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Novel Test Integral Quadrature Scheme for the Method of Moments

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***Abstract*—An extensive literature exists on the efficient and accurate evaluation of the double surface integrals that arise in the Method of Moments. Most papers have focused on the evaluation of the inner (source) integral with the idea that once that integral is evaluated, the test (outer) integral's integrand is sufficiently smooth that it should be much easier to integrate numerically. However, that affirmation is not always true. Here, we propose an integration scheme that improves the numerical evaluation of the test integral without affecting the treatment of the source integral. The method is numerically validated for static and dynamic kernels in the reaction integrals arising in electric field integral equations.**

***Index Terms*—integral equations, moment methods, numerical analysis, singular integrals.**

I. INTRODUCTION

A cost-effective numerical evaluation of double surface reaction integrals is required to achieve an accurate solution of direct or inverse electromagnetic problems using surface integral equation formulations. The inner integral is commonly known as *source* integral, and the outer one is the *test* integral. The most common approaches for dealing with the source integral are the *singularity subtraction* or *singularity cancellation* methods. For singularity subtraction [1]–[3], a simplified asymptotic form of the integrand is first identified and subtracted from integrand. The resulting difference integrand should be less singular than the original, and the subtracted term should be analytically integrable (or at least easily evaluated numerically); adding the analytical integral to the difference integral restores the original value of the integral. On the other hand, for singularity cancellation [3]–[7], variable transforms are chosen whose Jacobian cancels or regularizes any singularities. More recently, a paper has considered the possibility of using both methods to bring together the advantages of both previous approaches [8] handling the singular behavior that arise when evaluating the magnetic field integral equation (MFIE). As can be seen in the literature, the effort to evaluate the double integrals has been focused on the efficient evaluation of the *source* integral [1]–[7] and simply assuming that for the test integral any singular behavior has been smoothed by the source integral, making the test integral relatively easy to integrate numerically. In cases where the source and test domains share points in common, this affirmation does not hold, as shown in [9].

Here, we focus on the numerical evaluation of the outer test integral that arises in the reaction integrals. We will use the vertex functions described in [10] to discern the behavior of the test integral integrand. The vertex functions are element-independent functions describing the (possibly) singular behavior of potentials or their gradients near a given vertex or edge for constant or linear sources on a planar element. Once the behavior of the vertex functions is understood, we can design an efficient quadrature scheme for the test integral.

II. VERTEX FUNCTION

The evaluation of the electromagnetic interaction between a pair of triangles in the Method of Moments (MoM) leads to the evaluation of the double surface integral

$$\int_{S}\int_{S'} F(\mathbf{r},\mathbf{r}')\,dS'\,dS, \tag{1}$$

where typically $F(\mathbf{r},\mathbf{r}')$ takes the form

$$F(\mathbf{r},\mathbf{r}') = t(\mathbf{r})g(\mathbf{r},\mathbf{r}')\,b(\mathbf{r}'), \tag{2}$$

and where $t(\mathbf{r})$ is either a scalar or a vector component of a testing function, $b(\mathbf{r}')$ is similarly defined for a basis function, and $g(\mathbf{r},\mathbf{r}')$ is either a scalar or a scalar component of a vector or dyadic Green's function, with a $\mathcal{O}(|\mathbf{r}-\mathbf{r}'|^{-1})$ or $\mathcal{O}(\nabla|\mathbf{r}-\mathbf{r}'|^{-1})$ singularity.

We assume the inner integral in (1) may be evaluated to sufficiently high accuracy so its influence is dismissed. Its singular behavior follows from is static ($k=0$) form. This, in terms, can be represented as a sum over vertex function pairs, with the superposition describing the essential behavior of the various potentials (scalar and vector) involved. For a case of a pair of triangles with a common vertex, the static vertex potential functions for the common vertices associated with a pair of adjacent source triangle edges for a constant (unit) source density are given by

$$\begin{aligned} I_{V_1}^{l_3} =& R_1 \sin\alpha \left[\cos\beta \sinh^{-1}(\cot\alpha)\right. \\ &\left. -|\sin\beta|\tan^{-1}\left(\frac{\cos\alpha+\cos\beta}{\sin\alpha+|\sin\beta|}\right)\right], \end{aligned} \tag{3}$$

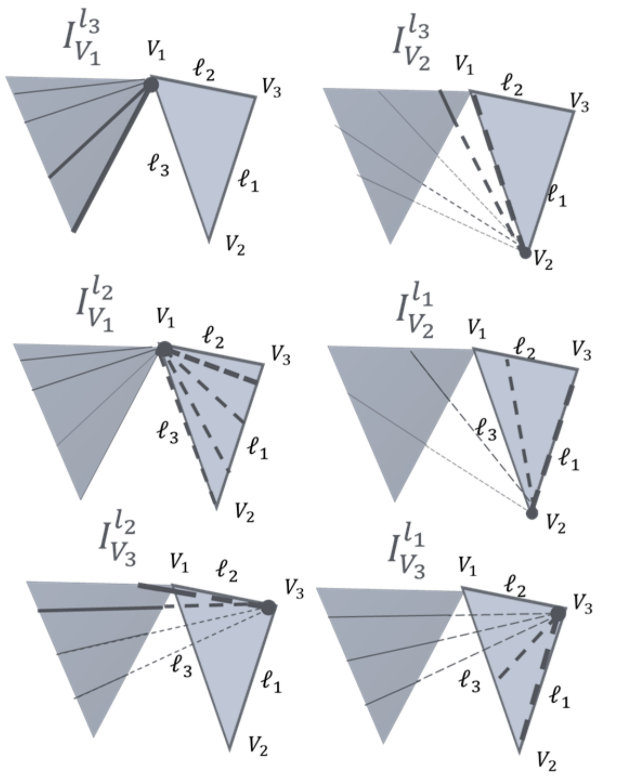

Fig. 1. The vertex function of each adjacent vertex-edge pairs shows a singular behavior close to the edge that defines the function (thicker lines) and smoother behavior far from the edge (thinner lines). Dashed lines are not in the test integral domain.

