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Numerical models comparison for fluid-viscous dampers: Performance investigations through Genetic Algorithm / Cucuzza, R.; Domaneschi, M.; Greco, R.; Marano, G. C.. - In: COMPUTERS & STRUCTURES. - ISSN 0045-7949. - 288:(2023), pp. 1-18. [10.1016/j.compstruc.2023.107122]

Availability:

This version is available at: 11583/2982815 since: 2023-10-06T10:56:07Z

Publisher:

Elsevier

Published

DOI:10.1016/j.compstruc.2023.107122

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<http://dx.doi.org/10.1016/j.compstruc.2023.107122>

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Numerical models comparison for fluid-viscous dampers: performance investigations through Genetic Algorithm

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Abstract

Fluid-viscous dampers played a crucial role in the protection of new or existing buildings against external actions as earthquakes and winds. In the last decade, several investigations have been conducted aiming to develop accurate numerical models. However, none has been focused on a comprehensive comparisons between the most used fluid-viscous damper models considering the variability of their parameters in a mass-production series.

In this paper, an identification procedure has been performed by comparing nine different existing literature models **with the objective of evaluating their ability to match experimental loops of mass-produced fluid-viscous devices**, both in terms of accuracy and robustness. **Indeed, the model that is most effective for reproducing the characteristic of a specific specimen may not be representative (i.e., showing larger parameters variability) of the mass production of the same device type.** For this purpose, dynamic tests have been developed in the laboratory and the experimental outputs have been adopted as the target function of the procedure. The identification scheme has been designed by implementing an optimization procedure via Genetic Algorithm. **Results demonstrate how differential laws better fit the experimental cycles with respect to algebraic ones, and also show how few models in the series can offer a high level of both accuracy and robustness.**

Keywords: Viscous Damper, Numerical model, Optimization, Genetic Algorithm, Statistical Analysis, Mass-production

1. Introduction

The significance of civil engineering structures in societal development cannot be overstated, as they form the backbone of transportation systems, utilities, and habitation. How-

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26 ever, these structures encounter formidable challenges arising from external actions, notably
27 seismic events, which pose substantial risks of damage and functional impairment [1, 2, 3, 4].
28 Consequently, the adoption of resilient design principles, specifically the integration of damp-
29 ing devices [5, 6, 7, 8, 9], assumes paramount importance in safeguarding these structures,
30 enabling their continued operation even amidst severe seismic events of greater intensity
31 [10, 11, 12].

32 Fluid-dynamic damper technology is a mature technology that has been extensively stud-
33 ied and incorporated into anti-seismic design standards and guidelines in many countries
34 worldwide [13, 14, 15, 16]. For example, the Japan Society of Civil Engineers recommends
35 the use of fluid-dynamic dampers in seismic-resistant design, and many buildings in Japan
36 are equipped with these devices [17]. Similarly, the Council of Tall Buildings and Urban
37 Habitat recommends the use of fluid-dynamic dampers in high-rise buildings located in
38 high-seismicity areas [18].

39 The ability of fluid viscous dampers (FVDs) to dissipate a large amount of energy, the
40 creation of forces that are out of phase with displacements, and the potential to improve
41 damping ratios without significantly changing stiffness make them appealing. This is not
42 the case with other devices as hysteretic ones. The military and aerospace industries were
43 the first to demonstrate the efficacy and reliability of FVDs. This paved the path to the
44 application of FVDs in civil engineering for both new and existing structures [19, 20].

45 Both experimental testing and numerical modelling are frequently used to characterize
46 the mechanical properties of novel systems or devices. Although the former approach is
47 more expensive than the latter, it directly leads to a physical understanding of the system
48 and can therefore be used for final verification. Numerical simulation, in contrast, employs
49 numerical techniques to quantitatively describe the development of a physical system and
50 enables more economically exploring a wide range of potential design options. A satisfactory
51 representation of the actual mechanical behavior of the understudied device can be obtained
52 from such simulations by employing realistic models [21, 22]. By simulating the behavior
53 of control devices, engineers can evaluate the qualities of different design solutions before
54 implementing them in the physical structure. This can save time and money by avoiding
55 costly trial-and-error processes and ensuring that the protection strategy is reliable and

56 efficient [23, 24].

57 The effectiveness of FVDs depends on various factors, such as the type and properties of
58 the materials used, the geometry and structural configuration, and the external forces acting
59 on the structure. **Therefore, numerical simulations can be used to evaluate their efficiency
60 and improve their performance [25, 26, 27, 28]**, also by reproducing their characteristic
61 behavior in complex structural systems [29, 30].

62 Furthermore, numerical simulations can provide valuable insights into the behavior of
63 control devices under extreme conditions that are difficult or impossible to replicate in phys-
64 ical experiments. For example, numerical simulations can be used to evaluate the behavior
65 of control devices under earthquakes of different magnitudes and frequencies, or under wind
66 speeds that are beyond the capabilities of physical experiments [31, 32].

67 Choosing the model to reproduce in operation the fluid-viscous device is a critical step
68 for reasons mainly related to mass production of the devices. Indeed, the model identified
69 on a single device could not be robust enough to represent the variability of the devices'
70 characteristics that belong to the same mass production. On the other hand, a less accurate
71 model identified on a tested device would more likely be reflective of the kind as a whole[33,
72 34].

73 Several papers can be found in the literature in which suitable numerical models for
74 reproducing the cyclic behavior of FVD devices are developed. A first class is that of
75 algebraic models that assemble spring-type stiffness elements and dashpot-type damping
76 elements, linearly or nonlinearly proportional to displacement and velocity. The simplicity
77 of these models favoured their widespread use [35, 36, 37], including within international
78 standards (e.g. [38]) and among FVD device manufacturers. However, these models although
79 having the advantage of being simple and requiring low computational efforts are not always
80 accurate in reproducing the hysteretic behavior of control devices as FVD ones. On the
81 contrary, a different class of models overcome the limitations of algebraic ones: they are the
82 differential models of hysteresis. The Bouc-Wen model [39] has been widely used to simulate
83 different hysteretic devices being able to display a broad range of cyclic behaviors (e.g., [40]).
84 Similarly, the LuGre model [41] although originally designed for friction hysteretic devices
85 has been developed to reproduce different dampers (e.g., [42]). Lu et al. [43] introduced

86 an unified differential model containing a number of variants to simulate different types of
87 devices with some applications in the literature (e.g., [44]).

88 Although fluid-viscous dampers have been extensively studied in the literature with var-
89 ious practical applications in the real world, the aspect described regarding mass production
90 and the identification of representative models has not yet been thoroughly investigated. The
91 objective of this research is to close this gap by contributing to the evaluation of a variety
92 of numerical models that have been used to replicate the characteristics of mass-produced
93 fluid-viscous devices. This comparison is based on some dynamic laboratory tests performed
94 on five samples of the same type of fluid-dynamic device. Parameter identification will be
95 carried out through an optimisation procedure based on the use of Genetic Algorithms (GA).
96 The result of the parameters identification for the different models, for the different tested
97 devices, will be then studied through a statistical investigation aimed at determining the
98 most robust and the most accurate numerical model.

99 The paper has the following organization. Section 2 collects the main numerical models
100 involved in the optimization problem. A brief description of each model has been introduced
101 and the main parameters discussed. In Section 3, the samples description and the laboratory
102 equipment used for the dynamic tests has been described. Section 4 is devoted to the
103 mathematical formulation of the problem and the description of the adopted optimization
104 algorithm with regard to the analysis settings and stopping criteria. Section 5 collects
105 the results of the optimization process and shows the optimal hysteresis cycles for each
106 investigated model. In Section 6, the authors provide a statistical analysis in order to identify
107 underlying trends and patterns. All the results derived from the analysis have been organized
108 in Section 7, in which suitable indices concerning with the accuracy and the robustness of
109 each model are adopted. The final discussion including future developments is provided in
110 Section 8. Finally, Appendix A and B report the outputs of the optimization process.

111 **2. Numerical models for fluid viscous dampers**

112 In the literature different numerical models to simulate FVD characteristics can be found,
113 with different formulations (e.g. differential or algebraic equations). Their parameters can
114 be tuned through the definition of a goal (or cost) function to be minimized, within an

115 identification process, until a suitable level of accuracy is achieved.

116 The present section addresses the selection of existing numerical models for FVD starting
117 with rheological models [33], moving up in complexity from the linear viscous damper to
118 generalized models. Next, differential models are presented for simulating the experimental
119 loops of FVDs in terms of force-displacement and force-velocity characteristics.

120 *2.1. Linear viscous model - Law #1*

121 The classic linear viscous model is the simplest way to represent a velocity-dependent
122 mechanical law. When the FVD is modeled in this fashion and given a cyclic input, the force
123 F produced by the relative velocity v with respect to the linear viscous damping parameter
124 C is:

$$F(t) = Cv(t) \quad (1)$$

125 The key benefit of this fundamental model is that it is very straightforward, although occa-
126 sionally it falls short of being an accurate description of actual mechanical behavior. The
127 generalized (nonlinear) viscous model, which relies on a fractional exponent of the velocity
128 rather than a straightforward linear relationship, has updated it as a result. The following
129 section describes the generalized nonlinear viscous model.

130 *2.2. Generalized viscous model - Law #2*

131 The fundamental distinction between the classical and generalized models is that the
132 latter incorporates nonlinearity in the spring and viscous components. Additionally, the
133 coefficients of the generalized models' resistive forces are fractional exponential. The gen-
134 eralized nonlinear viscous model consists of two parameters: the linear viscous damping
135 parameter C and the damping term exponent α , which is usually selected in the range $[0, 1]$.

$$F(t) = C \text{sign}(v(t)) |v(t)|^\alpha \quad (2)$$

136 Distinct values of α are related to different mechanical behaviour. For example, the linear
137 viscous damping law corresponds to $\alpha = 1$; if α assume values close to 0, the friction behavior
138 is seen (consequently, the force increases quickly for small velocity values and becomes almost
139 constant for large velocity values). Due to its effectiveness in predicting structural behavior,

140 this damping law has been widely adopted by different authors. For instance, Banazadeh et
 141 al. [45] research on a comparison of the seismic collapse performances of steel frames equipped
 142 with linear and nonlinear viscous dampers. Additionally, many structural computer routines
 143 and standard guidelines for reproducing FVDs reference to this law.

144 Experimental research, however, showed that the viscous dampers force varies on both
 145 damper relative displacement d and velocity v . Therefore, simulating such mechanical feature
 146 involves joining a spring element and a viscous element, respectively. The family of Kelvin-
 147 Voigt models is created if these two components are connected in parallel. The fundamental
 148 Kelvin-Voigt model, for instance, can be derived with a linear spring and a straightforward
 149 linear dashpot in parallel. Other behaviors are produced when nonlinear springs are coupled
 150 with generalized nonlinear viscous models.

151 Both linear and parabolic models for the elastic force F_e can be expressed as:

$$F_e(t) = K_1 d(t) \quad (3)$$

152

$$F_e(t) = K_2 d(t)^2 + K_1 d(t) + k_0 \quad (4)$$

153 where K_1 is the elastic stiffness, K_2 and K_0 are two constants. The parabolic function
 154 reproduces the test cycle shape better, but the linear function may be preferable because it
 155 is less complicated and generates a similar energy balance [46, 47].

156 2.3. Generalized viscous and linear elastic model - Law #3

157 The equation of motion for a generalized Kelvin-Voigt model subject to a time-varying
 158 force p is obtained by combining Equations 2 and 3:

$$F(t) = K_1 d(t) + C \text{sign}(v(t)) |v(t)|^\alpha \quad (5)$$

159 2.4. Generalized viscous and quadratic elastic model - Law #4

160 The combination of the generalized viscous model in Equation 2 with the parabolic
 161 model for the elastic force in Equation 4 (without considering the contribution of K_0) allows

162 to define the generalized viscous and quadratic elastic force:

$$F(t) = K_1 d(t) + K_2 d(t)^2 + C \text{sign}(v(t)) |v(t)|^\alpha \quad (6)$$

163 *2.5. Generalized Kelvin-Voigt model - Law #5*

164 The models previously described as function of displacement and relative velocity com-
 165 puted with respect to the connection points of the FVD device can be further generalized
 166 by defining the so-called Kelvin-Voigt model. Following this approach, the spring and the
 167 viscous components both involve nonlinearities. Consequently, in this generalized model, the
 168 device force is represented by two exponential components.

$$F(t) = K \text{sign}(d(t)) |d(t)|^\beta + C \text{sign}(v(t)) |v(t)|^\alpha \quad (7)$$

169 *2.6. Bouc-Wen model of hysteresis - Law #6*

170 The Bouc-Wen endochronic model has been chosen to simulate the hysteretic component
 171 of a broad class of control devices since it is perhaps the most eclectic phenomenological
 172 model of hysteretic mechanical systems given by the current literature. This decision is
 173 backed by the model's consistency in both mathematics and physics as well as by the out-
 174 standing agreement between experimental and numerical data [48, 28].

