6-D MoM Reaction Integrals Evaluated via the Divergence Theorem

6-D MoM Reaction Integrals Evaluated via the Divergence Theorem

J. Rivero[(1)], F. Vipiana*[(1)], D. R. Wilton[(2)] , and W. A. Johnson[(3)]
(1) Politecnico di Torino, 10129 Torino, Italy, e-mail: francesca.vipiana@polito.it; javier.rivero@polito.it
(2) University of Houston, Houston, TX, 77204-4005, USA, e-mail: wilton@uh.edu
(3) Consultant, Jemez Springs, NM 87025, USA, e-mail: w.johnson24@comcast.net

In this contribution we propose an accurate and efficient numerical evaluation of 6-D reaction integrals that appear in the Method of Moment (MoM) discretization of Volume Integral Equations (VIEs).

The usual numerical approach is to first carefully evaluate the inner 3-D integral, properly treating any singular or near-singular behavior of its integrand. Then, the outer integral is simply evaluated via a standard quadrature scheme. However, the inner integral (i.e. the integrand of the outer one) can have unbounded values or unbounded derivatives, in particular at its domain boundaries, and, hence, the straightforward numerical evaluation of the outer 3-D integral, via e.g. a Gauss-Legendre quadrature scheme, can be inefficient for source and test domains with a common face, edge or vertex.

Here, we propose a novel paradigm where the 6-D MoM reaction integral is rewritten in terms of a sum of double surface (4-D) integrals, each with two radial inner integrals, via the application of the divergence theorem to the initial volumetric inner and outer integrals. The proposed approach has been successfully applied by the authors to the MoM discretization of surface integral equations in [1] and [2]; it also has some similarities with the work reported in [3]. In fact, the two radial inner integrals can be evaluated analytically, reducing the integral dimensionality from 6-D to 4-D.

In this contribution, we investigate the efficient numerical evaluation of the double surface integrals resulting once the radial ones have been evaluated analytically. First of all, each surface integration domain is parametrized with respect to a variable z, along the axis of intersection of the two planes containing the domains, and a radial variable ρ along an axis orthogonal to the z-axis and in the domain plane. In this way, the double surface integrals are rewritten as four-line integrals, the outer ones in z and z' and the inner ones in ρ and ρ'. This novel integral expression allows reuse of the efficient variable transformation proposed in [1] to accelerate the numerical evaluation of reaction integrals in the case of coplanar surface elements.

The proposed approach has been extensively numerically tested and shows high accuracies also in the critical cases of source and test domains with a face, an edge or a vertex in common.

1. D. R. Wilton, F. Vipiana, W. A. Johnson, “Evaluation of 4-D Reaction Integrals in the Method of Moments: Co-planar Element Case”, IEEE Transactions on Antennas and Propagation, 65, 5, May 2017, pp. 2479-2493, doi: 10.1109/TAP.2017.2677916.

2. J. Rivero, F. Vipiana, D. R. Wilton, and W. A. Johnson, “Evaluation of 4-D Reaction Integrals Via Double Application of the Divergence Theorem”, IEEE Transactions on Antennas and Propagation, 67, 2, February 2019, pp. 1131-1142, doi: 10.1109/TAP.2018.2882589.

3. E. H. Bleszynski, M. K. Bleszynski, and T. Jaroszewicz, “Reduction of Volume Integrals to Nonsingular Surface Integrals for Matrix Elements of Tensor and Vector Green Functions of Maxwell Equations”, IEEE Transactions on Antennas and Propagation, 61, 7, July 2013, pp. 3642-3647, doi: 10.1109/TAP.2013.2254445