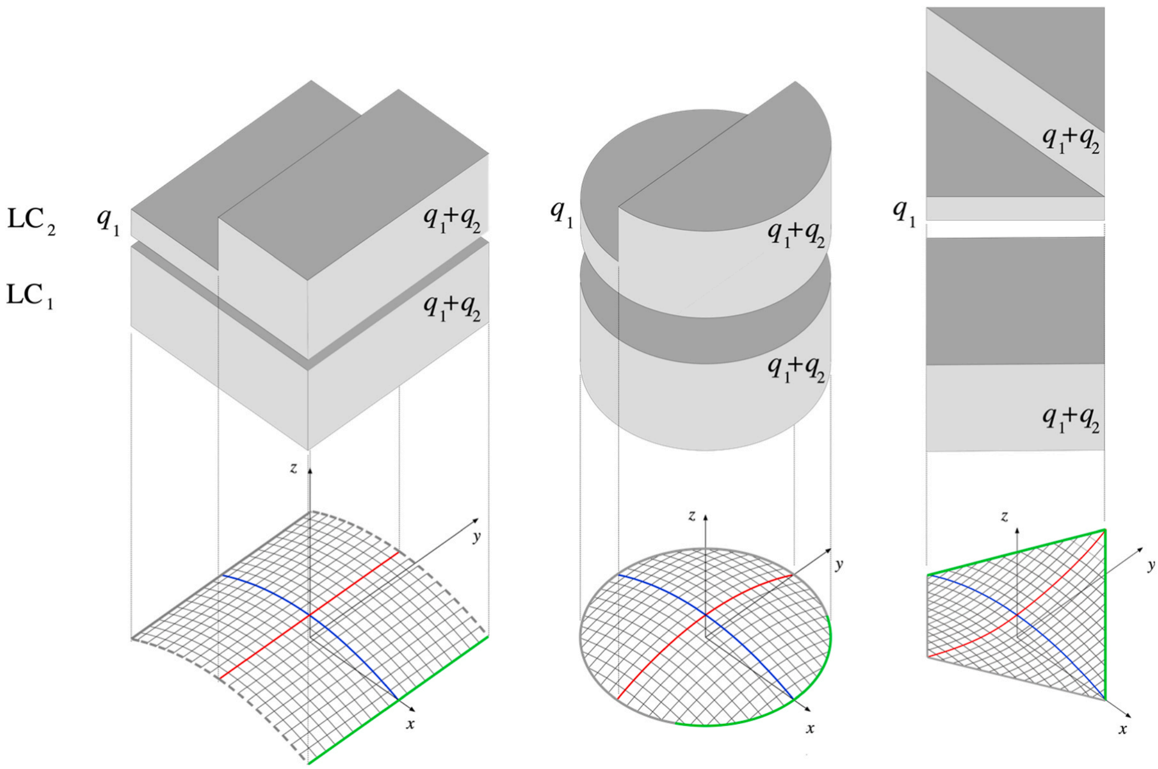

Fig. 2. FreeGrid Load Conditions.

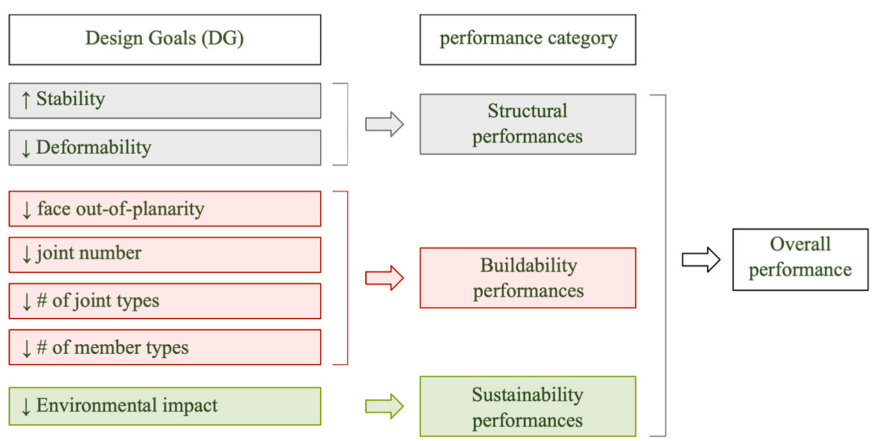

Fig. 3. Performance assessment framework.