175 The control force generated by the Bouc-Wen model of hysteresis can be defined as

$$F(t) = \alpha K d(t) + (1 - \alpha) K z(t) + C v(t) \quad (8)$$

176 Where $z(t)$ is the variable which allows to define the hysteretic behavior, K is the pre-
 177 yielding stiffness of the model, α is the pre-post yielding stiffness ratio ($\alpha = 1$ is associated
 178 to the elastic response, while $\alpha = 0$ to the perfectly plastic response. To introduce a linear
 179 viscous contribution, it is possible to define a component proportional to velocity according
 180 to the constant C . The hysteretic variable $z(t)$ is introduced by defining the differential
 181 component of the model as follows.

$$\frac{dz(t)}{dt} = A v(t) + \beta v(t) |z(t)|^n - \gamma |v(t)| |z(t)|^{n-1} \quad (9)$$

182 Where A , β , γ , n are constant with time parameters which regulate the the hysteresys cy-
 183 cle shape and amplitude (dissipation level), the unloading branch linearity and the transition
 184 smoothness between the pre-yielding and the post-yielding phase. In detail parameter A is
 185 associated to the initial stiffness, β drives the hysteresis amplitude (the dissipation intensity),
 186 γ regulates the unloading path in the cyclic behavior, while n the transition smoothness. The
 187 efficiency of the model (e.g., stability, consistency, passivity, admissibility) can be verified
 188 by assuming suitable values for the parameters, while the identification procedure for the
 189 characteristic parameters can be simplified assuming $\beta = \gamma$, $A = n = 1$ [49, 28].

190 2.7. Unified model for visco-elastic type devices - Law #7

191 For simulating the hysteretic behavior of various types of dissipative devices, Lu et al.
 192 [43] proposed a unified analysis model with the internal variable $z(t)$. The model may imitate
 193 many device kinds by giving the internal variable various physical meanings. The summary
 194 of the viscoelastic type is presented in this section, while friction dampers are the subject of
 195 the following one. The visco-elastic control force associated to the damper is computed as
 196 follows.

$$197 \quad F(t) = K \operatorname{sign}(z(t)) |z(t)|^\beta \quad (10)$$

$$\frac{dz(t)}{dt} = -\operatorname{sign}(z(t)) \left(\frac{K |z(t)|^\beta}{C} \right)^{\frac{1}{\alpha}} + v(t); \quad (11)$$

198 Where K is the stiffness of the elastic component, C the damping coefficient of the viscous
 199 element, α and β exponential coefficients.

200 2.8. Unified model for friction type devices - Law #8

201 The force-displacement characteristic of the device interested in reproducing in the present
 202 study, with a rapid transition between elastic and plastic phases, coupled with a rectangular-
 203 trapezoidal shape of hysteretic cycles, suggests that dissipative models normally used for
 204 friction control devices should also be considered in the identification process. So this sec-
 205 tion and the next are devoted to two types of models for friction type devices found in the
 206 literature. For simulating the friction dissipative control force, Lu et al. [43] reported the
 207 following unified analysis model.

$$F(t) = z(t) \mu N \quad (12)$$

$$\frac{dz(t)}{dt} = \left(\frac{K}{\mu N} \right) \text{sign}(1 - \text{INT}(|z(t)|) \text{sign}(z(t)v(t)))v(t) \quad (13)$$

209 Where μ is the friction coefficient, N the normal contact force, K the device axial stiffness,
210 $\text{INT}(x)$ is the integer part function of variable x .

211 2.9. LuGre model of hysteresis - Law #9

212 The so-called LuGre model was proposed by Canudas et al. [50] to explain friction
213 dynamics. This model has been then developed applied in a variety of contexts, also different
214 from the type of friction-related devices [41, 51]. The LuGre model is an effective solution
215 to model and control design problems due to its mathematical simplicity and accuracy. The
216 control force exerted by the damper can be computed as follows.

$$F(t) = \beta z(t) + CV(t) + Kd(t) + \epsilon \frac{dz(t)}{dt} + F_0 \quad (14)$$

217

$$\frac{dz(t)}{dt} = v(t) - \alpha |v(t)|z(t) \quad (15)$$

218 Where K is the stiffness parameter, C the damping coefficient, and α , β , ϵ allow to define
219 the cycles characteristics (i.e., amplitude and shape). For the aim of this identification
220 problem ϵ , K and F_0 can be assumed with zero values [51, 41].

221 3. Description of the laboratory tests

222 In the SISMALB srl laboratory in Taranto, Italy, five samples of the same 300kN FVD
223 series (Figure 1) was tested. Table 1 reports the FVDs design conditions.

Table 1: Fluid viscous damper design conditions

F[kN]	Stroke [mm]	d _{design} [mm]	V _{max} [mm/s]	α
292 ± 55	± 55	± 45	500	0.3

224 The SISMALAB test equipment consisted of a high-resistance steel frame that can sup-
225 port loads of 2200 kN in both tension and compression. The FVD was connected by pins to
226 the frame structure and to the actuator, which can be both force- or displacement-controlled
227 (Figure 2).

228 A 2500kN load cell was placed between the actuator and the FVD specimen to measure
 229 the cyclic forces throughout the experimental tests, while a transducer was used to measure
 230 the device movements. In a displacement-imposed test, the device movements are controlled
 231 by a transducer mounted on the device. The control and data acquisition system is able
 232 to generate real-time analysis of device displacements by instantaneous variation of applied
 233 forces by the servant cylinder, by means of a computer automatic control hydraulic pressure
 234 system. The displacement–time history should be imposed with different laws, from sinu-
 235 soidal to triangular as well as ramped or through a generator step of generic ones. Acquiring
 236 systems have 30 channels and can command two actuators at the same time.

237 The FVD specimens were tested using the transducer output as feedback for the closed-
 238 loop displacement control. More in detail, the database is composed of 24 tested devices
 239 with the same properties and set-up adopted during the test. The laboratory provided the
 240 results of the pressure test related to 5 devices that were identified with a specific ID serial
 241 number. For each device, the test has been performed by applying an internal pressure over
 242 275 bar and has been maintained for at least 120s. Moreover, all tests considered in the
 243 experimental campaign have been carried out by applying six sinusoidal cycles characterized
 244 by a specific ratio of the maximum velocity corresponding to low velocity (lower than 1%),
 245 1%, 25%, 50%, 75% and 100% of the maximum design value. Finally, for each device,
 246 the damping efficiency at 100% of the maximum velocity has been tested by applying 10
 247 sinusoidal cycles. Table 2 reports the test conditions for the five selected FVD specimens with
 248 specific regard to the last configuration in order to assess the damping capacity. Maximum,
 249 $Load_{max}$, and minimum load, $Load_{min}$, experienced during the test as well as maximum,
 250 d_{max} , and minimum displacement, d_{min} , have been reported. Consistent with what was said
 251 before, tests selected by the database consisted of 10 sinusoidal cycles with 40mm amplitude
 252 at 0.5m/s.

Table 2: Fluid viscous damper test conditions

No.	$Load_{max}/Load_{min}$ [kN]	d_{max}/d_{min} [mm]	Cycles
$Test_1$	327/-317	44/-43	10
$Test_2$	318/-308	45/-43	10
$Test_3$	267/-259	44/-43	10
$Test_4$	289/-282	43/-42	10
$Test_5$	283/-284	44/-42	10

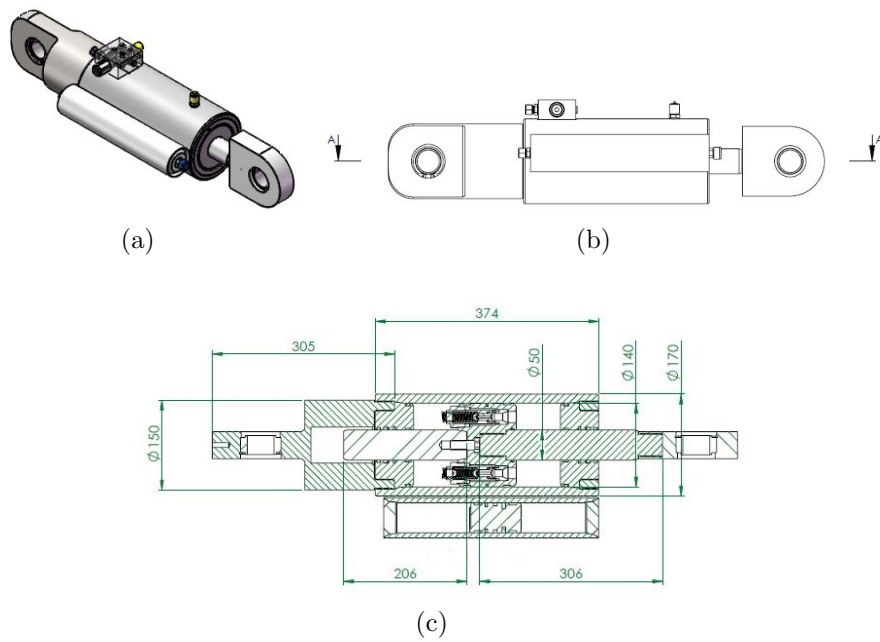


Figure 1: 3D model of FVD test device (a); Longitudinal view of the FVD test device (b); Trasversal section of FVD test device (c)



Figure 2: FVD specimen on the test facility at SISMALAB

253 4. Identification scheme and analysis setting

254 In this section, the mathematical formulation of the optimization problem is introduced.
 255 Specifically, the Objective Function (OF) and the design variable are defined. Therefore,
 256 both the setting criteria and the Genetic Algorithm (GA) for the optimization process are
 257 illustrated.

259 The parameters identification requires to solve an unconstrained optimization problem
260 and a suitable objective function (OF) has to be minimized.

261 All the selected constitutive models depend on different parameters, which become the com-
262 ponents of the design variable vector x . The model parameters are identified by solving the
263 following single-objective optimization problem:

$$\text{Minimize } W = f(x_i), \quad i = 1, \dots, n \quad (16)$$

$$\text{Subject to } x_i^l \leq x_i \leq x_i^u \quad (17)$$

265 where $x = \{x_1, \dots, x_j, \dots, x_n\}$ represents a set of real parameters which depends on the adopted
266 numerical model. The size of this vector, hence, is related to the number of variables which
267 govern the constitutive law. The $x^l = \{x_1^l, \dots, x_j^l, \dots, x_n^l\}$ and $x^u = \{x_1^u, \dots, x_j^u, \dots, x_n^u\}$ vectors
268 represent the lower and upper bounds of x , respectively. In this way, the domain of feasible
269 solutions is restricted to a sub-domain in which x can assume the values between x^l and x^u .
270 For the aim of this research, a suitable OF has been defined in the integral form as follows:

$$f(x) = \frac{1}{\sigma_{p_m}(t_{end} - t_{start})} \int_{t_{start}}^{t_{end}} abs(p_m - p_e(x)) dt \quad (18)$$

271 where t_{start} and t_{end} are the start and end time records, respectively, and $p_m(t)$ is the exper-
272 imentally measured force, while $p_e(t)$ is the estimated force.

273 The Matlab ODE45 function has been used for solving the (nonstiff) ordinary differential
274 problem. The procedure implemented by the solver is the Runge-Kutta method. The fixed
275 time step for the solution procedure is the sampling time step adopted during the laboratory
276 test. This procedure was adopted for only the differential laws (from law#6 up to law#9)

277 In order to reduce the computational effort during the optimization process or avoid
278 critical values generating numerical noises, lower, x^l , and upper, x^u , bounds for each de-
279 sign variable are carefully fixed, detecting more suitable ranges of variability. In this way,
280 no interest solutions are discarded, and the efficient exploration phase of the algorithm is
281 guaranteed.

283 In order to solve the problem stated in the previous section, a real-coded GA is adopted.
 284 This is a population-based stochastic optimization technique appropriate for global optimiza-
 285 tion, which does not require direct evaluation of gradients. Introduced by John Holland [52],
 286 it is inspired by Charles Darwin's theory of natural evolution. This algorithm reflects the
 287 process of natural selection where the fittest individuals, also called parents, are selected for
 288 reproduction in order to produce offspring of the next generation. At the end of the process,
 289 the best survival among all the fittest candidates found at each generation is selected as the
 290 best globally optimized solution. Although the native GA worked with binary values rep-
 291 resenting genes, encoded in string structures called chromosomes, in this paper the authors
 292 overcome the limits related to the decoding process by using a real-code GA in which genes
 293 and chromosomes represent directly the design variables and the solutions of the problem.

