Comparison of Different Approaches to Derive Global Safety Factors for Non-linear Analyses of Slender RC Members

Comparison of different approaches to derive global safety factors for non-linear analyses of slender RC members

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ABSTRACT: The present study relates to comparison between different approaches for definition of global safety factors for non-linear analysis of slender RC members with reference to new or existing structures. Firstly, a benchmark set of 40 experimental results on reinforced concrete columns is presented. After the description of the main features of the benchmark test sets the related non-linear numerical models have been realized using fiber-modelling as solution strategy. Then, appropriate assumptions concerning aleatoric and epistemic uncertainties have been performed with the aim to run probabilistic analysis of global resistance for each one of the 40 columns. The results of the probabilistic analysis are useful to define global safety factors in line to the global resistance method. Finally, the comparison between different approaches to derive global safety factors is presented and discussed.

KEYWORDS: reinforced concrete; global safety factors; non linear analysis

1 INTRODUCTION

The non-linear analysis (NLAs) is nowadays widely used to perform safety verifications of structures and infrastructures (Castaldo et al. 2017, Vecchi & Belletti 2021, Gino et al. 2021, Castaldo & Amendola 2021, Castaldo et al. 2020a, Castaldo et al. 2022, Troisi 2022, Troisi & Alfano 2022a, Troisi & Alfano 2022b). In this framework, different safety formats conceived for NLAs have been developed and investigated over the last decade (*fib* Model Code 2010, Castaldo et al. 2019). As a general concept, the mentioned above safety formats are based on the global resistance format introduced by *fib* Model Code 2010 and allow to estimate the design structural resistance R_d of a reinforced concrete (RC) system as:

$$R_d = \frac{R_{NLA,m}}{\gamma_R^{GL}} \tag{1}$$

where: $R_{NLA,m}$ is the mean value of global structural resistance estimated with NLA; γ_R^{GL} is the global safety factor that includes the influence of aleatoric uncertainties (i.e., materials and geometric properties) and epistemic uncertainty (i.e., model uncertainty) (Castaldo et al. 2020b, Castaldo et al 2020c, Engen et al. 2017).

This work is focused on the comparison of different approaches to derive the global safety factor γ_R^{GL} concerning the case of slender RC columns. As first, a benchmark set of 40 experimental results on reinforced concrete columns is presented according to Gino et al. (2021). The experimental tests have been reproduced by non-linear numerical (NLN) models using a validated solution strategy adopting the software OpenSees (McKenna et al., 2000). Then, probabilistic analysis of structural resistance has been performed using the Latin Hypercube Sampling (LHS) method to account for aleatoric and epistemic uncertainties (Gino et al. 2021, JCSS 2001). The results of the probabilistic investigation permit to compare the efficiency of three approaches (denoted as *Approach 1, 2* and *3*) useful to derive the global safety factor γ_R^{GL}. These outcomes can be useful to discuss about the best approach to define global safety factors in next generation of design codes (e.g., EN1992-1-1, *fib* Model Code 2020).

1.1 *Approaches to derive the global safety factor*

The global safety factor γ_R^{GL} can be derived with:

- *Approach 1:* this approach is the one adopted by *fib* Model Code 2010 and separates the partial safety factors accounting for aleatoric uncertainties γ_R and epistemic ones (i.e. model) γ_{Rd}. The γ_R^{GL} can be evaluated as the product:

$$\gamma_R^{GL} = \gamma_{Rd}\gamma_R \tag{2}$$

with:

$$\gamma_R = \exp(\alpha_R \beta_t V_R) \tag{3}$$

$$\gamma_{Rd} = \frac{\exp(\alpha'_R \beta_t V_\vartheta)}{\mu_\vartheta} \tag{4}$$

where V_R is the coefficient of variation of global resistance from the results of probabilistic analysis of global resistance including sampling of aleatoric uncertainties only; μ_ϑ is the mean value of model uncertainty; V_ϑ is the coefficient of variation of model uncertainty; α_R and α'_R are the FORM factors for dominant and non-dominant variables, fixed, respectively to 0.8 and 0.32 (Hasofer & Lind 1974) while β_t is the target reliability index (*fib* Model Code 2010).
In line to *fib* Model Code 2010 γ_R and γ_{Rd} are determined assuming a lognormal probabilistic distribution.