traditionally considered construction constraint for double curvature gridshells (e.g., [42,43]). Indeed, planar faces accommodate cheaper flat panels that are significantly less expensive than molded or cold bent doubly curved panels. The adopted metric $\overline{\Delta}$ to be decreased is the average over all gridshell faces F of the f-th face out-of-planarity metric Δ_f [44,45]:

$$\overline{\Delta} = \frac{\sum_{f=1}^{F} \Delta_f}{F} \tag{4}$$

The latter is defined as the average over the n_f incident vertices of the distance d_j between the j-th vertex and the best fitting plane π, further divided by the face half perimeter p_f (Fig. 5):

$$\Delta_f = \frac{\frac{\sum_{j=1}^{n_f} d_j}{n_f}}{0.5 p_f} \tag{5}$$

Joint number. The number of structural joints largely affects the overall cost and buildability. The adopted metric to be decreased is the cardinality of the ensemble of the structural joints (N).

Uniformity of structural joints. The DG is aimed at shortening the joints chart, but also affecting the gridshell aesthetics. The adopted metric [46] to be decreased is the cardinality (J) of the ensemble of the joint types, or in geometrical terms, the number of clusters of congruent vertices. Two vertices are congruent if they have the same valence v (the number of edges incident to a vertex) and similar shape [48]. The shape similarity results from the closeness of the layout of the edges incident to the two vertices, and it is quantified by the metric S_{jk}:

$$S_{jk} = \min_{|v!} \left(\sqrt{\frac{\sum_{i=1}^{v} q_i^2}{v}} \right). \tag{6}$$

S_{jk} is the minimum over all permutations among edges of the average distance q_i^2 between the corresponding normalized edges incident to the

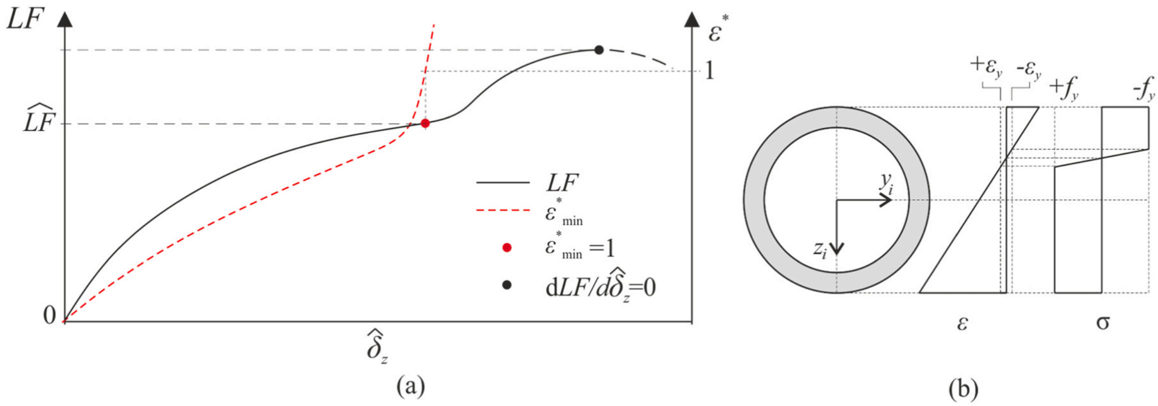

Fig. 4. Sketch for the determination of $\widehat{LF}$ (a); example of cross-section close to full plasticisation (b).

