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Holographic optical traps for atom-based topological Kondo devices

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Abstract

The topological Kondo (TK) model has been proposed in solid-state quantum devices as a way to realize non-Fermi liquid behaviors in a controllable setting. Another motivation behind the TK model proposal is the demand to demonstrate the quantum dynamical properties of Majorana fermions, which are at the heart of their potential use in topological quantum computation. Here we consider a junction of crossed Tonks–Girardeau gases arranged in a star-geometry (forming a Y-junction), and we perform a theoretical analysis of this system showing that it provides a physical realization of the TK model in the realm of cold atom systems. Using computer-generated holography, we experimentally implement a Y-junction suitable for atom trapping, with controllable and independent parameters. The junction and the transverse size of the atom waveguides are of the order of 5 μm, leading to favorable estimates for the Kondo temperature and for the coupling across the junction. Since our results show that all the required theoretical and experimental ingredients are available, this provides the demonstration of an ultracold atom device that may in principle exhibit the TK effect.

1. Introduction

The topological Kondo (TK) model is a device exploiting the degeneracy of the spectrum of a system in which a set of Majorana modes is present [1–4]. It has been recently introduced with the goal of realizing the physics of multichannel Kondo models in solid state devices [5]. Moreover, a major motivation in the quest of TK devices is the realization of hardware devices for quantum computation [6, 7]. The rationale is that the manipulation of anyonic excitations is at the heart of topological quantum computation [8]. Hence, proper control of a device with Majorana modes gives the possibility of performing a wide class of quantum information tasks [8–12]. In this respect, the TK device can be considered as the elementary unit of a variety of hardware setups. In general, if the states of Majorana zero modes can be measured in well-controlled experiments, this could pave the way towards the realization of a topological quantum computer [13], together with the possibility of manipulating the Majorana modes themselves, or performing measurement of certain observables involving the Majorana modes on special geometries. It appears that the control of the TK unit may provide an important resource to demonstrate the properties of Majorana modes and in turn control their dynamics, which is instrumental in their forseen use in topological quantum computation [8]. The possibility of such a control is currently one of the main goals of different lines of research in physics.

In this paper, we propose a new realization of the TK model in an highly controllable setup using cold atoms. We also present an experimental implementation of a suitably engineered holographic optical trap, which constitutes the central element of our realization.

The TK effect can be obtained, at low temperature, in a setup where a set of localized Majorana modes is hosted on a central island. One end of the (effectively one-dimensional) external wires is then connected to the central island, in such a way that the tunneling of electrons to or from the island can change the state of the set of Majorana degrees of freedom [14]. The Majorana modes can then be seen as non-locally encoding a degree of freedom, which is much alike that of a localized impurity interacting with a gas of fermions. In this solid state proposal, the actual realization of TK devices would provide a playground for the investigation of the properties of non-Fermi liquid critical points. The study of non-Fermi liquids plays a prominent role in the theory of strongly correlated solid-state systems [15].

To date, experimental realization of solid state TK devices has remained elusive. The reasons, in solid state proposals, are many. Among these, the fact that the single-particle tunneling onto the central island at temperatures which are not negligible compared to its charging energy may spoil the TK effect by mixing parity sectors [16]. Moreover, in realistic devices, the lifetime of localized Majorana modes is necessarily finite [17, 18]. The quasiparticle poisoning, connected to the presence of an unpaired electron within the superconducting substrate, is the phenomenon which most contributes to shorten the lifetime of localized Majorana modes. A considerable effort is in progress in this direction [19].

Complementary to such efforts, alternative approaches are highly desirable, in particular if an accurate control of the parameters characterizing the wires and their coupling with the Majorana modes can be achieved. For this reason, in this paper, we not only propose how to realize the TK Hamiltonian with cold atoms in a *Y-junction*, but we also show that it is possible to experimentally implement the Y-junction needed for this purpose. This junction may provide a basic component for connecting atom guides and more generally for atomtronics [20].

Cold atom setups emerge as a natural candidate to simulate low-energy Hamiltonians [21]. The architecture needed to implement the TK Hamiltonian is that of a suitably engineered quantum system trapped in a set of effectively one-dimensional waveguides, joined together in a central region. Such a geometry is often referred to as a Y-junction (or also T-junction). It is possible to map [5] the effective TK Hamiltonian to Y-junction systems [22, 23]. On the other hand Y-junctions of Ising chains are, in turn, a physical realization of the TK Hamiltonian [24]. In a cold atom realization, it would then be possible to tune the parameters of quantum one-dimensional systems merging in the junction, as well as the shape and characteristics of the junction, through the use of controllable traps. Moreover, in the proposed experimental setup, the Majorana modes will be non-locally encoded in some bosonic bulk fields and, therefore, will not be affected by any of the above-mentioned drawbacks characteristic of solid state devices.

Two difficulties have to be overcome in order to have a TK device in the realm of cold atoms: (i) one should find a low-dimensional cold atom system having an effective low-energy Hamiltonian which matches that of the TK model; (ii) one should have a reliable and controllable realization of the system geometry. It is clear that the 'wires' of solid state proposals correspond to 'waveguides' in cold atom setups in which the atoms are tightly confined: in the following, we will use both terms ('wires' and 'waveguides') to denote the different branches of the Y.

Among the different lines of research in the field of quantum simulations with cold gases, a very active one and relevant for our purposes is provided by the study of one-dimensional gases. There are several reasons for such an interest: for low dimensional systems in the last few decades powerful tools have been developed, ranging from bosonization to integrability methods like the Bethe ansatz [25, 26]. Furthermore these experimental configurations are ideal to test different approaches developed for systems where integrability is broken and to study controllable deviations from integrable models, e.g. achieved by connecting together several one-dimensional systems. One way of coupling one-dimensional systems is to glue them in a point or, more realistically, in a small part of them, creating a Y-like geometry, which is an essential ingredient to realize the TK effect with cold atoms. This way of coupling one-dimensional chains is possible in some solid state systems, e.g. carbon nanotubes [27].

The implementation of such a layout with cold atoms would open stimulating possibilities in the light of providing the Y-shape of the junction needed in the TK effect. Recently, part of a bosonic condensate in a quasi-one-dimensional optical trap has been split into two branches [20], generating the non-trivial geometry of an Y-junction. However, despite this and other considerable progress in the field, to date no experimental realization of a stable and controllable Y-geometry is known. In particular, it is required that several waveguides merge in a well defined region, that the tunnelings of atoms among different waveguides be tunable and that the low-energy Hamiltonian of the atoms in each waveguide be effectively one-dimensional.

