A non-conformal multi-resolution preconditioner in the MoM solution of large multi-scale structures

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***Abstract*—An efficient method to improve the convergence in non-conformal meshes including an automatic quasi-Helmholtz decomposition has been developed for the simulation of non-conforming meshes using the novel Multibranch Rao-Wilton-Glisson (MB-RWG) basis functions. Numerical experiments will be shown to illustrate the great flexibility of this approach for the solution of small-frequency and large multi-scale objects.**

I. Introduction

The extension of the surface integral equations (SIEs) [1] to non-conforming meshes has ignited intense research in the last years with the goal of finding a versatile and accurate method to address large and multi-scale complex problems, greatly simplifying computer-aided-design (CAD) generation and meshing processes.

Discontinuous Galerkin (DG) implementations of the SIEs [2] are the most popular approach to deal with this kind of problems. Other SIE non-conforming schemes alternative to DG are the monopolar-RWG [3] and the very recently presented Multibranch Rao-Wilton-Glisson (MB-RWG) [4]. The MB-RWG basis functions can be easily integrated into existing MoM codes without need of penalty terms, additional volumetric integrals or artificial surfaces. They are very convenient for h-refinement techniques and are div-conforming basis functions, allowing the construction of a solenoidal basis as linear combination of them.

SIE methods also have some inconveniences. They suffer from the ill-conditioning of MoM applied to realistic high-fidelity models that include multi-scale features. The physics-based preconditioners take advantage of the physical properties of the problem to improve the convergence in an iterative solver scheme. An example of dense-discretization stable physics-based preconditioner is the multiresolution preconditioner (MR) [5]. The MR preconditioner introduces a set of multi-level basis functions able to keep the different scales of variation of the solution, improving then the matrix conditioning in particular in the case of multi-scale structures [6]. This set of functions improves the spectral properties of the original MoM matrix system with a quasi-Helmholtz decomposition by splitting the current into solenoidal and non-solenoidal parts.

In this paper we present a multiresolution preconditioner realized with multibranch RWG functions for computing the electromagnetic solution of complex multi-scale problems discretized with non-conformal meshes, providing a method to automatically construct all solenoidal and non-solenoidal functions, including the topological (global) solenoidal ones. The proposed approach fully generalized the MR basis generation to non-conforming meshes without the need of any specific treatment of the mesh cells related to non-conforming triangles. Moreover, the obtained MR-MB preconditioner is a multiplicative preconditoner that can be easily inserted in any fast MoM code. To the best of authors' knowledge, this is the first work where a multi-level quasi-Helmholtz decomposition is applied to non-conforming meshes in SIE.

II. Multibranch RWG basis functions

A MB-RWG function is defined, analogous to a RWG function, in two domains, but in the case of MB-RWG the second domain can be made up of several triangles as:

$$\mathbf{f}_n^{MB}(\mathbf{r}) = \begin{cases} \frac{\boldsymbol{\rho}_n^+}{h_n^+}, \mathbf{r} \in T_n^+ \\ -\frac{\boldsymbol{\rho}_{n,i}^-}{h_{n,i}^-}, \mathbf{r} \in T_{n,i}^-, i = 1, \ldots, M_n \\ 0, \quad \text{otherwise} \end{cases} \tag{1}$$

where T_n^+ is the positive triangle and $T_{n,i}^-$ are the M_n triangles in the negative part of the nth function. $\boldsymbol{\rho}_n$, h_n are respectively the position vector relative to the free vertex and the height respect to the common edge of each triangle. The MB-RWGs have the same properties of the RWGs: null normal component in the external edges, unitary normal component in the common edge and analytic divergence.

III. The MR Preconditioner including MB-RWGs

The MR preconditioner generation is divided into two main steps. First, the input triangular mesh, supporting the structure discretization in terms of RWGs and MB-RWGs, is rearranged until getting a set of meshes with different mesh-element (cell) sizes. This is done via a multilevel algorithm in which the adjacent cells of the previous level are aggregated giving rise to macro-cells. A generalized version of the initial RWGs and MB-RWGs (gRWG) are then defined on each pair of adjacent macro-cells. Then, on each level mesh, the singular vectors in the null-space of the charge matrix [5] of each gRWG correspond to the solenoidal functions, while the non-zero singular vectors correspond to the non-solenoidal functions. The above scheme is applied recursively down to the quasi-Nyquist (coarsest) cell-size level, where gRWGs are defined completing the set of multilevel basis functions, or, in the case of an object with a small electrical size, down to when all initial level cells are completely included into one cell in order to split all functions into solenoidal and non-solenoidal parts. All the generated MR functions at the intermediate (detail) levels and gRWG functions at the coarsest level (if present) can be described as linear combinations of the initial RWG and MB-RWG functions. Due to the multilevel approach, the complexity of the generation of the MR basis is $O(Nlog(N))$.