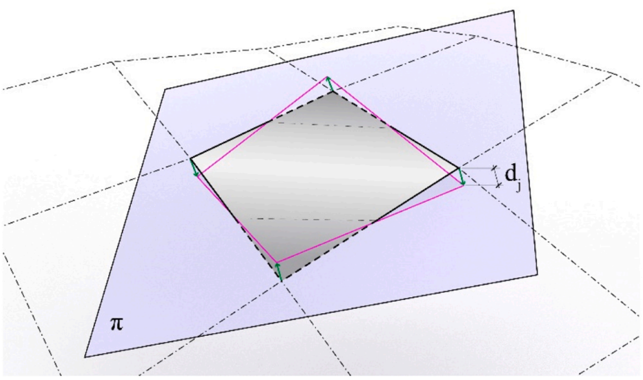

Fig. 5. Face out-of-planarity metric: d_j are the distances between the face vertices and their projection on the best fitting plane π (magenta line) [45].

vertices *j* and *k* (Fig. 6a). All edges are normalized in order to have unit length and the two vertices moved to overlap (Fig. 6b). As a result, all per-vertex normalized edges are included in a unit sphere (Fig. 6c).

The number of clusters is computed according to an iterative procedure with the farthest point sampling algorithm [48], assigning a vertex in a cluster as the reference vertex. Then, the similarity metric is computed for the reference vertex and every other vertex, verifying the condition:

$$\max(S_{jk}) \leq 0.01$$

If the condition is not verified, a new reference vertex is added. The procedure continues until the condition is verified for all vertices in the mesh. A joint belongs only to the cluster to whose reference vertex is closer, i.e., for which S_{jk} is minimum. The low model tolerance set equal to 1% is conventionally adopted to discern among different clusters. Different values can be adopted in the light of specific design requirements without impacting on the proposed modelling framework.

Uniformity of structural members. The DG is aimed at shortening the members chart, but also affecting the gridshell aesthetics. In what follows the member type results from its length and cross section. The corresponding metrics to be decreased are the coefficient of variation of member lengths $\tilde{l}$ [2,40] and the cardinality of the ensemble of the member cross-sections (C).

The buildability performance metric results from the average of the DG metrics as:

$$P_b = \frac{1}{\frac{1}{5}\left[\frac{1+\overline{\Delta}}{1+\Delta_0} + \frac{\#(N)}{\#(N_0)} + \frac{\#(J)}{\#(J_0)} + \frac{1+\tilde{l}}{1+l_0} + \frac{\#(C)}{\#(C_0)}\right]}. \quad (7)$$

3.3. Sustainability goals and metrics

Weight reduction is the traditional, widespread and largely emphasized design objective for structures in general, and for lightweight ones

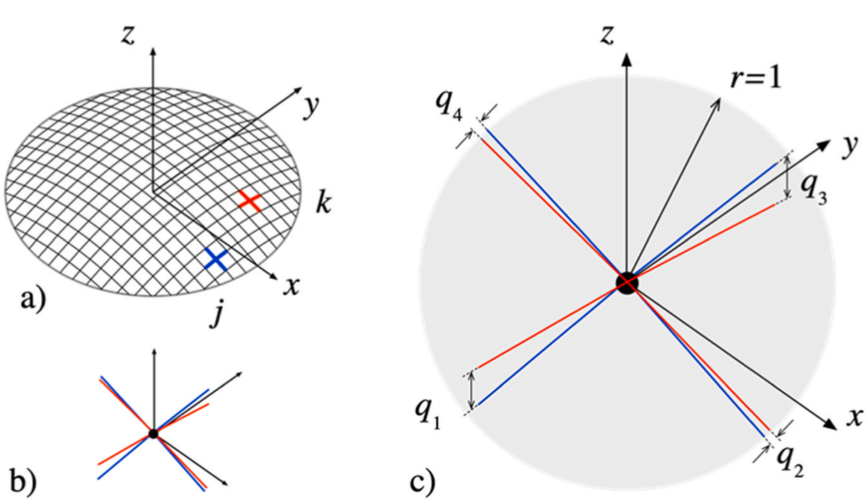

Fig. 6. Evaluation of the similarity metric S_{jk} between two vertices (in red and blue, a). The vertices are moved to the origin (b). The averaged distance of the normalized edges is evaluated (c) (after [47]).