- *Approach 2:* this approach consists in the evaluation of γ_R^{GL} in line to the assumption of lognormal probabilistic distribution for structural resistance as:

$$\gamma_R^{GL} = \frac{\exp(\alpha_R \beta_t V_R^{GL})}{\mu_\vartheta} \tag{5}$$

where the coefficient of variation of global resistance V_R^{GL} is inclusive of aleatoric and epistemic uncertainties and can be obtained in simplified manner as (Qianhui et al. 2021):

$$V_R^{GL} = \sqrt{(V_R)^2 + (V_\vartheta)^2} \tag{6}$$

- *Approach 3:* this approach is the more general and estimates the γ_R^{GL} in line to Eq.(5) but, as a difference, it determine V_R^{GL} from the results of probabilistic analysis of global resistance including sampling of both aleatoric and epistemic uncertainties. This approach can be adopted as the reference one.

Finally, with reference to the mentioned above approaches, the application of Eq.(1) can be performed using as for $R_{NLA,m}$ the outcome from NLAs of the 40 RC columns performed using experimental values (i.e. mean) of main properties according to Allaix et al. (2013). Eq. (3)-(6) are valid until the values of coefficient of variation of main variables are lower or equal than 0.3 (with an error lower than 5%).

2 BENCHMARK RESULTS AND MODELLING

In this study the benchmark experiments summarized in Gino et al. (2021) are adopted.

Table 1. Benchmark results collected by Gino et al., (2021) and results from numerical modelling using Opensees.

Exp. test	Spec.	Slenderness λ [-]	$R_{NLA,m}$ [kN]	R_{exp} [kN]
2L20-30	B	15	694.3	750.0
2L20-60			736.4	700.0
2L8-120R			1152.7	1092.0
4L8-30			1032.9	1100.0
4L20-120			830.7	900.0
4L8-120R			1319.5	1247.0
C000	A	17	560.6	559.6
C020			328.5	327.3
B020	B	52	263.7	271.5
RL300		56	423.3	474.3
A-17-0.25	B	48	1367.4	1181.4
C-31.7-0.25		94	280.1	333.4

3.3		59	856.4	782.6
5.1			810.8	735.5
4.1		88	391.7	367.7
N30-10.5-C0-3-30	C	68	16.6	16.6 (280)*
H60-10.5-C0-1-30			17.9	17.2 (412)*
III		74	347.3	343.2
Va			680.7	684.5
2		83	236.8	235.4
I	A	104	258.0	264.8
VI		106	363.2	392.3
15		136	560.3	549.2
3		137	563.4	666.9
8		83	236.8	235.4
9	B	135	205.9	205.9
12			112.2	112.8
6		137	227.6	225.6
24D-2	D	104	192.8	198.4
15E-2	A	139	129.3	161.0
S28		167	49.9	44.0
S30	B		53.4	48.0
S25		200	42.3	36.0
5		208	78.7	72.7
6			82.3	72.2
17A		225	37.1	31.9
20	A	243	39.8	37.9
18			39.8	33.9
8		274	31.0	31.9
20		15	32.3	29.9

* Value of the axial load applied to the RC column during the laboratory test.

The database collects results of tests in compression of 40 RC columns (Mehmel et al. 1969, Saenz & Martin, 1963, Foster & Attard 1997, Pancholi 1977, Dracos 1982, Iwai et al. 1986, Chuang & Kong 1997, Barrera et al. 2011, Baumann 1935) characterized by wide range of slenderness, materials properties and reinforcement configuration and fulfils the limits of applicability of EN1992-1-1 and *fib* Model Code 2010.

Figure 1 shows tests sets configurations and also related assumptions for numerical modelling according to Gino et al. (2021). The test sets configuration presents both compression tests with incremental axial load (type A, D), compression tests with eccentric incremental load (type B), test with incremental horizontal force with constant axial load (type C).