294 At each iteration, traditional GA phases were adopted during the optimization as follow-
 295 ing:

- 296 • **Initial population:** in this phase, individuals with a set of random genes (x_i) com-
 297 posing chromosomes (x) are created by observing lower and upper bounds reported in
 298 table 4. Gene represents, at each generation, the candidate value of a specific design
 299 variable involved in the identification procedure. A set of genes (vector form) represent
 300 a solution of the problem for the current generation. In this way, the best solution is
 301 selected and the optimal set of parameters which govern that specific law is detected.
- 302 • **Fitness function:** in this phase, the fitness of the candidate solutions is evaluated by
 303 calculating the OF introduced in the previous section 4.1.
- 304 • **Selection:** During this phase, a Roulette Wheel Selection was implemented in order
 305 to guarantee that the two fittest parents are selected for the next steps. Adopting this
 306 technique, a probability to each parent is assigned and the parents with higher fitness
 307 are more likely to be chosen for the next steps.
- 308 • **Crossover:** in this phase, a uniform random crossover was performed in which re-
 309 combination of gene pool between parents is performed in a random way. Lower and
 310 upper bounds are imposed at this stage such that if only a gene of the new offspring

311 is not ranged within the imposed interval (higher than the maximum value or lower
 312 than the minimum value of that specific parameter), it is forced to assume maximum
 313 or minimum value, respectively.

314 • **Mutation:** aiming to improve the exploration and exploitation ability of the algo-
 315 rithm, a mutation rate of 1% is assigned. In this way, new genes are introduced into
 316 the population by modifying the gene pools of parents in a random way.

317 At the end of these stages, a sorted function was implemented aiming to store survivors
 318 with the best fitness among the generated offspring, at each generation. For this purpose,
 319 an elitism algorithm was adopted in order to copy the best chromosome of the previous
 320 population to the new one. The identification procedure can be considered ended when
 321 the stopping criteria of the algorithm are satisfied and the optimal set of parameters for
 322 each law is found. The entire procedure was run n times in order to check the reliability
 323 and robustness of the algorithm. Specifically, the authors observed that stagnation usually
 324 occurred for the algebraic models while no differential ones needed to use all the available
 325 200 generations.

326 For clarity and repetitive purposes, the settings of the adopted algorithm and the stopping
 criteria are listed in table 3.

Table 3: Setting of the GA for each law

Pop. size	N.Generation	Evaluation	N. Run	Stagnation
200	200	40.000	20	20

327
 328 Though a maximum of generation equal to 200 is set, stopping criteria occurs if OF value
 329 remains the same for 20 subsequent generations. In this way, the computational effort was
 330 reduced if stagnation is recognized.

331 In Table 4, the design variables of the optimization problem with their corresponding
 332 lower and upper bounds are reported.

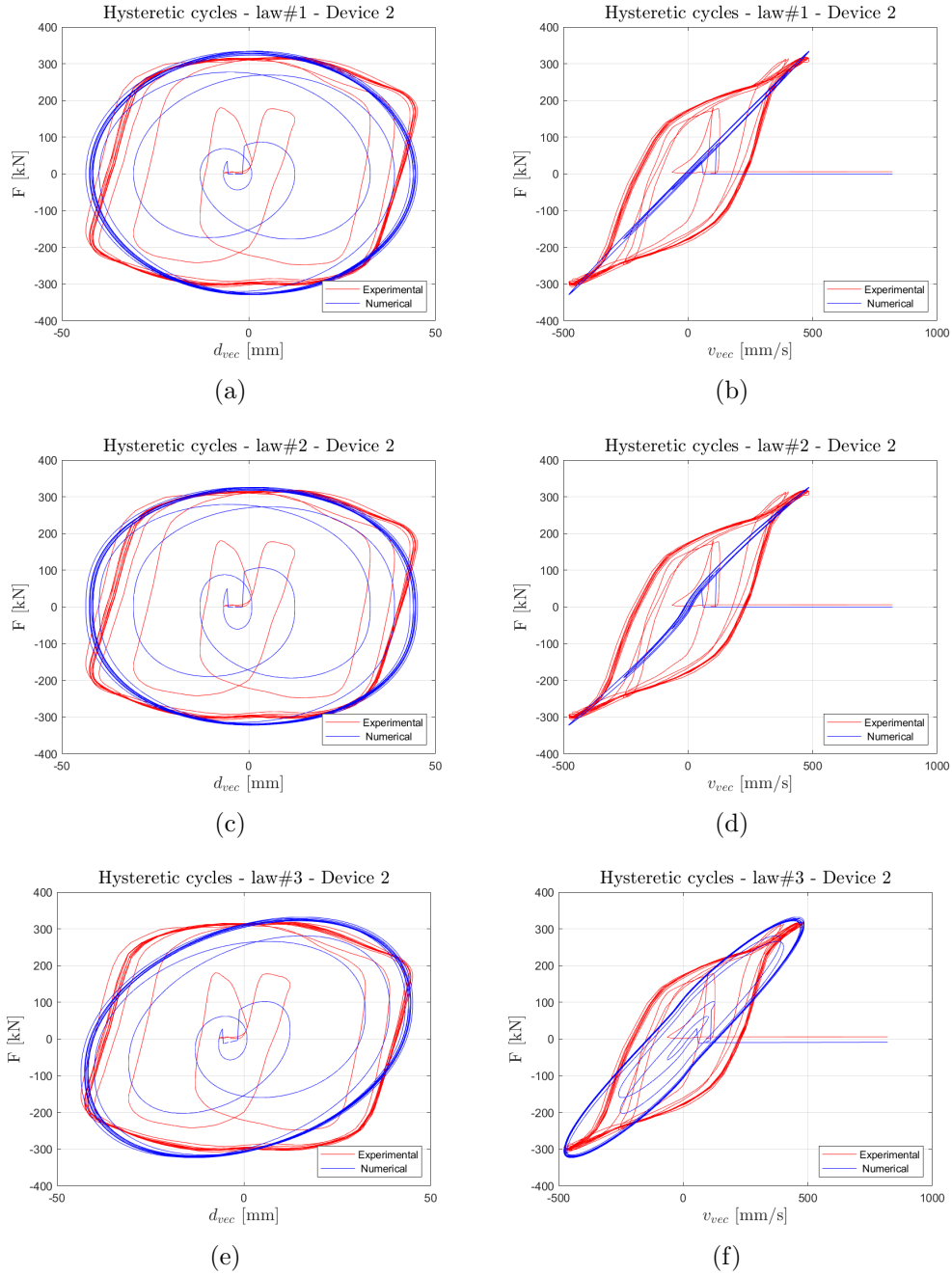
333 5. Parameter identification results and cyclic behavior reproduction

334 In this section, the performance of the selected numerical models in reproducing the
 335 cyclic behavior of the tested FVDs has been analysed. Appendix A reports the mean and
 336 the standard deviation values of the design variables and the OFs for each investigated law.

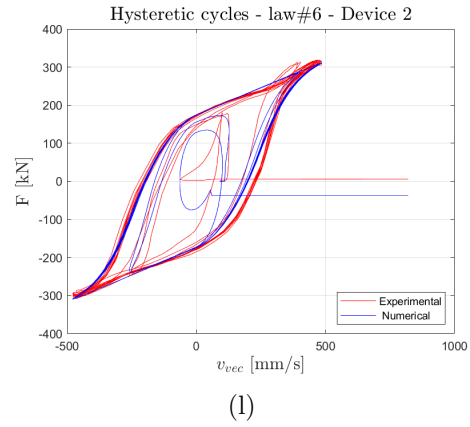
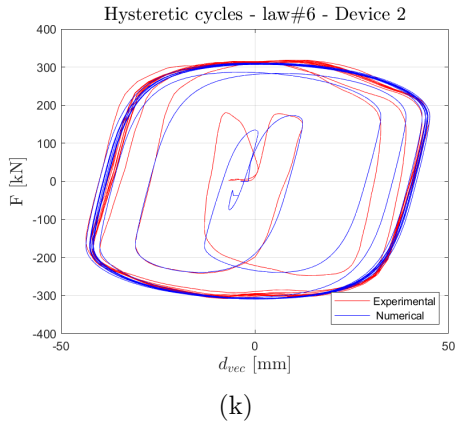
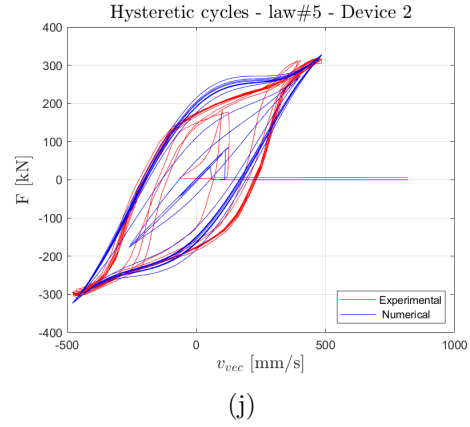
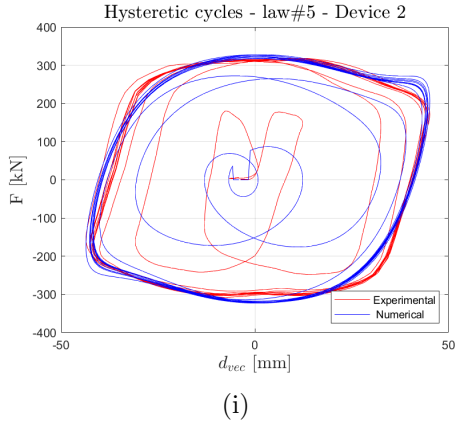
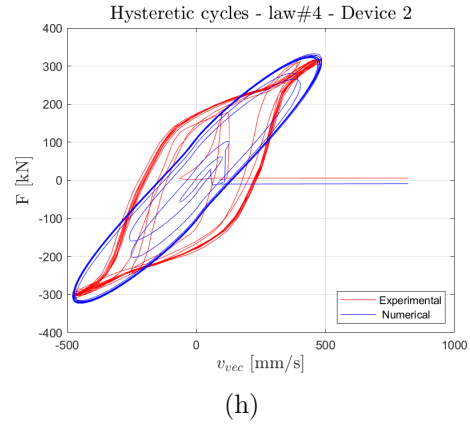
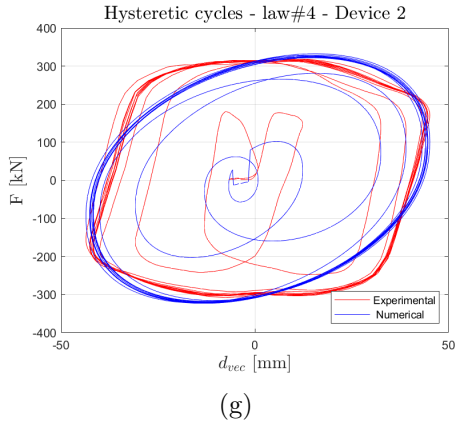
Table 4: Lower and upper bounds of models' parameters

		x	
		x^l	x^u
Law #1	C [kN/(mm/s)]	0.0	2.0
Law #2	C [kN/(mm/s)]	0.0	2.0
	α [-]	0.0	1.0
Law #3	C [kN/(mm/s)]	0.0	2.0
	α [-]	0.0	1.0
	K_1 [kN/mm]	0.0	200.0
Law #4	C [kN/(mm/s)]	0.0	2.0
	α [-]	0.0	1.0
	K_1 [kN/mm]	0.0	300.0
	K_2 [(kN/mm) ²]	0.0	300.0
Law #5	C [kN/(mm/s)]	0.0	2.0
	α [-]	0.0	1.0
	K [kN/mm]	0.0	1000.0
	β [-]	0.0	5.0
Law #6	C [kN/(mm/s)]	0.0	0.3
	K [kN/mm]	30.0	100.0
	α [-]	0.0	1.0
	β [1/mm]	0.0	5.0
Law #7	C [kN/(mm/s)]	3.0	10.0
	α [-]	0.05	0.3
	K [kN/mm]	50.0	200.0
	β [-]	0.2	0.4
Law #8	C [kN/(mm/s)]	0.1	0.3
	K [kN/mm]	10	50
	μ [-]	0.1	1
	N [kN]	250	450
Law #9	C [kN/(mm/s)]	0.0	0.3
	α [1/mm]	0.0	2.0
	β [kN/mm]	20	100

337 Because of the reasonably high accuracy of the results in terms of OF, Test 2 has been
 338 considered to compare the hysteresis cycles of the different models against the laboratory
 339 tests. To avoid burdening the results description of the plots carried out by the optimization
 340 process, Appendix B reports the comparison between experimental and numerical force time-
 341 histories (F vs t_{vec}) for each model with specific regards to device 2.