in particular, gridshells included, e.g., [49]. In a Life Cycle Assessment (LCA) perspective, steel weight reduction nevertheless may be a misleading criterion not necessarily delivering the most sustainable solution. For the scope of this study, it has been proposed to reduce the complex analysis linked with the LCA by assigning a single non-dimensional parameter to the unitary mass of the element, which will be called environmental impact coefficient. This coefficient is used as a weighting factor of the steel weight of the elements. Clearly it is not easy to express such as a complex topic in one single parameter and this choice may depend on the approach and assumptions taken. In order to have a producer independent database and a direct comparison, this parameter has been based on the embodied carbon coefficient for the A1-A3 phase proposed in [50]. The values may be questioned but their advantage is that they are averaged with a large base, have been used since some years in real LCA, and are freely available. Since the structural performance is based also on design strength, it was important to attribute also a gradient in function of the yield strength. The technical literature proposes some values for some steel grades and products [51]; they were improperly extrapolated over the range of products in order to have a homogeneous and consistent approach.

The adopted metric to be decreased is $W = \sum_{i=1}^{M} g_i \cdot l_i \cdot \alpha_i$, where the summation over the M structural members includes the weight per unit length g, the length l of the i-th member, and an environmental impact correction coefficient α that depends on the steel grade and the type of member cross section, and that is normalized with respect to hollow sections made of S355 (Fig. 7).

The linear fitting laws of the environmental impact correction coefficient take the form $\alpha = a + 0.0002 \cdot f_y$, where f_y is the steel grade expressed in [MPa], and a= 0.475, 0.641, 0.792, 0.939 for I/H/C/L sections, round bars and rods, plates and flats, hollow and welded sections, respectively.

The sustainability metric applied is clearly simplistic and debatable, but it has the ambition to identify a coherent trend amongst different solutions. In particular it raises the designer awareness that weight reduction alone is not sufficient, as the choice of the product has become of primary importance in view of sustainable structures.

The sustainability performance metric simply reads as

$$P_{su} = \frac{1}{\frac{W}{W_0}} \tag{8}$$

3.4. Bulk performance metric

A bulk performance metric P results from the three partial performance metrics above:

$$P = \gamma_s \cdot P_s + \gamma_b \cdot P_b + \gamma_{su} \cdot P_{su}, \tag{9}$$

where γ_s, γ_b, γ_{su} are partial weighting factors constrained by $\gamma_s + \gamma_b + \gamma_{su} = 1$, so that $P_0 = 1$. In the FreeGrid benchmark, the partial weighting factors are uniformly set equal to $\gamma_s = \gamma_b = \gamma_{su} = 1/3$ in order to offer to the participants a common term of reference. In a broader perspective, designers are in charge to discuss the overall performance of DSG(s) by setting other values of the partial weighting factors.

Being P a sortable performance metric, the ranking of the DSG issued from the benchmark for each DBG does not depend on a subjective and *a posteriori* assessment, so as to guarantee fair comparison between the proposed design solutions.

4. Design constraints

DSGs shall fulfil the following Geometrical (GC) and Mechanical (MC) Constraints. They are listed in the following by making general reference to a design solution with generic shape and structural features:

1. the single-layer gridshell structural type cannot be changed, i.e., the mesh is 2-manifold, that is non-manifold vertices and edges are not permitted [52];
2. the position, shape and length of the continuous spring line and of the head arches (for barrel vault only) cannot be modified;
3. the gridshell spans along the x and y directions B_x and B_y, respectively, shall not be shorter than 30 m, being B_x and B_y generally defined as the maximum span free of external constraints;
4. the extent of the projection of the overall gridshell surface on the horizontal reference plane shall be no smaller than 0.95S;
5. the rise shall be kept equal to f=B/8, being f generally defined as the distance between the horizontal reference plane and the horizontal tangent plane to the shell surface having the minimum height;
6. the height h shall be no longer than B/4, being h generally defined as the distance between the horizontal reference plane and the

