Proposals Within the Safety Formats for Non-linear Finite Element Analysis of Reinforced Concrete Structures

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Proposals within the Safety Formats for Non-Linear Finite Element Analysis of Reinforced Concrete Structures

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ABSTRACT: The study analyses the different safety formats for non-linear finite element analyses to assess the design strength of reinforced concrete members. In detail, non-linear finite element analyses are developed to reproduce various experimental tests in compliance with the different safety formats analysing the results in terms of resistance and failure mode. Then, some proposals are discussed. Specifically, two preliminary non-linear finite element analyses are suggested to verify which safety format can be used. In addition, in order to always apply all the safety formats for their reduced computational effort, an additional failure mode-based safety factor is presented to assess the design global resistance of reinforced concrete structures.

KEYWORDS: NLFEAs; safety formats; design global resistance; reinforced concrete structure; structural reliability; safety factor; failure mode.

1 INTRODUCTION

The use of non-linear finite elements numerical codes represents an efficient tool in order to reproduce the response of reinforced concrete (RC) members (Castaldo et al. 2018a, Castaldo et al. 2018b, Di Trapani & Malavisi 2019, Di Trapani et al. 2020, Gino et al. 2016). Engineers and analysts adopt these instruments in order to model structures and infrastructures under extreme loading conditions at ultimate limit state and also to analyse serviceability conditions in order to improve the safety and resilience of infrastructure systems (Troisi and Alfano 2019, 2020). Over the years, several methodologies (Allaix et al. 2013, Castaldo et al. 2020) focused to account for different sources of uncertainties and prescribed target reliability levels (CEN EN 1992-2, *fib* Model Code 2010, *fib* Bulletin 80 2016, Gino et al. 2019, Castaldo and Alfano 2020) also for extreme loads have been presented (La Mazza et al. 2017). For instance, the recommendations deriving regarding the safety formats for non-linear finite element analyses (NLFEAs) are fundamental for the analysts with the purpose to design or assess new or existing RC structures (*fib* Model Code 2010, Castaldo et al. 2019, Allaix et al. 2013).

The mentioned above safety formats (SFs) for NLFEAs proposed by *fib* Model Code 2010 are based on the global resistance format approach (i.e., GR Format). In line with the GR Format, the evaluation of the structural safety may be performed in terms of the global structural response as follows:

$$F_d \leq R_d \;\; with \;\; R_d = \frac{R_{NLFEA,rep}}{\gamma_R \cdot \gamma_{Rd}} \tag{1}$$

where F_d is the design value of the actions evaluated in line with EN 1990 (2002); R_d is the design value of the global resistance; $R_{NLFEA,rep}$ is the value of the global resistance assessed in line to safety formats for NLFEAs; γ_R is the global resistance safety factor which takes into account the aleatory uncertainties associated to material properties variability; γ_{Rd} is the resistance model uncertainty safety factor (which includes the influence of epistemic uncertainties). This approach adopts the global structural verification based on NLFEAs and is very efficient since takes into account the redistribution of internal forces due to the damaging of concrete and reinforcements.

The purpose of this study is to compare the different safety formats based on the GRFormat as illustrated by *fib* Model Code 2010, Allaix et al. 2013, CEN EN 1992-2) in order to investigate whether the global strength expressions are able to reproduce the structural response (i.e., failure mode) of RC structures with appropriate safety levels. For instance, plane stress NLFEAs are developed using appropriate modelling hypotheses with the aim to reproduce the experimental tests of four RC beams with web openings Aykac et al. (2013) and one beam without web openings that has been designed in line with EN1992-1 (2004). The applicability of the different safety formats to the outcomes deriving from the plane stress NLFEAs is commented in function of the values assumed for the material properties (i.e., mean, characteristic and design values) and of the predicted failure mode. Finally, useful recommendations in relation to the general applicability of the safety formats for non-linear numerical analyses of RC structures are provided.

2 SHORT NOTES RELATED TO SAFETY FORMATS FOR NLFEAs OF RC STRUCTURES

As discussed in the previous section, the present investigation will investigate and compare the following safety formats (SFs):

- Partial factor method (PFM), *fib* Model Code (2010);
- Estimation of the coefficient of variation (ECOV), *fib* Model Code 2010;
- Global resistance factor (GRF), *fib* Model Code (2010);
- Global safety format (GSF), Allaix et al. (2013);
- Probabilistic method (PM), *fib* Model Code (2010).