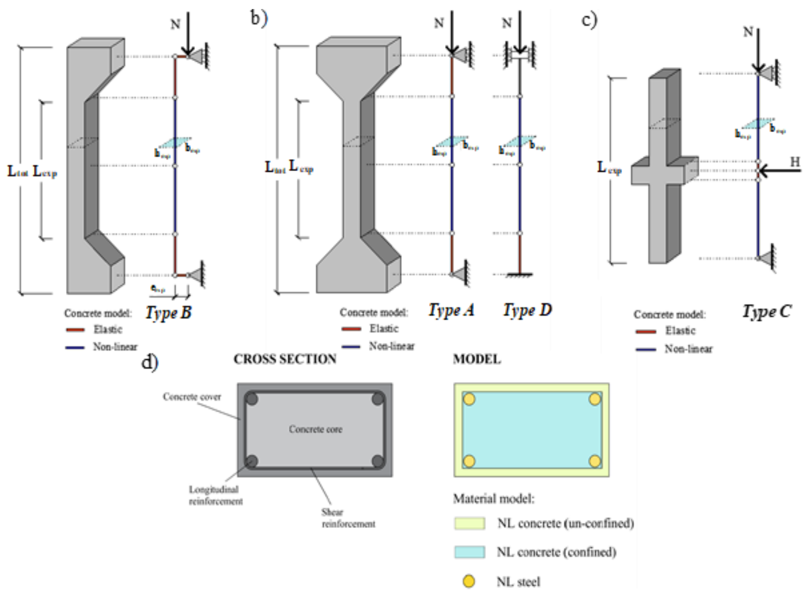

Figure 1. Test sets configuration and strategy for numerical modelling of the 40 RC columns (Gino et al. 2021).

The NLN models of the 40 RC columns have been realized using force-based fiber elements implemented in Opensees. The solution strategy herein adopted correspond to the optimum one that minimizes the model uncertainty in line to the "*Model 8*" as described by Gino et al. (2021) and is inclusive of both geometric and materials non-linearities. The NLN models consider the effect of confinement provided by stirrups (Saatcioglu et al. 1992) and of instability of reinforcement bars loaded in compression (Dhakal & Maekawa 2022). The ultimate axial load reached within NLA is established as the one associated to the last load-step able to fulfil the convergence criteria (Gino et al. 2021). The loading process have been defined according to the procedure of execution of the experimental tests including the influence of self-weight.

Table 1 reports general information associated to the tests sets and the results in terms of comparison between NLAs performed using the experimental values of both materials and geometrical properties $R_{NLA,m}$ and experimental evidence R_{exp}. Further and detailed information about the NLN models can be acknowledged in Gino et al. (2021).

3 PROBABILISTIC ANALYSIS OF STRUCTURAL RESISTANCE

The NLN models determined according to previous section has been used to perform probabilistic analysis of structural resistance of the 40 RC columns.

Table 2. Probabilistic model for material properties random variables (aleatoric) in line to JCSS 2001.

Random variable	Distr.	Mean value	Coefficient of variation [-]	Statistical correlation*
Concrete cylinder compressive strength f_c [MPa]	LN	$f_{c,exp}$	0.15	-
Reinforcements yielding strength f_y [MPa]	LN	$f_{y,exp}$	0.05	f_u (0.75), ε_u(-0.45)
Reinforcements yielding strength f_u [MPa]	LN	$f_{u,exp}$	0.05	f_y (0.75), ε_u(-0.60)
Reinforcements Young's modulus E_s [MPa]	LN	210000	0.03	-
Reinforcements ultimate strain ε_u [-]	LN	0.075	0.09	f_y (-0.45), f_u (-0.60)

* (-) coefficient of linear correlation with respect to other material random variables.

Table 3. Probabilistic model for geometric properties random variables (aleatoric) in line to JCSS 2001.

Random variable	Distr.	Mean value	Standard deviation
Concrete cover (C) $Y_c=C-C_{exp}$ [mm]	N	0	5
Cross section (b) $Y_b=b-b_{exp}$ [mm]	N	$0 \leq 0.003b_{exp} \leq 3$	$4+0.006\ b_{exp} \leq 10$
Cross section (h) $Y_h=h-h_{exp}$ [mm]	N	$0 \leq 0.003h_{exp} \leq 3$	$4+0.006\ h_{exp} \leq 10$
Column length (L) $Y_L=L-L_{exp}$[mm]	N	$0 \leq 0.003L_{exp} \leq 3$	$4+0.006\ L_{exp} \leq 10$
Eccentricity e [mm]	N	e_{exp}	$L_{exp}/1000$

In Tables 2-4 it is reported the summary of the probabilistic model adopted for the main involved random variables in according to JCSS 2001 and Gino et al. (2021).

