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# Numerical models comparison for fluid-viscous dampers: performance investigations through Genetic Algorithm

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## Abstract

Fluid-viscous dampers played a crucial role in the protection of new or existing buildings against external actions as earthquakes and winds. In the last decade, several investigations have been conducted aiming to develop accurate numerical models. However, none has been focused on a comprehensive comparisons between the most used fluid-viscous damper models considering the variability of their parameters in a mass-production series.

In this paper, an identification procedure has been performed by comparing nine different existing literature models **with the objective of evaluating their ability to match experimental loops of mass-produced fluid-viscous devices**, both in terms of accuracy and robustness. **Indeed, the model that is most effective for reproducing the characteristic of a specific specimen may not be representative (i.e., showing larger parameters variability) of the mass production of the same device type.** For this purpose, dynamic tests have been developed in the laboratory and the experimental outputs have been adopted as the target function of the procedure. The identification scheme has been designed by implementing an optimization procedure via Genetic Algorithm. **Results demonstrate how differential laws better fit the experimental cycles with respect to algebraic ones, and also show how few models in the series can offer a high level of both accuracy and robustness.**

*Keywords:* Viscous Damper, Numerical model, Optimization, Genetic Algorithm, Statistical Analysis, Mass-production

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## 1. Introduction

**The significance of civil engineering structures in societal development cannot be overstated, as they form the backbone of transportation systems, utilities, and habitation. How-**

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26 ever, these structures encounter formidable challenges arising from external actions, notably  
27 seismic events, which pose substantial risks of damage and functional impairment [1, 2, 3, 4].  
28 Consequently, the adoption of resilient design principles, specifically the integration of damp-  
29 ing devices [5, 6, 7, 8, 9], assumes paramount importance in safeguarding these structures,  
30 enabling their continued operation even amidst severe seismic events of greater intensity  
31 [10, 11, 12].

32 Fluid-dynamic damper technology is a mature technology that has been extensively stud-  
33 ied and incorporated into anti-seismic design standards and guidelines in many countries  
34 worldwide [13, 14, 15, 16]. For example, the Japan Society of Civil Engineers recommends  
35 the use of fluid-dynamic dampers in seismic-resistant design, and many buildings in Japan  
36 are equipped with these devices [17]. Similarly, the Council of Tall Buildings and Urban  
37 Habitat recommends the use of fluid-dynamic dampers in high-rise buildings located in  
38 high-seismicity areas [18].

39 The ability of fluid viscous dampers (FVDs) to dissipate a large amount of energy, the  
40 creation of forces that are out of phase with displacements, and the potential to improve  
41 damping ratios without significantly changing stiffness make them appealing. This is not  
42 the case with other devices as hysteretic ones. The military and aerospace industries were  
43 the first to demonstrate the efficacy and reliability of FVDs. This paved the path to the  
44 application of FVDs in civil engineering for both new and existing structures [19, 20].

45 Both experimental testing and numerical modelling are frequently used to characterize  
46 the mechanical properties of novel systems or devices. Although the former approach is  
47 more expensive than the latter, it directly leads to a physical understanding of the system  
48 and can therefore be used for final verification. Numerical simulation, in contrast, employs  
49 numerical techniques to quantitatively describe the development of a physical system and  
50 enables more economically exploring a wide range of potential design options. A satisfactory  
51 representation of the actual mechanical behavior of the understudied device can be obtained  
52 from such simulations by employing realistic models [21, 22]. By simulating the behavior  
53 of control devices, engineers can evaluate the qualities of different design solutions before  
54 implementing them in the physical structure. This can save time and money by avoiding  
55 costly trial-and-error processes and ensuring that the protection strategy is reliable and

56 efficient [23, 24].

57 The effectiveness of FVDs depends on various factors, such as the type and properties of  
58 the materials used, the geometry and structural configuration, and the external forces acting  
59 on the structure. **Therefore, numerical simulations can be used to evaluate their efficiency  
60 and improve their performance [25, 26, 27, 28]**, also by reproducing their characteristic  
61 behavior in complex structural systems [29, 30].

62 Furthermore, numerical simulations can provide valuable insights into the behavior of  
63 control devices under extreme conditions that are difficult or impossible to replicate in phys-  
64 ical experiments. For example, numerical simulations can be used to evaluate the behavior  
65 of control devices under earthquakes of different magnitudes and frequencies, or under wind  
66 speeds that are beyond the capabilities of physical experiments [31, 32].

67 Choosing the model to reproduce in operation the fluid-viscous device is a critical step  
68 for reasons mainly related to mass production of the devices. Indeed, the model identified  
69 on a single device could not be robust enough to represent the variability of the devices'  
70 characteristics that belong to the same mass production. On the other hand, a less accurate  
71 model identified on a tested device would more likely be reflective of the kind as a whole[33,  
72 34].

73 Several papers can be found in the literature in which suitable numerical models for  
74 reproducing the cyclic behavior of FVD devices are developed. A first class is that of  
75 algebraic models that assemble spring-type stiffness elements and dashpot-type damping  
76 elements, linearly or nonlinearly proportional to displacement and velocity. The simplicity  
77 of these models favoured their widespread use [35, 36, 37], including within international  
78 standards (e.g. [38]) and among FVD device manufacturers. However, these models although  
79 having the advantage of being simple and requiring low computational efforts are not always  
80 accurate in reproducing the hysteretic behavior of control devices as FVD ones. On the  
81 contrary, a different class of models overcome the limitations of algebraic ones: they are the  
82 differential models of hysteresis. The Bouc-Wen model [39] has been widely used to simulate  
83 different hysteretic devices being able to display a broad range of cyclic behaviors (e.g., [40]).  
84 Similarly, the LuGre model [41] although originally designed for friction hysteretic devices  
85 has been developed to reproduce different dampers (e.g., [42]). Lu et al. [43] introduced

86 an unified differential model containing a number of variants to simulate different types of  
87 devices with some applications in the literature (e.g., [44]).

88 Although fluid-viscous dampers have been extensively studied in the literature with var-  
89 ious practical applications in the real world, the aspect described regarding mass production  
90 and the identification of representative models has not yet been thoroughly investigated. The  
91 objective of this research is to close this gap by contributing to the evaluation of a variety  
92 of numerical models that have been used to replicate the characteristics of mass-produced  
93 fluid-viscous devices. This comparison is based on some dynamic laboratory tests performed  
94 on five samples of the same type of fluid-dynamic device. Parameter identification will be  
95 carried out through an optimisation procedure based on the use of Genetic Algorithms (GA).  
96 The result of the parameters identification for the different models, for the different tested  
97 devices, will be then studied through a statistical investigation aimed at determining the  
98 most robust and the most accurate numerical model.

99 The paper has the following organization. Section 2 collects the main numerical models  
100 involved in the optimization problem. A brief description of each model has been introduced  
101 and the main parameters discussed. In Section 3, the samples description and the laboratory  
102 equipment used for the dynamic tests has been described. Section 4 is devoted to the  
103 mathematical formulation of the problem and the description of the adopted optimization  
104 algorithm with regard to the analysis settings and stopping criteria. Section 5 collects  
105 the results of the optimization process and shows the optimal hysteresis cycles for each  
106 investigated model. In Section 6, the authors provide a statistical analysis in order to identify  
107 underlying trends and patterns. All the results derived from the analysis have been organized  
108 in Section 7, in which suitable indices concerning with the accuracy and the robustness of  
109 each model are adopted. The final discussion including future developments is provided in  
110 Section 8. Finally, Appendix A and B report the outputs of the optimization process.

## 111 **2. Numerical models for fluid viscous dampers**

112 In the literature different numerical models to simulate FVD characteristics can be found,  
113 with different formulations (e.g. differential or algebraic equations). Their parameters can  
114 be tuned through the definition of a goal (or cost) function to be minimized, within an

115 identification process, until a suitable level of accuracy is achieved.

116 The present section addresses the selection of existing numerical models for FVD starting  
117 with rheological models [33], moving up in complexity from the linear viscous damper to  
118 generalized models. Next, differential models are presented for simulating the experimental  
119 loops of FVDs in terms of force-displacement and force-velocity characteristics.

### 120 *2.1. Linear viscous model - Law #1*

121 The classic linear viscous model is the simplest way to represent a velocity-dependent  
122 mechanical law. When the FVD is modeled in this fashion and given a cyclic input, the force  
123  $F$  produced by the relative velocity  $v$  with respect to the linear viscous damping parameter  
124  $C$  is:

$$F(t) = Cv(t) \quad (1)$$

125 The key benefit of this fundamental model is that it is very straightforward, although occa-  
126 sionally it falls short of being an accurate description of actual mechanical behavior. The  
127 generalized (nonlinear) viscous model, which relies on a fractional exponent of the velocity  
128 rather than a straightforward linear relationship, has updated it as a result. The following  
129 section describes the generalized nonlinear viscous model.

### 130 *2.2. Generalized viscous model - Law #2*

131 The fundamental distinction between the classical and generalized models is that the  
132 latter incorporates nonlinearity in the spring and viscous components. Additionally, the  
133 coefficients of the generalized models' resistive forces are fractional exponential. The gen-  
134 eralized nonlinear viscous model consists of two parameters: the linear viscous damping  
135 parameter  $C$  and the damping term exponent  $\alpha$ , which is usually selected in the range  $[0, 1]$ .

$$F(t) = C \text{sign}(v(t))|v(t)|^\alpha \quad (2)$$

136 Distinct values of  $\alpha$  are related to different mechanical behaviour. For example, the linear  
137 viscous damping law corresponds to  $\alpha = 1$ ; if  $\alpha$  assume values close to 0, the friction behavior  
138 is seen (consequently, the force increases quickly for small velocity values and becomes almost  
139 constant for large velocity values). Due to its effectiveness in predicting structural behavior,

140 this damping law has been widely adopted by different authors. For instance, Banazadeh et  
 141 al. [45] research on a comparison of the seismic collapse performances of steel frames equipped  
 142 with linear and nonlinear viscous dampers. Additionally, many structural computer routines  
 143 and standard guidelines for reproducing FVDs reference to this law.

144 Experimental research, however, showed that the viscous dampers force varies on both  
 145 damper relative displacement  $d$  and velocity  $v$ . Therefore, simulating such mechanical feature  
 146 involves joining a spring element and a viscous element, respectively. The family of Kelvin-  
 147 Voigt models is created if these two components are connected in parallel. The fundamental  
 148 Kelvin-Voigt model, for instance, can be derived with a linear spring and a straightforward  
 149 linear dashpot in parallel. Other behaviors are produced when nonlinear springs are coupled  
 150 with generalized nonlinear viscous models.

151 Both linear and parabolic models for the elastic force  $F_e$  can be expressed as:

$$F_e(t) = K_1 d(t) \quad (3)$$

152

$$F_e(t) = K_2 d(t)^2 + K_1 d(t) + k_0 \quad (4)$$

153 where  $K_1$  is the elastic stiffness,  $K_2$  and  $K_0$  are two constants. The parabolic function  
 154 reproduces the test cycle shape better, but the linear function may be preferable because it  
 155 is less complicated and generates a similar energy balance [46, 47].

### 156 2.3. Generalized viscous and linear elastic model - Law #3

157 The equation of motion for a generalized Kelvin-Voigt model subject to a time-varying  
 158 force  $p$  is obtained by combining Equations 2 and 3:

$$F(t) = K_1 d(t) + C \text{sign}(v(t)) |v(t)|^\alpha \quad (5)$$

### 159 2.4. Generalized viscous and quadratic elastic model - Law #4

160 The combination of the generalized viscous model in Equation 2 with the parabolic  
 161 model for the elastic force in Equation 4 (without considering the contribution of  $K_0$ ) allows

162 to define the generalized viscous and quadratic elastic force:

$$F(t) = K_1 d(t) + K_2 d(t)^2 + C \text{sign}(v(t)) |v(t)|^\alpha \quad (6)$$

163 *2.5. Generalized Kelvin-Voigt model - Law #5*

164 The models previously described as function of displacement and relative velocity com-  
 165 puted with respect to the connection points of the FVD device can be further generalized  
 166 by defining the so-called Kelvin-Voigt model. Following this approach, the spring and the  
 167 viscous components both involve nonlinearities. Consequently, in this generalized model, the  
 168 device force is represented by two exponential components.

$$F(t) = K \text{sign}(d(t)) |d(t)|^\beta + C \text{sign}(v(t)) |v(t)|^\alpha \quad (7)$$

169 *2.6. Bouc-Wen model of hysteresis - Law #6*

170 The Bouc-Wen endochronic model has been chosen to simulate the hysteretic component  
 171 of a broad class of control devices since it is perhaps the most eclectic phenomenological  
 172 model of hysteretic mechanical systems given by the current literature. This decision is  
 173 backed by the model's consistency in both mathematics and physics as well as by the out-  
 174 standing agreement between experimental and numerical data [48, 28].