The SFs are herein applied in compliance with Eq. (1) and with the hypotheses listed in Table 1. The hypotheses performed for probabilistic models adopted for GSF and PM safety formats, are listed in Table 2 in line with JCSS (2001). The 5% characteristic and design values for material properties have been calculated from the experimental/mean values EN1992-1 (2004). The resistance model uncertainty safety factor γ_{Rd} is assumed to be equal to 1.00 to not affect the results and evaluate the influence of aleatory uncertainty on the structural response evaluating the failure mode.

Table 1. Hypotheses for application of safety formats for NLFEAs.

Safety format	$R_{NLFEA,rep}$	γ_R	γ_{Rd}
PFM	$R_{NLFEA}(f_{cd},f_{yd})$	1.00	
ECOV	$R_{NLFEA}(f_{cm},f_{ym})$	$exp(\alpha_R\beta V_R)^{*1}$	
GRF	$R_{NLFEA}(f_{cmd}{}^{*2},f_{ym})$	1.27	1.00
GSF	$R_{NLFEA}(f_{cm},f_{ym})$	$exp(\alpha_R\beta V_R)^{*3}$	
PM	$R_{NLFEA,m}{}^{*3}$	$exp(\alpha_R\beta V_R)^{*3}$	

*1 Coefficient of variation of global resistance V_R is estimated with a simplified approach according to the hypothesis of lognormal distribution performing two NLFEAs using mean ($R_{NLFEA}(f_{cm},f_{ym})$) and characteristic values ($R_{NLFEA}(f_{ck},f_{ym})$) of material properties.

*2 The representative value for concrete compressive strength is evaluated as: $f_{cmd}=0.85\,f_{ck}$.

*3 The mean value $R_{NLFEA,m}$ and coefficient of variation V_R of global resistance can be estimated performing a reduced Monte Carlo simulation such as the Latin Hypercube Sampling with at least 30 samples. Lognormal distribution has been assumed to represent the global structural resistance R_{NLFEA}.

Discussions related to γ_{Rd} may be found in Castaldo et al. (2020) and Castaldo et al. (2018a) for both cyclic and static loading configurations.

The safety level is quantified by means of the reliability index β which is set equal to 3.8 in the assumption of ordinary structure with moderate consequences for a structural failure with a reference life of 50 years *fib* Model Code (2010), EN 1990 (2002) and the FORM sensitivity factor α_R=0.8 (i.e., for dominant resistance variables, *fib* Model Code 2010).

Table 2. Probabilistic models for application of GSF and PM safety formats.

Random variable	Probabilistic distribution	Mean value*	Coefficient of variation
Concrete cylinder compressive strength f_c [MPa]	*log-normal*	f_{cm}	0.15
Reinforcement steel yielding strength f_y [MPa]	*log-normal*	f_{ym}	0.05
Reinforcement steel Young modulus E_s [MPa]	*log-normal*	200000	0.03

*Mean value = experimental value for reference Aykac et al. 2013.