One method to realize unusual optical trapping geometries is computer generated holography, in which a phase modulation is applied to the trapping light such that a desired intensity distribution is realized in the far field. Holographic optical traps provide a flexible tool to tailor the potential experienced by neutral atoms, and have been employed in experiments ranging from single atoms [28–30] to Bose–Einstein condensates [31, 32]. The development of numerical [33, 34] and experimental [35–38] techniques which allow the creation of smooth light profiles has given significant freedom in engineering different trapping geometries, including proposals in investigations of superfluidity [35], atomtronics [36] and entropy-engineering of ultracold Fermi gases [38].

For the implementation of the TK model in the framework of atomtronics, two preliminary obstacles have to be tackled. The first is the realization of the junction itself, with fabrication parameters that allow functionality and tunability. The second is the understanding of the low-temperature properties of this junction, when bosonic atoms are loaded in the trap. In this paper, we provide a path to overcome both these obstacles and we show that holographic optical traps can be exploited for a systematic implementation of the junction geometry for TK devices. Our results are the following:

- We show that when the interacting Bose gases in the wires are in the Tonks–Girardeau (TG) limit, the Y-junction provides a physical realization of the TK Hamiltonian [5, 16, 24, 39, 40]. This identification is fruitful for two reasons. Firstly, it opens the possibility of studying the TK model and excitations at the junction with cold bosons in an highly controllable experimental setup. Secondly, recent results obtained from the theory of the TK model allow to write exact expressions for thermodynamical quantities, as well as to characterize the transport across the central region. In all these results the formalism associated with Majorana modes plays a central role (see also the discussion in section 5) and we will describe how to detect the experimental signatures of the Majorana fermion physics in the system under study.
- We experimentally demonstrate the possibility of creating Y-junctions for cold bosons using holographic techniques. We show that one can realize junction widths of the order of 5 μm, with tunable hopping parameters, using blue- and red-detuned laser potentials (respectively providing repulsive and attractive potentials on the atoms [41]). Given that the one-dimensional regime for cold bosons is reachable [42–44] and that TG gases have been widely studied in experiments [45, 46], the realization of holography-based Y-junctions provides a proof-of-principle of an atom-based holographic device that potentially may exhibit the TK effect.

2. The TK model

The Hamiltonian of the TK model (or Coulomb–Majorana box) is:

$$H = -\mathrm{i}\frac{\hbar v_{\mathrm{F}}}{2\pi}\sum_{\alpha=1}^{M}\int \mathrm{d}x\psi_\alpha^\dagger(x)\partial_x\psi_\alpha(x) + \sum_{\alpha\neq\beta}\lambda_{\alpha\beta}\gamma_\alpha\gamma_\beta\psi_\beta^\dagger(0)\psi_\alpha(0). \tag{1}$$

Here $\psi_\alpha(x)$ are the (complex) Fermi fields describing electrons in the wires $\alpha = 1, \ldots, M$ and satisfying canonical anticommutation relations

$$\{\psi_\alpha(x), \psi_\beta(y)\} = 0 \qquad \{\psi_\alpha(x), \psi_\beta^\dagger(y)\} = \delta_{\alpha,\beta}\delta(x-y), \tag{2}$$

while the $\gamma_\alpha = \gamma_\alpha^\dagger$ are Majorana fields constrained in a box connected with the wires and satisfying the Clifford algebra

$$\{\gamma_\alpha, \gamma_\beta\} = 2\delta_{\alpha,\beta}. \tag{3}$$

The symmetric matrix $\lambda_{\alpha,\beta} > 0$ is the analogue of the coupling with the magnetic impurity in the usual Kondo problem. This model coincides with the one studied in [40]. A related model, with real spinless fermions in the bulk, has been solved in [47].

Solid-state proposals of TK devices are based on the fact that a set of nanowires with strong Rashba coupling (InAs, InSb), laid on a BCS superconductor (Al, Nb) and subject to a suitably tuned magnetic field, can develop Majorana ending modes [48–50]. The TK model is obtained connecting a set of M effectively one-dimensional wires to a set of nanowires supporting Majorana modes at their ends, hosted on a mesoscopic superconducting substrate with a large charging energy, subject to an applied electrostatic potential [5]. Interactions on the wires are not essential [39]. The TK effect takes place at temperatures much smaller than the charging energy and the pairing parameter of the substrate, when only the massless Majorana modes are the relevant degrees of freedom on the island and all the processes which change the charge of the island (hence the fermion parity) are virtual. Under these conditions [5, 51–53], the effective low-energy Hamiltonian describing the junction is the one of the TK model (1).

This Hamiltonian has received considerable attention in recent years, as it can provide a realization of a non-Fermi liquid fixed point (at temperatures much below the Kondo temperature T_{K} [15]), which can be described by an $SO(M)_2$ Wess–Zumino–Witten conformal field theory [26]. Remarkably, such fixed point is stable against anisotropy in the couplings $\lambda_{\alpha,\beta}$ [5], which in principle implies that a very accurate fine tuning of these parameters is not needed in experiments.

Conversely, it has been observed [40] that the direct coupling among Majorana modes [39] which originates from the overlap of their wavefunctions [48, 49], can split the energies of a pair of Majorana modes. Such a coupling is modeled by adding a term

$$H_h = \mathrm{i} \sum_{\alpha \neq \beta} h_{\alpha\beta} \gamma_\alpha \gamma_\beta \tag{4}$$

to the Hamiltonian (1), and its effect is that of effectively removing the pair of Majorana modes from the zero-energy sector of the spectrum and driving the system towards another fixed point, generated by the remaining $M - 2$ modes. This implies that in any experimental setup, the parameters $h_{\alpha,\beta}$ must be carefully controlled. In condensed matter realizations, they may be controlled by having the Majorana modes sufficiently distant from one another, given that their wavefunctions are exponentially localized. A feature of the implementation proposed in this article is that terms involving such direct coupling do not appear without further operations on the system.

An exact solution of the TK Hamiltonian was proposed in [40] for a fixed number of (non-interacting) electrons, fermion number parity, external wires and for arbitrary coupling strength λ (taken to be the same for all pairs of wires). In [16], using the thermodynamic Bethe ansatz (TBA), the thermodynamics of the TK Hamiltonian (1) with an arbitrary number of wires M was investigated.

