An Integral-Equation Kernel for Glide Symmetric Structures

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***Abstract*—Glide-symmetric structures can improve properties of periodic structures for a wide variety of applications, such as lenses, filters and gap waveguides. Therefore, fast and accurate tools are needed to facilitate their use. In this work, we present a modelling approach based on a method of moments with a novel Green's function. The solutions are found as singularities of the impedance matrix. The results are shown to be in good agreement with a well-established method.**

I. Introduction

Utilizing glide symmetry in metasurfaces can lead to improved electromagnetic performance in terms of reducing dispersion, and increasing bandgap and magnetic response [1]. As opposed to mirror-symmetric structures, where the bottom and top parts are invariant to a mirroring operation, for glide-symmetric structures, the top part is additionally translated by half of a period [2]. Recently, glide-symmetric unit cells have been used in wideband lens antennas [3], gap waveguides [4] and filters [5]. Common methods of analysis include commercial software, mode matching [6] and multi-modal transfer matrix method [7].

In this work, we propose a method-of-moment based modelling approach, building on [8], to obtain full dispersion diagrams of 2-D periodic glide-symmetric structures. A novel Green's function for glide- and mirror-symmetric structures is also presented.

II. Modelling

To find the modes which exist in a given fully metallic periodic structure, we start by writing the tangential component of the electrical field on the surface, described by perfect electric conductor (PEC):

$$\mathbf{E}_{\mathrm{t}} = -j\omega\mathbf{A} - \nabla\Phi = \mathbf{0}, \tag{1}$$

where the incident field is zero, as no impressed field is necessary to support a mode in the structure. Here, j is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, with f the frequency. The vector potential $\mathbf{A}$ is

$$\mathbf{A} = \int_S \mathbf{J}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,\mathrm{d}S' \tag{2}$$

and Φ is the scalar potential:

$$\Phi = -\frac{1}{j\omega\varepsilon}\int_S \nabla'\cdot\mathbf{J}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,\mathrm{d}S'. \tag{3}$$

Here, $\mathbf{J}$ is the surface current density, ε is the permittivity of the surrounding medium, G is the Green's function and $\mathbf{r}$ and $\mathbf{r}'$ are vectors pointing to the observation and source points. The integration is done over the entire surface of the structure S with the primed variables.

In this work, we present a novel Green's function, tailored for glide-symmetric structures. By combining the free space Green's function [9], the mirror image theorem, and the generalized Floquet theorem [2], we obtain:

$$G(\mathbf{r},\mathbf{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} G_{mn} \pm e^{-j\mathbf{k}_{\mathrm{t}00}\cdot\rho_g}G_{mn,t}, \tag{4}$$

where G_{mn} is computed by

$$G_{mn} = e^{-j\mathbf{k}_{\mathrm{t}00}\cdot\rho_{mn}}\frac{e^{-jkR_{mn}}}{4\pi R_{mn}}. \tag{5}$$

In (5), $\mathbf{k}_{\mathrm{t}00}$ is the transverse wave vector, $k = \omega/c$ is the magnitude of the wave vector, R_{mn} is the distance from observation and source point. The vector ρ_{mn} obtained by $\rho_{mn} = m\mathbf{s}_1 + n\mathbf{s}_2$, where $\mathbf{s}_1$ and $\mathbf{s}_2$ are lattice periodicity vectors. The additional phase factor is included for the top part of the structure due to the translation of the top plate by $\rho_g = 0.5\mathbf{s}_1 + 0.5\mathbf{s}_2$ for glide-symmetric structures. For mirror-symmetric structures, $\rho_g = \mathbf{0}$. The choice of the subtracting or summing is done based on whether the mode can be described by mirroring with a PEC or perfect magnetic conductor plane at $z = 0$. As we are interested in all possible modes in the structure, both branches are necessary to be evaluated for a complete solution. Using (4) allows us to reduce the computational domain to the bottom part of the unit cell only.

Next, we expand the surface current density $\mathbf{J}$ into Rao-Wilton-Glisson basis functions [10]. Then, Galerkin testing is applied to obtain the homogeneous system

$$\mathbf{ZI} = \mathbf{0}, \tag{6}$$

where $\mathbf{Z}$ is the impedance matrix and $\mathbf{I}$ contains the basis function coefficients. Solutions can then be found as pairs of f and $\mathbf{k}_{\mathrm{t}00}$ which result in a singular impedance matrix.