3 EXAMPLES OF APPLICATION OF THE SAFETY FORMATS FOR NLFEAs

3.1 *Cases study and numerical models*

The present investigation reports the outcomes of application of the mentioned above safety formats to five reinforced concrete structural members. In detail, four beams having simply supported static scheme with transverse web openings tested in laboratory by Aykac et al. (2013) and one simply supported beam (i.e., T-beam) designed with reference to EN1992-1. Focusing on Aykac et al. (2013) results, four beams having a 150 x 400 mm rectangular section and a span length of 3900 mm have been selected. In particular, three beams are realized with n°12 200x200 mm square openings and are casted with increasing reinforcement ratios (e.g., SL, SM, SH where S=square openings, L=low reinforcement ratio, M=medium reinforcement ratio, H=high reinforcement ratio). The cylinder concrete compressive strength of beams SL, SM and SH was experimentally evaluated equal to 22, 20 and 21 MPa, respectively. The last beam of Aykac et al. (2013) is denoted as CLX and presents n°12 circular openings with a diameter of 200 mm. Diagonal reinforcements between the openings are provided. The experimental value of cylinder concrete compressive strength of beam CLX is 22 MPa. The grade 500 reinforcements has been used in order to realize the different beams with a tensile yielding strength of 480 MPa (for bars with diameters 8, 10, 12 mm) and 520 MPa (for bars diameters 4 and 6 mm). The specimens have been realized with simply supported static scheme and symmetrically loaded at four points with the loading points located at 0.3 m and 1.2 m on either side of the midspan. The geometry of the beams is reported in Figure A1 of the Annex with the explanation of the test set configuration. The last beam, denoted as “T-Beam”, has been designed by the authors with reference to EN1992-1. The beam presents a bending failure mode with a design load of 335 kN. The beam has a “T” shaped cross section with total height of 500 mm, top flange width of 500 mm and web width of 150 mm. The top flange thickness is 100 mm. The mean value of the cylinder concrete compressive strength is 28 MPa while reinforcements have a mean tensile yielding strength of 495 MPa. The details of the beam are represented in Figure A2 of the Annex.

The definition of the non-linear finite element structural models of the beams SL, SM, SH, CLX and of the T-Beam have been carried out within the numerical code ATENA 2D (2014). Specifically, the four-node quadrilateral iso-parametric plane stress finite elements based on a linear shape functions and 2x2 Gauss point’s integration scheme has been adopted. The finite elements meshes have been defined after a specific sensitivity analysis for each structural member. The non-linear system of equations is solved by means of the standard Newton-Raphson method. With reference to the material models, non-linear behavior of concrete in compression has been reproduced with the SBeta Model available in ATENA 2D (2014). The SBeta model considers the compression softening behavior with a reduction of the compression strength and of the concrete shear stiffness (shear retention factor equal to 0.2) after cracking. The tensile concrete behavior has been modelled with a linear tension softening response which is able to take into account the “tension stiffening effect” (Castaldo et al., 2018a). The NLFE structural models have been defined considering half beam due to the symmetry of tests configuration. In compliance to laboratory tests developed by Aykac et al. (2013) and with the loading process selected for the T-Beam, the following loading history has been adopted with the purpose to perform the NLFEAs: application of dead weight; application of incremental loading up to failure. The Young modulus of the different materials as well as the concrete tensile strength have been evaluated from the experimental parameters in line to EN1992-1 and *fib* Model Code (2010).

3.2 *Results from numerical simulations and application of the safety formats for NLFEAs*

In the following, the global structural resistances achieved from the incremental loading processes of the experimental tests are reported in Table 3 with a description of the corresponding failure modes.

Table 3. Comparison between the NLFEAs results and the experimental outcomes

Beam	Exp. ultimate load R_{Exp} kN	Exp. failure mode	Material properties NLFEA	Ultimate load NLFEA R_m kN	NLFEA failure mode
SL	92.2	*Bending*	*Mean*	85.1	*Bending*
SM	117.0	*Vierendeel*	*Mean*	100.7	*Vierendeel*
SH	123.0	*Vierendeel*	*Mean*	109.9	*Vierendeel*
CLX	116.0	*Bending*	*Mean*	94.2	*Bending*
T-Beam	435.4*	*Bending**	*Mean*	458.6	*Bending*

* evaluated in line to EN1992-1 using mean the values of the material properties

The terms “the failure mode” are used to mean a specific resistance mechanism developed due to the failure of a specific material in a certain location of the structural member. The beams with square opening show an increasing ultimate loads with the increase of the reinforcement ratio (i.e., SL, SM and SH) with different resistance mechanisms. In fact, the beam SL is characterized by a bending failure whereas beams SM and SH show Vierendeel failure mechanisms (i.e., different local plastic hinges formation), respectively. The Beam CLX presents a bending failure mode with bar yielding in tensed chord and concrete crushing in the top chord. The T-Beam is characterized by a bending failure mode in line to the design assumptions. Table 3 reports the experimental ultimate loads for beams SL, SM, SH and CLX, the ultimate loads evaluated according to EN1992-1 using the mean values of the material properties for the T-Beam and the NLFEAs ultimate loads using experimental/mean values of material properties. The discrepancies in terms of the global resistance between the finite elements results and the experimental outcomes are due to the model uncertainties that in practice have to be accounted for adopting appropriate values of model uncertainty safety factor γ_{Rd} (Castaldo et al., 2018a, Castaldo et al., 2020).