The TBA analysis allowed to compute exactly the change in free energy due to the presence of the coupling with Majorana modes, when the whole system is in contact with a heat bath at temperature T. For an even number of wires, the free energy reads

$$F_J^{(e)}(T) = -T \sum_{j=1}^{\lfloor M/2 \rfloor} \int_{\mathbb{R}} \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{\mathrm{i}\omega/\lambda} \frac{\cosh \frac{\omega}{2}}{\cosh \frac{(M-2)\omega}{4}} \frac{\sinh\left(\frac{j\omega}{2}\right)}{\sinh\left(\frac{\omega}{2}\right)} \hat{L}_{-,1}^{(j)}(\omega) \tag{5}$$

while for an odd number of wires, the free energy is instead given by

$$F_J^{(o)}(T) = F_J^{(e)}(T) + \int_{\mathbb{R}} \frac{\mathrm{d}\omega}{2\pi} \frac{\mathrm{e}^{\mathrm{i}\omega/\lambda} \sinh \frac{(M-3)\omega}{4}}{2\cosh \frac{(M-2)\omega}{4} \sinh \frac{\omega}{2}} \hat{L}_{+,1}^{((M-1)/2)}(\omega). \tag{6}$$

In these expressions, there appears the Fourier transform $\hat{\mathrm{L}}$ of the quantity $L_{\pm,n}^{(j)}(x) = \ln(1 + \mathrm{e}^{\pm\phi_n^{(j)}(x)})$. The functions $\phi_n^{(j)}(x)$ (with 'level' index $j = 1, 2, \ldots, \lfloor M/2 \rfloor$ and 'length' index $n = 1, 2, \ldots$) satisfy a system of coupled nonlinear integral equations which are called TBA equations and have been written in [16].

In addition to that, the ground state energy shift due to the tunneling was computed to be

$$E_J^{(0)}(\lambda, M) = \mathrm{i} \ln \frac{\mathrm{i}\Gamma\left(\frac{M+2}{4(M-2)} + \frac{\mathrm{i}}{(M-2)\lambda}\right)\Gamma\left(\frac{3M-2}{4(M-2)} - \frac{\mathrm{i}}{(M-2)\lambda}\right)}{\Gamma\left(\frac{M+2}{4(M-2)} - \frac{\mathrm{i}}{(M-2)\lambda}\right)\Gamma\left(\frac{3M-2}{4(M-2)} + \frac{\mathrm{i}}{(M-2)\lambda}\right)} \tag{7}$$

valid for both even and odd values of M.

The entropy introduced by the junction reduces to simple formulas both for $T \to 0$ and $T \to \infty$ [16, 40]. The zero-temperature limit is particularly noteworthy, as there appears a residual ground state degeneracy [54], which is non-integer in the thermodynamic limit and is related to the symmetry of the fixed point. The entropy reads

$$S_J^{(0)} = \ln \sqrt{\frac{M}{2}} \qquad (\text{even } M), \qquad\qquad S_J^{(0)} = \ln \sqrt{M} \qquad (\text{odd } M). \tag{8}$$

In order to better illustrate the peculiarity of the TK effect obtained using Y-junctions of TG gases, it is useful to discuss the junction entropy for non-interacting fermions on the same star geometry. The reason is that TG and ideal Fermi gases share the same local properties and one may wonder whether the change in the thermodynamical properties due to the *junction* may be also obtained with ideal (polarized) Fermi gases in the same geometry. The result is that, as expected, the entropy tends to 0 for vanishing temperature also when the junction is present. This, as will be clear in the next section, confirms the different junction entropy in the hard-core and ideal Fermi cases, as a consequence of the *non-local* nature of the couplings at the junction for the TG gases.

Finally, it was shown in [16] that the system exhibits non-Fermi liquid behavior at low temperatures, by computing the temperature dependence of the variation of specific heat c_v due to the tunneling among the ends of the guides. It was found that, beside a term which is extensive in the number of particles and linear in the temperature, the power expansion exhibits a term which is intensive and is proportional to $T^{2\frac{M-2}{M}}$, where the characteristic power originates only from the symmetry of the low-temperature fixed point. For the particular case $M = 4$, the TBA equations yield [55] the dependence $c_v \propto -T \ln \mathrm{T}$.

If one prepares the system in an initial state characterized by different chemical potentials on different wires, a current I_α will flow from wire α through the junction [56]. This can be characterized by the conductance tensor $G_{\alpha,\beta} = -\frac{\partial I_\alpha}{\partial \mu_\beta}$. At low temperatures $T \ll T_{\mathrm{K}}$, in the proximity of the TK fixed point one finds [39, 51, 52]

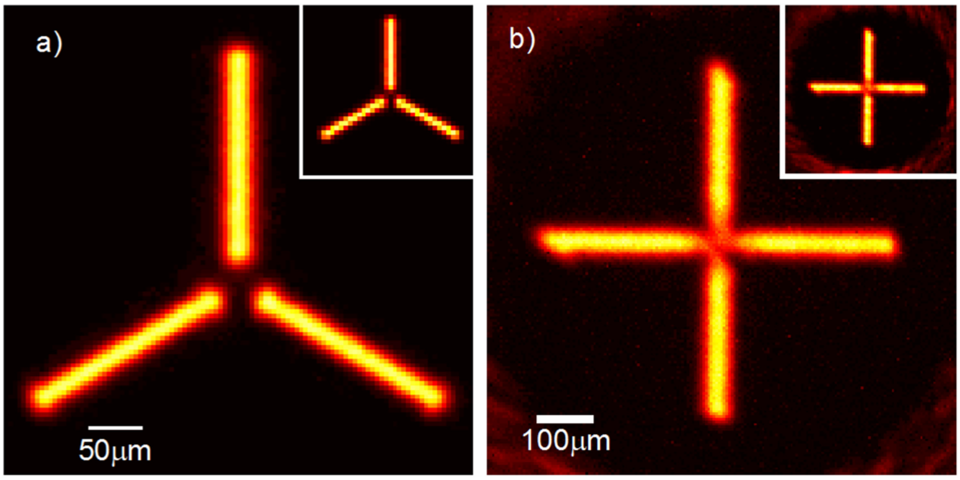

Figure 1. Holographic realization of Y-junctions in the case of red-detuned light, with (a) three waveguides and (b) four waveguides. The large patterns are taken in the focal plane of a lens with focal length $f = 150$ mm, while the insets are smaller patterns taken with $f = 50$ mm. The same scale bar applies to both large and small patterns.