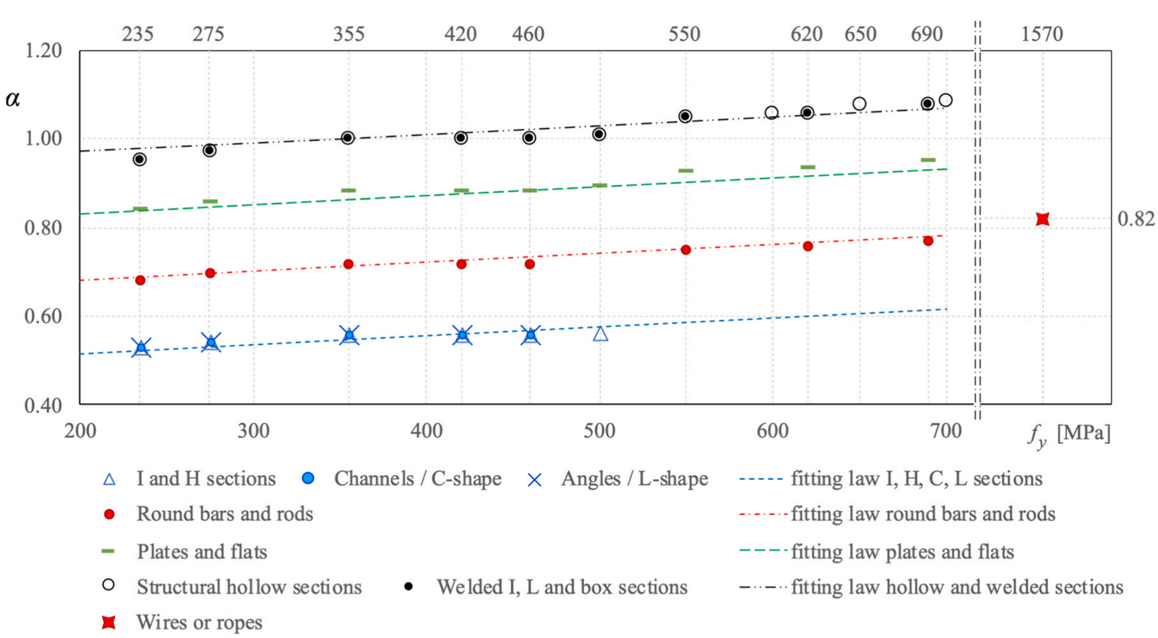

Fig. 7. Environmental impact correction coefficient α as a function of the steel grade and the type of member cross section.

horizontal plane passing through the shell vertex having the maximum height;
7. geometrical vertices and structural joints cannot lie below the horizontal reference plane;

1. along the spring line L, x-, y- and z-displacements of all the structural joints resulting from mesh generation shall be externally constrained (perfect hinges);
2. along the head arches (barrel vault), x- and z-displacements of all the structural joints resulting from mesh generation shall be externally constrained;
3. additional structural external constraints are not allowed anywhere;
4. the structural members shall have commercial cross sections;
5. the structural material shall be steel.

Whichever design parameter can be varied if not explicitly excluded above, for instance geometry (e.g., overall shape of the gridshell, node position, grid density and topology), number and properties of the structural members (cross section, length), number and type of the structural joints, steel grade, prestressing magnitude, et cetera.

5. Methods for structural performance assessment

Any kind of structural model can be used by the participants during the conceptual design/optimization of DSGs. Conversely, final structural performance assessment of the retained DSG(s) shall be carried out through FEM according to the specifications provided in the following paragraphs, in order to guarantee comparability among results. The proposed specifications come from preliminary sensitivity studies carried out using the codes Ansys® Mechanical APDL [53] and SAP2000® [54]. Specifications refer to (i) governing equations, (ii) FEM discretization and (iii) numerical solver.

(i) Structural performance metrics are obtained through Geometrically and Materially Non-linear Analysis (GMNA). The adopted material model for steel is elastic-perfectly plastic. The Distributed Plasticity (DP) modelling approach [53,55] is adopted, with nonlinear behavior modelled along the element and over the element cross section. If a Concentrated Plasticity (CP) approach [54,56] is adopted, the plastic regions shall be conventionally placed at the structural joints only, and each plastic region length shall be set equal to two times the maximum dimension of the cross section of the structural member.
(ii) The adopted Finite Elements are 3D beams based on Timoshenko model both for DP and CP modeling approach; in the case of DP approaches, a cubic shape function and 3 points of integration along the length are employed. Each structural member is discretized by 4 Finite Elements (FEs), in order to account for member buckling.
(iii) The load control path-following procedure is applied. The load step magnitude is set equal to 1/1000 the magnitude of the load condition LC_k. Numerical solution is obtained by means of the standard Newton-Raphson iterative method, with a tolerance on weighted residuals of the variables set equal to 5e-3.