175 The control force generated by the Bouc-Wen model of hysteresis can be defined as

$$F(t) = \alpha K d(t) + (1 - \alpha) K z(t) + C v(t) \quad (8)$$

176 Where  $z(t)$  is the variable which allows to define the hysteretic behavior,  $K$  is the pre-  
 177 yielding stiffness of the model,  $\alpha$  is the pre-post yielding stiffness ratio ( $\alpha = 1$  is associated  
 178 to the elastic response, while  $\alpha = 0$  to the perfectly plastic response. To introduce a linear  
 179 viscous contribution, it is possible to define a component proportional to velocity according  
 180 to the constant  $C$ . The hysteretic variable  $z(t)$  is introduced by defining the differential  
 181 component of the model as follows.

$$\frac{dz(t)}{dt} = A v(t) + \beta v(t) |z(t)|^n - \gamma |v(t)| |z(t)|^{n-1} \quad (9)$$

182 Where  $A$ ,  $\beta$ ,  $\gamma$ ,  $n$  are constant with time parameters which regulate the the hysteresys cy-  
 183 cle shape and amplitude (dissipation level), the unloading branch linearity and the transition  
 184 smoothness between the pre-yielding and the post-yielding phase. In detail parameter  $A$  is  
 185 associated to the initial stiffness,  $\beta$  drives the hysteresis amplitude (the dissipation intensity),  
 186  $\gamma$  regulates the unloading path in the cyclic behavior, while  $n$  the transition smoothness. The  
 187 efficiency of the model (e.g., stability, consistency, passivity, admissibility) can be verified  
 188 by assuming suitable values for the parameters, while the identification procedure for the  
 189 characteristic parameters can be simplified assuming  $\beta = \gamma$ ,  $A = n = 1$  [49, 28].

### 190 2.7. Unified model for visco-elastic type devices - Law #7

191 For simulating the hysteretic behavior of various types of dissipative devices, Lu et al.  
 192 [43] proposed a unified analysis model with the internal variable  $z(t)$ . The model may imitate  
 193 many device kinds by giving the internal variable various physical meanings. The summary  
 194 of the viscoelastic type is presented in this section, while friction dampers are the subject of  
 195 the following one. The visco-elastic control force associated to the damper is computed as  
 196 follows.

$$197 \quad F(t) = K \operatorname{sign}(z(t)) |z(t)|^\beta \quad (10)$$

$$\frac{dz(t)}{dt} = -\operatorname{sign}(z(t)) \left( \frac{K |z(t)|^\beta}{C} \right)^{\frac{1}{\alpha}} + v(t); \quad (11)$$

198 Where  $K$  is the stiffness of the elastic component,  $C$  the damping coefficient of the viscous  
 199 element,  $\alpha$  and  $\beta$  exponential coefficients.

### 200 2.8. Unified model for friction type devices - Law #8

201 The force-displacement characteristic of the device interested in reproducing in the present  
 202 study, with a rapid transition between elastic and plastic phases, coupled with a rectangular-  
 203 trapezoidal shape of hysteretic cycles, suggests that dissipative models normally used for  
 204 friction control devices should also be considered in the identification process. So this sec-  
 205 tion and the next are devoted to two types of models for friction type devices found in the  
 206 literature. For simulating the friction dissipative control force, Lu et al. [43] reported the  
 207 following unified analysis model.

$$F(t) = z(t) \mu N \quad (12)$$

$$\frac{dz(t)}{dt} = \left( \frac{K}{\mu N} \right) \text{sign}(1 - \text{INT}(|z(t)|) \text{sign}(z(t)v(t)))v(t) \quad (13)$$

Where  $\mu$  is the friction coefficient,  $N$  the normal contact force,  $K$  the device axial stiffness,  $\text{INT}(x)$  is the integer part function of variable  $x$ .

### 2.9. LuGre model of hysteresis - Law #9

The so-called LuGre model was proposed by Canudas et al. [50] to explain friction dynamics. This model has been then developed applied in a variety of contexts, also different from the type of friction-related devices [41, 51]. The LuGre model is an effective solution to model and control design problems due to its mathematical simplicity and accuracy. The control force exerted by the damper can be computed as follows.

$$F(t) = \beta z(t) + CV(t) + Kd(t) + \epsilon \frac{dz(t)}{dt} + F_0 \quad (14)$$

$$\frac{dz(t)}{dt} = v(t) - \alpha |v(t)|z(t) \quad (15)$$

Where  $K$  is the stiffness parameter,  $C$  the damping coefficient, and  $\alpha$ ,  $\beta$ ,  $\epsilon$  allow to define the cycles characteristics (i.e., amplitude and shape). For the aim of this identification problem  $\epsilon$ ,  $K$  and  $F_0$  can be assumed with zero values [51, 41].

## 3. Description of the laboratory tests

In the SISMALB srl laboratory in Taranto, Italy, five samples of the same 300kN FVD series (Figure 1) was tested. Table 1 reports the FVDs design conditions.

Table 1: Fluid viscous damper design conditions

| F[kN]    | Stroke [mm] | d <sub>design</sub> [mm] | V <sub>max</sub> [mm/s] | $\alpha$ |
|----------|-------------|--------------------------|-------------------------|----------|
| 292 ± 55 | ± 55        | ± 45                     | 500                     | 0.3      |

The SISMALAB test equipment consisted of a high-resistance steel frame that can support loads of 2200 kN in both tension and compression. The FVD was connected by pins to the frame structure and to the actuator, which can be both force- or displacement-controlled (Figure 2).

228 A 2500kN load cell was placed between the actuator and the FVD specimen to measure  
 229 the cyclic forces throughout the experimental tests, while a transducer was used to measure  
 230 the device movements. In a displacement-imposed test, the device movements are controlled  
 231 by a transducer mounted on the device. The control and data acquisition system is able  
 232 to generate real-time analysis of device displacements by instantaneous variation of applied  
 233 forces by the servant cylinder, by means of a computer automatic control hydraulic pressure  
 234 system. The displacement–time history should be imposed with different laws, from sinu-  
 235 soidal to triangular as well as ramped or through a generator step of generic ones. Acquiring  
 236 systems have 30 channels and can command two actuators at the same time.

237 The FVD specimens were tested using the transducer output as feedback for the closed-  
 238 loop displacement control. More in detail, the database is composed of 24 tested devices  
 239 with the same properties and set-up adopted during the test. The laboratory provided the  
 240 results of the pressure test related to 5 devices that were identified with a specific ID serial  
 241 number. For each device, the test has been performed by applying an internal pressure over  
 242 275 bar and has been maintained for at least 120s. Moreover, all tests considered in the  
 243 experimental campaign have been carried out by applying six sinusoidal cycles characterized  
 244 by a specific ratio of the maximum velocity corresponding to low velocity (lower than 1%),  
 245 1%, 25%, 50%, 75% and 100% of the maximum design value. Finally, for each device,  
 246 the damping efficiency at 100% of the maximum velocity has been tested by applying 10  
 247 sinusoidal cycles. Table 2 reports the test conditions for the five selected FVD specimens with  
 248 specific regard to the last configuration in order to assess the damping capacity. Maximum,  
 249  $Load_{max}$ , and minimum load,  $Load_{min}$ , experienced during the test as well as maximum,  
 250  $d_{max}$ , and minimum displacement,  $d_{min}$ , have been reported. Consistent with what was said  
 251 before, tests selected by the database consisted of 10 sinusoidal cycles with 40mm amplitude  
 252 at 0.5m/s.

Table 2: Fluid viscous damper test conditions

| No.      | $Load_{max}/Load_{min}$ [kN] | $d_{max}/d_{min}$ [mm] | Cycles |
|----------|------------------------------|------------------------|--------|
| $Test_1$ | 327/-317                     | 44/-43                 | 10     |
| $Test_2$ | 318/-308                     | 45/-43                 | 10     |
| $Test_3$ | 267/-259                     | 44/-43                 | 10     |
| $Test_4$ | 289/-282                     | 43/-42                 | 10     |
| $Test_5$ | 283/-284                     | 44/-42                 | 10     |

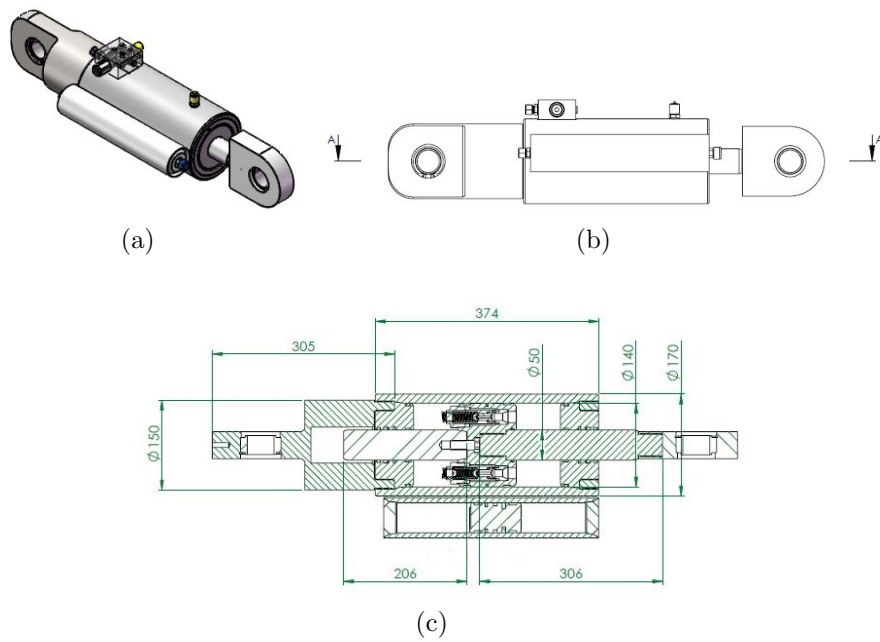


Figure 1: 3D model of FVD test device (a); Longitudinal view of the FVD test device (b); Trasversal section of FVD test device (c)



Figure 2: FVD specimen on the test facility at SISMALAB

#### 253 4. Identification scheme and analysis setting

254 In this section, the mathematical formulation of the optimization problem is introduced.  
 255 Specifically, the Objective Function (OF) and the design variable are defined. Therefore,  
 256 both the setting criteria and the Genetic Algorithm (GA) for the optimization process are  
 257 illustrated.

259 The parameters identification requires to solve an unconstrained optimization problem  
260 and a suitable objective function (OF) has to be minimized.

261 All the selected constitutive models depend on different parameters, which become the com-  
262 ponents of the design variable vector  $x$ . The model parameters are identified by solving the  
263 following single-objective optimization problem:

$$\text{Minimize } W = f(x_i), \quad i = 1, \dots, n \quad (16)$$

$$\text{Subject to } x_i^l \leq x_i \leq x_i^u \quad (17)$$

265 where  $x = \{x_1, \dots, x_j, \dots, x_n\}$  represents a set of real parameters which depends on the adopted  
266 numerical model. The size of this vector, hence, is related to the number of variables which  
267 govern the constitutive law. The  $x^l = \{x_1^l, \dots, x_j^l, \dots, x_n^l\}$  and  $x^u = \{x_1^u, \dots, x_j^u, \dots, x_n^u\}$  vectors  
268 represent the lower and upper bounds of  $x$ , respectively. In this way, the domain of feasible  
269 solutions is restricted to a sub-domain in which  $x$  can assume the values between  $x^l$  and  $x^u$ .  
270 For the aim of this research, a suitable OF has been defined in the integral form as follows:

$$f(x) = \frac{1}{\sigma_{p_m}(t_{end} - t_{start})} \int_{t_{start}}^{t_{end}} abs(p_m - p_e(x)) dt \quad (18)$$

271 where  $t_{start}$  and  $t_{end}$  are the start and end time records, respectively, and  $p_m(t)$  is the exper-  
272 imentally measured force, while  $p_e(t)$  is the estimated force.

273 The Matlab ODE45 function has been used for solving the (nonstiff) ordinary differential  
274 problem. The procedure implemented by the solver is the Runge-Kutta method. The fixed  
275 time step for the solution procedure is the sampling time step adopted during the laboratory  
276 test. This procedure was adopted for only the differential laws (from law#6 up to law#9)

277 In order to reduce the computational effort during the optimization process or avoid  
278 critical values generating numerical noises, lower,  $x^l$ , and upper,  $x^u$ , bounds for each de-  
279 sign variable are carefully fixed, detecting more suitable ranges of variability. In this way,  
280 no interest solutions are discarded, and the efficient exploration phase of the algorithm is  
281 guaranteed.

283 In order to solve the problem stated in the previous section, a real-coded GA is adopted.  
284 This is a population-based stochastic optimization technique appropriate for global optimiza-  
285 tion, which does not require direct evaluation of gradients. Introduced by John Holland [52],  
286 it is inspired by Charles Darwin's theory of natural evolution. This algorithm reflects the  
287 process of natural selection where the fittest individuals, also called parents, are selected for  
288 reproduction in order to produce offspring of the next generation. At the end of the process,  
289 the best survival among all the fittest candidates found at each generation is selected as the  
290 best globally optimized solution. Although the native GA worked with binary values rep-  
291 resenting genes, encoded in string structures called chromosomes, in this paper the authors  
292 overcome the limits related to the decoding process by using a real-code GA in which genes  
293 and chromosomes represent directly the design variables and the solutions of the problem.