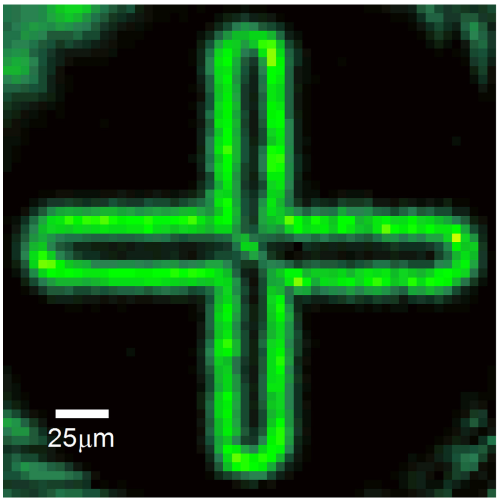

Figure 2. Y-junction with four waveguides in the case of blue-detuned light. The pattern is obtained with $f = 50$ mm. The central light spot provides a barrier between the guides.

$$G_{\alpha,\beta} = \frac{2Ke^2}{h}\left(\delta_{\alpha,\beta} - \frac{1}{M}\right) + c_{\alpha,\beta}T^{\frac{4}{M}}, \tag{9}$$

where the $c_{\alpha,\beta}$ are non universal constants and K is the Luttinger parameter characterizing the wires [57].

Correlation functions of bulk operators can also be approached via bosonization. Very close to the boundary, details of the experimental realization will be essential and no universal prediction can be made. However, at larger distances, but when both operators are still close to the boundary $x \ll \hbar v_F/T_K$, correlation functions can be computed as in [58, 59].

3. The Y-junction of TG gases and the TK Hamiltonian

We now prove that (1) can be obtained from a model Hamiltonian for interacting bosons in the TG limit, confined in M one-dimensional waveguides arranged in a star geometry, i.e. a Y-junction. Such Y-junctions of low-dimensional systems have been studied in a variety of physical systems. For Luttinger liquids crossing at a point, the fixed points [22, 23, 60] and the transport have also been investigated [61]. Lattice Y-junctions with non-interacting Bose gases were also investigated in [62, 63], while in [64] the dynamics of one-dimensional Bose liquids in Y-junctions and the reflection at the center of the star was studied.

In figures 1 and 2, we show optical trap geometries for ultracold bosonic atoms, which are stable and controllable Y-junctions of three or four waveguides connected at a point. These traps have been created using computer-generated holography, where a spatial light modulator (SLM—Boulder P256, 256 $\times$ 256 pixels, 8 bit phase control) imparts a phase pattern on a 1064 nm laser beam, which is then focused by an achromatic doublet lens to create the desired intensity distribution (to achieve trapping in all directions, a light sheet can be added to provide confinement along the optical axis). The phase modulation required to realize this pattern was calculated using the mixed-region amplitude freedom algorithm [33], and the pattern was optimized with a feedback algorithm as described in [37]. The trap geometries in figure 1 should be implemented with red-detuned light. In this case the guides are defined by the lines of maximum intensity, and the reduced light level at the center is a potential barrier across which atoms can tunnel, thereby providing a junction term H_J in the Hamiltonian. Figure 2 on the other hand should be implemented with blue-detuned light so that the atoms will be guided along the lines of intensity minimum, with the light spot in the center forming the potential barrier. We observe that for the four-legged patterns shown here, the tunneling between a given pair of guides depends on whether the guides are adjacent, or on opposite sides of the junction. However this is not a limitation, and if required it will be possible to fine-tune the tunneling, for instance by creating multi-wavelength patterns as shown in [38].

In each waveguide $\alpha = 1, \cdots, M$ the Lieb–Liniger Hamiltonian describing interacting bosons in one-dimensional guides of length $\mathcal{L}$ reads [25, 65, 66]:

$$H^{(\alpha)} = \int_0^{\mathcal{L}} \mathrm{d}x \left[\frac{\hbar^2}{2m}\partial_x \Psi_\alpha^\dagger(x)\partial_x \Psi_\alpha(x) + \frac{c}{2}\Psi_\alpha^\dagger(x)\Psi_\alpha^\dagger(x)\Psi_\alpha(x)\Psi_\alpha(x)\right]. \tag{10}$$

The parameter m is the mass of the bosons and $c > 0$ is the repulsion strength, as determined by the s-wave scattering length [67]. The bosonic fields Ψ_α satisfy canonical commutation relations $[\Psi_\alpha(x), \Psi_\alpha^\dagger(y)] = \delta(x - y)$.

The coupling of the Lieb–Liniger Hamiltonian, denoted by γ, is proportional to $c/n_{1\mathrm{D}}$ where $n_{1\mathrm{D}} \equiv \mathcal{N}/\mathcal{L}$ is the density of bosons and $\mathcal{N}$ is the number of bosons per waveguide. More specifically we have $\gamma = mc/\hbar^2 n_{1\mathrm{D}}$. The limit of vanishing $c/n_{1\mathrm{D}}$ corresponds to an ideal one-dimensional Bose gas, while the limit of infinite $c/n_{1\mathrm{D}}$ corresponds to the TG gas [68, 69], which generally has the expectation values and thermodynamic quantities of a one-dimensional ideal Fermi gas [25, 42–44]. The experimental realization of the TG gas with cold atoms [45, 46] triggered intense activity in the last decade, reviewed in [42–44].

In our experimental implementation in figure 2, the width of the junction and the transverse width of the guides are close to the diffraction limit of the optical system, which is 11 μm at 1064 nm. Given that the diffraction limit is proportional to wavelength, if the trap is created with light at 532 nm, all dimensions are halved. In particular, the full width at half maximum of the barrier becomes d $\approx$ 5 μm. With regard to the guide parameters, the transverse radius of the light profile is $w_\perp = 4$ μm and the length is $\mathcal{L} \sim 50$ μm. This transverse radius is sufficiently small for the gas to be in the TG regime. For instance, an optical power of the order of mWs creates a trap depth $D = 500$ nK and a transverse trapping frequency $\omega_\perp = 2\sqrt{D/m}/w_\perp = 2\pi \times 444$ Hz. This is sufficient to enter the TG regime at a density $n_{1\mathrm{D}} \sim 1$ atom $\mu\mathrm{m}^{-1}$ with bosonic ^{133}Cs atoms, given that their scattering length can be enhanced with Feshbach resonances [70]. In particular γ is given by:

$$\gamma = \frac{2a_{3\mathrm{D}}}{n_{1\mathrm{D}}a_\perp^2}\frac{1}{1 - Ca_{3\mathrm{D}}/a_\perp}, \tag{11}$$

where $\mathrm{C} = 1.0326$, $a_{3\mathrm{D}}$ is the scattering length and $a_\perp = \sqrt{\hbar/(m\omega_\perp)}$ is the harmonic oscillator length [71]. With $a_{3\mathrm{D}} = 4000\ a_0$ and our guide parameters above, we obtain $\gamma = 5.3$, which has been shown to be within the Tonks regime both from experimental signatures [45, 46] and from the computation of three-body recombination rate [72].