Table A1 of the Annex reports the outcomes of the application of the different SFs described in previous sections. The beams SL, SH and CLX have not showed different failure modes with the adoption of different combinations of values for the material properties in order to perform NLFEAs. The beams SM and T-beam showed a tendency to modify their failure mechanisms (Vierendeel-A/B and bending/shear, respectively) when the probabilistic analysis using 30 Latin Hypercube samples is performed (Golzio and Troisi 2013, Garzillo and Troisi 2015). The failure mechanisms Vierendeel-A/B differ in the location of the plastic hinges along the beams realized with square holes. With reference to the global design resistance evaluated according to the PM as the reference one, the latter two beams presented unsafe results for the estimation of design global resistance using the simplified SFs (i.e., GSF, GRF, PFM and ECOV). This does not happen when the predicted failure modes are the same for all the simulations as for the beams SL, SH and CLX. Table A1 also shows the results in terms of coefficient of variation of global resistance V_R and global resistance safety factor γ_R for the different safety formats. Differences are recognized between the V_R estimated by means of Latin Hypercube sampling and ECOV method when different failure modes may be present (beams SM and T-beam).

4 DISCUSSION

As also proposed in (Castaldo et al. 2019), grounding on the results of the previous section, preliminary evaluation of the possible modifications of the failure mode can be performed by means of two preliminary NLFEAs in order to verify the applicability of the PFM, GRF, ECOV and GSF with reference to the PM. The two preliminary non-linear finite elements simulations consist of:

- one NLFEA using the mean values for the concrete properties and the design values for the reinforcement properties;
- one NLFEA using the design values for the concrete properties and the mean values for the reinforcement properties.

In order to apply PFM, GRF, ECOV and GSF, due to their reduced computational effort, modified values of the global resistance safety factor are proposed for the assessment of the design global resistance. With reference to the results of the present investigation, a new failure mode-based safety factor denoted as γ_{FM} is proposed. The failure mode-based safety factor γ_{FM} has been calibrated in order to achieve the results, in terms of design global resistance R_d evaluated with SFs reported in Table A1 (i.e., GSF, ECOV, GRF and PFM), in compliance with the design global resistance R_d estimated with the probabilistic method (PM). This assessment leads to values of γ_{FM} varying in the range about 1.00-1.18. Finally, the assessment of the design global resistance R_d can be performed, according to the selected safety format, modifying Eq. (1) as follows:

$$R_d = \frac{R_{rep}}{\gamma_R \cdot \gamma_{Rd,sf} \cdot \gamma_{Rd}} \quad (2)$$

Then, according to Table A1, if the two preliminary analyses provide the same failure mode, γ_{FM} set equal to 1.00 can be adopted. Differently, the γ_{FM} set equal to 1.15 is suggested.

5 CONCLUSIONS

The purpose of the present study is to compare the different safety formats within the global resistance format (GRFormat) for NLFEAs to estimate the global design strength of different RC structures. The different SFs are investigated to verify their effectiveness to evaluate the design global resistance of the RC structure and capture the failure mode. The PM is assumed as the reference safety format. From the outcomes, it is possible to observe that for some beams (i.e., Beam SL, Beam SH and Beam CLX) the failure mode does not present any modification and the GRF and PFM safety formats always provide design ultimate loads lower the one evaluated with the PM. Moreover, for the Beam CLX, the ECOV safety format also provides a design ultimate load lower than the estimation of the PM. Instead, for the other beams (i.e., Beam SM and T-Beam) the failure modes change in function of the SF. In fact, for the Beam SM, only the GRF safety format provides a design ultimate load lower the one of the PM; for the T-Beam, all the simplified SF (i.e., PFM, ECOV, GRF and GSF) estimate design ultimate loads higher than the one of the PM with lower reliability levels because they are not able to capture the modifications in the failure mode.

The mechanical response of a structure is a fundamental requirement to assess the structural safety. For instance, a preliminary evaluation based on two preliminary NLFEAs under the hypothesis of one material (i.e., concrete/steel) stronger than the other one and vice versa, is proposed to understand which SF can be used. In fact, if the failure mode does not change in the two preliminary analyses it follows that PFM, GRF, ECOV, GSF methods can be adopted to estimate the design ultimate load, whereas if the resisting mechanism are different, a failure mode-based safety factor γ_{FM} set equal to 1.15 is recommended.