294 At each iteration, traditional GA phases were adopted during the optimization as follow-  
295 ing:

- 296 • **Initial population:** in this phase, individuals with a set of random genes ( $x_i$ ) com-  
297 posing chromosomes ( $x$ ) are created by observing lower and upper bounds reported in  
298 table 4. Gene represents, at each generation, the candidate value of a specific design  
299 variable involved in the identification procedure. A set of genes (vector form) represent  
300 a solution of the problem for the current generation. In this way, the best solution is  
301 selected and the optimal set of parameters which govern that specific law is detected.
- 302 • **Fitness function:** in this phase, the fitness of the candidate solutions is evaluated by  
303 calculating the OF introduced in the previous section 4.1.
- 304 • **Selection:** During this phase, a Roulette Wheel Selection was implemented in order  
305 to guarantee that the two fittest parents are selected for the next steps. Adopting this  
306 technique, a probability to each parent is assigned and the parents with higher fitness  
307 are more likely to be chosen for the next steps.
- 308 • **Crossover:** in this phase, a uniform random crossover was performed in which re-  
309 combination of gene pool between parents is performed in a random way. Lower and  
310 upper bounds are imposed at this stage such that if only a gene of the new offspring

311 is not ranged within the imposed interval (higher than the maximum value or lower  
 312 than the minimum value of that specific parameter), it is forced to assume maximum  
 313 or minimum value, respectively.

314 • **Mutation:** aiming to improve the exploration and exploitation ability of the algo-  
 315 rithm, a mutation rate of 1% is assigned. In this way, new genes are introduced into  
 316 the population by modifying the gene pools of parents in a random way.

317 At the end of these stages, a sorted function was implemented aiming to store survivors  
 318 with the best fitness among the generated offspring, at each generation. For this purpose,  
 319 an elitism algorithm was adopted in order to copy the best chromosome of the previous  
 320 population to the new one. The identification procedure can be considered ended when  
 321 the stopping criteria of the algorithm are satisfied and the optimal set of parameters for  
 322 each law is found. The entire procedure was run  $n$  times in order to check the reliability  
 323 and robustness of the algorithm. Specifically, the authors observed that stagnation usually  
 324 occurred for the algebraic models while no differential ones needed to use all the available  
 325 200 generations.

326 For clarity and repetitive purposes, the settings of the adopted algorithm and the stopping  
 criteria are listed in table 3.

Table 3: Setting of the GA for each law

| Pop. size | N.Generation | Evaluation | N. Run | Stagnation |
|-----------|--------------|------------|--------|------------|
| 200       | 200          | 40.000     | 20     | 20         |

327  
 328 Though a maximum of generation equal to 200 is set, stopping criteria occurs if OF value  
 329 remains the same for 20 subsequent generations. In this way, the computational effort was  
 330 reduced if stagnation is recognized.

331 In Table 4, the design variables of the optimization problem with their corresponding  
 332 lower and upper bounds are reported.

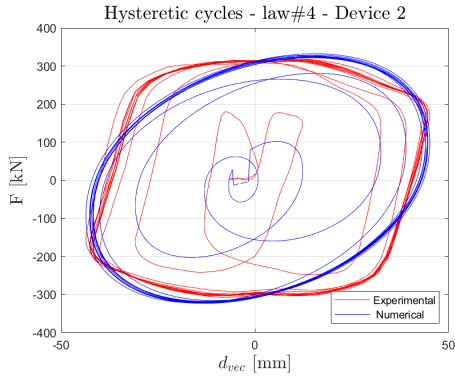
### 333 5. Parameter identification results and cyclic behavior reproduction

334 In this section, the performance of the selected numerical models in reproducing the  
 335 cyclic behavior of the tested FVDs has been analysed. Appendix A reports the mean and  
 336 the standard deviation values of the design variables and the OFs for each investigated law.

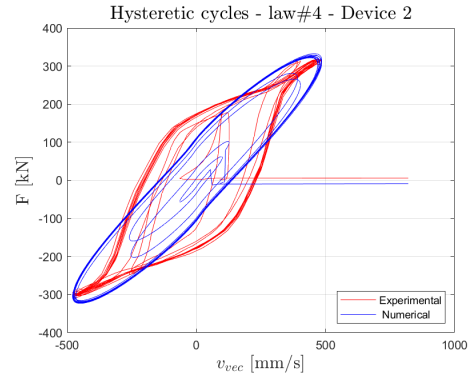
Table 4: Lower and upper bounds of models' parameters

|               |                               | $x$   |        |
|---------------|-------------------------------|-------|--------|
|               |                               | $x^l$ | $x^u$  |
| <b>Law #1</b> | $C$ [kN/(mm/s)]               | 0.0   | 2.0    |
| <b>Law #2</b> | $C$ [kN/(mm/s)]               | 0.0   | 2.0    |
|               | $\alpha$ [-]                  | 0.0   | 1.0    |
| <b>Law #3</b> | $C$ [kN/(mm/s)]               | 0.0   | 2.0    |
|               | $\alpha$ [-]                  | 0.0   | 1.0    |
|               | $K_1$ [kN/mm]                 | 0.0   | 200.0  |
| <b>Law #4</b> | $C$ [kN/(mm/s)]               | 0.0   | 2.0    |
|               | $\alpha$ [-]                  | 0.0   | 1.0    |
|               | $K_1$ [kN/mm]                 | 0.0   | 300.0  |
|               | $K_2$ [(kN/mm) <sup>2</sup> ] | 0.0   | 300.0  |
| <b>Law #5</b> | $C$ [kN/(mm/s)]               | 0.0   | 2.0    |
|               | $\alpha$ [-]                  | 0.0   | 1.0    |
|               | $K$ [kN/mm]                   | 0.0   | 1000.0 |
|               | $\beta$ [-]                   | 0.0   | 5.0    |
| <b>Law #6</b> | $C$ [kN/(mm/s)]               | 0.0   | 0.3    |
|               | $K$ [kN/mm]                   | 30.0  | 100.0  |
|               | $\alpha$ [-]                  | 0.0   | 1.0    |
|               | $\beta$ [1/mm]                | 0.0   | 5.0    |
| <b>Law #7</b> | $C$ [kN/(mm/s)]               | 3.0   | 10.0   |
|               | $\alpha$ [-]                  | 0.05  | 0.3    |
|               | $K$ [kN/mm]                   | 50.0  | 200.0  |
|               | $\beta$ [-]                   | 0.2   | 0.4    |
| <b>Law #8</b> | $C$ [kN/(mm/s)]               | 0.1   | 0.3    |
|               | $K$ [kN/mm]                   | 10    | 50     |
|               | $\mu$ [-]                     | 0.1   | 1      |
|               | $N$ [kN]                      | 250   | 450    |
| <b>Law #9</b> | $C$ [kN/(mm/s)]               | 0.0   | 0.3    |
|               | $\alpha$ [1/mm]               | 0.0   | 2.0    |
|               | $\beta$ [kN/mm]               | 20    | 100    |

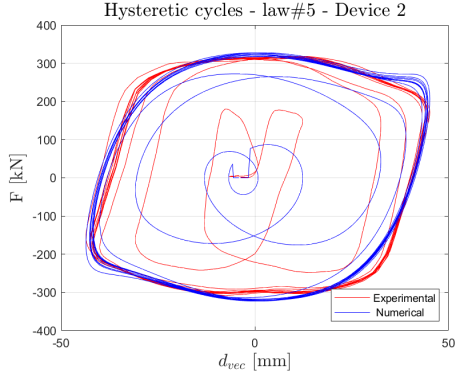




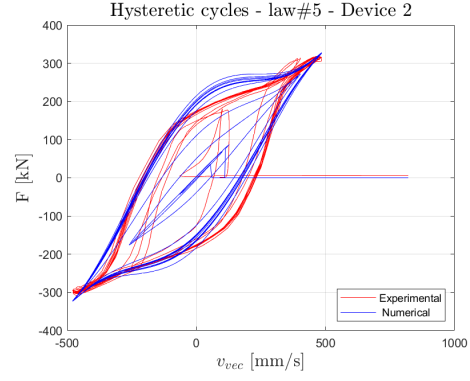
(g)



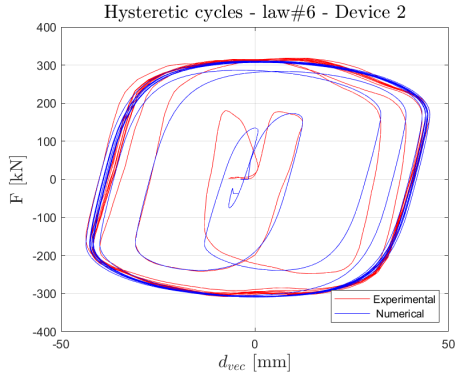
(h)



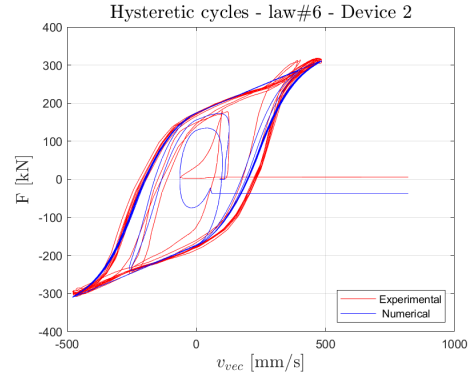
(i)



(j)



(k)



(l)

345 phenomenon is highlighted.

346 The first five algebraic models composed by different elementary features (elastic and  
 347 damping elements with different characteristics) are the simplest and the most computation-  
 348 ally inexpensive. The first two models (Fig.s 3 a-d) are evidently the least efficient in repro-  
 349 ducing the cyclic response of the device: the exclusive presence of the damping component in  
 350 the laws does not allow to reproduce the asymmetric shape of the force-displacement char-  
 351 acteristics. Focusing on the force-velocity cycles, again the exclusive velocity proportional  
 352 component of the force, with linear or nonlinear expression, still fails greatly in reproducing

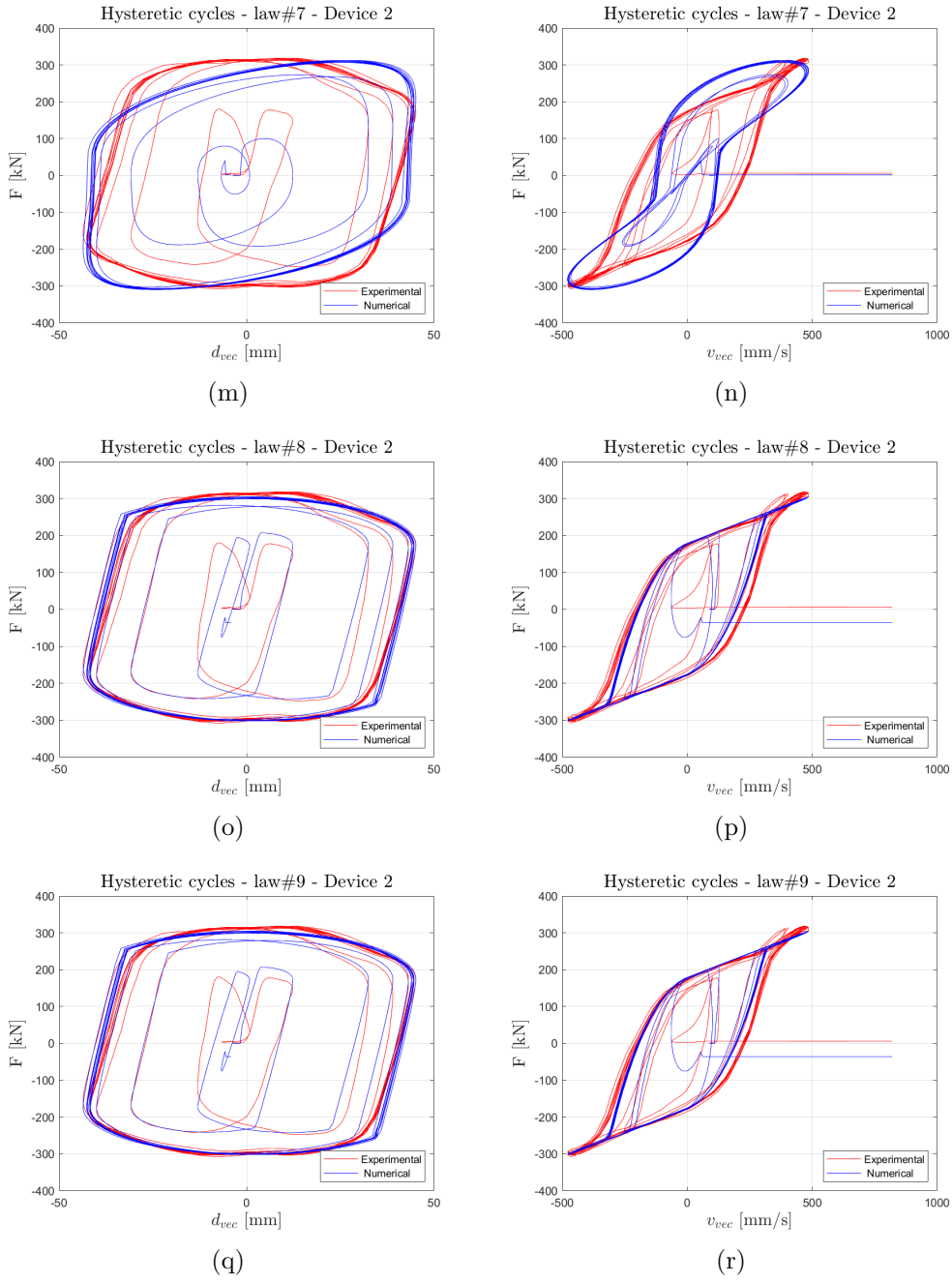


Figure 3: Comparison between theoretical and experimental force-displacement relationships (referred to Fig.s (a)-(c)-(e)-(g)-(i)-(k)-(m)-(o)-(q)) and force-velocity relationships (referred to Fig.s (b)-(d)-(f)-(h)-(i)-(l)-(n)-(p)-(r)) for law 1 up to law 9 with specific regard to Test 2

353 the response.