342 Figure 3 depicts the comparison of the force-displacement and force-velocity cycles for
 343 the nine numerical models considered against the device 2 laboratory results. A macroscopic
 344 difference between the first five models and the next four ones in reproducing the physical



345 phenomenon is highlighted.

346 The first five algebraic models composed by different elementary features (elastic and
 347 damping elements with different characteristics) are the simplest and the most computation-
 348 ally inexpensive. The first two models (Fig.s 3 a-d) are evidently the least efficient in repro-
 349 ducing the cyclic response of the device: the exclusive presence of the damping component in
 350 the laws does not allow to reproduce the asymmetric shape of the force-displacement char-
 351 acteristics. Focusing on the force-velocity cycles, again the exclusive velocity proportional
 352 component of the force, with linear or nonlinear expression, still fails greatly in reproducing

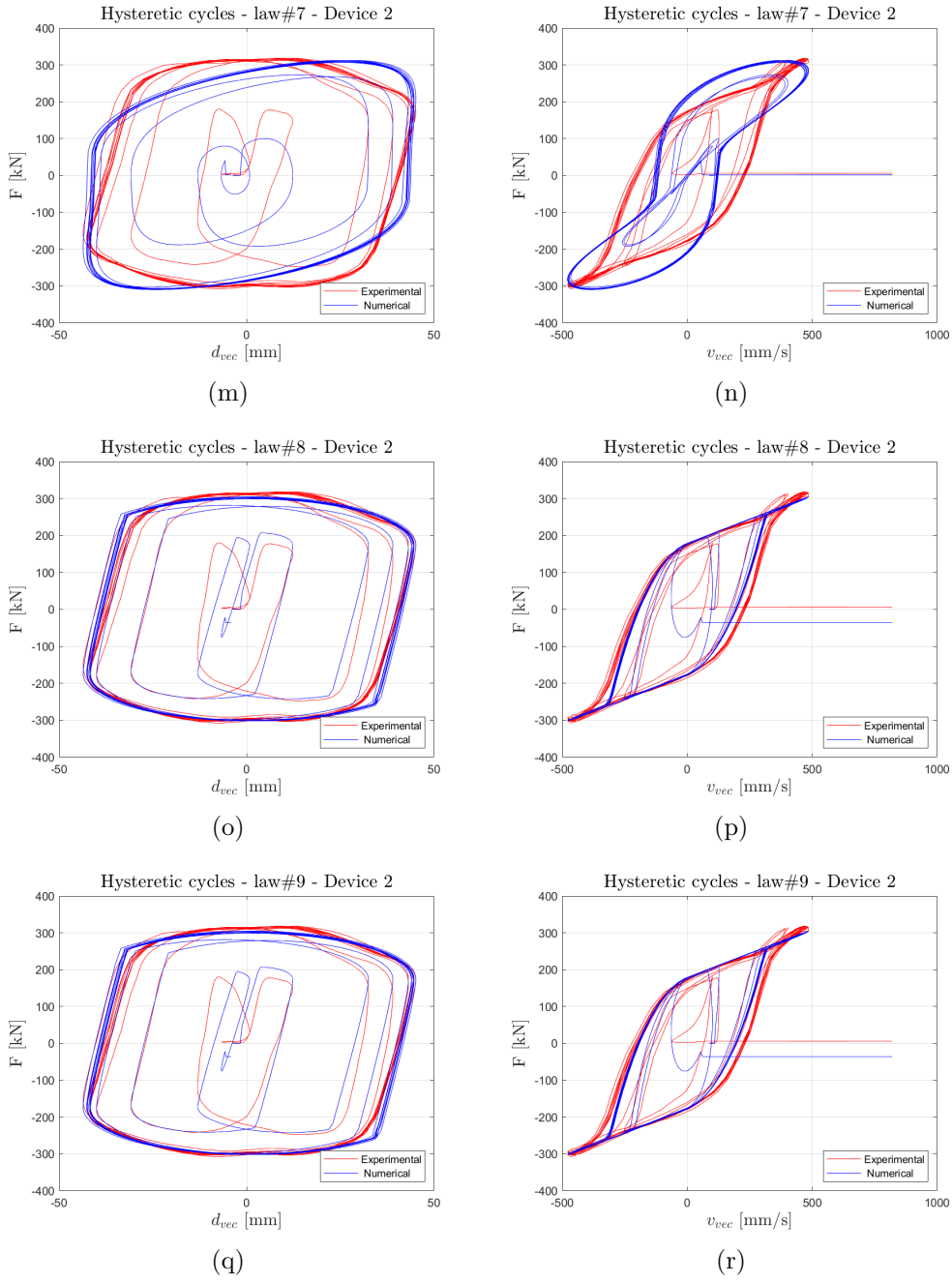


Figure 3: Comparison between theoretical and experimental force-displacement relationships (referred to Fig.s (a)-(c)-(e)-(g)-(i)-(k)-(m)-(o)-(q)) and force-velocity relationships (referred to Fig.s (b)-(d)-(f)-(h)-(i)-(l)-(n)-(p)-(r)) for law 1 up to law 9 with specific regard to Test 2

353 the response.

354 From the third model to the fifth (Fig.s 3 e-j), due to the introduction of the elastic
355 elements, an improvement in reproducing the cycles is observed. However, only model num-
356 ber five (Fig.s 3 i-j) succeeds for the first time in qualitatively following the shape of both
357 force-displacement and force-velocity cycles.

358 Focusing on the response of the last four models (Fig.s 3 k-r), an improved efficiency
359 in simulating the behaviour of FVD devices is highlighted. However, this positive outcome
360 is achieved at the price of a higher computational burden. Indeed, differential models are
361 substantially more complex to process in terms of computational time than algebraic models.

362 However, the model related to Law #7 (Fig.s 3 m-n) does not appear satisfactory in its
363 ability to reproduce the characteristic of FVDs, even from a qualitative observation, either as
364 force-displacement or velocity-displacement. The optimization process has never been able
365 to improve this result, despite different attempts.

366 The friction model (Fig.s 3 o-r) is qualitatively the most accurate in simulating the
367 cyclic response of the FVDs. The Bouc-Wen model, probably the most versatile among the
368 literature options (Figures 3 k-l), also produces a satisfactory qualitative result, equivalent
369 to the ones from the friction models.

370 Generally, it is evident that none of the algebraic models is capable of reproducing the
371 transient branch of hysteresis cycles at the beginning and final phases. On the contrary,
372 among differential models, law#6, law#8 and law#9 exhibit a good grade of match not only
373 at the full-capacity phase but also during the transient one. This result is demonstrated by
374 the comparisons between experimental and numerical F vs t_{vec} curves reported in Appendix
375 B.

376 6. Statistic analysis and discussion

377 In this section, the collected data from the optimization process for the adopted numerical
378 models have been examined in order to identify underlying trends and patterns.

379 Fig.4 reports the bar graphs of the OF mean distribution and standard deviation for
380 each tested FVD device considering the nine numerical models of hysteresis. In detail,
381 the graphs represent the accuracy (Fig.4-(a)) of the models in reproducing the laboratory

382 tests, and their robustness (Fig. 4-(b)), respectively. In detail, Fig.4-(a) highlights the
383 laws that are able to reduce the difference, or the error, between the variable that has
384 been measured in the laboratory and that one computed as the optimal numerical solution.
385 A low value of OF for a specified law means that the corresponding model is capable of
386 fitting the experimentally measured force with reasonable accuracy. The best performers
387 are represented by the differential laws, in particular Laws #8 and #9. However, the higher
388 accuracy is obtained at the cost of higher computational burden, as already discussed, but
389 mostly at the cost of lower robustness (higher standard deviation of, e.g., Laws #8 and #9,
390 with respect to Law #1). This means that the law that is most accurate in reproducing the
391 single physical phenomenon (i.e., the specific FVD) is also the one that shows the highest
392 variability among samples of the devices in the same production series. However, it is worth
393 noting that the intensity of the standard deviation is still small compared to the mean
394 intensity. In order to emphasize the trend of standard deviation σ among the investigated
395 laws, a logarithmic scale was adopted in Fig.4-(b). As expected, by increasing the complexity
396 of the model from the algebraic to differential models, the dispersion index σ becomes higher
397 except for Law #9 for which standard deviation, especially for Device #3, #4 and #5, assumes
quite moderate values. Once that preliminary considerations are pointed out concerning

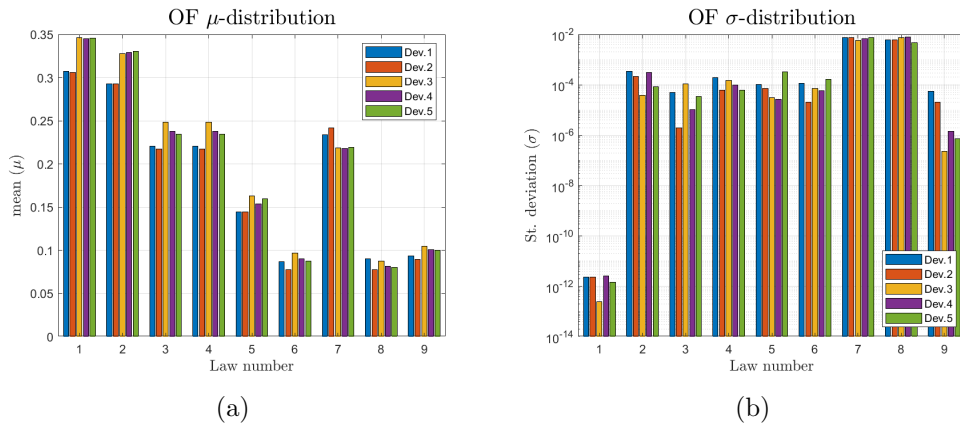


Figure 4: OF mean (μ) distribution for each test devices (a); OF standard deviation (σ) distribution for each test devices (b). For each law, ($X - axis$), different colours are used for detecting the corresponding device.

398
399 with the global accuracy and robustness of each model, the authors focused on identifying
400 the dispersion degree of each parameter for each law. In this way, a measure of the parameter
401 which mostly affects the overall robustness can be assessed.

402 The models performance observed in terms of OF in Fig. 4 can be deepened by Figs 5-

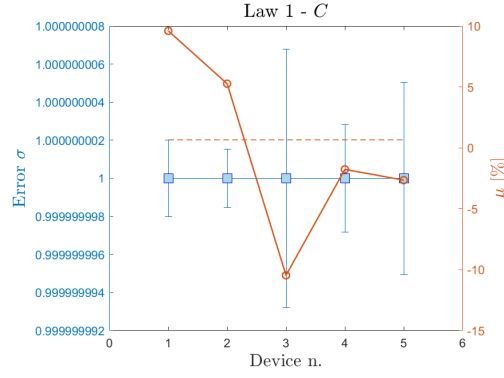


Figure 5: Normalized standard deviation and mean distribution of the identified damping C for the law 1. The left $y - axis$ shows the error bar of the standard deviation (σ), while the right $y - axis$ the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

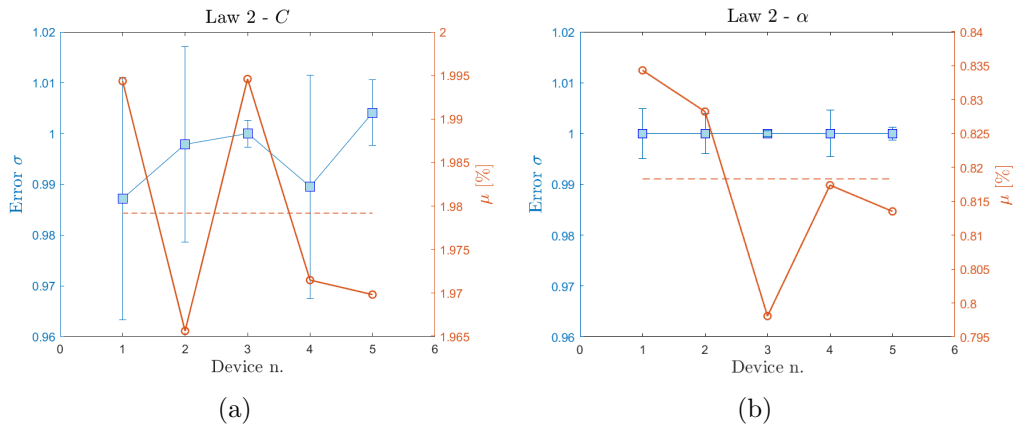


Figure 6: Normalized standard deviation and mean distribution of the identified damping C (a) and control parameter α (b) for the law 2. The left $y - axis$ shows the error bar of the standard deviation (σ), while the right $y - axis$ the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

403 13. They present the statistics of the identified parameters, for the different models adopted
 404 and for the tested FVDs. In particular, each graph shows a double vertical axis, on the left
 405 the standard deviation of the considered parameter with respect to the mean (error bar of
 406 the standard deviation - $Error \sigma$) is reported, while on the right axis each figure shows the
 407 percentage variation of the mean of the considered parameter (calculated over the range of
 408 20 performed optimization runs) with respect to the reference mean value (red dotted line)
 409 evaluated as the mean of all means value for each device.