We now consider M copies of this one-dimensional Bose gas and join them together by the ends of the segments, in such a way that the bosons can tunnel from one waveguide to the others. The bosonic fields in different legs commute:

$$[\Psi_\alpha(x), \Psi_\beta^\dagger(y)] = \delta_{\alpha,\beta}\,\delta(x - y),$$

and the total Hamiltonian has the form $H = \sum_{\alpha=1}^{M} H^{(\alpha)} + H_J$ where the junction term H_J describes the tunneling process among legs.

As a tool for performing computations, as well as to give a precise meaning to the tunneling processes at the edges of the legs, in each leg we discretize space into a lattice of L sites with lattice spacing a (where $La = \mathcal{L}$ and the total number of sites N_S of the star lattice is $N_S \equiv LM$). This discretization can be physically realized by superimposing optical lattices on the legs [73]. One can then perform a tight-binding approximation [74, 75] and write the bosonic fields as $\Psi_\alpha(x) = \sum_{\alpha,j} w_{\alpha,j}(x) b_{\alpha,j}$ where $b_{\alpha,j}$ is the operator destroying a particle in the site $j = 1, \cdots, L$ of the leg α and $w_{\alpha,j}(x)$ is the appropriate Wannier wavefunction localized in the same site.

The resulting lattice Bose–Hubbard Hamiltonian on each leg then reads [74, 76]

$$H_U^{(\alpha)} = -t\sum_{j=1}^{L-1}(b_{\alpha,j}^{\dagger}b_{\alpha,j+1} + b_{\alpha,j+1}^{\dagger}b_{\alpha,j}) + \frac{U}{2}\sum_{j=1}^{L}b_{\alpha,j}^{\dagger}b_{\alpha,j}^{\dagger}b_{\alpha,j}b_{\alpha,j}, \tag{12}$$

where the interaction coefficient is $U = c\int|w_\alpha(x)|^4\,\mathrm{d}x$, the hopping coefficient is $t = -\int w_{\alpha,j}\hat{T}w_{\alpha,j+1}\,\mathrm{d}x$, and $\hat{T} = -(\hbar^2/2m)\partial^2/\partial x^2$ is the kinetic energy operator.

The total lattice Hamiltonian, in which M copies of the system are connected to one another by a hopping term, is then written as:

$$H_U = \sum_{\alpha=1}^{M}H_U^{(\alpha)} + H_J, \tag{13}$$

where the junction term has the form

$$H_J = -\lambda\sum_{1\leqslant\alpha<\beta\leqslant M}(b_{\alpha,1}^{\dagger}b_{\beta,1} + b_{\beta,1}^{\dagger}b_{\alpha,1}) \tag{14}$$

with λ being the hopping between the first site of a leg and the first sites of the others. Typically one has $\lambda > 0$, which corresponds to an *antiferromagnetic* Kondo model, as shown in the following. Nevertheless, we observe that the sign of t and λ could be changed by shaking the trap [77].

The total number of bosons in the system, $N = \mathcal{N}M$, is a conserved quantity in the lattice model and can be tuned in experiments. In the canonical ensemble $N = \sum_{\alpha,j}\langle b_{\alpha,j}^{\dagger}b_{\alpha,j}\rangle$. The phase diagram of the bulk Hamiltonian (12) in each leg undergoes quantum phase transitions between superfluid and Mott insulating phases [78]: notice that in the canonical ensemble the system is superfluid as soon as the filling N/N_S is not integer.

We are interested in the limit $U \to \infty$, so that after the continuous limit is taken back again, the TG gas is retrieved in the bulk. It is well known that this limit brings substantial simplifications in the computation: it was shown in [79] that, on each leg, the spectrum and the scattering matrix are equivalent to a system of spins in the $s = 1/2$ representation. As customary, we can map the hard-core bosons to 1/2 spins by the mapping:

$$b_{\alpha,j} \to \sigma_{\alpha,j}^{-} \qquad b_{\alpha,j}^{\dagger} \to \sigma_{\alpha,j}^{+} \qquad 2b_{\alpha,j}^{\dagger}b_{\alpha,j} - 1 \to \sigma_{\alpha,j}^{z}. \tag{15}$$

The Hamiltonian (13) can be then written in spin form

$$H_\infty^{(\alpha)} = -t\sum_{j=1}^{N-1}(\sigma_{\alpha,j}^{+}\sigma_{\alpha,j+1}^{-} + \sigma_{\alpha,j+1}^{+}\sigma_{\alpha,j}^{-}), \tag{16}$$

$$H_J = -\lambda\sum_{\alpha<\beta}^{M}(\sigma_{\alpha,1}^{+}\sigma_{\beta,1}^{-} + \sigma_{\beta,1}^{+}\sigma_{\alpha,1}^{-}) \tag{17}$$

which coincides with a junction of XX-type spin chains [80].

In general, only for one-dimensional systems the Jordan–Wigner transformation straightforwardly gives rise to a fermionic Hamiltonian which is both quadratic and local. In order to apply the transformation also to the system at hand, the introduction of an auxiliary spin-1/2 spin degree of freedom has been proposed in the case $M = 3$ [80]. This allows to write (17) as an impurity Hamiltonian, effectively equivalent to a four-channel Kondo model [81–85]. For an arbitrary number of legs, the corresponding generalization of the Jordan–Wigner transformation is [47]:

$$\sigma_\alpha^{-}(j) = \gamma_\alpha\prod_{l<j}\mathrm{e}^{\mathrm{i}\pi c_{\alpha,l}^{\dagger}c_{\alpha,l}}c_{\alpha,j} \qquad \sigma_\alpha^{z}(j) = 2c_{\alpha,j}^{\dagger}c_{\alpha,j} - 1, \tag{18}$$

where the fermions $c_{\alpha,j}$ satisfy canonical anticommutation relations

$$\{c_{\alpha,j}, c_{\beta,k}^{\dagger}\} = \delta_{\alpha,\beta}\delta_{j,k} \qquad \{c_{\alpha,j}, c_{\beta,k}\} = 0 \qquad \forall\,\alpha,\,\beta \quad \forall\,j,\,k \tag{19}$$

and the Klein factors γ_α satisfy the Clifford algebra

$$\{\gamma_\alpha, \gamma_\beta\} = 2\delta_{\alpha,\beta} \tag{20}$$

Using (18) in the Hamiltonian (16) and (17), one obtains

$$H = -t\sum_{j=1}^{N-1}\sum_{\alpha=1}^{M}(c_{\alpha,j}^{\dagger}c_{\alpha,j+1} + c_{\alpha,j+1}^{\dagger}c_{\alpha,j}) + H_J, \tag{21}$$

$$H_J = -\lambda\sum_{1\leqslant\alpha<\beta\leqslant M}\gamma_\alpha\gamma_\beta(c_{\alpha,1}^{\dagger}c_{\beta,1} + c_{\alpha,1}c_{\beta,1}^{\dagger}). \tag{22}$$