IV. Numerical Results

A first numerical example is introduced to validate the proposed approach to automatically construct all solenoidal and non-solenoidal functions of a non-conformal mesh for the solution of a small object that contains global loops. A toroid with $\lambda/10$ of diameter is considered. The toroid structure (shown in Fig.1(b)) is divided into eight octants applying different refinements to each pair of neighbour octants providing a total of 36151 RWGs and 256 MBs. The first level of MR grouping is shown in Fig.1(b), where each color represents different macro-cells. The toroid has a total of 11965 inner nodes (V_{int}), 24442 triangles (M) and one handle (H). The number of solenoidal (N_s) and non-solenoidal (N_{ns}) functions generated with the Multiresolution scheme are 11966 and 24441 respectively obtained in the 7 levels of grouping, that match with the relation between the number of nonsolenoidal ($N_{ns} = M - 1$) and solenoidal ($N_s = V_{int} - 1 + 2H$) functions described in [7], including the topological (global) loop correspond to the handle.

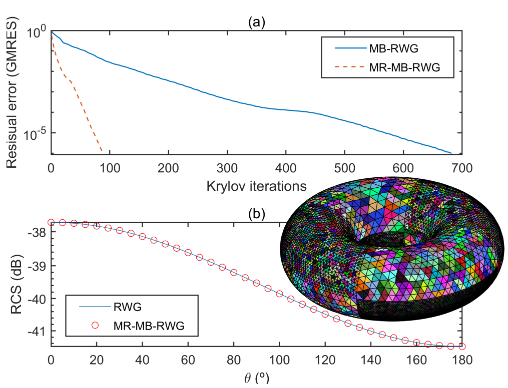

Fig. 1. (a) Iteration count for the toroid of the figure considering a plane wave excitation. (b) Bistatic radar cross section of the toroid.

Fig.1 shows the bistatic radar cross section (RCS) calculated using the MR-MB, compared to the reference MoM solution (b) and the improvement of convergence of the proposed approach (a) for small objects.

A second numerical example is introduced to highlight the capacity and versatility of the proposed approach to solve complex multi-scale non-conformal meshed problems. The radiation at 600MHz of four patch antennas embedded into a challenging structure consisting of a realistic vessel is considered. The mesh is adapted to the fine detail features of the antenna allowing non-conforming triangles in the connections with the structure (mesh details in Fig2), providing a total of 13782364 RWGs and 3468 MBs. In this example, the grouping is stopped at the quasi-Nyquist level ($\lambda/4$) providing a total of 3514716 solenoidal, 8571690 non-solenoidal and 1699426 gRWG basis functions. The problem is solved via the multilevel fast multiple algorithm applying a diagonal preconditioner and appling also the proposed MR-MB preconditioner.

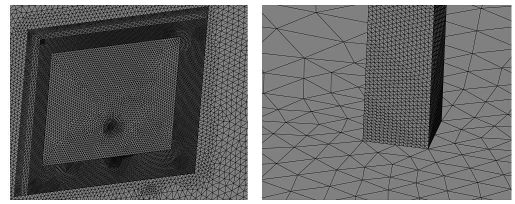

Fig. 2. Non-conformal mesh details of the feeding point and the connections of antennas with the structure.

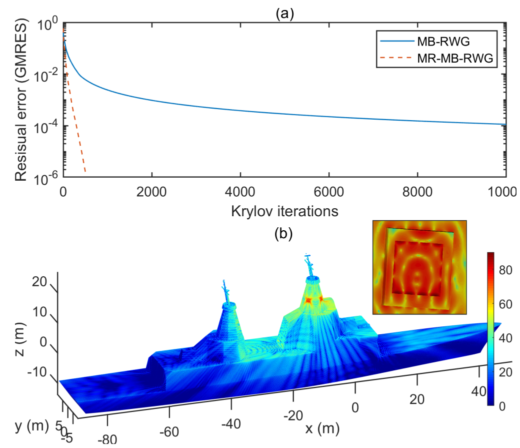

Fig. 3. (a) Iteration count for the proposed vessel considering a delta-gap excitation. (b) Equivalent electric currents induced on the vessel surfaces.

Finally, Fig.3(a) shows the number of needed iterations to reach a residual error of 10^{-6}: it is very evident the converge acceleration due to the applied MR-MB preconditioner. Fig.3(b) reports the obtained equivalent electric currents induced on the ship surfaces.

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