410 Due to the fact that optimal parameters values obtained among the 20 runs show null
 411 variability, the authors decided to calculate the standard deviation σ with respect to the
 412 mean value of all runs for each device. This is demonstrated by the fact that all $Error$
 413 σ bars are centred to value 1 with the exception of Law#2 in which a tiny variability of

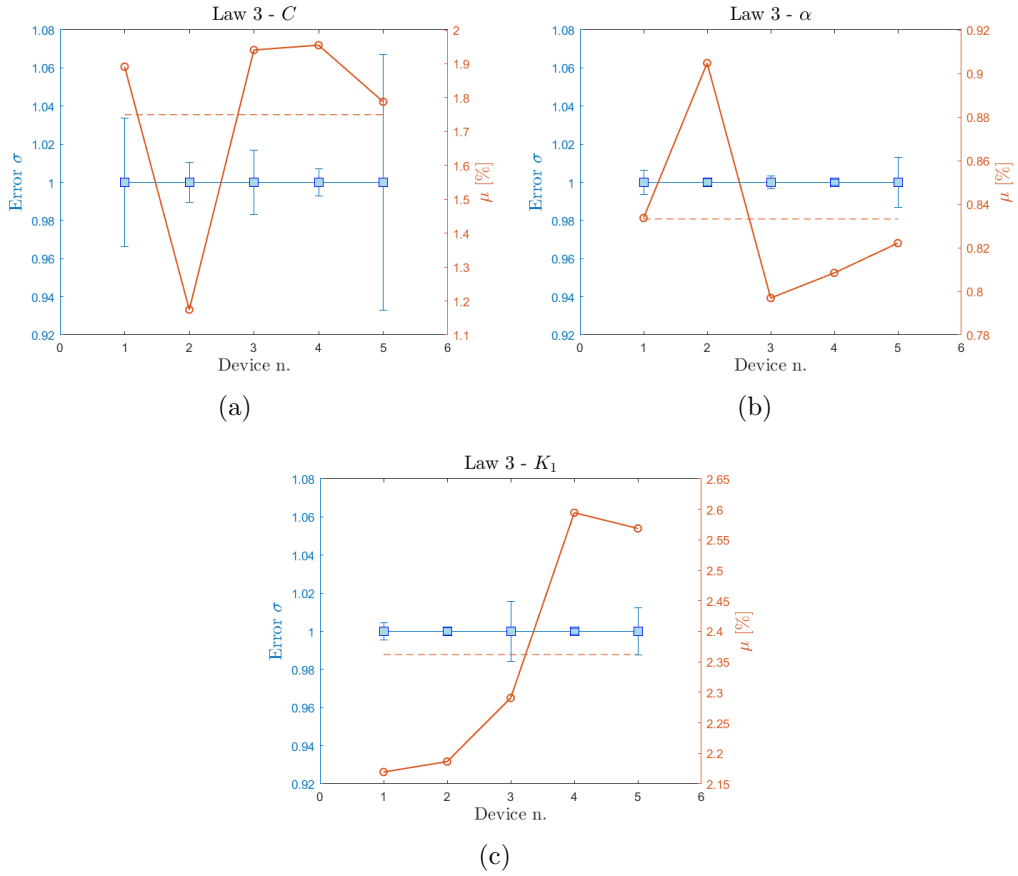


Figure 7: Normalized standard deviation and mean distribution of the identified damping C (a), control parameter α (b) and stiffness parameter k_0 (c) for the law 3. The left y - axis shows the error bar of the standard deviation (σ), while the right y - axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

414 parameter C can be recognized (see Fig.6-(a))

415 Therefore, the intensity scale of $Error \sigma$ belonging to the same law has been uniformed.

416 However, because of the high variability of μ parameter, right vertical axis, among different

417 plots of the same law, it has not been possible to uniform. In this way, the identification of

418 parameter(s) that mostly affects the overall dispersion of the law has been facilitated.

419 According to the trends observed in Fig.4-(b), $Error \sigma$ range of values, represented by

420 maximum and minimum values, increases from 1.0/0.92 (Law#1) up to 2.5/-0.5 (Law#5)

421 with specific regard to algebraic laws.

422 The same trend is observed with respect to differential laws (Law#6 up to Law#9).

423 However, differently from algebraic laws in which only one parameter is responsible for the

424 entire dispersion of the law, several parameters show significant variability. In this sense,

425 interesting considerations can be revealed by comparing Fig.10 and Fig.11. Though the range

426 variability decreases from Law#6 to Law#7, the number of parameters with a significant

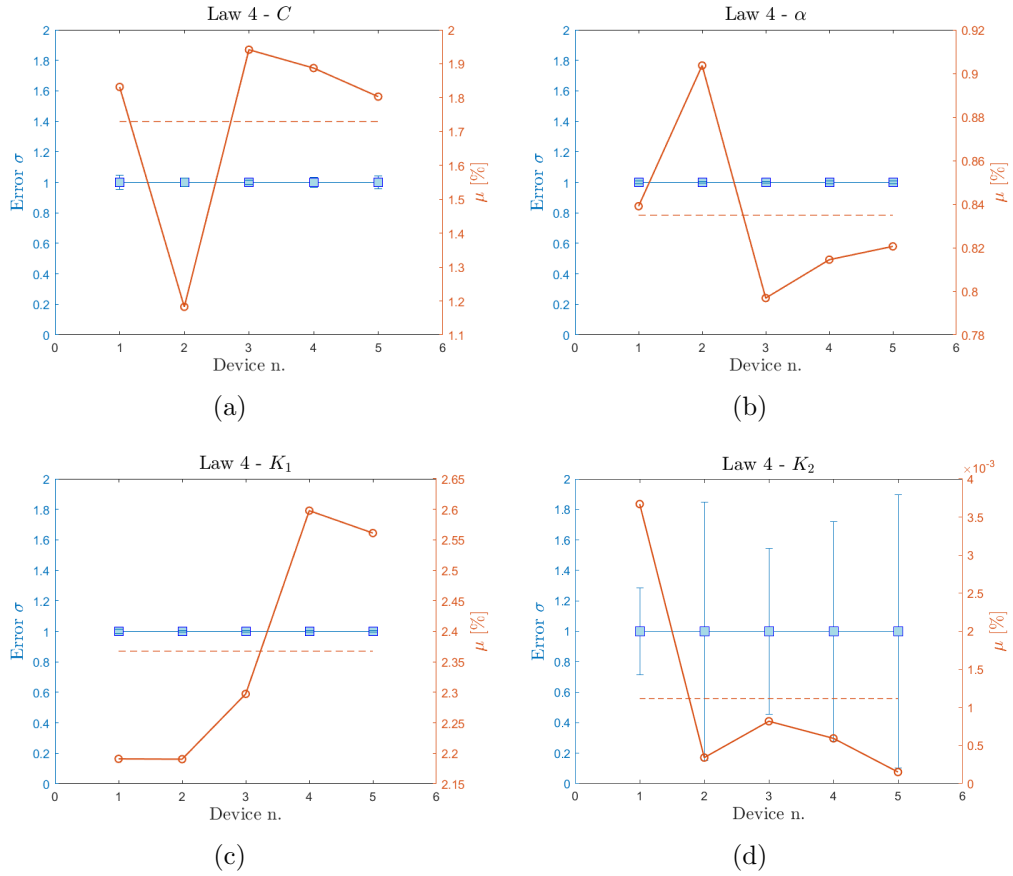


Figure 8: Normalized standard deviation and mean distribution of the identified damping C (a), control parameter α (b), stiffness parameter k_0 (c) and constant coefficient k_1 (d) for the law 4. The left y – axis shows the error bar of the standard deviation (σ), while the right y – axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

427 error bar, for each device, increases. Compared with Law#6 in which the dispersion is
 428 entirely concentrated into parameter C , in Law#7 each parameter exhibits not-negligible
 429 value of the $Error \sigma$. That is also reflected by observing the growth in Fig.4-(b) in which
 430 an overall increase of σ is detected for each device. Finally, growth trend is interrupted by
 431 Law#9 for which dispersion among parameters has been reduced dramatically.

432 If the $Error \sigma$ trend was clearly identified by the authors, several discrepancies between
 433 the general behaviour of OF bar histograms (Fig.s 4(a),(b)) and the percentage variation μ
 434 plots for each device (law Fig.s 5-13) have been detected. However, the authors are convinced
 435 that the high variability of parameters can be addicted to the capability of a model to fit
 436 with experimental outputs. High is the variability μ of a specific parameter, high is the
 437 ability to capture differences between devices. As proof of what has been said, the highest
 438 variability is recognized for differential models, especially for Law#7, Law#8 and Law#9.

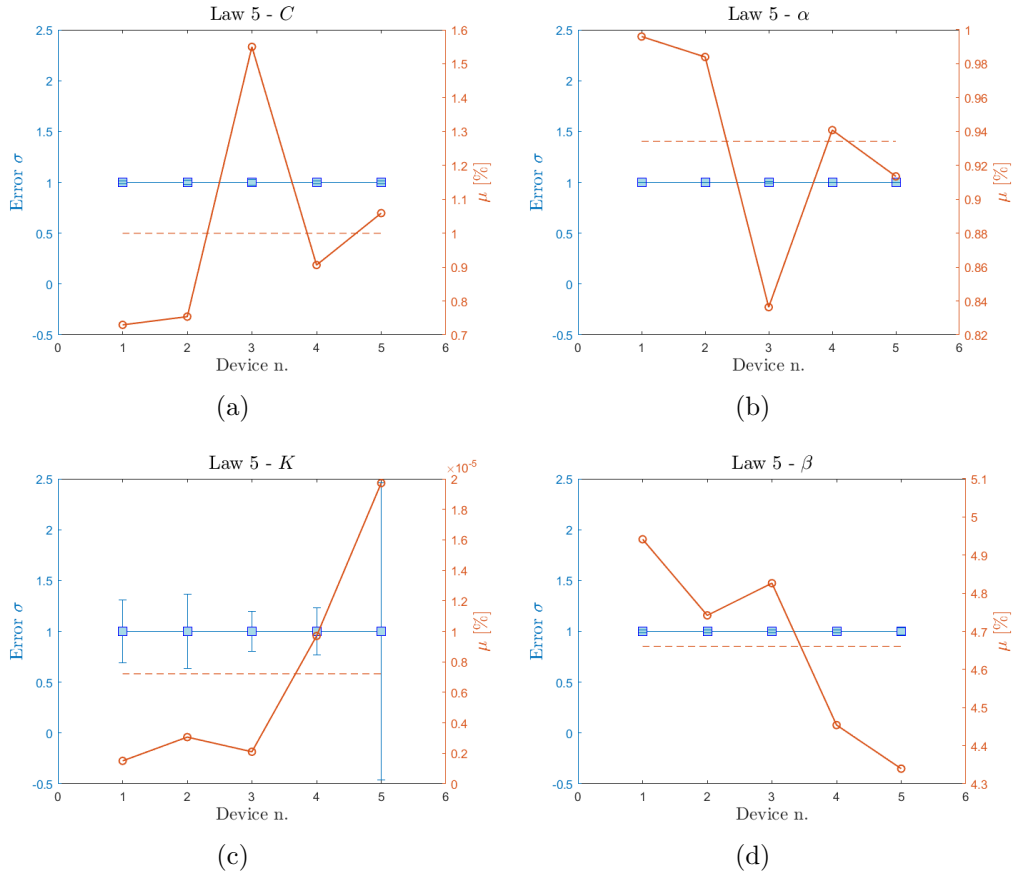


Figure 9: Normalized standard deviation and mean distribution of the identified damping C (a), exponent parameter α (b) and β (d) and stiffness parameter k_0 (c) for the law 5. The left y – axis shows the error bar of the standard deviation (σ), while the right y – axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

439 The high variability of μ , conceived as a good feature for a model in capturing changes in
 440 behaviour among the same devices, seems to fail when algebraic laws are treated. Indeed,
 441 though the variability of μ , especially for Law#1, results to be one of the highest among
 442 all sets of investigated models, the accuracy is not affected by sensible variation of this
 443 variable since the natively poor complexity of the models. In other words, the variability of
 444 parameters must not be allowed to outweigh the inner inefficiency of the model.