354 From the third model to the fifth (Fig.s 3 e-j), due to the introduction of the elastic  
355 elements, an improvement in reproducing the cycles is observed. However, only model num-  
356 ber five (Fig.s 3 i-j) succeeds for the first time in qualitatively following the shape of both  
357 force-displacement and force-velocity cycles.

358 Focusing on the response of the last four models (Fig.s 3 k-r), an improved efficiency  
359 in simulating the behaviour of FVD devices is highlighted. However, this positive outcome  
360 is achieved at the price of a higher computational burden. Indeed, differential models are  
361 substantially more complex to process in terms of computational time than algebraic models.

362 However, the model related to Law #7 (Fig.s 3 m-n) does not appear satisfactory in its  
363 ability to reproduce the characteristic of FVDs, even from a qualitative observation, either as  
364 force-displacement or velocity-displacement. The optimization process has never been able  
365 to improve this result, despite different attempts.

366 The friction model (Fig.s 3 o-r) is qualitatively the most accurate in simulating the  
367 cyclic response of the FVDs. The Bouc-Wen model, probably the most versatile among the  
368 literature options (Figures 3 k-l), also produces a satisfactory qualitative result, equivalent  
369 to the ones from the friction models.

370 Generally, it is evident that none of the algebraic models is capable of reproducing the  
371 transient branch of hysteresis cycles at the beginning and final phases. On the contrary,  
372 among differential models, law#6, law#8 and law#9 exhibit a good grade of match not only  
373 at the full-capacity phase but also during the transient one. This result is demonstrated by  
374 the comparisons between experimental and numerical  $F$  vs  $t_{vec}$  curves reported in Appendix  
375 B.

## 376 6. Statistic analysis and discussion

377 In this section, the collected data from the optimization process for the adopted numerical  
378 models have been examined in order to identify underlying trends and patterns.

379 Fig.4 reports the bar graphs of the OF mean distribution and standard deviation for  
380 each tested FVD device considering the nine numerical models of hysteresis. In detail,  
381 the graphs represent the accuracy (Fig.4-(a)) of the models in reproducing the laboratory

382 tests, and their robustness (Fig. 4-(b)), respectively. In detail, Fig.4-(a) highlights the  
383 laws that are able to reduce the difference, or the error, between the variable that has  
384 been measured in the laboratory and that one computed as the optimal numerical solution.  
385 A low value of OF for a specified law means that the corresponding model is capable of  
386 fitting the experimentally measured force with reasonable accuracy. The best performers  
387 are represented by the differential laws, in particular Laws #8 and #9. However, the higher  
388 accuracy is obtained at the cost of higher computational burden, as already discussed, but  
389 mostly at the cost of lower robustness (higher standard deviation of, e.g., Laws #8 and #9,  
390 with respect to Law #1). This means that the law that is most accurate in reproducing the  
391 single physical phenomenon (i.e., the specific FVD) is also the one that shows the highest  
392 variability among samples of the devices in the same production series. However, it is worth  
393 noting that the intensity of the standard deviation is still small compared to the mean  
394 intensity. In order to emphasize the trend of standard deviation  $\sigma$  among the investigated  
395 laws, a logarithmic scale was adopted in Fig.4-(b). As expected, by increasing the complexity  
396 of the model from the algebraic to differential models, the dispersion index  $\sigma$  becomes higher  
397 except for Law #9 for which standard deviation, especially for Device #3, #4 and #5, assumes  
quite moderate values. Once that preliminary considerations are pointed out concerning

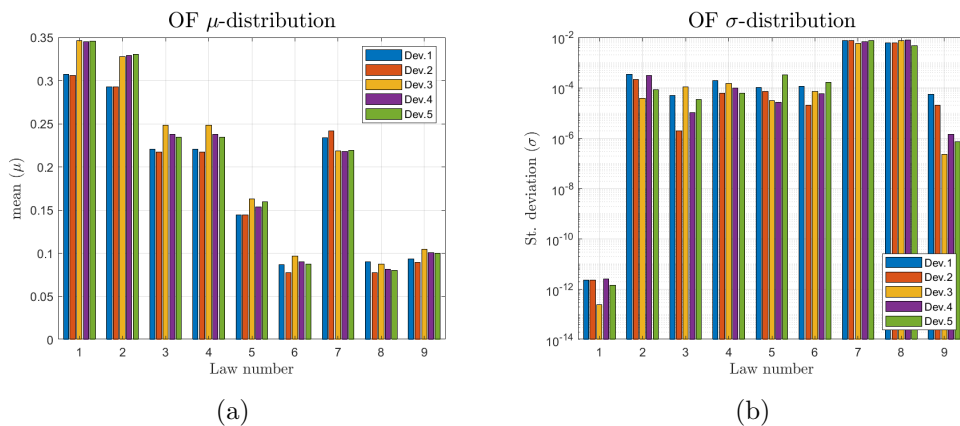


Figure 4: OF mean ( $\mu$ ) distribution for each test devices (a); OF standard deviation ( $\sigma$ ) distribution for each test devices (b). For each law, ( $X - axis$ ), different colours are used for detecting the corresponding device.

398  
399 with the global accuracy and robustness of each model, the authors focused on identifying  
400 the dispersion degree of each parameter for each law. In this way, a measure of the parameter  
401 which mostly affects the overall robustness can be assessed.

402 The models performance observed in terms of OF in Fig. 4 can be deepened by Figs 5-

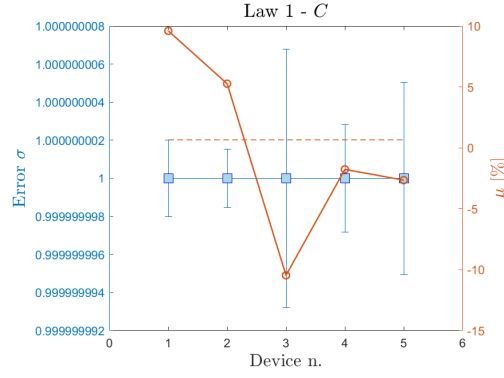


Figure 5: Normalized standard deviation and mean distribution of the identified damping  $C$  for the law 1. The left  $y - axis$  shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y - axis$  the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

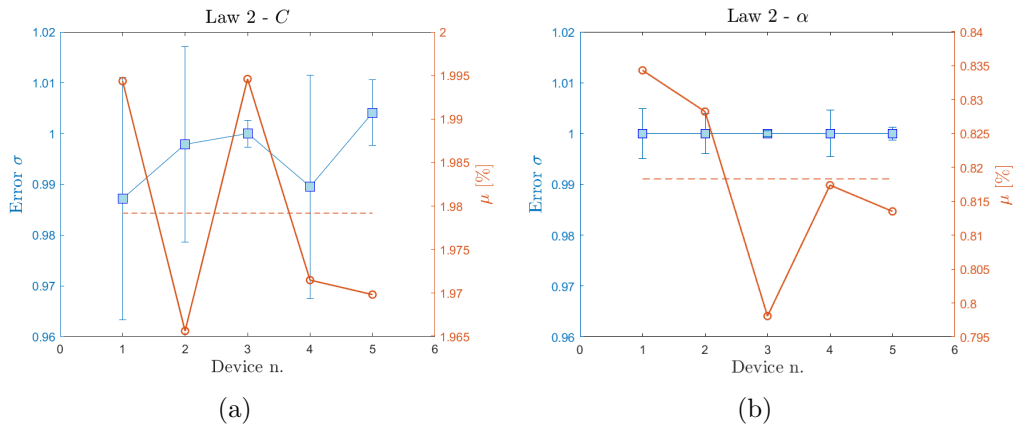


Figure 6: Normalized standard deviation and mean distribution of the identified damping  $C$  (a) and control parameter  $\alpha$  (b) for the law 2. The left  $y - axis$  shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y - axis$  the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

403 13. They present the statistics of the identified parameters, for the different models adopted  
 404 and for the tested FVDs. In particular, each graph shows a double vertical axis, on the left  
 405 the standard deviation of the considered parameter with respect to the mean (error bar of  
 406 the standard deviation -  $Error \sigma$ ) is reported, while on the right axis each figure shows the  
 407 percentage variation of the mean of the considered parameter (calculated over the range of  
 408 20 performed optimization runs) with respect to the reference mean value (red dotted line)  
 409 evaluated as the mean of all means value for each device.

410 Due to the fact that optimal parameters values obtained among the 20 runs show null  
 411 variability, the authors decided to calculate the standard deviation  $\sigma$  with respect to the  
 412 mean value of all runs for each device. This is demonstrated by the fact that all  $Error$   
 413  $\sigma$  bars are centred to value 1 with the exception of Law#2 in which a tiny variability of

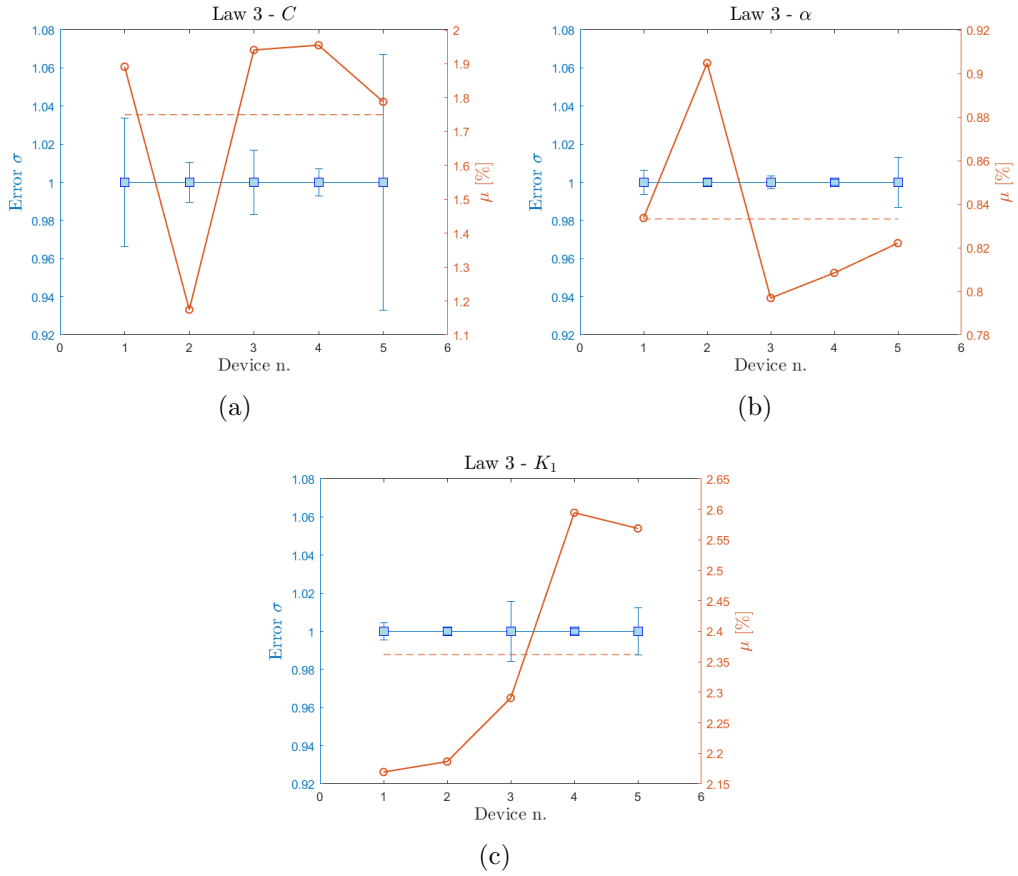


Figure 7: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), control parameter  $\alpha$  (b) and stiffness parameter  $k_0$  (c) for the law 3. The left  $y$  - axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$  - axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

414 parameter  $C$  can be recognized (see Fig.6-(a))

415 Therefore, the intensity scale of  $Error \sigma$  belonging to the same law has been uniformed.

416 However, because of the high variability of  $\mu$  parameter, right vertical axis, among different

417 plots of the same law, it has not been possible to uniform. In this way, the identification of

418 parameter(s) that mostly affects the overall dispersion of the law has been facilitated.