6. Performances of DBGs

Figs. 8–11 summarize the DG metrics described in Section 3 for both BGs and DBGs.

Looking at the Structural performance metrics (Fig. 8-Fig. 9), the following synthetic comments can be outlined:

- The free edge dramatically reduces the structural performances of all the gridshells at both SLS and ULS: the critical load factors $\widehat{LF}$ of the DBGs are from 2 to 20 times lower than the ones of the corresponding BG; the maximum vertical displacements $\widehat{\delta}_{z,0}$ of the DBG are from 13 to 460 times higher than the ones of the corresponding BG. In particular, the hyperbolic paraboloid is the gridshell with the highest difference in terms of $\widehat{LF}$ under both LCs, while the barrel vault is the most sensitive in terms of $\widehat{\delta}_{z,0}$, namely under uniform LC_1;
- All DBGs do not satisfy performance levels at both SLS ($|\widehat{\delta}_{z,0}|/\widehat{\delta}_{z,l} > 1$) and ULS ($\widehat{LF_0} < 1$), while BG do, as intended. In short, the design challenge is set;
- The critical Load Factor approximately takes the same value for all DBGs ($\widehat{LF} \approx 0.8$), as desired. In other terms, the design challenge is analogous for all DBGs at ULS;
- Conversely, the DGBs perform differently at SLS, in the light of their geometrical (i.e., simple or double curvature) and mechanical (i.e., structural members in tension or compression) specific features. In particular, the barrel vault DBG, due to its single curvature, is particularly sensitive to serviceability issues: $|\widehat{\delta}_{z,0}|/\widehat{\delta}_{z,l} \approx 12$ under both LCs. Parabolic dome DBG ($|\widehat{\delta}_{z,0}|/\widehat{\delta}_{z,l} \approx 2.3$) and hyperbolic paraboloid DBG ($|\widehat{\delta}_{z,0}|/\widehat{\delta}_{z,l} \approx 1.1$) are increasingly stiffer.
- The structural performances of both BGs and DBGs at both SLS and ULS are analogous under the two different LCs. The exception is the barrel vault BG case, where the not uniform LC_2 induces more critical serviceability and ultimate behavior. Such difference does not hold for the corresponding barrel vault DBG, where the free-edge effects largely prevail over the ones of the LCs.

Fig. 10 summarizes the buildability DG metrics for the three DBGs. The barrel vault covers the widest surface, therefore it has the highest number of joints. Conversely, it has the lowest number of joint types,

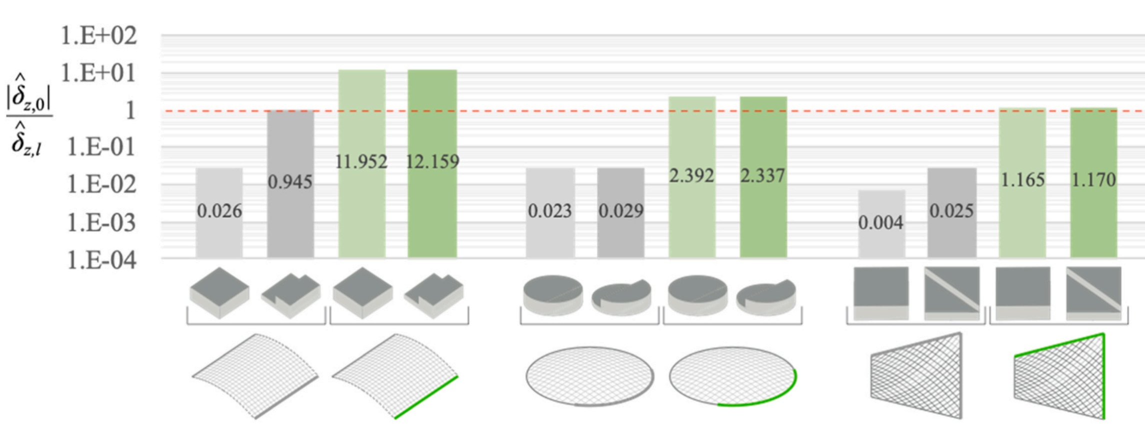

Fig. 8. BG (in gray) and DBG (in green) structural DG metrics: maximum normalized vertical displacement.