In particular, Tables 2-3 illustrates the assumptions for probabilistic modelling of material and geometric uncertainties (i.e., aleatoric) whereas Table 4 reports the probabilistic idealization for model uncertainty according to Gino et al. (2021) (i.e., epistemic). The data of table 4 relates to the herein adopted solution strategy and are determined assuming limited influence of experimental uncertainty (Gino et al. 2021).

The Latin Hypercube Sampling method (Mckey et al. 1979, Castaldo et al. 2019) have been adopted to realize 100 samples for each NLN numerical model related to the RC columns of Table 1.

Instead, the *set II* reports the characterization of the resistance random variable $R_{NLA}{}^{GL}$ as a function of both aleatoric and epistemic (i.e., model) uncertainties with reference to Table 2, 3 and 4. The Eq.(8) is consistent with the multiplicative approach of JCSS (2001) for determination of model uncertainty. The outcomes from the probabilistic analysis are reported in Figure 2. In particular, the ratio δ between mean value μ of structural resistance derived from probabilistic investigation (i.e., μ_R for *set I* and $\mu_R{}^{GL}$ for *set II*) and the $R_{NLA,m}$ determined using experimental values (i.e., mean values) of geometric and material properties is illustrated in Figure 2(a). It can be observed that these ratios, in average, are below the unit. This result is a confirmation that the first order approximation, which consists in the equality between $R_{NLA,m}$ and the mean value derived from probabilistic analysis, is a safe approximation (Allaix et al. 2013, Castaldo et al. 2019). Figure 2(b) reports the summary of the coefficient of variation of structural resistance estimated by *set I* (V_R) and *set II* LHS samplings. In both cases, the coefficient of variation of structural resistance is strongly influenced by geometric uncertainty when the slenderness grows. The contribution of model uncertainty to overall variability is more significant for slenderness values lower that 150 with respect to slenderness higher than 150. This is due to the assumption related to the model uncertainty random variable (Table 4) and to the increasing influence of geometric uncertainty for high values of slenderness.

The statistical parameters related to mean values μ and coefficients of variation V reported in Figure 2 are the maximum likelihood estimators (MLE) (Faber 2012) assuming that the structural resistance pertains to lognormal probabilistic distribution.

Table 4. Probabilistic model for model uncertainty (epistemic) in line to Gino et al 2021.

Random variable	Distr.	Mean value	Standard deviation
Model uncertainty ϑ $\lambda\leq150$	LN	1.01	0.07
Model uncertainty ϑ $\lambda>150$	LN	0.91	0.03

In particular, two sets of probabilistic analyses have been carried out investigating the following resistance random variables:

- *Set I:* $$R_{NLA} = R_{NLA}\left(f_c, f_y, f_u, \mathrm{E}_s, \varepsilon_u, Y_C, Y_b, Y_h, e\right) \tag{7}$$

- *Set II:* $$R_{NLA}^{GL} = \vartheta \cdot R_{NLA}\left(f_c, f_y, f_u, \mathrm{E}_s, \varepsilon_u, Y_C, Y_b, Y_h, e\right) \tag{8}$$

The *set I* of proposes the evaluation of the resistance random variable R_{NLA} as a function of the main aleatoric random variables according to Table 2 and 3.