419 According to the trends observed in Fig.4-(b),  $Error \sigma$  range of values, represented by

420 maximum and minimum values, increases from 1.0/0.92 (Law#1) up to 2.5/-0.5 (Law#5)

421 with specific regard to algebraic laws.

422 The same trend is observed with respect to differential laws (Law#6 up to Law#9).

423 However, differently from algebraic laws in which only one parameter is responsible for the

424 entire dispersion of the law, several parameters show significant variability. In this sense,

425 interesting considerations can be revealed by comparing Fig.10 and Fig.11. Though the range

426 variability decreases from Law#6 to Law#7, the number of parameters with a significant

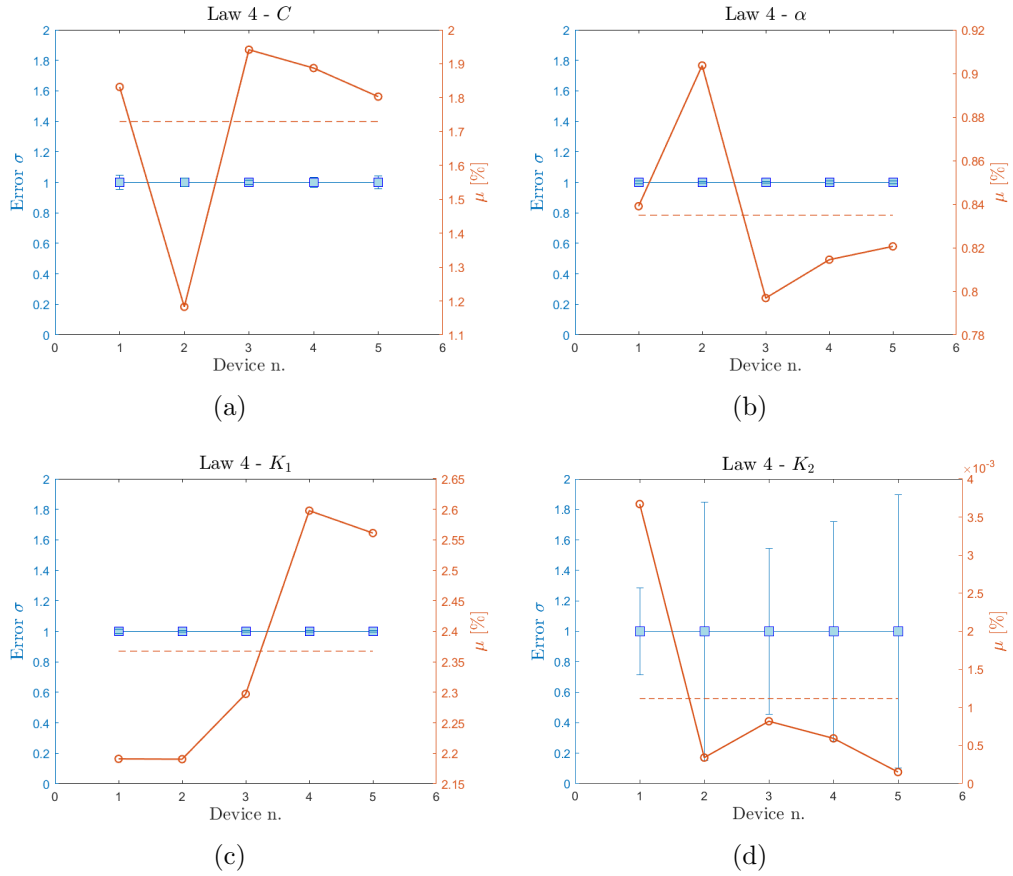


Figure 8: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), control parameter  $\alpha$  (b), stiffness parameter  $k_0$  (c) and constant coefficient  $k_1$  (d) for the law 4. The left  $y$  – axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$  – axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

427 error bar, for each device, increases. Compared with Law#6 in which the dispersion is  
 428 entirely concentrated into parameter  $C$ , in Law#7 each parameter exhibits not-negligible  
 429 value of the  $Error \sigma$ . That is also reflected by observing the growth in Fig.4-(b) in which  
 430 an overall increase of  $\sigma$  is detected for each device. Finally, growth trend is interrupted by  
 431 Law#9 for which dispersion among parameters has been reduced dramatically.

432 If the  $Error \sigma$  trend was clearly identified by the authors, several discrepancies between  
 433 the general behaviour of OF bar histograms (Fig.s 4(a),(b)) and the percentage variation  $\mu$   
 434 plots for each device (law Fig.s 5-13) have been detected. However, the authors are convinced  
 435 that the high variability of parameters can be addicted to the capability of a model to fit  
 436 with experimental outputs. High is the variability  $\mu$  of a specific parameter, high is the  
 437 ability to capture differences between devices. As proof of what has been said, the highest  
 438 variability is recognized for differential models, especially for Law#7, Law#8 and Law#9.

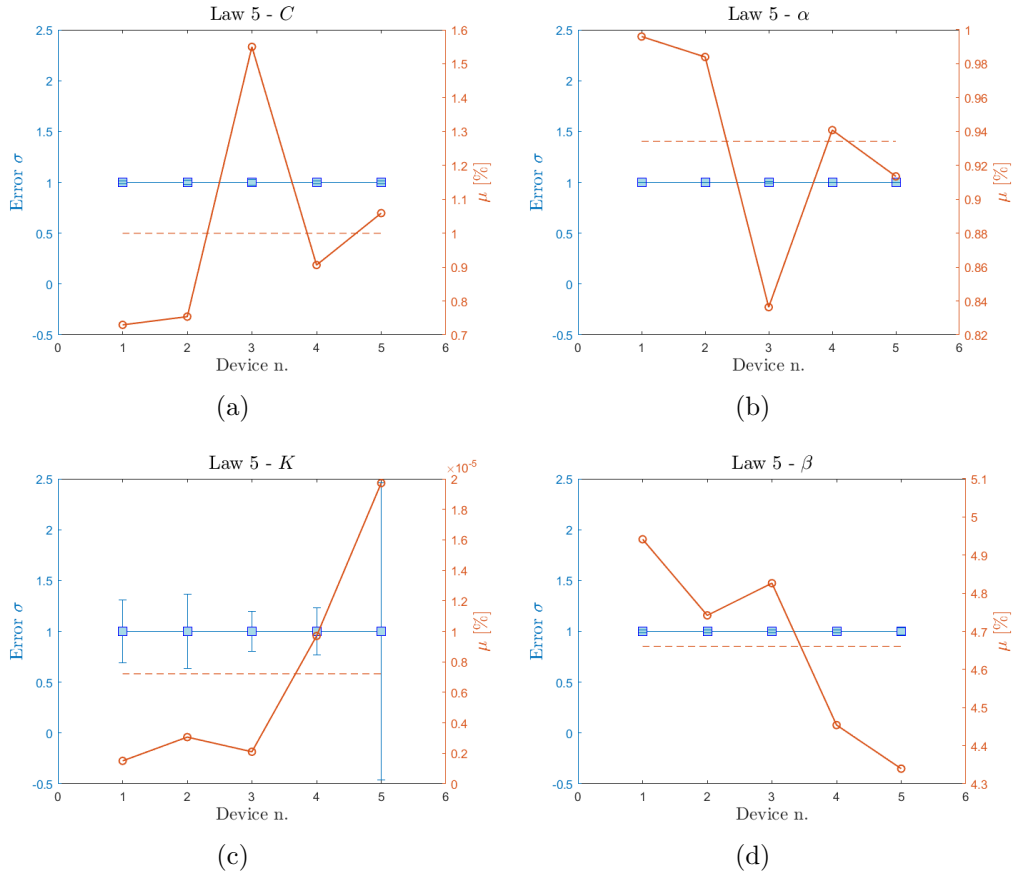


Figure 9: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), exponent parameter  $\alpha$  (b) and  $\beta$  (d) and stiffness parameter  $k_0$  (c) for the law 5. The left  $y$  – axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$  – axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

439 The high variability of  $\mu$ , conceived as a good feature for a model in capturing changes in  
 440 behaviour among the same devices, seems to fail when algebraic laws are treated. Indeed,  
 441 though the variability of  $\mu$ , especially for Law#1, results to be one of the highest among  
 442 all sets of investigated models, the accuracy is not affected by sensible variation of this  
 443 variable since the natively poor complexity of the models. In other words, the variability of  
 444 parameters must not be allowed to outweigh the inner inefficiency of the model.

## 445 7. Pareto Front representation

446 In this section, the authors point out mathematical considerations to identify the best  
 447 performer among the investigated models. As introduced in the previous Section 6, the  
 448 goodness of each model has been evaluated in terms of accuracy (OF) and robustness ( $\sigma$ ).  
 449 If the former can be still assumed as representative of the global capability of a model to fit  
 450 with experimental outputs, the latter gives a measure of how a single parameter can affect

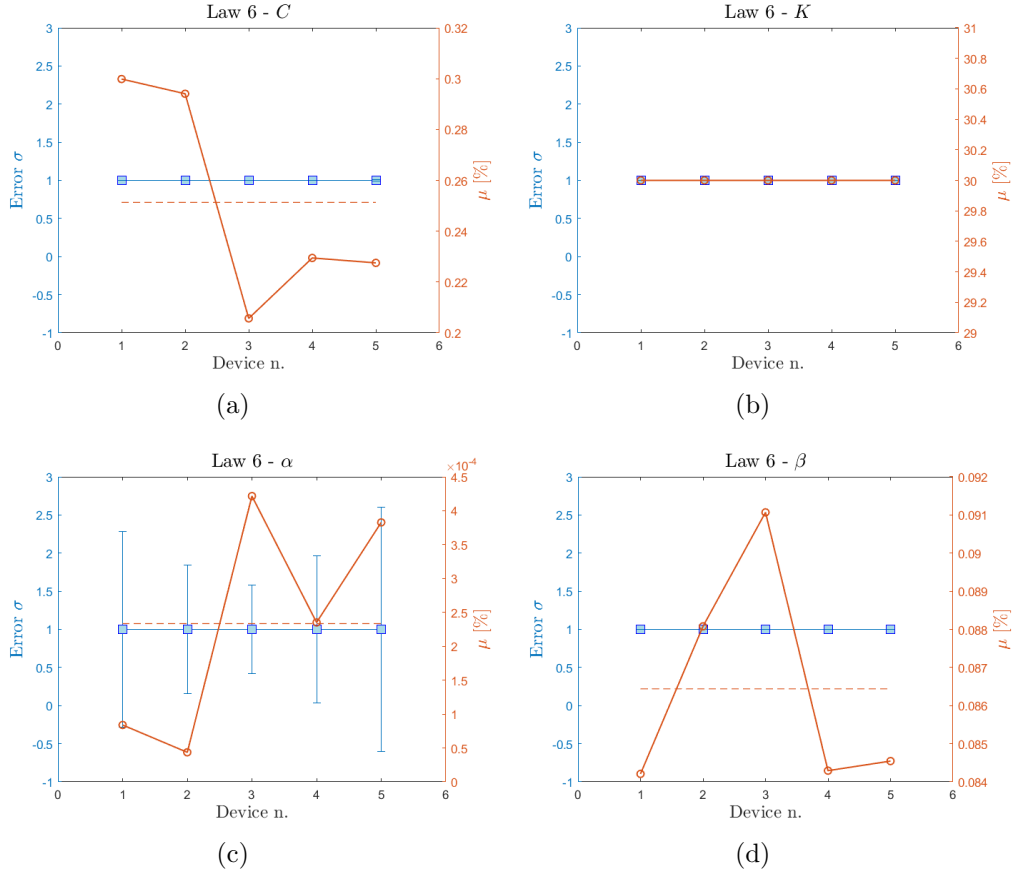


Figure 10: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), stiffness parameter  $k_0$  (b) and exponent parameter  $\alpha$  (c) and  $\beta$  (d) for the law 6. The left  $y$  – axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$  – axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

451 a specific model without any information about the overall robustness of that model.

452 For this reason, the authors introduce a new index  $\beta$  and the following mathematical  
 453 procedure is adopted for its calculation.