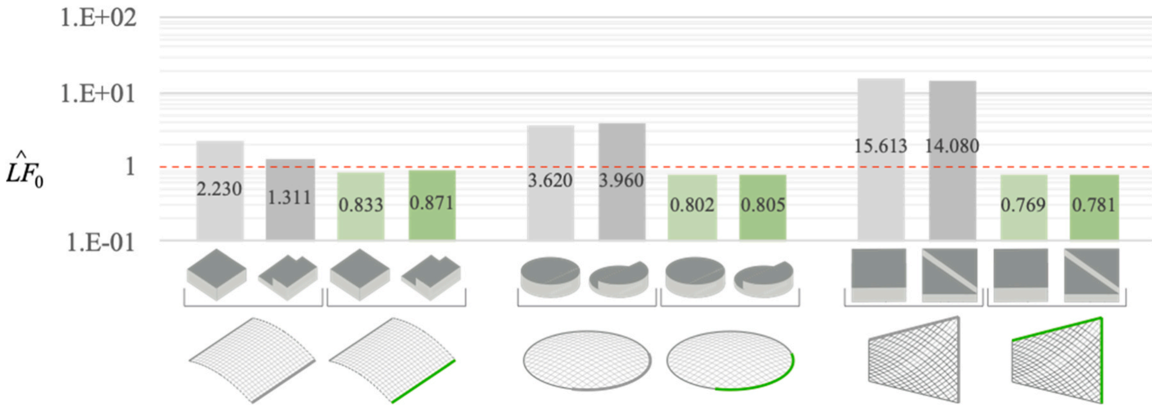

Fig. 9. BG (in gray) and DBG (in green) structural DG metrics: Load Factor.

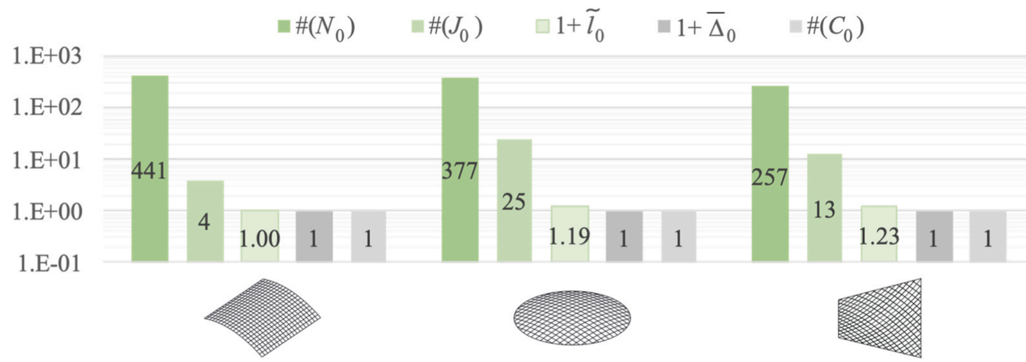

Fig. 10. BG and DBG buildability DG Metrics.

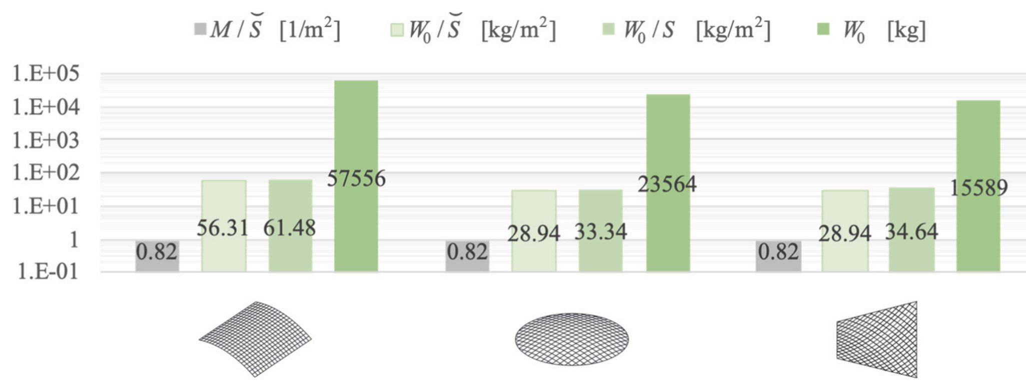

Fig. 11. BG and DBG sustainability DG Metric W_0, embodied carbon horizontal surface density W_0/S, embodied carbon surface density $W_0/\breve{S}$, mesh surface density $M/\breve{S}$.