ACKNOWLEDGEMENTS

This work is part of the collaborative activity developed by the authors within the framework of the Commission 3 – Task Group 3.1: “Reliability and safety evaluation: full-probabilistic and semi-probabilistic methods for existing structures” of the International Federation for Structural Concrete (*fib*).

This work is also part of the collaborative activity developed by the authors within the framework of the WP 11 – Task 11.4 – ReLUIS.

REFERENCES

Allaix, D.L., Carbone, V.I. & Mancini G. 2013. Global safety format for non-linear analysis of reinforced concrete structures. *Structural Concrete,*14(1): 29-42.

ATENA 2D v5. 2014. Cervenka Consulting s.r.o. Prague, Czech Republic.

Aykac, B., Kalkan, I., Aykac, S. & Egriboz, E.M. 2013. Flexural behaviour of RC beams with regular square or circular web openings, *Engineering Structures*, 56, pp. 2165-2174.

Castaldo P., Alfano G. 2020. Seismic reliability-based design of hardening and softening structures isolated by double concave sliding devices, *Soil Dynamics and Earthquake Engineering*, 129,105930.

Castaldo, P., Gino, D. & Mancini, G. 2019. Safety formats for non-linear analysis of reinforced concrete structures: discussion, comparison and proposals. *Engineering Structures*, 193,136-153.

Castaldo, P., Gino, D., Bertagnoli, G. & Mancini, G. 2018a. Partial safety factor for resistance model uncertainties in 2D non-linear finite element analysis of reinforced concrete structures. *Engineering Structures,* 176:746-762.

Castaldo, P., Gino, D., Bertagnoli, G. & Mancini, G. 2020. Resistance model uncertainty in non-linear finite element analyses of cyclically loaded reinforced concrete systems, *Engineering Structures*, 211(2020), 110496, https://doi.org/10.1016/j.engstruct.2020.110496

Castaldo, P., Gino, D., Carbone, V.I. & Mancini, G. 2018b. Framework for definition of design formulations from empirical and semi-empirical resistance models, *Structural Concrete*, 19(4): 980-987.

CEN EN 1992-2 Eurocode 2 *Design of concrete structures, Part 2: concrete bridges*. CEN 2005. Brussels.

Di Trapani, F. & Malavisi, M., 2019. Seismic fragility assessment of infilled frames subject to mainshock/aftershock sequences using a double incremental dynamic analysis ap-proach*, Bull. Earthquake Eng*, 17(1): 211–235.

Di Trapani, F., Ferro, G.A., Malavisi, M. Definition of inelastic displacement demand spectra for precast industrial facilities with friction and fixed beam-to-column joints. *Soil Dynamics and Earthquake Engineering*. Volume 128, January 2020, Article number 105871.

EN 1990, 2002. *Eurocode - Basis of structural design*. Brussels: CEN.

EN 1992-1-1. 2004. *Eurocode 2 – Design of concrete structures. Part 1-1: general rules and rules for buildings*. CEN. Brussels.

fib Bulletin N°80. 2016. Partial factor methods for existing concrete structures, Lausanne, Switzerland.

fib Model Code for Concrete Structures 2010. 2013. Lausanne.

Garzillo C.; Troisi R. 2015. Le decisioni dell'EMA nel campo delle medicine umane. pp.85-133. In EMA e le relazioni con le Big Pharma - I profili organizzativi della filiera del farmaco - ISBN:9788892102279 - G. Giappichelli

Gino, D., Bertagnoli, G. & Mancini, G. 2016. Effect of endogenous deformations on composite bridges. *Recent Progress in Steel and Composite Structures - Proceedings of the 13th International Conference on Metal Structures*, ICMS 2016, *15-17 June 2016, Zielona Gora, Poland,* pp. 287-298.

Gino, D., Castaldo, P., Bertagnoli, G., Giordano, L. & Mancini, G. 2019. Partial factor methods for existing structures according to *fib* Bulletin 80: Assessment of an existing prestressed concrete bridge. *Structural Concrete*, 2019: 1–17. https://doi.org/10.1002/suco.201900231.