454 At first, a matrix representation of parameters set for each law is derived in the following  
 455 form:

$$\alpha = \left( \begin{array}{c} \left[ \begin{array}{cccc} \alpha_{11}^{(1)} & \alpha_{12}^{(1)} & \dots & \alpha_{1n}^{(1)} \\ \alpha_{21}^{(1)} & \alpha_{22}^{(1)} & \dots & \alpha_{2n}^{(1)} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{m1}^{(1)} & \alpha_{m2}^{(1)} & \dots & \alpha_{mn}^{(1)} \end{array} \right] \left[ \begin{array}{cccc} \alpha_{11}^{(2)} & \alpha_{12}^{(2)} & \dots & \alpha_{1n}^{(2)} \\ \alpha_{21}^{(2)} & \alpha_{22}^{(2)} & \dots & \alpha_{2n}^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{m1}^{(2)} & \alpha_{m2}^{(2)} & \dots & \alpha_{mn}^{(2)} \end{array} \right] \dots \left[ \begin{array}{cccc} \alpha_{11}^{(j)} & \alpha_{12}^{(j)} & \dots & \alpha_{1n}^{(j)} \\ \alpha_{21}^{(j)} & \alpha_{22}^{(j)} & \dots & \alpha_{2n}^{(j)} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{m1}^{(j)} & \alpha_{m2}^{(j)} & \dots & \alpha_{mn}^{(j)} \end{array} \right] \end{array} \right) \quad (19)$$

456 Vector  $\alpha$  collects all parameters ( $\alpha_{m,n}$ ) in matrices. Each matrix is representative of a specific  
 457 law ( $j$ ).

458 The first index,  $m$ , and the second index,  $n$ , of the generic identified parameter  $\alpha_{m,n}$  represent

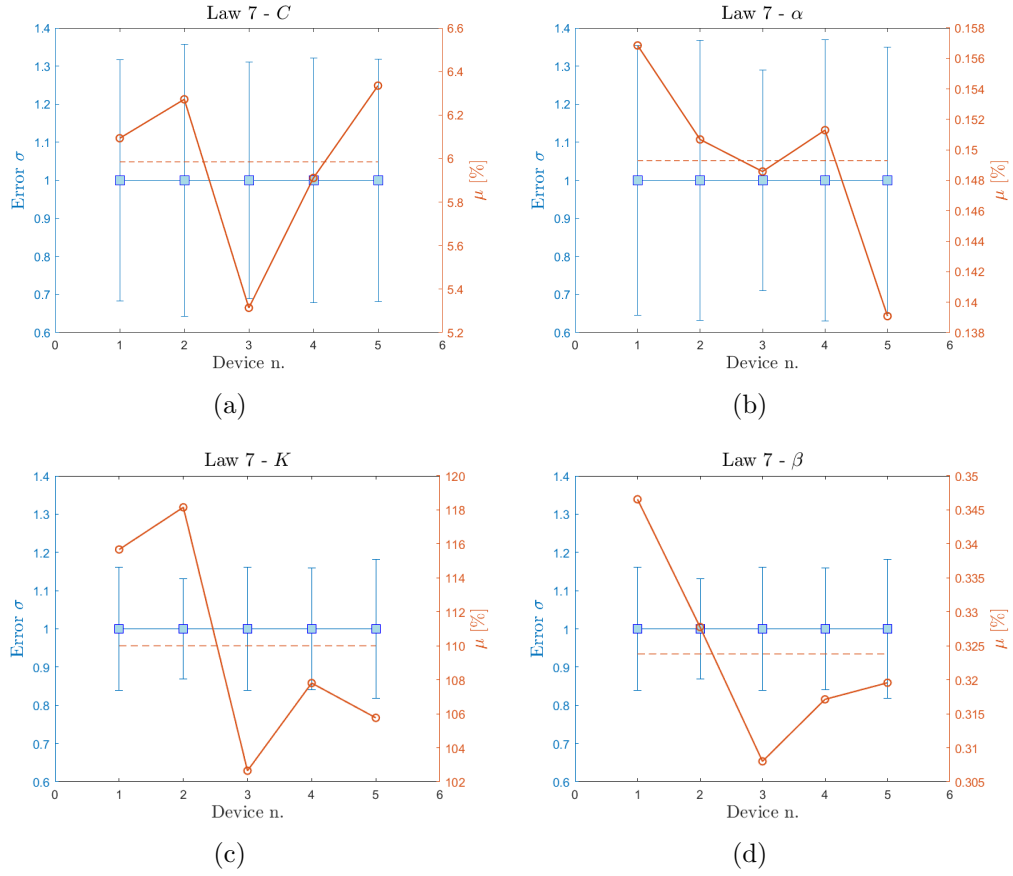


Figure 11: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), exponent parameter  $\alpha$  (b) and  $\beta$  (d) and stiffness parameter  $k_0$  (c) for the law 7. The left  $y$  - axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$  - axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

459 the device number and the parameter number, respectively. Dimensions of  $n$ , hence, the size  
 460 of each component of the vector  $\alpha$ , depends on the considered constitutive law. Moreover,  
 461 apex  $j = [1, 2, \dots, 9]$  represents the different constitutive laws of the investigated numerical  
 462 models.  
 463 For instance, an explicit representation of vector  $\alpha$ , according to the symbology adopted for  
 464 each parameter's law and introduced in Table 4, assumes the following form:

$$\alpha = \left( \begin{array}{c} \left[ \begin{array}{ccc} C_{dev.1}^{(1)} & C_{dev.1}^{(2)} & \alpha_{dev.1}^{(2)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.2}^{(1)} & C_{dev.2}^{(2)} & \alpha_{dev.2}^{(2)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.3}^{(1)} & C_{dev.3}^{(2)} & \alpha_{dev.3}^{(2)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.4}^{(1)} & C_{dev.4}^{(2)} & \alpha_{dev.4}^{(2)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.5}^{(1)} & C_{dev.5}^{(2)} & \alpha_{dev.5}^{(2)} \end{array} \right] \end{array} \right) \cdots \left( \begin{array}{c} \left[ \begin{array}{ccc} C_{dev.1}^{(9)} & \alpha_{dev.1}^{(9)} & \beta_{dev.1}^{(9)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.2}^{(9)} & \alpha_{dev.2}^{(9)} & \beta_{dev.2}^{(9)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.3}^{(9)} & \alpha_{dev.3}^{(9)} & \beta_{dev.3}^{(9)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.4}^{(9)} & \alpha_{dev.4}^{(9)} & \beta_{dev.4}^{(9)} \end{array} \right] \\ \left[ \begin{array}{ccc} C_{dev.5}^{(9)} & \alpha_{dev.5}^{(9)} & \beta_{dev.5}^{(9)} \end{array} \right] \end{array} \right) \quad (20)$$

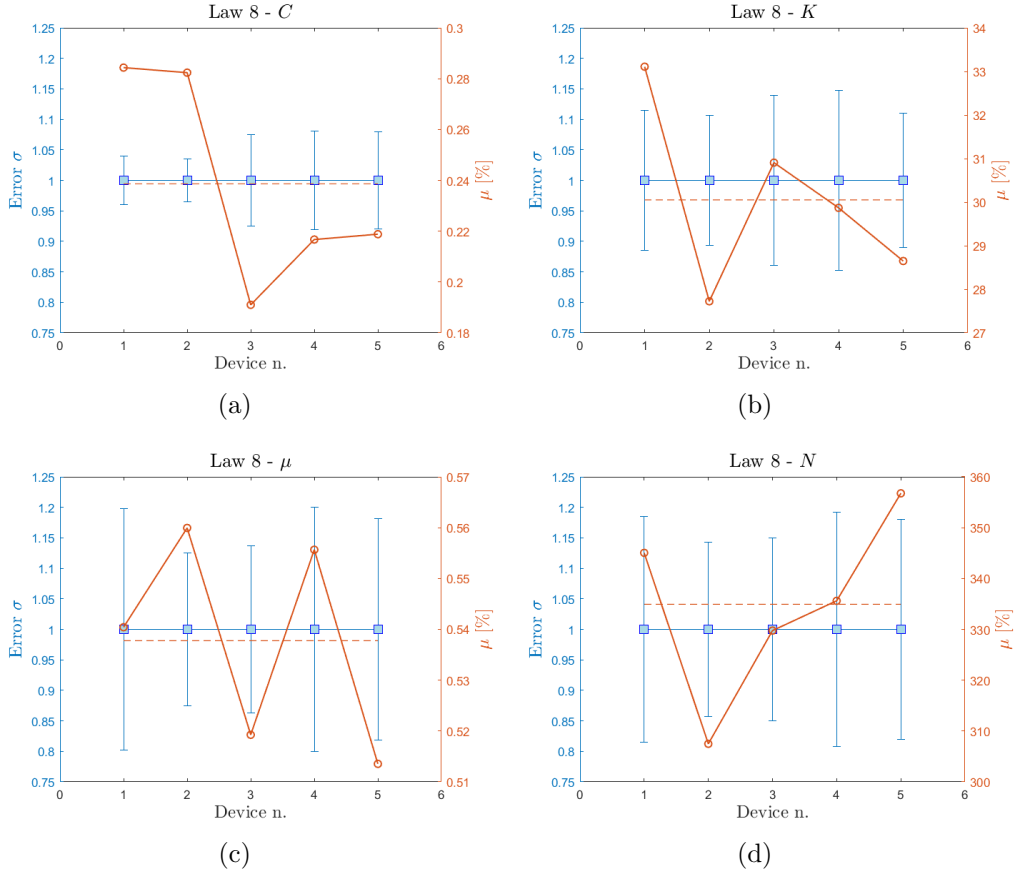


Figure 12: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), stiffness parameter  $k_0$  (b), friction coefficient  $\mu$  and normal contact force  $N$  for the law 8. The left  $y$ -axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$ -axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

465 At first, percentage variations of each parameter, with respect to a reference value, are eval-  
 466 uated for each considered law. For example, the percentage variation,  $\Delta_1^{(j)}$ , of the parameter  
 467  $n = 1$  of the generic constitutive law  $j$  will be written as following:

$$\Delta_1^{(j)} = \max(\text{abs}(\{\Delta_{11}^{(j)}, \Delta_{21}^{(j)}, \dots, \Delta_{m1}^{(j)}\})) \quad (21)$$

468

$$\text{where } \Delta_{m1} = \frac{\alpha_{m1} - \alpha_{\mu 1}}{\alpha_{\mu 1}}. \quad (22)$$

469 For clarity purposes, the maximum percentage variation,  $\Delta$ , evaluated for each device, is  
 470 expressed for the parameter  $n = 1$  as reported in Eq.21. The generic percentage variation,  
 471  $\Delta_{m1}^{(j)}$ , is a measure of the dispersion of the parameter  $\alpha$  of a specific device  $m$  with respect to a  
 472 reference value expressed by the mean,  $\alpha_{\mu 1}$ , of all  $\alpha$  values evaluated for each device. Though  
 473 each device exhibits similar properties, the authors focused on investigating the variability  
 474 of a specific parameter with respect to the average value evaluated among all the devices.

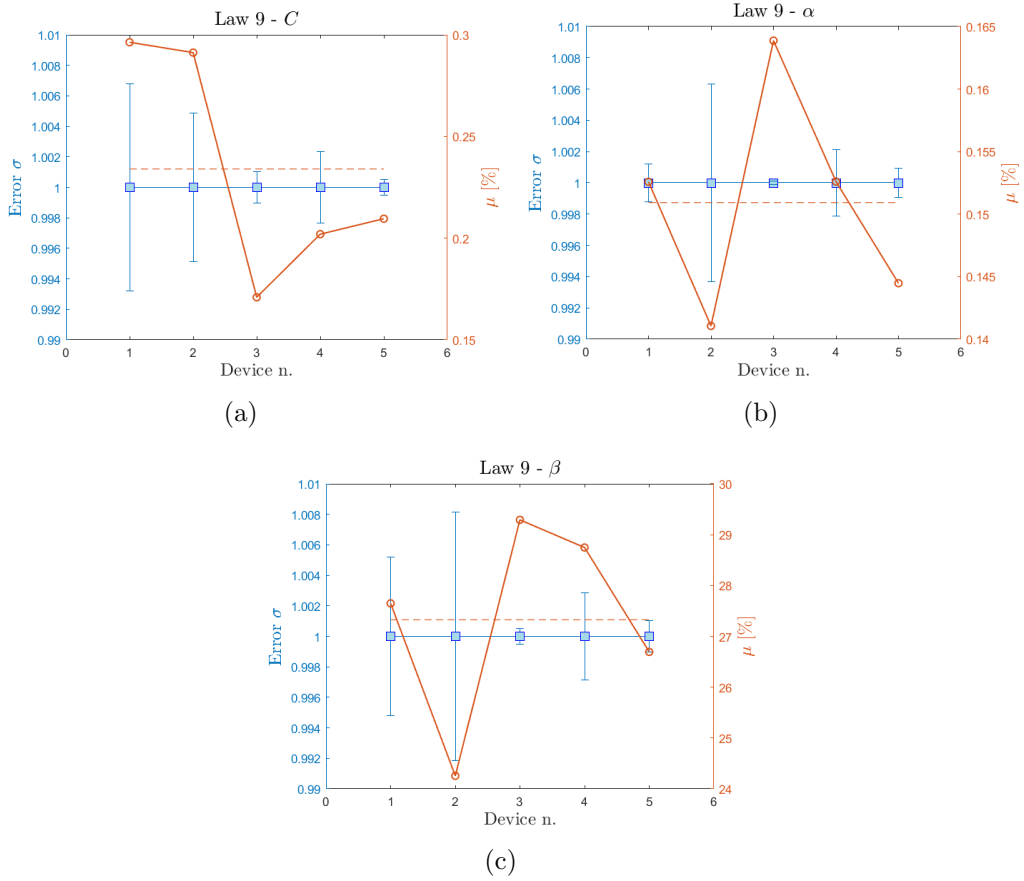


Figure 13: Normalized standard deviation and mean distribution of the identified damping  $C$  (a), exponent parameter  $\alpha$  (b) and  $\beta$  (c) for the law 9. The left  $y$  - axis shows the error bar of the standard deviation ( $\sigma$ ), while the right  $y$  - axis the percentage variation of mean distribution ( $\mu$ ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

475 In this way, a measure of the accuracy provided by that parameter in a specific law can be  
 476 detected. This procedure is applied for each parameter of each investigated constitutive law.  
 477 In this way, for the generic law  $j$ , the value  $\beta_j$  can be evaluated as:

$$\beta_j = \frac{\sum \Delta_n^{(j)}}{n_n}. \quad (23)$$

478 The no-dimensional value  $\beta$  can be assumed as a robustness index corresponding to the  
 479 generic law  $j$ . This value is obtained, for each law, by summing all the percentage variations  
 480 of the  $n$  parameter which govern that specific constitutive law.