In conclusion, we have mapped the Hamiltonian (17), acting on N_S spin variables, onto another one, defined in terms of N_S spinless fermionic degrees of freedom plus one Klein factor per leg. In other words, the hard-core boson Hamiltonian (13) in the limit $U \to \infty$ is mapped onto the fermionic Hamiltonian (21), given by the sum of the M non-interacting wires and the highly non-trivial junction term H_J. The Fermi energy of the non-interacting fermions in (each of) the external wires is denoted by E_F.

We can further manipulate the coupling in the central region and conclude the mapping between the ferromagnetic star junction of XX spin chains and the TK model (see also [24, 47, 86] for comparison with Ising spin chains). In particular, denoting by $C = (c_{1,1}, \ldots, c_{M,1})^T$ the fermionic operators in the sites on the junction, we can rewrite the interaction term for $M = 3$ as an antiferromagnetic coupling between three-flavor fermions and a localized magnetic impurity [80]

$$H_J = \lambda\sigma^3 C^\dagger S^3 C + \lambda\sigma^2 C^\dagger S^2 C + \lambda\sigma^1 C^\dagger S^1 C, \tag{23}$$

where $\sigma^a = 1/(2\mathrm{i})\varepsilon^{abc}\gamma_b\gamma_c$, with ε^{abc} being the completely antisymmetric tensor, satisfy the commutation relations of the Pauli matrices. The matrices S^a are the generators of $su(2)$ in the spin-1 representation:

$$S^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix} \qquad S^2 = \begin{pmatrix} 0 & 0 & \mathrm{i} \\ 0 & 0 & 0 \\ -\mathrm{i} & 0 & 0 \end{pmatrix} \qquad S^3 = \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{24}$$

Finally, for a general number of waveguides M, after defining the impurity operators as

$$\Gamma_{\alpha,\beta} = -\frac{i}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha) \tag{25}$$

we obtain that the interaction can be written in the form

$$H_J = \lambda \sum_{1 \leqslant \alpha < \beta \leqslant M} \Gamma_{\alpha,\beta} C^\dagger T_{\alpha,\beta} C. \tag{26}$$

The operators $\Gamma_{\alpha,\beta}$ and $T_{\alpha,\beta}$ satisfy the $so(M)$ algebra

$$[T_{\alpha,\beta}, T_{\mu,\nu}] = -\mathrm{i}(\delta_{\beta,\mu}T_{\alpha,\nu} + \delta_{\alpha,\nu}T_{\beta,\mu} - \delta_{\beta,\nu}T_{\alpha,\mu} - \delta_{\alpha,\mu}T_{\beta,\nu}). \tag{27}$$

Once again, this interaction has the form of an antiferromagnetic Kondo term. In extended form, the latter is just the part containing Majorana fermions of (1). The thermodynamic limit is taken in a standard way (see e.g. [87]). At low temperature, one goes back to a continuous description. The relevant physics will take place around the Fermi edges of the spectrum, so one writes a relativistic effective Hamiltonian containing only left- and right-moving chiral fields on M half-infinite lines. The next step is to map the system onto an equivalent system in which only either left- or right-moving fields appear and are defined between $-\infty$and ∞. The junction term is located in the origin. The resulting effective Hamiltonian is (1). Note that the interaction among Majorana modes, proportional to $h_{\alpha,\beta}$, is absent by construction. We conclude that a Y-junction of TG gases provides an experimentally realizable system in which the TK effect may be observed.

We also remark that a mechanism through which an unwanted change of the state encoded in the Majorana modes can occur is the loss of bosonic atoms by the trap. With our parameters, we found this possibility negligible on the time scales needed for the experiment, since the energy barrier for the atom loss is ~500 nK, much larger than the Fermi energy of the TG gases.

Another useful experimental consideration is that for the patterns shown in figures 1 and 2, the residual intensity fluctuations at the bottom of the guides are of the order of a percent of the trap depth. This poses an upper limit to the trap depth and hence to the transverse trapping frequency that can be achieved with our off-the-shelf optics. However the trapping frequency could be increased by using custom-made higher resolution optics, such as in quantum gas microscopes [88–90], enabling us to go deeper in the TG regime. We also note that some arbitrary potentials have already been implemented in [90], which suggests that holographic traps are feasible in quantum gas microscope experiments. Moreover, we could improve the signal to noise ratio in atomic images by adding an optical lattice along the axis of the optical system, therefore creating many copies of the Y-junction. This is similar to the technique used in the first experimental realizations of the TG limit in cold atoms [45, 46], where many one-dimensional tubes are created with an optical lattice.

We finally observe that our goal was to show a method to obtain the TK Hamiltonian within a cold atom setup, and not to have a physical realization of the Kitaev chain and the associated Majorana modes [91]. Indeed, in our setup, the Majorana fermions are not physical objects, since the observable quantities are the ones related to the hard-core bosons. A related example of Y-junction of bosonic systems with repulsive interaction has also appeared in [92], and connection with the TK model has been hinted through the low-temperature behavior of the conductance.

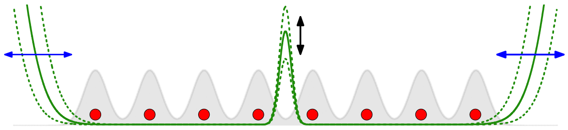

Figure 3. Pictorial representation of the parametric excitation procedure to impart energy to the TG gas, with the purpose of measuring the specific heat. The height of the barrier or the length of the guides can be modulated by changing the phase pattern on the SLM. Alternatively, the intensity of the entire pattern can be varied with an acousto-optic modulator.

4. Estimates for the detection of the TK regime

Next, we assess the feasibility of an experiment in which ultracold Cs atoms are loaded into the Y-junction presented in this paper. We provide estimates of the tunneling coefficient λ and the Kondo temperature T_K given a barrier width of d $\sim$ 5 μm and a barrier height V_0 which can be as low as ~15–20 nK.