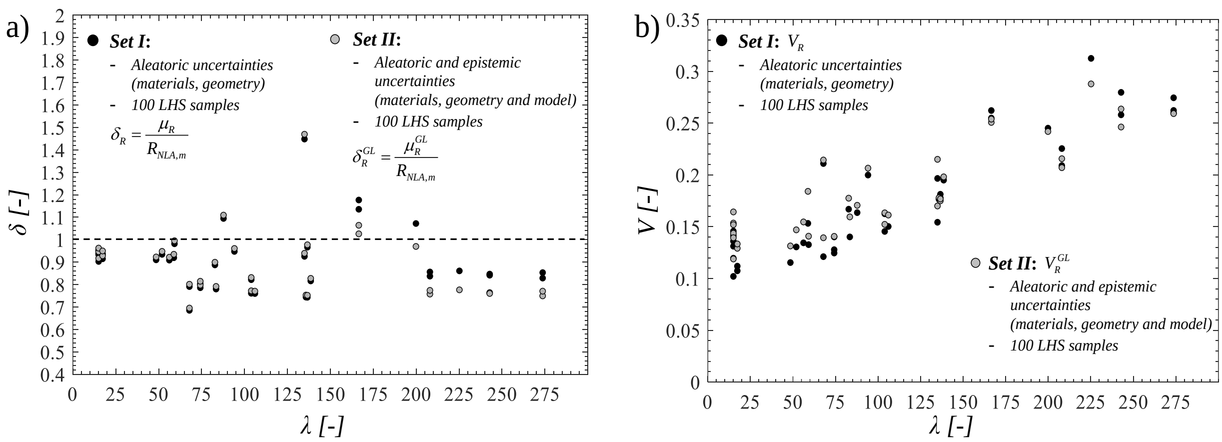

Figure 2. Results from probabilistic analysis of structural resistance of the 40 RC column: δ ratio a); coefficient of variation of structural resistance V b) for both *set I* and *set II*.

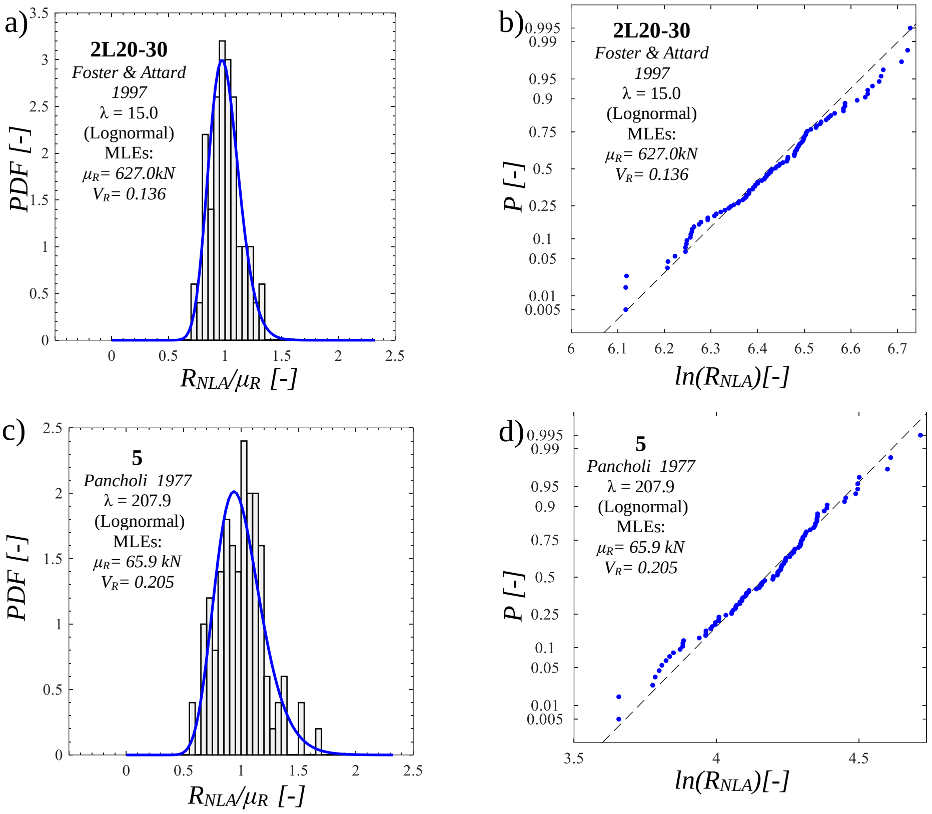

Figure 3. Frequency histograms, lognormal fitting distributions a),c) and probabilistic plot b),d) for 2 columns (Gino et al. 2021) considering the LHS – *set I* (only aleatoric uncertainty).