481 With the aim to summarize general outcomes in a single plot where a measure of accuracy  
 482 and robustness of each law is given, Fig.14 depicts the OF parameter, evaluated as the mean  
 483 of all twenty optimization runs and among all devices, and index  $\beta$  on the Y and X-axis,  
 484 respectively. Increasing the accuracy of a model (lower values of OF), a decrease in the

485 overall robustness (high values of  $\beta$ ) is observed. Due to the opposite trend of these indices,  
 486 a Pareto Front plot is developed and the best performer among all investigated models is  
 identified.

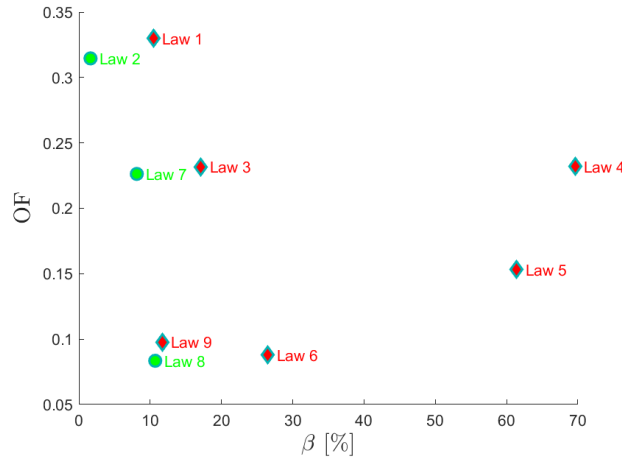


Figure 14: Accuracy (OF) vs robustness ( $\beta$ ) Pareto front. The best constitutive laws as a good compromise between accuracy and robustness are marked in green while the dominated solutions are reported in red. The trend realized by connecting the green points represents the Pareto front of the non-dominated solutions.

487

488 In the scatter representation, data are clustered into two groups by using green and  
 489 red colours. Green points represent the good compromise between accuracy and robustness  
 490 features, while red points are the dominated-solution of the optimization problem. In other  
 491 words, by selecting one of the red marked points, it is always possible to identify whatever  
 492 solutions, among the green ones, which can be preferred in terms of accuracy or robustness  
 493 (or both). Therefore, those models corresponding to green points lie in a Pareto Front, and  
 494 all performers, identified by the couple  $[\beta, OF]$ , which belong the right side can be considered  
 495 dominated solutions of the optimization problem.

496 For instance, Law#2 represents the best performer in terms of dispersion, hence, it represents  
 497 the best model in which identified parameters contribute to obtaining the lowest value of  $\beta$   
 498 among the investigated models. On the contrary, due to the algebraic nature of Law#2, it  
 499 is not the most effective with respect to the capability of reproducing the real behaviour of  
 500 experimental outputs. Indeed, solutions, such as Law#7 or Law#8, can be preferred for the  
 501 lower OF.

502 An interesting consideration can be pointed out by comparing the gain and loss in terms  
 503 of OF and  $\beta$  from Law#2 to Law#8. Focusing on Fig.14, it is quite evident that the loss  
 504 in terms of parameters variability is negligible if compared with the gain in terms of OF

505 of Law#8. This is evident in the comparison between experimental and numerical cycles  
506 depicted in Fig.s3-(i),(l) and Fig.s3-(o),(p).

## 507 8. Conclusions and future developments

508 In the present paper, an identification procedure for the evaluation of the optimal pa-  
509 rameters of nine different FVD numerical models has been implemented. The procedure  
510 has been performed via metaheuristic technique and Genetic Algorithm suitably modified  
511 and adapted to the final scope. Data derived from an experimental campaign has been  
512 adopted and five FVD devices, with the same characteristics, have been tested in dynamic  
513 conditions. An identification scheme has been developed to compare the experimental and  
514 analytical values of the forces experienced by the device series under investigation. A com-  
515 parison of the optimal numerical hysteresis cycles against the laboratory outcomes, for each  
516 adopted law, has been shown. Moreover, a statistic analysis has been conducted and trends  
517 and patterns have been highlighted. Generally, results shown how differential laws better  
518 fit with experimental curves. Specifically, Law#8 and Law#9 exhibited a high level of ac-  
519 curacy with a moderate dispersion among identified parameters. Finally, the Pareto front  
520 representation has been adopted aiming to identify the set of best performers both in terms  
521 of laws' accuracy and robustness.

522 Law#2 has been identified as that one with the highest robustness with a significant loss in  
523 terms of accuracy. However, differential models as Law#7 and, especially, Law#8, can be  
524 considered the most promising models for their low OF values even though a minimal loss  
525 in terms of overall dispersion has been assessed.

526 Extending the experimental campaign could be useful to consolidate the obtained results.  
527 However, it would involve obvious implementation difficulties. Therefore, future develop-  
528 ment of this study will involve the procedure generalization to a higher number of devices  
529 by numerical simulation, introducing parameters' variability (e.g., fluid characteristics, tol-  
530 erances in the mechanical components). This virtual test campaign, supported by a limited  
531 number of real tests, would also allow the study of multiple loading patterns to analyze the  
532 effectiveness of the identified models under different operating conditions.

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## 707 **Appendix A.**

708 This section is devoted to the results derived by the optimization process. Mean ( $\mu$ ) and  
709 standard deviation ( $\sigma$ ) for all design variables involved into the optimization process are  
710 calculated. Values of OF and identified value of parameters, for each law, are reported in  
711 table A.7 for completeness purposes.

Table A.7: Means ( $\mu$ ) and standard deviations ( $\sigma$ ) of design variables and OFs

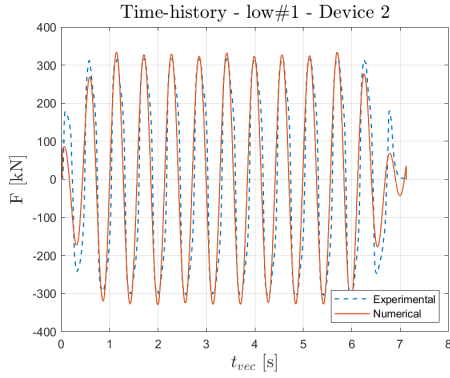
|              |           | Device 1 |                        | Device 2 |                        | Device 3 |                        | Device 4 |                        | Device 5 |                        |
|--------------|-----------|----------|------------------------|----------|------------------------|----------|------------------------|----------|------------------------|----------|------------------------|
|              |           | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               |
| <b>Law 1</b> | <b>OF</b> | 0.3074   | $2.36 \times 10^{-12}$ | 0.3057   | $2.30 \times 10^{-12}$ | 0.3463   | $2.50 \times 10^{-12}$ | 0.3450   | $2.67 \times 10^{-12}$ | 0.3454   | $1.50 \times 10^{-12}$ |
|              | <i>C</i>  | 0.7248   | $1.47 \times 10^{-09}$ | 0.6962   | $1.07 \times 10^{-09}$ | 0.5922   | $4.01 \times 10^{-09}$ | 0.6497   | $1.84 \times 10^{-09}$ | 0.6438   | $3.26 \times 10^{-09}$ |
| <b>Law 2</b> | <b>OF</b> | 0.2925   | $3.46 \times 10^{-04}$ | 0.2928   | $2.19 \times 10^{-04}$ | 0.3279   | $3.85 \times 10^{-04}$ | 0.3295   | $3.04 \times 10^{-04}$ | 0.3303   | $8.19 \times 10^{-04}$ |
|              | <i>C</i>  | 1.9944   | 0.0476                 | 1.9657   | 0.0378                 | 1.9946   | 0.0052                 | 1.9715   | 0.0434                 | 1.9698   | 0.0128                 |
|              | $\alpha$  | 0.8343   | $4.10 \times 10^{-03}$ | 0.8282   | $3.30 \times 10^{-03}$ | 0.7981   | $4.33 \times 10^{-04}$ | 0.8174   | $3.70 \times 10^{-03}$ | 0.8135   | $1.10 \times 10^{-03}$ |
| <b>Law 3</b> | <b>OF</b> | 0.2205   | $5.09 \times 10^{-05}$ | 0.2173   | $1.95 \times 10^{-05}$ | 0.2484   | $1.08 \times 10^{-05}$ | 0.2379   | $1.03 \times 10^{-05}$ | 0.2348   | $3.37 \times 10^{-05}$ |
|              | <i>C</i>  | 1.8913   | $6.36 \times 10^{-02}$ | 1.1743   | $1.22 \times 10^{-02}$ | 1.9405   | $3.26 \times 10^{-02}$ | 1.9548   | $1.37 \times 10^{-02}$ | 1.7877   | 0.12                   |
|              | $\alpha$  | 0.8337   | $5.40 \times 10^{-03}$ | 0.9047   | $1.70 \times 10^{-03}$ | 0.7969   | $2.70 \times 10^{-03}$ | 0.8084   | $1.10 \times 10^{-03}$ | 0.8221   | $1.09 \times 10^{-02}$ |
|              | $K_1$     | 2.1694   | $9.80 \times 10^{-03}$ | 2.1865   | $3.80 \times 10^{-03}$ | 2.2908   | $3.60 \times 10^{-02}$ | 2.5945   | $3.90 \times 10^{-03}$ | 2.5687   | $3.15 \times 10^{-02}$ |
| <b>Law 4</b> | <b>OF</b> | 0.2204   | $1.91 \times 10^{-04}$ | 0.2173   | $6.11 \times 10^{-05}$ | 0.2486   | $1.47 \times 10^{-04}$ | 0.2380   | $9.99 \times 10^{-05}$ | 0.2348   | $6.24 \times 10^{-05}$ |
|              | <i>C</i>  | 1.8315   | $8.73 \times 10^{-02}$ | 1.1827   | $3.15 \times 10^{-02}$ | 1.9414   | $3.17 \times 10^{-02}$ | 1.8873   | $6.38 \times 10^{-02}$ | 1.8029   | $7.75 \times 10^{-02}$ |
|              | $\alpha$  | 0.8391   | $4.10 \times 10^{-03}$ | 0.9036   | $3.30 \times 10^{-03}$ | 0.7969   | $4.33 \times 10^{-04}$ | 0.8145   | $3.70 \times 10^{-03}$ | 0.8206   | $1.10 \times 10^{-03}$ |
|              | $K_1$     | 2.1910   | $3.05 \times 10^{-02}$ | 2.1904   | $6.30 \times 10^{-03}$ | 2.2975   | $1.52 \times 10^{-02}$ | 2.5978   | $9.80 \times 10^{-03}$ | 2.5611   | $1.50 \times 10^{-02}$ |
|              | $K_2$     | 0.0037   | $11.0 \times 10^{-04}$ | 0.0003   | $2.90 \times 10^{-04}$ | 0.0008   | $4.47 \times 10^{-04}$ | 0.0006   | $4.28 \times 10^{-04}$ | 0.0002   | $1.37 \times 10^{-04}$ |