which reaches the highest value for the parabolic dome, due to the higher variety of joint types close to the boundary. All the gridshells have been conceived to have all members with the same cross section, planar quad faces, and almost all structural members with the same length. Actually, the metrics $(1+\overline{\Delta}_0)$ and (C_0) are equal to unity for all DBGs, while the metric $(1+\tilde{l}_0)$ equals unity only for the barrel vault, but is slightly above unity for the other DBGs. In other words, DBGs have been conceived to have quite good performances from the buildability perspective. Anyway, they have not intentionally been optimized along the boundaries, i.e., the mesh is not boundary fitted, leaving room for improvements of the buildability performances, e.g., by reducing the number of joint types or the standard deviation of the members' length.

Fig. 11 depicts the sustainability DG metric W_0, together with the embodied carbon horizontal surface density W_0/S, the embodied carbon surface density $W_0/\breve{S}$, and the mesh surface density $M/\breve{S}$. Despite the 3 gridshells have the same mesh surface density, their different overall dimensions and adopted cross sections result in quite different values of W_0: in particular, W_0 is the highest for the barrel vault, which has the highest overall dimension and cross-section dimension, while the parabolic dome and hyperbolic paraboloid share the same cross section but have different W_0, being the cumulative length of structural members higher in the case of the parabolic dome. The parabolic dome and hyperbolic paraboloid have almost the same values of embodied carbon horizontal surface and surface densities, while the barrel vault values almost double. The member cross-sections of the DBGs have not been chosen according to sustainability criteria, but only according to structural criteria, thus allowing Participants to improve the sustainability performance metric.

7. Conclusions and perspectives

This study describes in detail the FreeGrid benchmark on design and optimization of free-edge gridshells (https://sites.google.com/view/freegrid), which has been launched at the IASS 2023 Annual Symposium in Melbourne [57]. FreeGrid proposes three Design Baseline Gridshells, whose performances should be improved by participants by proposing Design Solution Gridshells. This paper fully describes the Design Baseline Gridshell geometrical and structural setups and the metrics introduced to quantitatively evaluate their structural, buildability and sustainability performances. Moreover, the analysis methods for structural assessment are presented and the values of the performance metrics obtained for the Design Baseline Gridshells are discussed.

The hopefully wide participation to the benchmark will allow to build an Open Access database of both the results of the DBG analysis, and of the DSG proposals obtained by a number of different design approaches. In a wider perspective, the FreeGrid benchmark potentially paves the way to the performance assessment of different types of gridshells (e.g., bending active) and structural materials (e.g. wood, reinforced concrete) thanks to the general form of the proposed framework. For instance, it provides a practical way to include environmental impact considerations into structural design by tuning the environmental impact correction coefficient on the basis of the chosen material. Future improvements of the approach will be certainly focused on this last topic, which may assume a central role for comparing among them design solutions made of different materials and manufacturing/construction processes.

FreeGrid Technical Specifications can be downloaded by participants at https://sites.google.com/view/freegrid/docs, while data and tools for the automatic calculations of performance metrics are available at https://sites.google.com/view/freegrid/data-tools.

The FreeGrid future activities include *in itinere* special sessions possibly organized at national and international conferences. The benchmark first milestone is tentatively set within the IASS Annual Symposium 2026, when the FreeGrid ArcelorMittal Steligence Awards (https://sites.google.com/view/freegrid/award) will be remitted according to given specific regulation to the young Author(s) of the most performing design solution for each type of DGS, together with an additional award remitted to the author(s) who will deal with all the three DGS and obtain the highest performances in average.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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