445 7. Pareto Front representation

446 In this section, the authors point out mathematical considerations to identify the best
 447 performer among the investigated models. As introduced in the previous Section 6, the
 448 goodness of each model has been evaluated in terms of accuracy (OF) and robustness (σ).
 449 If the former can be still assumed as representative of the global capability of a model to fit
 450 with experimental outputs, the latter gives a measure of how a single parameter can affect

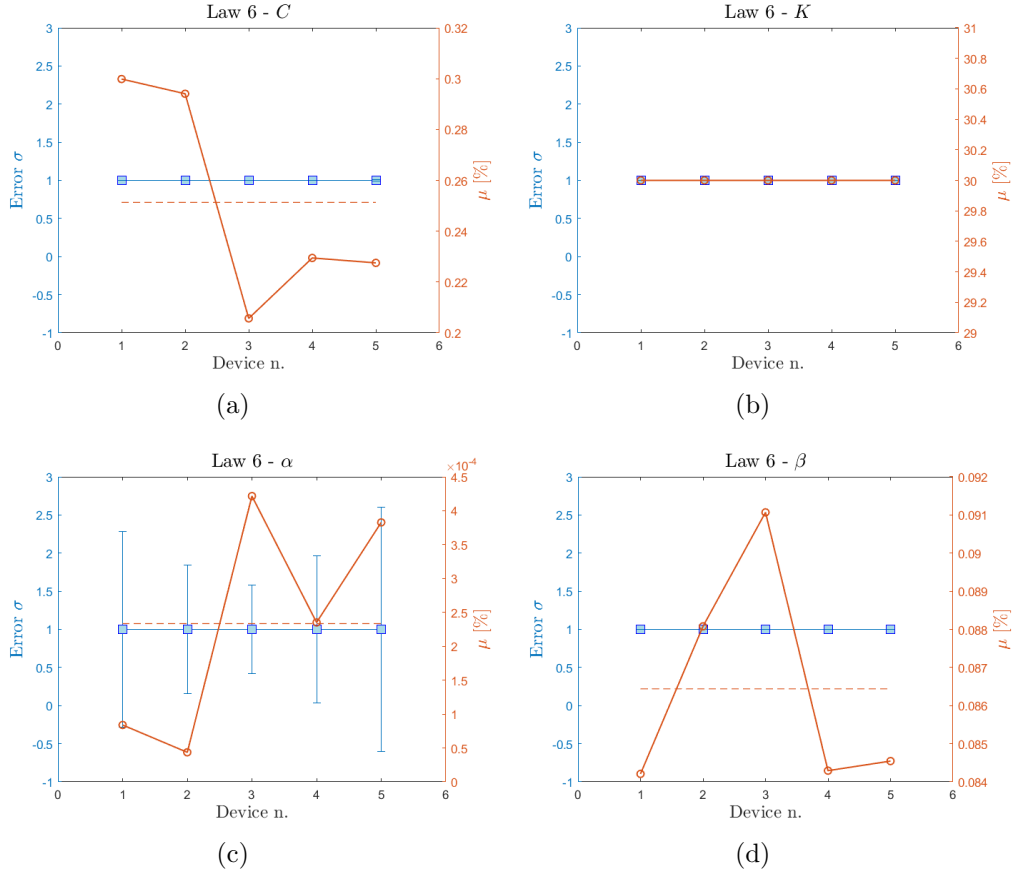


Figure 10: Normalized standard deviation and mean distribution of the identified damping C (a), stiffness parameter k_0 (b) and exponent parameter α (c) and β (d) for the law 6. The left y – axis shows the error bar of the standard deviation (σ), while the right y – axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

451 a specific model without any information about the overall robustness of that model.

452 For this reason, the authors introduce a new index β and the following mathematical
 453 procedure is adopted for its calculation.

454 At first, a matrix representation of parameters set for each law is derived in the following
 455 form:

$$\alpha = \left(\begin{array}{c} \left[\begin{array}{cccc} \alpha_{11}^{(1)} & \alpha_{12}^{(1)} & \dots & \alpha_{1n}^{(1)} \\ \alpha_{21}^{(1)} & \alpha_{22}^{(1)} & \dots & \alpha_{2n}^{(1)} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{m1}^{(1)} & \alpha_{m2}^{(1)} & \dots & \alpha_{mn}^{(1)} \end{array} \right] \left[\begin{array}{cccc} \alpha_{11}^{(2)} & \alpha_{12}^{(2)} & \dots & \alpha_{1n}^{(2)} \\ \alpha_{21}^{(2)} & \alpha_{22}^{(2)} & \dots & \alpha_{2n}^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{m1}^{(2)} & \alpha_{m2}^{(2)} & \dots & \alpha_{mn}^{(2)} \end{array} \right] \dots \left[\begin{array}{cccc} \alpha_{11}^{(j)} & \alpha_{12}^{(j)} & \dots & \alpha_{1n}^{(j)} \\ \alpha_{21}^{(j)} & \alpha_{22}^{(j)} & \dots & \alpha_{2n}^{(j)} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{m1}^{(j)} & \alpha_{m2}^{(j)} & \dots & \alpha_{mn}^{(j)} \end{array} \right] \end{array} \right) \quad (19)$$

456 Vector α collects all parameters ($\alpha_{m,n}$) in matrices. Each matrix is representative of a specific
 457 law (j).

458 The first index, m , and the second index, n , of the generic identified parameter $\alpha_{m,n}$ represent

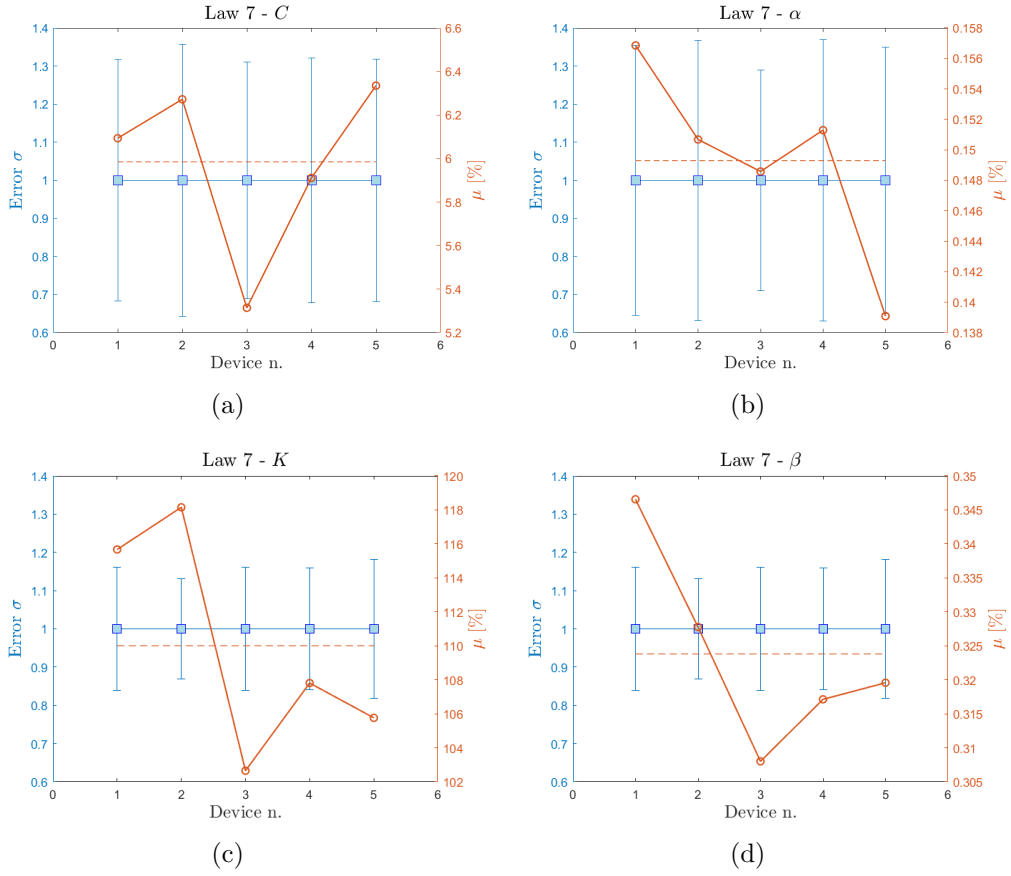


Figure 11: Normalized standard deviation and mean distribution of the identified damping C (a), exponent parameter α (b) and β (d) and stiffness parameter k_0 (c) for the law 7. The left y – axis shows the error bar of the standard deviation (σ), while the right y – axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

459 the device number and the parameter number, respectively. Dimensions of n , hence, the size
460 of each component of the vector α , depends on the considered constitutive law. Moreover,
461 apex $j = [1, 2, \dots, 9]$ represents the different constitutive laws of the investigated numerical
462 models.
463 For instance, an explicit representation of vector α , according to the symbology adopted for
464 each parameter's law and introduced in Table 4, assumes the following form:

$$\alpha = \left(\begin{array}{c} \left[\begin{array}{c} C_{dev.1}^{(1)} \\ C_{dev.2}^{(1)} \\ C_{dev.3}^{(1)} \\ C_{dev.4}^{(1)} \\ C_{dev.5}^{(1)} \end{array} \right] \left[\begin{array}{c} C_{dev.1}^{(2)} \\ C_{dev.2}^{(2)} \\ C_{dev.3}^{(2)} \\ C_{dev.4}^{(2)} \\ C_{dev.5}^{(2)} \end{array} \right] \alpha_{dev.1}^{(2)} \\ \dots \\ \left[\begin{array}{c} C_{dev.1}^{(9)} \\ C_{dev.2}^{(9)} \\ C_{dev.3}^{(9)} \\ C_{dev.4}^{(9)} \\ C_{dev.5}^{(9)} \end{array} \right] \left[\begin{array}{c} \alpha_{dev.1}^{(9)} \\ \alpha_{dev.2}^{(9)} \\ \alpha_{dev.3}^{(9)} \\ \alpha_{dev.4}^{(9)} \\ \alpha_{dev.5}^{(9)} \end{array} \right] \beta_{dev.1}^{(9)} \end{array} \right) \quad (20)$$

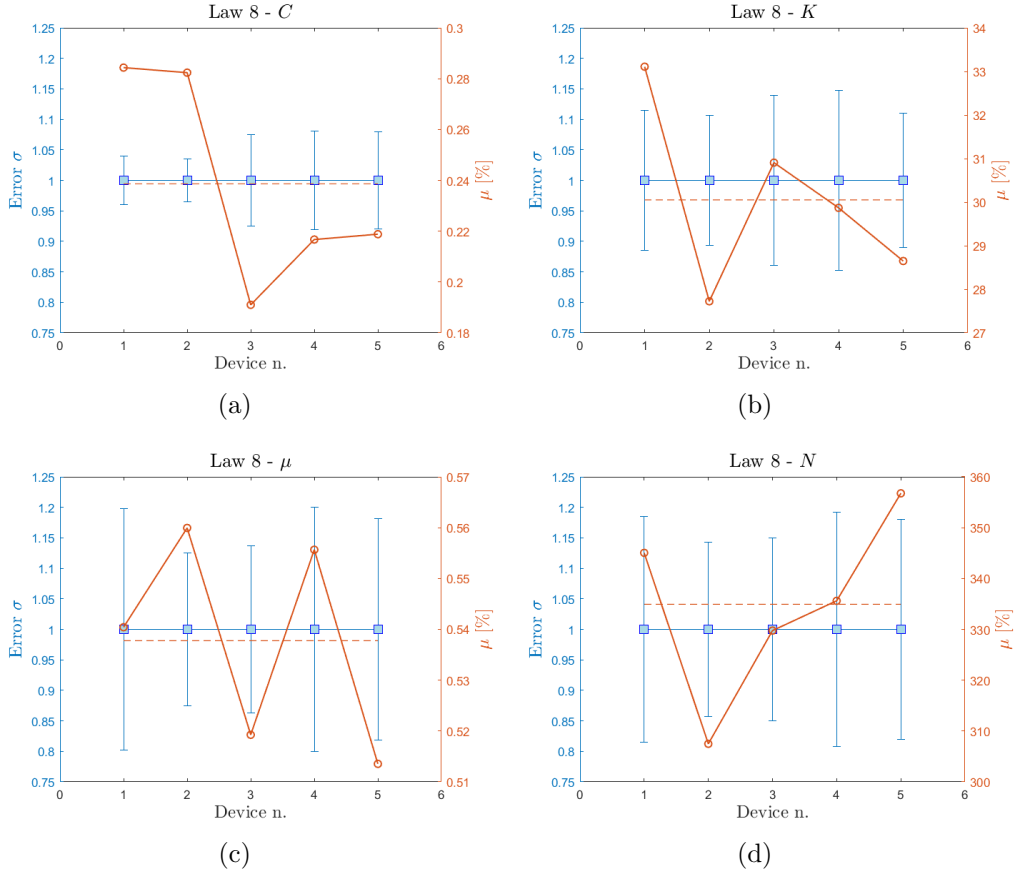


Figure 12: Normalized standard deviation and mean distribution of the identified damping C (a), stiffness parameter k_0 (b), friction coefficient μ and normal contact force N for the law 8. The left y -axis shows the error bar of the standard deviation (σ), while the right y -axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