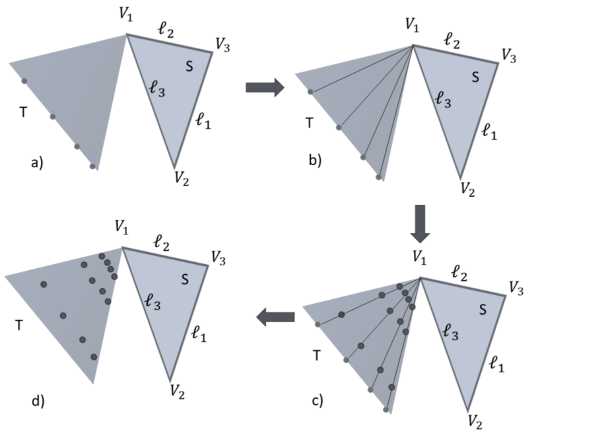

Fig. 2. Procedure to obtain the proposed scheme, a) parametrize the edge oposite to the common vertex, b) define radial integration dimension, c) parametrize radial integration dimension, d) obtain quadrature points.

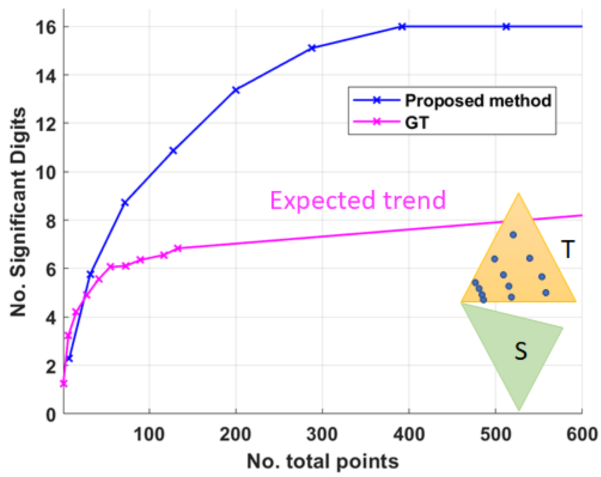

Fig. 3. Near-field convergence of test integrals. Inset: Orientation of a pair of triangle elements in space.

where R_1 is the distance from the obs. point to the vertex, α is the angle between the edge l_3 and R_1 and β is the angle between the two triangles (or the extended planes in which they lie). Analyzing (3), we find that the vertex function has a linear dependence with distance R_1 and exhibits a singular behavior as $\alpha \rightarrow 0$. The remaining vertex functions, though with more complex expressions, exhibit similar singular behavior, that is, a linear dependence with respect to the distance to the vertex and a singular behavior with respect to angle. Figure 1 shows graphically this behavior, where the vertex functions exhibit more nearly- singular along the thicker lines (with dashed lines indicating points outside the test integral domain). Considering this behavior, we can better handle the singularities if we concentrate more sample points close to the common vertex as well as closer to the source triangle. The vertex functions also reveal a logarithmic angular singularity, hence the proposed scheme uses MWR [11] quadrature with logarithmically weighted sample points for the angular integration and a parameterization of the edge opposite the common vertex to form an angular scheme. Using these points as endpoints, we now parameterize radially from the common vertex also using the logarithmic MWR scheme. This procedure is described graphically in Fig. 2. The new quadrature points ξ_i can be mathematically expressed as

$$\xi_i = V_1^T(1-\xi_j^r) + V_2^T(\xi_j^r - \xi_j^r\xi_k^a) + V_3^T(\xi_j^r\xi_k^a) \quad (4)$$

where $V_{i=1,2,3}^T$ are the vertices of the test triangle and ξ_j^r and ξ_k^a are the 1-D MRW quadrature points used to parameterize the radial and angular directions, respectively.

III. Preliminary Numerical results

To demonstrate the accuracy of the proposed scheme, we analyze the convergence behavior of the test scalar potential integral in (1). We consider a pair of triangles with a common vertex as shown in the Fig. 3 inset. We compare the proposed method with the standard Gauss-triangle (GT) quadrature scheme [12]. The reference for each of the plot is evaluated with the highest number of points we have available. In the case of GT, we do not have more than 166 points so the behavior over that number of points is estimated with the plot trend. The proposed method reaches machine precision at a faster rate in the medium accuracy range than GT.

IV. Conclusion and Perspectives

Preliminary results show better accuracy and efficiency of the proposed method with respect to the GT quadrature scheme.With small modifications, the method can also be applied to test triangles with a test triangle edge in common with the source triangle and self-term triangles for all the reaction integrals arising in the electric field integral equation and magnetic field integral equation.

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