Golzio L. E., Troisi R. 2013. 'The value of interdisciplinary research: a model of interdisciplinarity between legal research and research in organizations". pp.23-38. In *Journal For Development And Leadership* , vol. 2.

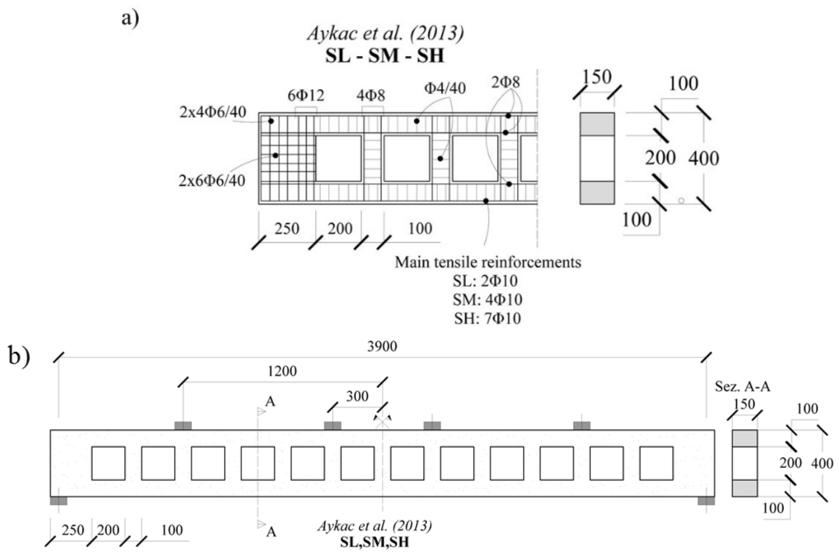
JCSS. 2001. *JCSS Probabilistic Model Code.*

La Mazza, D., Giordano, L., Castaldo, P. & Gino, D. 2017. Assessment of the efficiency of seismic design for structural robustness of rc structures , *Ingegneria Sismica*, 34(3-4): 63-77.

Troisi R., Alfano G. 2019. Towns as Safety Organizational Fields: An Institutional Framework in Times of Emergency. *Sustainability* 2019, 11, 7025, doi:10.3390/su11247025.

Troisi R., Alfano G. 2020. Firms'crimes and land use in Italy. An exploratory data analysis. *New Metropolitan Perspectives, International Symposium – 4th edition*, 27-30 May 2020, pp 10.

ANNEX



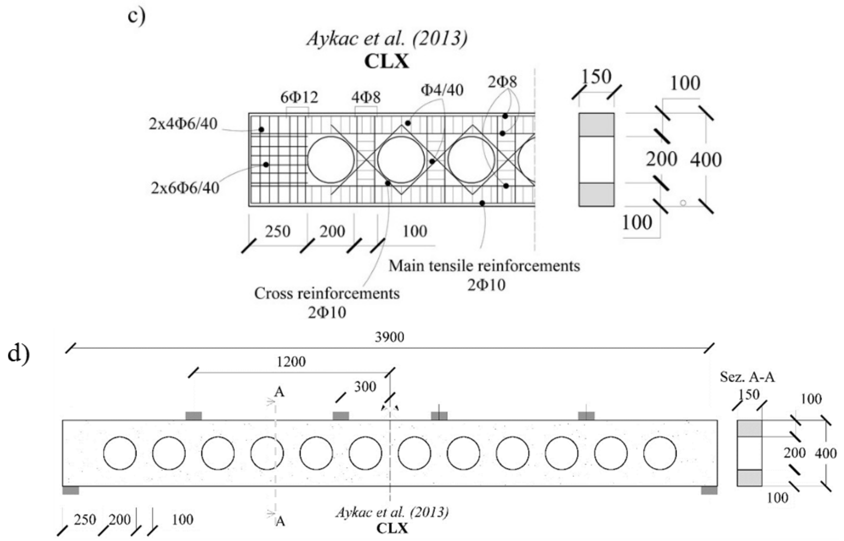


Figure A1. Geometry for beams with square openings SL, SM, SH (a) (Aykac et al., 2013) and CLX (c) (Aykac et al., 2013); representation of the tests set and loading configuration (b) and (d). (Dimensions in [mm]).