465 At first, percentage variations of each parameter, with respect to a reference value, are eval-
 466 uated for each considered law. For example, the percentage variation, $\Delta_1^{(j)}$, of the parameter
 467 $n = 1$ of the generic constitutive law j will be written as following:

$$\Delta_1^{(j)} = \max(\text{abs}(\{\Delta_{11}^{(j)}, \Delta_{21}^{(j)}, \dots, \Delta_{m1}^{(j)}\})) \quad (21)$$

468

$$\text{where } \Delta_{m1} = \frac{\alpha_{m1} - \alpha_{\mu 1}}{\alpha_{\mu 1}}. \quad (22)$$

469 For clarity purposes, the maximum percentage variation, Δ , evaluated for each device, is
 470 expressed for the parameter $n = 1$ as reported in Eq.21. The generic percentage variation,
 471 $\Delta_{m1}^{(j)}$, is a measure of the dispersion of the parameter α of a specific device m with respect to a
 472 reference value expressed by the mean, $\alpha_{\mu 1}$, of all α values evaluated for each device. Though
 473 each device exhibits similar properties, the authors focused on investigating the variability
 474 of a specific parameter with respect to the average value evaluated among all the devices.

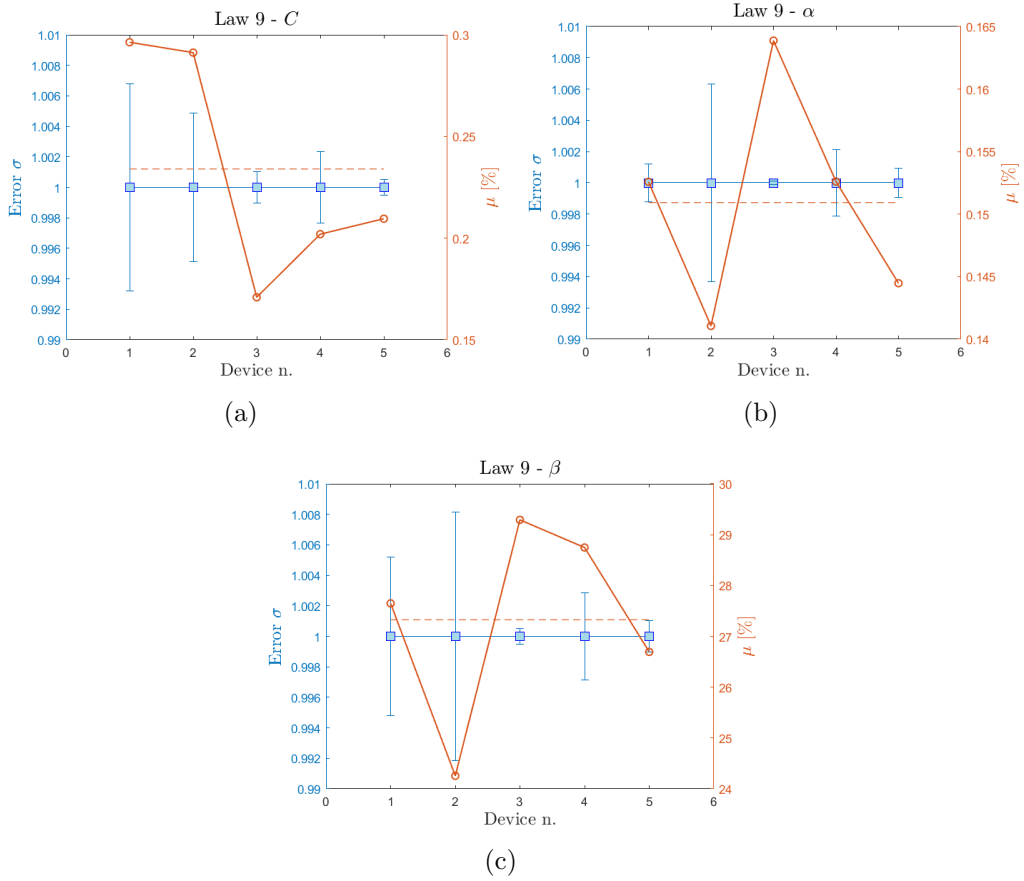


Figure 13: Normalized standard deviation and mean distribution of the identified damping C (a), exponent parameter α (b) and β (c) for the law 9. The left y – axis shows the error bar of the standard deviation (σ), while the right y – axis the percentage variation of mean distribution (μ). Dotted line represents the mean reference value among all five mean values evaluated for each Device.

475 In this way, a measure of the accuracy provided by that parameter in a specific law can be
 476 detected. This procedure is applied for each parameter of each investigated constitutive law.
 477 In this way, for the generic law j , the value β_j can be evaluated as:

$$\beta_j = \frac{\sum \Delta_n^{(j)}}{n_n}. \quad (23)$$

478 The no-dimensional value β can be assumed as a robustness index corresponding to the
 479 generic law j . This value is obtained, for each law, by summing all the percentage variations
 480 of the n parameter which govern that specific constitutive law.

481 With the aim to summarize general outcomes in a single plot where a measure of accuracy
 482 and robustness of each law is given, Fig.14 depicts the OF parameter, evaluated as the mean
 483 of all twenty optimization runs and among all devices, and index β on the Y and X-axis,
 484 respectively. Increasing the accuracy of a model (lower values of OF), a decrease in the

485 overall robustness (high values of β) is observed. Due to the opposite trend of these indices,
 486 a Pareto Front plot is developed and the best performer among all investigated models is
 identified.

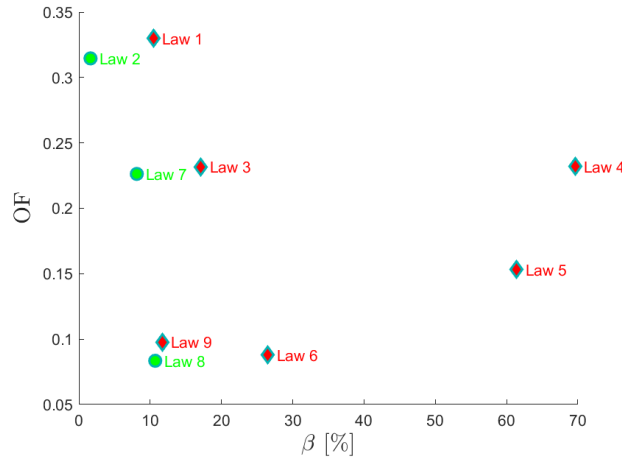


Figure 14: Accuracy (OF) vs robustness (β) Pareto front. The best constitutive laws as a good compromise between accuracy and robustness are marked in green while the dominated solutions are reported in red. The trend realized by connecting the green points represents the Pareto front of the non-dominated solutions.

487

488 In the scatter representation, data are clustered into two groups by using green and
 489 red colours. Green points represent the good compromise between accuracy and robustness
 490 features, while red points are the dominated-solution of the optimization problem. In other
 491 words, by selecting one of the red marked points, it is always possible to identify whatever
 492 solutions, among the green ones, which can be preferred in terms of accuracy or robustness
 493 (or both). Therefore, those models corresponding to green points lie in a Pareto Front, and
 494 all performers, identified by the couple $[\beta, OF]$, which belong the right side can be considered
 495 dominated solutions of the optimization problem.

496 For instance, Law#2 represents the best performer in terms of dispersion, hence, it represents
 497 the best model in which identified parameters contribute to obtaining the lowest value of β
 498 among the investigated models. On the contrary, due to the algebraic nature of Law#2, it
 499 is not the most effective with respect to the capability of reproducing the real behaviour of
 500 experimental outputs. Indeed, solutions, such as Law#7 or Law#8, can be preferred for the
 501 lower OF.

502 An interesting consideration can be pointed out by comparing the gain and loss in terms
 503 of OF and β from Law#2 to Law#8. Focusing on Fig.14, it is quite evident that the loss
 504 in terms of parameters variability is negligible if compared with the gain in terms of OF

505 of Law#8. This is evident in the comparison between experimental and numerical cycles
506 depicted in Fig.s3-(i),(l) and Fig.s3-(o),(p).

507 8. Conclusions and future developments

508 In the present paper, an identification procedure for the evaluation of the optimal pa-
509 rameters of nine different FVD numerical models has been implemented. The procedure
510 has been performed via metaheuristic technique and Genetic Algorithm suitably modified
511 and adapted to the final scope. Data derived from an experimental campaign has been
512 adopted and five FVD devices, with the same characteristics, have been tested in dynamic
513 conditions. An identification scheme has been developed to compare the experimental and
514 analytical values of the forces experienced by the device series under investigation. A com-
515 parison of the optimal numerical hysteresis cycles against the laboratory outcomes, for each
516 adopted law, has been shown. Moreover, a statistic analysis has been conducted and trends
517 and patterns have been highlighted. Generally, results shown how differential laws better
518 fit with experimental curves. Specifically, Law#8 and Law#9 exhibited a high level of ac-
519 curacy with a moderate dispersion among identified parameters. Finally, the Pareto front
520 representation has been adopted aiming to identify the set of best performers both in terms
521 of laws' accuracy and robustness.

522 Law#2 has been identified as that one with the highest robustness with a significant loss in
523 terms of accuracy. However, differential models as Law#7 and, especially, Law#8, can be
524 considered the most promising models for their low OF values even though a minimal loss
525 in terms of overall dispersion has been assessed.

526 Extending the experimental campaign could be useful to consolidate the obtained results.
527 However, it would involve obvious implementation difficulties. Therefore, future develop-
528 ment of this study will involve the procedure generalization to a higher number of devices
529 by numerical simulation, introducing parameters' variability (e.g., fluid characteristics, tol-
530 erances in the mechanical components). This virtual test campaign, supported by a limited
531 number of real tests, would also allow the study of multiple loading patterns to analyze the
532 effectiveness of the identified models under different operating conditions.

533 Acknowledgments

534 The research leading to these results has received funding from the European Research
535 Council under the Grant agreement ID: 101007595 of the project ADDOPTML, MSCA RISE
536 2020 Marie Skłodowska Curie Research and Innovation Staff Exchange (RISE). SISMALAB
537 in Crispiano TA is gratefully acknowledged for providing the test data.

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707 **Appendix A.**

708 This section is devoted to the results derived by the optimization process. Mean (μ) and
709 standard deviation (σ) for all design variables involved into the optimization process are
710 calculated. Values of OF and identified value of parameters, for each law, are reported in
711 table A.7 for completeness purposes.