An important preliminary remark is on the role of the density n_{1D}: as n_{1D} is increased, so too is the Fermi energy E_F, which is given by $E_F = \hbar^2\pi^2 n_{1D}^2/2m$ in the one-dimensional limit (this formula is valid because the transverse degrees of freedom, having energies $\hbar\omega_\perp$, are not excited). The Kondo temperature T_K is given by $T_K \approx E_F \exp\left\{-\frac{E_F}{(M-2)\lambda}\right\}$: for given values of λ and M, T_K increases with E_F/λ up to a certain value of E_F/λ (which is $\sim$1 for $M = 3$), and afterwards it decreases with E_F/λ. At the same time, n_{1D} should be not too large since then γ would decrease (as discussed in section 3) and the condition of large γ, i.e. to have TG gases, would not be satisfied.

With these constraints in mind, we find that a density $n_{1D} \sim 1$ atom/μm, giving $\gamma \approx 5$ and $E_F \approx 20$ nK, leads to a favorable estimate for T_K. The tunneling coefficient λ is estimated by assuming that the relevant processes for the tunneling between different TG gases only happen around E_F and that the other particles only behave like a background Fermi sea. The single-particle tunneling energy λ is the coefficient of the term $\propto \psi_\beta^\dagger(0)\psi_\alpha(0)$ representing the tunneling between the hard-core bosons at or near the point 0 of the different bulk chains. In presence of lattices, which could be created by superimposing lattices in each guide and merging them at the junction, if ℓ is the spacing of the lattice one can think of taking $\ell \approx$ d; in the continuous limit it is $\ell \sim \mathcal{L}/\mathcal{N}$. It is clear that λ depends on the form of the barrier, on the barrier length d, and on V_0 and E_F. Clearly, the closer E_F is to V_0 (with $E_F < V_0$), the larger is λ. With $V_0 - E_F$ of the order of 1–2 nK, we obtain $\lambda \sim 1 - 2$ nK and $T_K \sim 10$ nK. (We observe that tunneling through distances of $\sim$5 μm has been observed with Rb atoms in [93] with an energy barrier $V_0/h \gtrsim 500$ Hz$\sim$20 nK.) We also remark that V_0 should be compared with the residual intensity fluctuations at the bottom of the waveguides which are $\sigma_V \lesssim 5$ nK. Even if the two quantities are still comparable, we are reasonably within the range $V_0 \gg \sigma_V$ and the central barrier can be clearly resolved. Furthermore, with $E_F \approx 20$ nK we also satisfy the condition $E_F \gg \sigma_V$, i.e. the residual intensity fluctuations do not significantly alter the tunneling of hard-core bosons at the Fermi energy scale.

The above estimates of λ and T_K are obtained with $d \approx 5$ μm, so larger values may be obtained for smaller values of d. It is realistic to think that a width d $\gtrsim 2$ μm can be obtained experimentally: as an example we mention the recent work [94] in which a potential barrier was created with a strongly anisotropic laser beam at 532 nm focussed to a $1/e^2$ beam waist of 2 μm, corresponding to a full width at half maximum $d \approx 2.3$ μm. Moreover, a combined use of recently developed quantum gas microscope techniques and of holographic traps may give the possibility to further reduce d, which in turn would result in even more favorable estimates for the Kondo temperature.

We finally comment on the detection of the TK regime, using the theoretical results reviewed in section 2. A direct way to observe the TK effect is to measure the temperature of the system after parametric heating, from which the specific heat can be obtained. For this purpose, energy can be transferred to the system by modulating (with a frequency $\sim E_F$) the length of the guides and/or the height of the barrier, as shown in figure 3. An important point is that measurements should be done with and without the impurity (i.e., $\lambda = 0$ or $V_0 \to \infty$). Then, comparison of the two allows to isolate the contribution of the impurity to the specific heat. The low-temperature dependence has been computed using Bethe ansatz and has a characteristic non-Fermi-liquid contribution. Another promising method is to extract from *in situ* measurements the decay of the static density–density correlation function around the center of the Y-junction, and to compare this decay with that of the correlation functions far from the center. One could also perform a measurement of conductivity, where a wavepacket is created in one of the guides and allowed to evolve across the trap. Finally, the entropy exhibits a clear even–odd effect in M for vanishing temperature. Provided some signatures of this effect are still present at very low—yet finite—temperatures, one can detect these signatures from independent measurements of the temperature of the gas (extracted from time-of-flight images) and of its energy (possibly extracted from *in situ* images). With the results of section 2, giving in practice the equation of state, it is then possible to extract the entropy as a function of temperature and internal energy.

The concrete implementation of these methods is certainly non-trivial, however we are confident that with present-day technology a combined use of them will give clear evidence of the presence of the TK effect.

5. Conclusions

The physics of the TK effect is based on the interaction of localized Majorana modes with external one-dimensional wires, merging in a star-like geometry. In the original formulation, fermionic wires were coupled to a high-charging energy superconducting island, hosting a set of nanowires with strong spin–orbit coupling which, in appropriate magnetic field, developed localized Majorana edge modes. Conversely, in the present paper, one-dimensional wires containing strongly interacting bosons are coupled among them at one end. In particular, we considered a junction of TG gases arranged in a star geometry (forming a Y-junction). We showed that this system provides a physical realization of the TK model.

We then presented experimental results for Y-junctions using holographic optical traps. We estimated that it is possible to have controllable and independent tunnelings of atoms between the different waveguides. Since the one-dimensional regime for cold bosons is routinely reached in ultracold atom experiments, both in optical traps and in atom-chip traps [42–44], and since TG gases have been widely studied in experiments [45, 46], our holographic Y-junctions provide a proof-of-principle of the possibility of realizing TK devices in cold atoms experiments. By proof-of-principle we mean that all the theoretical and experimental ingredients (the model, the Y-junction and the one-dimensional TG gases) are available. Even though the completion of an ultracold TK device in the realm of atomtronics is certainly challenging, research in the near future will clarify whether cold atoms may be used as a platform for studying the TK effect complementary to the solid-state realizations.

We finally comment on the complementarity between the approach proposed in the present paper and the realization of the TK effect in solid state devices. Pros and cons of the present approach can be summarized this way: in the cold-atoms architecture, the interacting terms among Majorana modes are absent, which is an advantage for the stability of the TK effect, but at the same time the manipulation of information may be more difficult in view of the implementation of quantum information tasks.