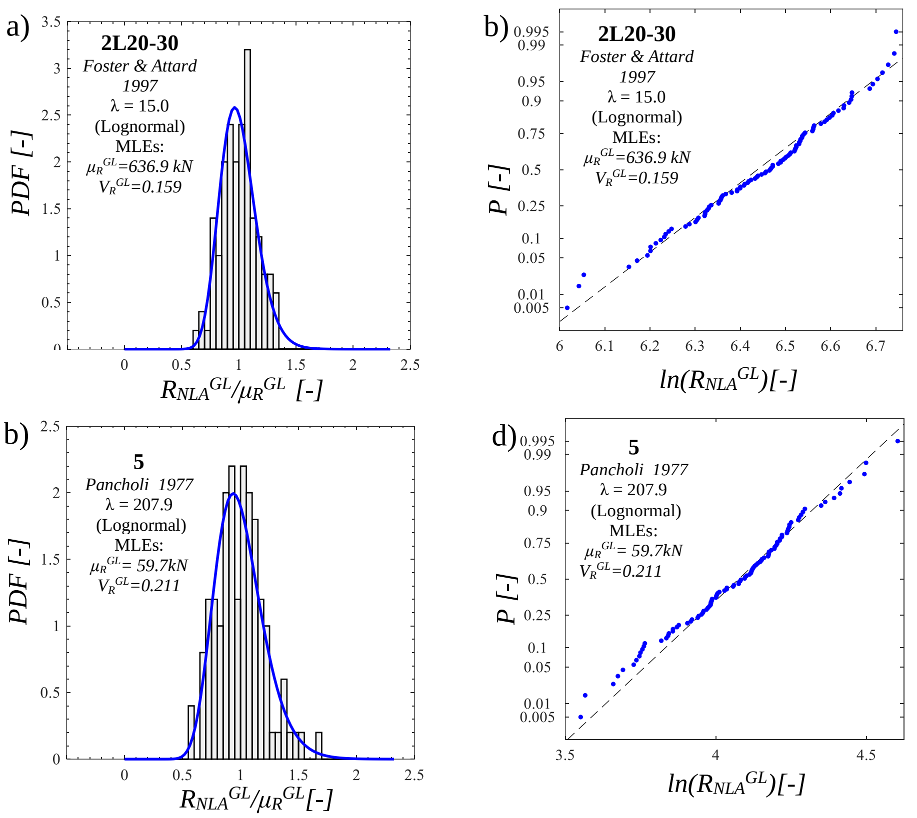

Figure 4. Frequency histograms, lognormal fitting distributions a),c) and probabilistic plot b),d) for 2 columns (Gino et al. 2021) considering the LHS – *set II* (aleatoric and epistemic uncertainty).

The assumption of lognormal probabilistic model for structural resistance have been verified according to Anderson-Darling statistical test with a level of significance 5% (Anderson & Darling 1953).

Figures 3 and 4 illustrate the frequency histograms, lognormal fits and probabilistic plots (lognormal) related to two columns of the 40 considered for probabilistic investigation.

The results of the probabilistic analysis can be used to estimate the global safety factor γ_R^{GL} according to description of Section 1.1.

4 COMPARISON BETWEEN GLOBAL SAFETY FACTORS

In the next, the comparison between the Approaches 1, 2 and 3 to derive the global safety factor γ_R^{GL} is proposed. The comparison is performed between global safety factors derived for the same target level of reliability.

In detail, concerning reinforced concrete structures of new realization and reference period set equal to 50 years (*fib* Model Code 2010, *fib* Bulletin 80 2016 and ISO 2394 2015), the target value of reliability index can be set equal to β_t=3.8. In case of existing structures, reference to *fib* Bulletin 80 can be made with aim to determine the most appropriate value of the target reliability index.

With reference to the *Approach 1*, the derivation of the global resistance factor γ_R according to Eq.(3) have been carried out using the results in terms of coefficient of variation V_R which derives from probabilistic results of LHS sampling *Set I*.