|              |           | Device 1               |                        | Device 2               |                        | Device 3               |                        | Device 4               |                        | Device 5               |                        |
|--------------|-----------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
|              |           | $\mu$                  | $\sigma$               | $\mu$                  | $\sigma$               | $\mu$                  | $\sigma$               | $\mu$                  | $\sigma$               | $\mu$                  | $\sigma$               |
| <b>Law 5</b> | <b>OF</b> | 0.1443                 | $1.02 \times 10^{-04}$ | 0.1447                 | $7.03 \times 10^{-05}$ | 0.1631                 | $3.13 \times 10^{-05}$ | 0.1540                 | $2.67 \times 10^{-05}$ | 0.1599                 | $3.29 \times 10^{-04}$ |
|              | <i>C</i>  | 0.7295                 | 0.0120                 | 0.7538                 | 0.0149                 | 1.5499                 | 0.0439                 | 0.9060                 | 0.0140                 | 1.0592                 | 0.0125                 |
|              | $\alpha$  | 0.9961                 | 0.0027                 | 0.9840                 | 0.0032                 | 0.8364                 | 0.0047                 | 0.9408                 | 0.0025                 | 0.9135                 | 0.0021                 |
|              | <i>K</i>  | $1.51 \times 10^{-06}$ | $4.64 \times 10^{-07}$ | $3.07 \times 10^{-06}$ | $0.11 \times 10^{-07}$ | $2.11 \times 10^{-06}$ | $4.14 \times 10^{-07}$ | $9.70 \times 10^{-06}$ | $0.22 \times 10^{-07}$ | $0.20 \times 10^{-06}$ | $0.03 \times 10^{-07}$ |
|              | $\beta$   | 4.9415                 | 0.0727                 | 4.7419                 | 0.0813                 | 4.8263                 | 0.0557                 | 4.4542                 | 0.0655                 | 4.3398                 | 0.1647                 |
| <b>Law 6</b> | <b>OF</b> | 0.0872                 | $1.14 \times 10^{-04}$ | 0.0776                 | $2.05 \times 10^{-05}$ | 0.0971                 | $7.22 \times 10^{-05}$ | 0.0902                 | $5.85 \times 10^{-05}$ | 0.0880                 | $1.65 \times 10^{-04}$ |
|              | <i>C</i>  | 0.2999                 | $3.05 \times 10^{-04}$ | 0.2941                 | $7.51 \times 10^{-04}$ | 0.2057                 | $8.23 \times 10^{-04}$ | 0.2295                 | $3.86 \times 10^{-04}$ | 0.2275                 | $14.0 \times 10^{-04}$ |
|              | <i>K</i>  | 30.0                   | 0.0081                 | 30.0                   | 0.0042                 | 30.0                   | 0.0027                 | 30.0                   | 0.0058                 | 30.0                   | 0.0072                 |
|              | $\alpha$  | $83.9 \times 10^{-04}$ | 0.0305                 | $43.6 \times 10^{-04}$ | 0.0063                 | $4.22 \times 10^{-04}$ | 0.0152                 | $2.35 \times 10^{-04}$ | 0.0098                 | $3.83 \times 10^{-04}$ | 0.0150                 |
|              | $\beta$   | 0.0842                 | $53.7 \times 10^{-04}$ | 0.0881                 | $1.27 \times 10^{-04}$ | 0.0911                 | $1.36 \times 10^{-04}$ | 0.0843                 | $1.02 \times 10^{-04}$ | 0.0845                 | $3.04 \times 10^{-04}$ |
| <b>Law 7</b> | <b>OF</b> | 0.2336                 | $7.5 \times 10^{-03}$  | 0.2419                 | $7.7 \times 10^{-03}$  | 0.2188                 | $5.8 \times 10^{-03}$  | 0.2179                 | $6.8 \times 10^{-03}$  | 0.2192                 | $7.6 \times 10^{-03}$  |
|              | <i>C</i>  | 6.0935                 | 1.9256                 | 6.2724                 | 2.2382                 | 5.3142                 | 1.6503                 | 5.9110                 | 1.9019                 | 6.3351                 | 2.0116                 |
|              | $\alpha$  | 0.1569                 | 0.0556                 | 0.1507                 | 0.0554                 | 0.1486                 | 0.0431                 | 0.1513                 | 0.0558                 | 0.1391                 | 0.0486                 |
|              | <i>K</i>  | 115.67                 | 18.726                 | 118.15                 | 15.562                 | 102.65                 | 16.546                 | 107.81                 | 17.240                 | 105.76                 | 19.306                 |
|              | $\beta$   | 0.3466                 | 0.0531                 | 0.3278                 | 0.0466                 | 0.3080                 | 0.0548                 | 0.3171                 | 0.0552                 | 0.3196                 | 0.0604                 |

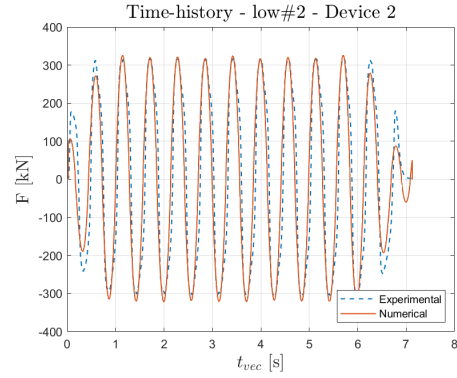
|              |           | Device 1 |                        | Device 2 |                        | Device 3 |                        | Device 4 |                        | Device 5 |                        |
|--------------|-----------|----------|------------------------|----------|------------------------|----------|------------------------|----------|------------------------|----------|------------------------|
|              |           | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               | $\mu$    | $\sigma$               |
| <b>Law 8</b> | <b>OF</b> | 0.0902   | $6.2 \times 10^{-03}$  | 0.0779   | $6.0 \times 10^{-03}$  | 0.0875   | $7.5 \times 10^{-03}$  | 0.0814   | $7.7 \times 10^{-03}$  | 0.0802   | $4.8 \times 10^{-03}$  |
|              | <i>C</i>  | 0.2844   | 0.0114                 | 0.2824   | 0.0099                 | 0.1910   | 0.0144                 | 0.2167   | 0.0175                 | 0.2188   | 0.0175                 |
|              | <i>K</i>  | 33.108   | 3.7877                 | 27.724   | 2.9511                 | 30.907   | 4.3087                 | 29.569   | 4.3901                 | 28.649   | 3.1568                 |
|              | $\mu$     | 0.5404   | 0.1069                 | 0.5599   | 0.0703                 | 0.5193   | 0.0710                 | 0.5557   | 0.1116                 | 0.5135   | 0.0934                 |
|              | <i>N</i>  | 345.05   | 63.713                 | 307.48   | 43.920                 | 329.76   | 49.245                 | 335.62   | 64.503                 | 356.76   | 64.199                 |
| <b>Law 9</b> | <b>OF</b> | 0.0934   | $5.43 \times 10^{-05}$ | 0.0896   | $2.07 \times 10^{-05}$ | 0.1047   | $2.25 \times 10^{-07}$ | 0.1006   | $1.42 \times 10^{-06}$ | 0.1002   | $7.31 \times 10^{-07}$ |
|              | <i>C</i>  | 0.2964   | $20.0 \times 10^{-04}$ | 0.2913   | $14.0 \times 10^{-04}$ | 0.1710   | $1.78 \times 10^{-04}$ | 0.2020   | $4.75 \times 10^{-04}$ | 0.2096   | $1.12 \times 10^{-04}$ |
|              | $\alpha$  | 0.1526   | $1.83 \times 10^{-04}$ | 0.1411   | $8.93 \times 10^{-04}$ | 0.1639   | $0.19 \times 10^{-04}$ | 0.1526   | $3.23 \times 10^{-04}$ | 0.1445   | $1.33 \times 10^{-04}$ |
|              | $\beta$   | 27.649   | 0.1439                 | 24.256   | 0.1972                 | 29.292   | 0.0150                 | 28.748   | 0.0821                 | 26.695   | 0.0276                 |

713 **Appendix B.**

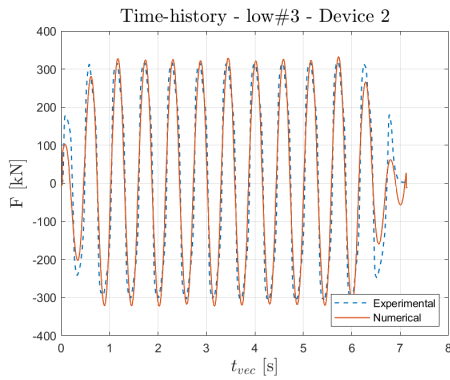
714 In this section, numerical and experimental Time-history curves are reported for com-  
 715 parison purposes. All plots are referred to device 2 which exhibits the best results in terms  
 716 of accuracy (OF). In Figs 3 (a)-(i), experimental ( $F_{experimental}$  vs  $t_{vec}$ ) and numerical curves  
 ( $F_{numerical}$  vs  $t_{vec}$ ) are depicted for each law, respectively. Generally, as recognized for



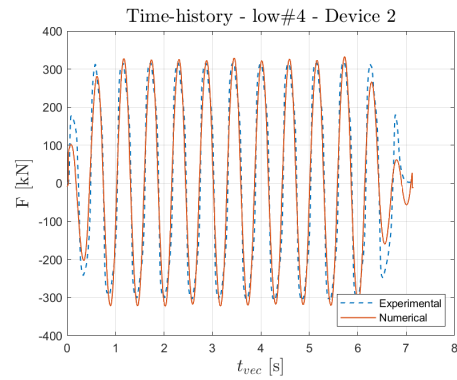
(a)



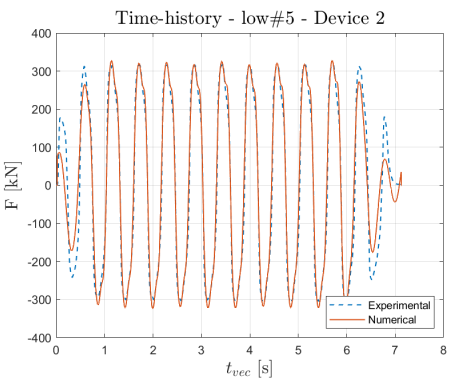
(b)



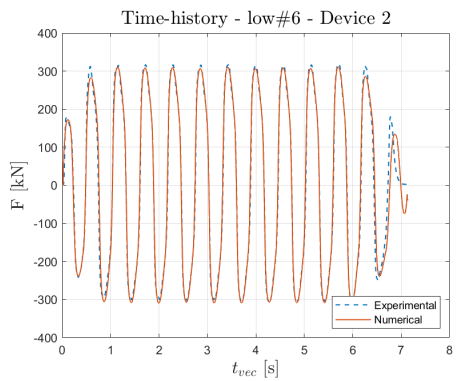
(c)



(d)

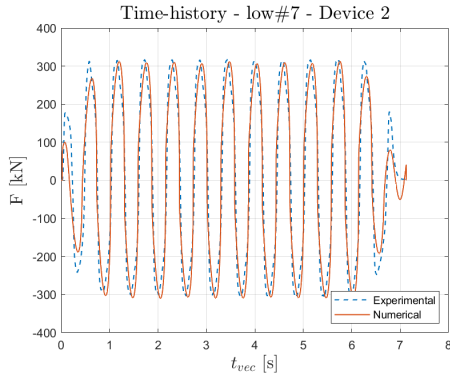


(e)

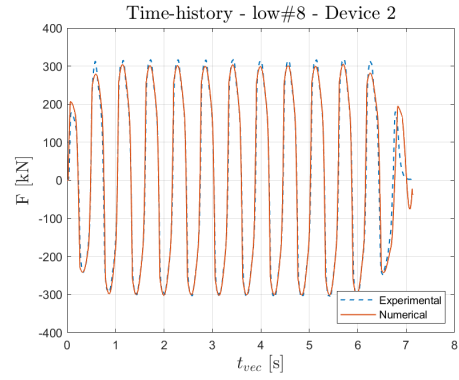


(f)

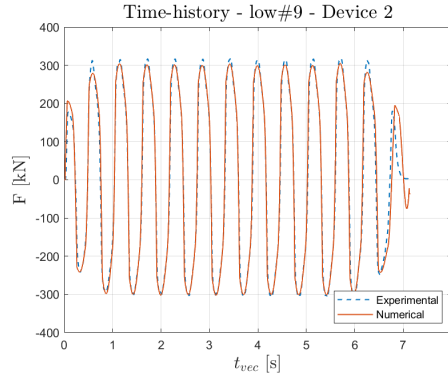
717  
 718 the optimal hysteresis cycles discussed in section 5, best matches between experimental and  
 719 numerical curves are observed by differential law (from law#6 up to law#9). Despite to  
 720 the first five models, differential models fit better specifically at the peaks of each cycle. In



(g)



(h)



(i)

Figure B.15: Comparison between experimental (dotted blu line) and numerical (solid red line) time-history is depicted for each investigated law. Recorded experimental load,  $F_{experimental}$ , and numerical load,  $F_{numerical}$ , are reported along  $Y - axis$  while  $t_{vec}$ , along  $X - axis$ , is representative of both time-step acquisition and numerical integration.

721 particular, law#8 and law#9 show a good grade of match not only at the stage of maximum  
722 load but during the transient phase too.

723 On the other hand, though numerical curves derived by no differential model (from law#1  
724 up to law#5) show a generic good grade of match with experimental curve, numerical noises  
725 are recognized at the peak value of each cycle. By observing the plot related to law#5, this  
726 phenomenon becomes more evident. Moreover, all these models are not capable of taking  
727 the initial and final transient phases.