III. Numerical results

The method of moments model is now compared it to CST Studio Suite's eigenvalue solver (CST ES). A unit cell with a rectangular hole is analyzed for a coarser and refined mesh, presented in Fig. 1. The comparison is presented in Figs. 2

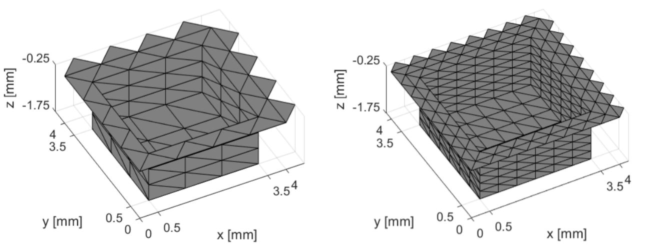

Fig. 1. The two meshes used in this paper. Current continuity between adjacent unit cells is ensured by extra triangles added at two boundaries. Left: coarse mesh. Right: refined mesh.

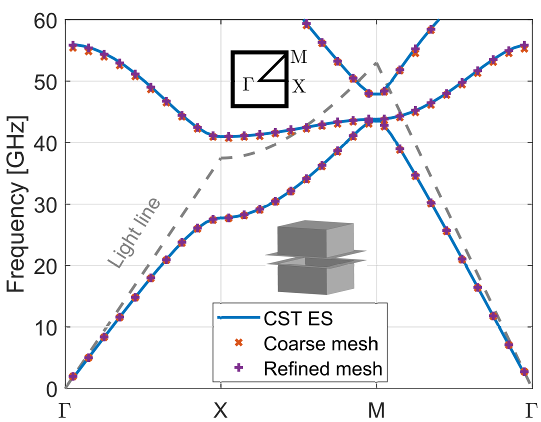

Fig. 2. Dispersion diagram of mirror-symmetric unit cell. The insets depict the irreducible Brillouin zone and the unit cell.

and 3 for mirror- and glide-symmetric configurations of the unit cell. Good agreement is observed for both coarse and fine meshes, although it can be observed that the coarse mesh is less accurate at higher frequencies.

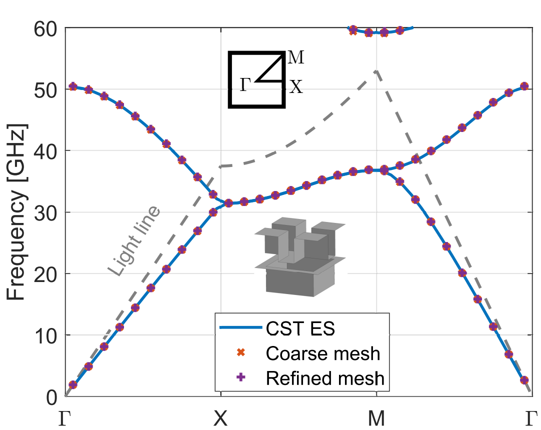

Fig. 3. Dispersion diagram of glide-symmetric unit cell. The insets depict the irreducible Brillouin zone and the unit cell.

IV. Conclusion and Perspectives

In this paper, a modelling approach for obtaining modes in mirror- and glide-symmetric structures was proposed and tested. The results show good agreement with reference and improve with a mesh refinement. The model can be used for the analysis of arbitrary geometries as long as the structure is fully metallic and can be extended to include lossy metals and dielectric materials. For future work, we aim to verify the evolution of the mode attenuation in the stopband.

Acknowledgement

This work is supported in part by COST Action Symat (CA18223), the Horizon Europe Research and Innovation Program under the GENIUS Project, Marie Skłodowska-Curie Grant under Agreement 101072560, and by the project PON Research and Innovation ”Microwave Imaging and Detection powered by Artificial Intelligence for Medical and Industrial Applications (DM 1062/21),” funded by the Italian Ministry of University and Research (MUR). It is also supported by Unite! – University Network for Innovation, Technology and Engineering.

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