The model uncertainty safety factor γ_{Rd} have been determined through the Eq.(4) using the statistical properties reported din Table 4. In this way, the global safety factor γ_R^{GL} can be obtained as the product between γ_R and γ_{Rd} (Eq.(2)). Both Eq.(3) and Eq.(4) grounds to the assumption of lognormal probabilistic distribution. As introduced in Section 1.1, this approach separates the aleatoric uncertainties from the epistemic ones and adopts fixed values of FORM factors (Hasofer & Lind 1974) able to cover the major part of loading and structural configurations related to practical cases. Within the Approach 1, as commented in *fib* Model Code 2010, the aleatoric uncertainties are, in general, considered as dominant variables while the epistemic ones as non-dominant. This assumption is clearly a simplification and can turns out to be not proper in cases where the model uncertainty mostly contributes to probability of structural failure with respect to aleatoric ones.

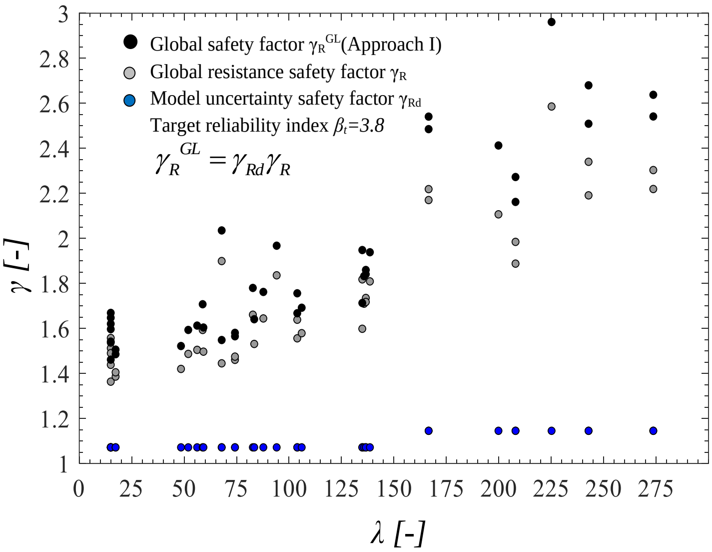

Figure 5. Results from the determination of global safety factor γ_R^{GL} according to the *Approach 1.*

This situation verifies in cases where, as an example, the coefficient of variation describing the probabilistic distribution of model uncertainty is higher than the ones associated to probabilistic distribution of materials and geometric random variables (*fib* Bulletin 80). This approach is the one that have been implemented within the *fib* Model Code 2010. Figure 5 illustrates the results of the application of *Approach 1* to the estimation of γ_R^{GL}. Due to the relevant influence of geometric uncertainties for high values of slenderness, the maximum value of γ_R^{GL} is close to 3.0 whereas the minimum one is close to 1.45 for slenderness values around to 15.

The *Approach 2* consist in the determination of γ_R^{GL} considering together sources of uncertainty of both aleatoric (i.e., material and geometric) and epistemic (i.e., model) nature. This approach leads to the evaluation of the γ_R^{GL} global safety factor according to Eq.(5) in line to assumption of lognormal probabilistic distribution. The Eq.(5) can be used adopting statistical parameters for model uncertainty according to Table 4 and the outcomes from probabilistic analysis related to *Set I* of LHS. In particular, the Eq.(6) allow to estimate the coefficient of variation V_R^{GL} inclusive of both aleatoric and epistemic uncertainty with a simplified approach depending from the coefficient of variation V_R and V_ϑ. Note that with the *Approaches 1* and *2*, as an alternative and in absence of probabilistic analysis of structural resistance, the term V_R can be estimated in simplified manner in line to estimation of coefficient of variation methods as described by Cervenka (2013) and Novak & Novak (2021).

In analogy with the *Approach 2*, the *Approach 3* is able to include directly in γ_R^{GL} the random variability of both aleatoric and epistemic uncertainties. The global safety factor γ_R^{GL} can be determined always using the Eq.(5) but, this time, the outcomes from the probabilistic analysis related to *Set II* LHS are used to determine the coefficient of variation V_R^{GL}. This is the most general approach but, in practice, it is not of simple application as it requires necessarily a probabilistic evaluation of structural resistance including both epistemic and aleatoric uncertainties according to Eq.(8). In the case of the present investigation, the *Approach 3* is considered as the reference one as descends from refined probabilistic analysis of structural resistance. The comparison between the *Approaches 1*, *2* and *3* is reported in Figure 6.