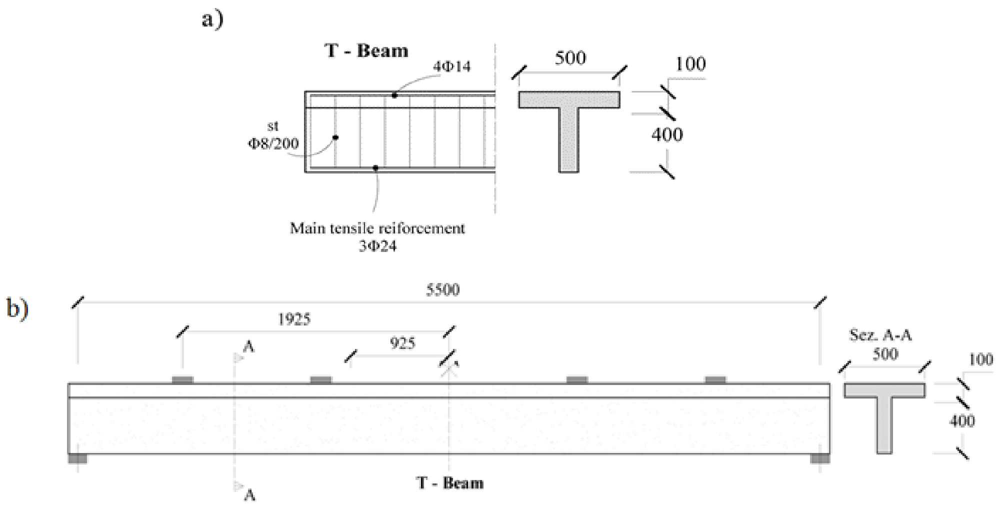


Figure A2. Geometry for the T-Beam (a) and representation of the static scheme and loading configuration (b). (Dimensions in [mm]).

Table A1. Results design ultimate load calculated according to the different safety formats for NLFEAs.

Safety Format	Ultimate load R kN	Failure mode	Coefficient of variation V_R	Global resistance factor γ_R	Design ultimate load R_d kN
Beam SL (Aykac et al., 2013)					
PFM	65.1	*Bending*	-	-	65.1
ECOV*	85.1 - 77.1	*Bending*	0.060	1.20	70.9
GRF	79.9	*Bending*	-	1.27	62.8
GSF	30 results	*Bending*	0.054	1.18	72.2
PM	30 results	*Bending*	0.054	1.24	68.5
Beam SM (Aykac et al., 2013)					
PFM	73.1	*Vierendeel - A*	-	-	73.1
ECOV*	100.7 - 85.5	*Vierendeel - A*	0.099	1.35	74.5
GRF	84.3	*Vierendeel - A*	-	1.27	66.4
GSF	30 results	*Vierendeel - A;Vierendeel - B*	0.107	1.38	72.8
PM	30 results	*Vierendeel - A;Vierendeel - B*	0.107	1.42	70.7
Beam SH (Aykac et al., 2013)					
PFM	69.1	*Vierendeel - A*	-	-	69.1
ECOV*	109.9 - 91.9	*Vierendeel - A*	0.108	1.39	79.0
GRF	89.5	*Vierendeel - A*	-	1.27	70.5
GSF	30 results	*Vierendeel - A*	0.102	1.37	80.5
PM	30 results	*Vierendeel - A*	0.102	1.46	75.4
Beam CLX (Aykac et al., 2013)					
PFM	72.0	*Bending*	-	-	72.0
ECOV*	94.2 - 85.6	*Bending*	0.058	1.19	79.0
GRF	90.0	*Bending*	-	1.27	70.9
GSF	30 results	*Bending*	0.052	1.17	80.5
PM	30 results	*Bending*	0.052	1.19	79.1
T-Beam					
PFM	349.4	*Bending*	-	-	349.4
ECOV*	458.6 - 411.8	*Bending*	0.065	1.22	376.1
GRF	443.0	*Bending*	-	1.27	348.9
GSF	30 results	*Bending;Shear*	0.086	1.30	352.9
PM	30 results	*Bending;Shear*	0.086	1.44	318.3

*For ECOV method, the two values of the ultimate load correspond, respectively, to R_m and R_k