Table A.7: Means (μ) and standard deviations (σ) of design variables and OFs

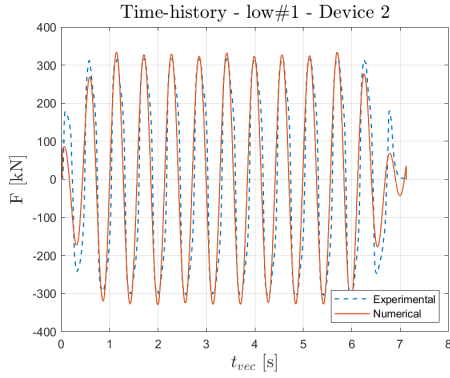
		Device 1		Device 2		Device 3		Device 4		Device 5	
		μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Law 1	OF	0.3074	2.36×10^{-12}	0.3057	2.30×10^{-12}	0.3463	2.50×10^{-12}	0.3450	2.67×10^{-12}	0.3454	1.50×10^{-12}
	C	0.7248	1.47×10^{-09}	0.6962	1.07×10^{-09}	0.5922	4.01×10^{-09}	0.6497	1.84×10^{-09}	0.6438	3.26×10^{-09}
Law 2	OF	0.2925	3.46×10^{-04}	0.2928	2.19×10^{-04}	0.3279	3.85×10^{-04}	0.3295	3.04×10^{-04}	0.3303	8.19×10^{-04}
	C	1.9944	0.0476	1.9657	0.0378	1.9946	0.0052	1.9715	0.0434	1.9698	0.0128
	α	0.8343	4.10×10^{-03}	0.8282	3.30×10^{-03}	0.7981	4.33×10^{-04}	0.8174	3.70×10^{-03}	0.8135	1.10×10^{-03}
Law 3	OF	0.2205	5.09×10^{-05}	0.2173	1.95×10^{-05}	0.2484	1.08×10^{-05}	0.2379	1.03×10^{-05}	0.2348	3.37×10^{-05}
	C	1.8913	6.36×10^{-02}	1.1743	1.22×10^{-02}	1.9405	3.26×10^{-02}	1.9548	1.37×10^{-02}	1.7877	0.12
	α	0.8337	5.40×10^{-03}	0.9047	1.70×10^{-03}	0.7969	2.70×10^{-03}	0.8084	1.10×10^{-03}	0.8221	1.09×10^{-02}
	K_1	2.1694	9.80×10^{-03}	2.1865	3.80×10^{-03}	2.2908	3.60×10^{-02}	2.5945	3.90×10^{-03}	2.5687	3.15×10^{-02}
Law 4	OF	0.2204	1.91×10^{-04}	0.2173	6.11×10^{-05}	0.2486	1.47×10^{-04}	0.2380	9.99×10^{-05}	0.2348	6.24×10^{-05}
	C	1.8315	8.73×10^{-02}	1.1827	3.15×10^{-02}	1.9414	3.17×10^{-02}	1.8873	6.38×10^{-02}	1.8029	7.75×10^{-02}
	α	0.8391	4.10×10^{-03}	0.9036	3.30×10^{-03}	0.7969	4.33×10^{-04}	0.8145	3.70×10^{-03}	0.8206	1.10×10^{-03}
	K_1	2.1910	3.05×10^{-02}	2.1904	6.30×10^{-03}	2.2975	1.52×10^{-02}	2.5978	9.80×10^{-03}	2.5611	1.50×10^{-02}
	K_2	0.0037	11.0×10^{-04}	0.0003	2.90×10^{-04}	0.0008	4.47×10^{-04}	0.0006	4.28×10^{-04}	0.0002	1.37×10^{-04}

		Device 1		Device 2		Device 3		Device 4		Device 5	
		μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Law 5	OF	0.1443	1.02×10^{-04}	0.1447	7.03×10^{-05}	0.1631	3.13×10^{-05}	0.1540	2.67×10^{-05}	0.1599	3.29×10^{-04}
	<i>C</i>	0.7295	0.0120	0.7538	0.0149	1.5499	0.0439	0.9060	0.0140	1.0592	0.0125
	α	0.9961	0.0027	0.9840	0.0032	0.8364	0.0047	0.9408	0.0025	0.9135	0.0021
	<i>K</i>	1.51×10^{-06}	4.64×10^{-07}	3.07×10^{-06}	0.11×10^{-07}	2.11×10^{-06}	4.14×10^{-07}	9.70×10^{-06}	0.22×10^{-07}	0.20×10^{-06}	0.03×10^{-07}
	β	4.9415	0.0727	4.7419	0.0813	4.8263	0.0557	4.4542	0.0655	4.3398	0.1647
Law 6	OF	0.0872	1.14×10^{-04}	0.0776	2.05×10^{-05}	0.0971	7.22×10^{-05}	0.0902	5.85×10^{-05}	0.0880	1.65×10^{-04}
	<i>C</i>	0.2999	3.05×10^{-04}	0.2941	7.51×10^{-04}	0.2057	8.23×10^{-04}	0.2295	3.86×10^{-04}	0.2275	14.0×10^{-04}
	<i>K</i>	30.0	0.0081	30.0	0.0042	30.0	0.0027	30.0	0.0058	30.0	0.0072
	α	83.9×10^{-04}	0.0305	43.6×10^{-04}	0.0063	4.22×10^{-04}	0.0152	2.35×10^{-04}	0.0098	3.83×10^{-04}	0.0150
	β	0.0842	53.7×10^{-04}	0.0881	1.27×10^{-04}	0.0911	1.36×10^{-04}	0.0843	1.02×10^{-04}	0.0845	3.04×10^{-04}
Law 7	OF	0.2336	7.5×10^{-03}	0.2419	7.7×10^{-03}	0.2188	5.8×10^{-03}	0.2179	6.8×10^{-03}	0.2192	7.6×10^{-03}
	<i>C</i>	6.0935	1.9256	6.2724	2.2382	5.3142	1.6503	5.9110	1.9019	6.3351	2.0116
	α	0.1569	0.0556	0.1507	0.0554	0.1486	0.0431	0.1513	0.0558	0.1391	0.0486
	<i>K</i>	115.67	18.726	118.15	15.562	102.65	16.546	107.81	17.240	105.76	19.306
	β	0.3466	0.0531	0.3278	0.0466	0.3080	0.0548	0.3171	0.0552	0.3196	0.0604

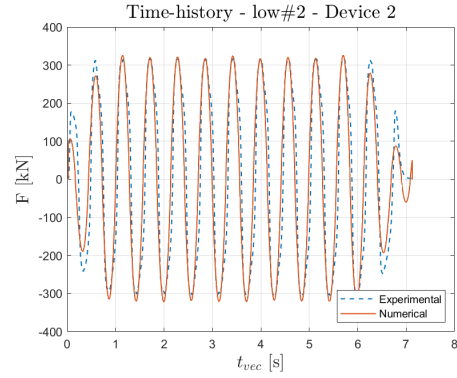
		Device 1		Device 2		Device 3		Device 4		Device 5	
		μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Law 8	OF	0.0902	6.2×10^{-03}	0.0779	6.0×10^{-03}	0.0875	7.5×10^{-03}	0.0814	7.7×10^{-03}	0.0802	4.8×10^{-03}
	<i>C</i>	0.2844	0.0114	0.2824	0.0099	0.1910	0.0144	0.2167	0.0175	0.2188	0.0175
	<i>K</i>	33.108	3.7877	27.724	2.9511	30.907	4.3087	29.569	4.3901	28.649	3.1568
	μ	0.5404	0.1069	0.5599	0.0703	0.5193	0.0710	0.5557	0.1116	0.5135	0.0934
	<i>N</i>	345.05	63.713	307.48	43.920	329.76	49.245	335.62	64.503	356.76	64.199
Law 9	OF	0.0934	5.43×10^{-05}	0.0896	2.07×10^{-05}	0.1047	2.25×10^{-07}	0.1006	1.42×10^{-06}	0.1002	7.31×10^{-07}
	<i>C</i>	0.2964	20.0×10^{-04}	0.2913	14.0×10^{-04}	0.1710	1.78×10^{-04}	0.2020	4.75×10^{-04}	0.2096	1.12×10^{-04}
	α	0.1526	1.83×10^{-04}	0.1411	8.93×10^{-04}	0.1639	0.19×10^{-04}	0.1526	3.23×10^{-04}	0.1445	1.33×10^{-04}
	β	27.649	0.1439	24.256	0.1972	29.292	0.0150	28.748	0.0821	26.695	0.0276

713 **Appendix B.**

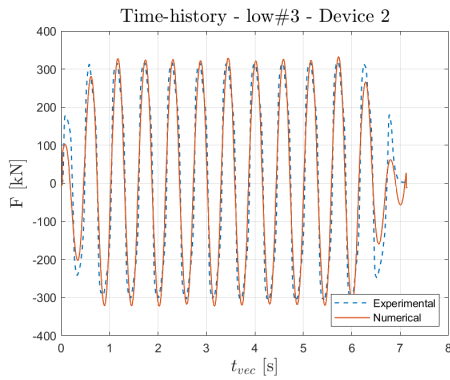
714 In this section, numerical and experimental Time-history curves are reported for com-
 715 parison purposes. All plots are referred to device 2 which exhibits the best results in terms
 716 of accuracy (OF). In Figs 3 (a)-(i), experimental ($F_{experimental}$ vs t_{vec}) and numerical curves
 ($F_{numerical}$ vs t_{vec}) are depicted for each law, respectively. Generally, as recognized for



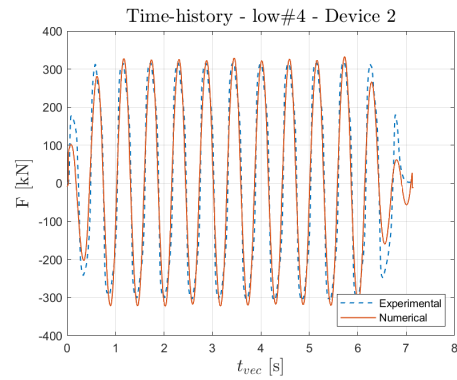
(a)



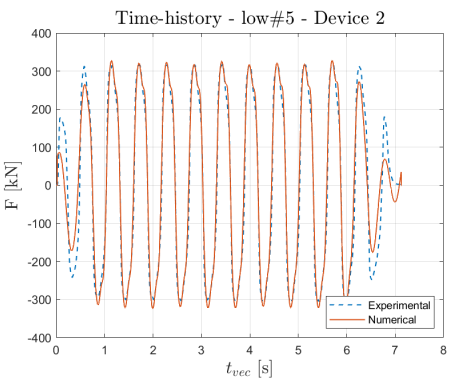
(b)



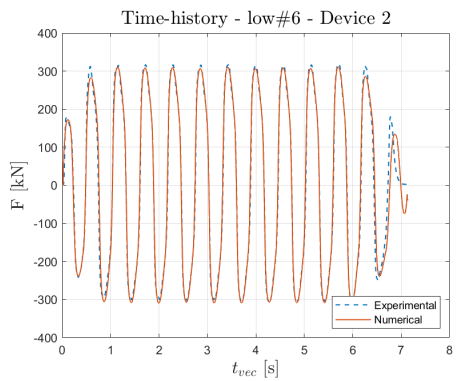
(c)



(d)

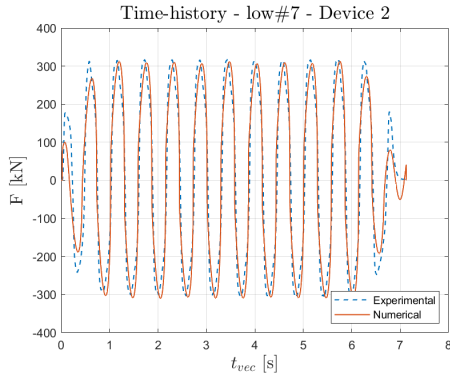


(e)

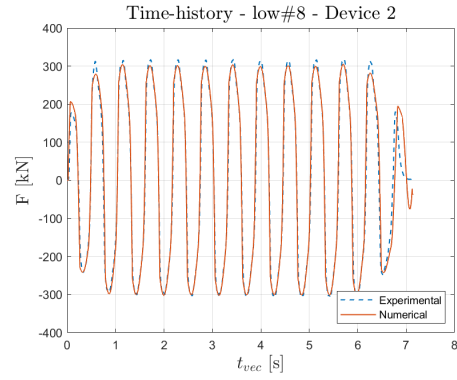


(f)

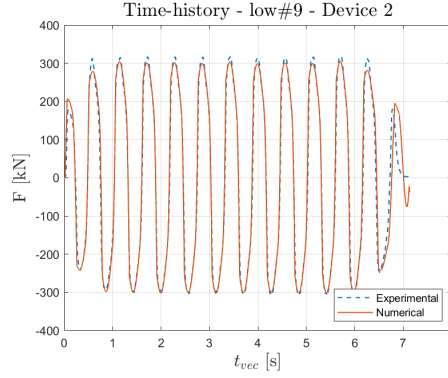
717
 718 the optimal hysteresis cycles discussed in section 5, best matches between experimental and
 719 numerical curves are observed by differential law (from law#6 up to law#9). Despite to
 720 the first five models, differential models fit better specifically at the peaks of each cycle. In



(g)



(h)



(i)

Figure B.15: Comparison between experimental (dotted blu line) and numerical (solid red line) time-history is depicted for each investigated law. Recorded experimental load, $F_{experimental}$, and numerical load, $F_{numerical}$, are reported along $Y - axis$ while t_{vec} , along $X - axis$, is representative of both time-step acquisition and numerical integration.

721 particular, law#8 and law#9 show a good grade of match not only at the stage of maximum
 722 load but during the transient phase too.

723 On the other hand, though numerical curves derived by no differential model (from law#1
 724 up to law#5) show a generic good grade of match with experimental curve, numerical noises
 725 are recognized at the peak value of each cycle. By observing the plot related to law#5, this
 726 phenomenon becomes more evident. Moreover, all these models are not capable of taking
 727 the initial and final transient phases.