As first, it can be recognized that the *Approach 1* is the most conservative and leads to the determination of global safety factors γ_R^{GL} in average 5% higher with respect to the one of the *Approach 3* and *3%* with respect to the *Approach 2*.

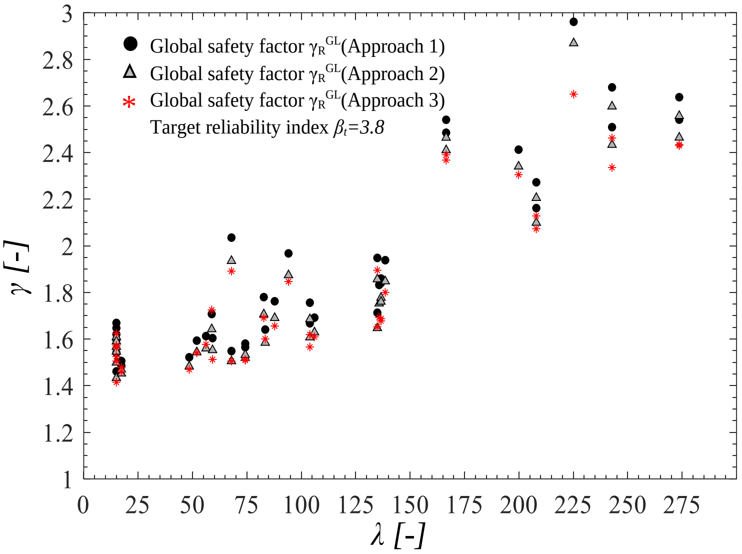

Figure 6. Comparison of the results in terms of γ_R^{GL} achieved using the *Approach 1*, *2* and *3*.

The adoption of fixed values for FORM factors α_R and α'_R relating to dominant and non-dominant variables (respectively 0.8 and 0.32 in line to Hasofer & Lind (1974) lead to achieve extra safety that justifies the choice performed by *fib* Model Code 2010. However, too much extra safety may become inconvenient in design processes where the economic aspect is of crucial relevance.

The *Approach 2*, in comparison to *Approach 1*, requires the same input data and allow to take into account each random variable affecting structural resistance with its actual variability without differentiating between dominant or non-dominant variables. The *Approach 2* leads to global safety factors γ_R^{GL} that are, in average, higher than 2% with respect to the ones obtained with the *Approach 3*. This highlight that the *Approach 2* keep a certain level of extra-safety with respect to pure probabilistic approach but that is less that the one associated to the *Approach 1*. The results of the investigation may suggest that the *Approach 2* can be considered as the most consistent to be adopted for implementation in next generation design codes, changing the road traced, for example, by *fib* Model Code 2010.

5 CONCLUSIONS

The present study relates to comparison between different approaches for definition of global safety factors for non-linear analysis of slender RC columns with reference to new or existing structures. A set of 40 benchmark test on RC columns has been considered in line to current design codes. The solution strategy for NLN modelling have been selected with the aim to minimize the epistemic uncertainty related to the definition of numerical model. The numerical models have been useful to perform two different sets of probabilistic analysis considering aleatoric (Set I) and both aleatoric and epistemic uncertainty (Set II) using the Latin Hypercube Sampling. The outcomes of the probabilistic investigation have been useful to compare three approaches (*Approach 1*, *2* and *3*) to determine the global safety factor γ_R^{GL} related to the 40 RC columns. The following conclusions can be drawn:

- the *Approach 1* is the safer one as it ground on the assumption of fixed values for FORM factors differentiating between dominant (aleatoric) and non-dominant (epistemic) variables;
- the *Approach 2* proposes intermediate results with respect to *Approaches 1* and *3* having the same input variable as required by *Approach 1*. The *Approach 2* seems to be the most convenient to be implemented in drafting of next generation of design codes.

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