To be more specific, in solid state proposals, the charging energy of the island must be the largest energy scale as single-particle tunneling onto the central island may mix fermion number parity sectors and spoil the TK effect [16]. Other difficulties arise from the necessity of controlling relatively strong magnetic fields on a mesoscopic scale. Finally, one has tight bounds on the lifetime of the Majorana modes, arising from the poisoning of quasiparticles in the superconducting substrate, whenever the device is part of a circuit [17, 18]. In contrast, in the proposed experimental setup, the Majorana modes are non-locally encoded in the bosonic field and therefore do not suffer any decoherence. However, this implies that Majorana fermions cannot be directly manipulated, which, incidentally, is a problem common to many solid-state candidate devices for topological quantum computation. Indeed, in our proposal, the effective Hamiltonian is that of the TK model, but the physical quantities to be measured are the observables related to the trapped atoms and not directly the Majorana modes. This motivates the very interesting quest of suitable schemes and algorithms for quantum information tasks on devices constructed from the strongly interacting bosonic Y-junction.

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References

[1] Leijnse M and Flensberg K 2012 Introduction to topological superconductivity and majorana fermions *Semicond. Sci. Technol.* **27** 124003
[2] Beenakker C W J 2013 Search for Majorana fermions in superconductors *Annu. Rev. Con. Mat. Phys.* **4** 113–36
[3] Alicea J, Oreg Y, Refael G, von Oppen F and Fisher M P A 2011 Non-abelian statistics and topological quantum information processing in 1 d wire networks *Nat. Phys.* **7** 412–7

[4] Stanescu T D and Tewari S 2013 Majorana fermions in semiconductor nanowires: fundamentals, modeling, and experiment *J. Phys.: Condens. Matter* **25** 233201

[5] Béri B and Cooper N R 2012 Topological kondo effect with majorana fermions *Phys. Rev. Lett.* **109** 156803

[6] Preskill J 1998 *Lecture Notes for Physics 229: Quantum Information and Computation*

[7] Nielsen M and Chuang I 2000 *Quantum Computation and Quantum Information* (*Cambridge Series on Information and the Natural Sciences*) (Cambridge: Cambridge University Press)

[8] Nayak C, Simon S, Stern A, Freedman M and Das Sarma S 2008 Non-Abelian anyons and topological quantum computation *Rev. Mod. Phys.* **80** 1083–159

[9] Hyart T, van Heck B, Fulga I C, Burrello M, Akhmerov A R and Beenakker C W J 2013 Flux-controlled quantum computation with majorana fermions *Phys. Rev.* B **88** 035121

[10] van Heck B, Akhmerov A R, Hassler F, Burrello M and Beenakker C W J 2012 Coulomb-assisted braiding of Majorana fermions in a Josephson junction array *New J. Phys.* **14** 035019

[11] Sau J D, Clarke D J and Tewari S 2011 Controlling non-Abelian statistics of Majorana fermions in semiconductor nanowires *Phys. Rev.* B **84** 094505

[12] Aasen D *et al* 2015 Milestones toward Majorana-based quantum computing arXiv:1511.05153

[13] Das Sarma S, Freedman M and Nayak C 2015 Majorana Zero Modes and Topological Quantum Computation *npj Quantum Inf.* **1** 15001

[14] Fu L 2010 Electron teleportation via Majorana bound states in a mesoscopic superconductor *Phys. Rev. Lett.* **104** 056402

[15] Hewson A 1997 *The Kondo Problem to Heavy Fermions* (*Cambridge Studies in Magnetism*) (Cambridge: Cambridge University Press)

[16] Buccheri F, Babujian H, Korepin V E, Sodano P and Trombettoni A 2015 Thermodynamics of the topological Kondo model *Nucl. Phys.* B **896** 52–79

[17] Higginbotham A P, Albrecht S M, Kirsanskas G, Chang W, Kuemmeth F, Krogstrup P, Jespersen T S, Nygard J, Flensberg K and Marcus C M 2015 Parity lifetime of bound states in a proximitized semiconductor nanowire *Nat. Phys.* **11** 1017–21

[18] Rainis D and Loss D 2012 Majorana qubit decoherence by quasiparticle poisoning *Phys. Rev.* B **85** 174533

[19] Plugge S, Zazunov A, Eriksson E, Tsvelik A M and Egger R 2016 Kondo physics from quasiparticle poisoning in majorana devices *Phys. Rev.* B **93** 104524

[20] Ryu C and Boshier M G 2015 Integrated coherent matter wave circuits *New J. Phys.* **17** 092002

[21] Levin K and Fetter A L 2012 *Ultracold Bosonic and Fermionic Gases* (Amsterdam: Elsevier)

[22] Chamon C, Oshikawa M and Affleck I 2003 Junctions of three quantum wires and the dissipative Hofstadter model *Phys. Rev. Lett.* **91** 206403

[23] Oshikawa M, Chamon C and Affleck I 2006 Junctions of three quantum wires *J. Stat. Mech.* **2006** P02008

[24] Tsvelik A 2014 Topological kondo effect in star junctions of ising magnetic chains. Exact solution *New J. Phys.* **16** 033003

[25] Korepin V E, Bogoliubov N M and Izergin A G 1997 *Quantum Inverse Scattering Method and Correlation Functions* (*Cambridge Monographs on Mathematical Physics*) (Cambridge: Cambridge University Press)

[26] Gogolin A O, Nersesyan A A and Tsvelik A M 1998 *Bosonization and Strongly Correlated Systems* (Cambridge: Cambridge University Press)

[27] Fuhrer M S *et al* 2000 Crossed nanotube junctions *Science* **288** 494–7

[28] Bergamini S, Darquie B, Jones M, Jacubowiez L, Browaeys A and Grangier P 2004 Holographic generation of microtrap arrays for single atoms by use of a programmable phase modulator *J. Opt. Soc. Am.* B **21** 1889–94

[29] He X, Xu P, Wang J and Zhan M 2009 Rotating single atoms in a ring lattice generated by a spatial light modulator *Opt. Express* **17** 21007–14

[30] Nogrette F, Labuhn H, Ravets S, Barredo D, Béguin L, Vernier A, Lahaye T and Browaeys A 2014 Single-atom trapping in holographic 2D arrays of microtraps with arbitrary geometries *Phys. Rev.* X **4** 021034

[31] Boyer V, Godun R M, Smirne G, Cassettari D, Chandrashekar C M, Deb A B, Laczik Z J and Foot C J 2006 Dynamic manipulation of Bose–Einstein condensates with a spatial light modulator *Phys. Rev.* A **73** 031402

[32] Gaunt A L, Schmidutz T F, Gotlibovych I, Smith R P and Hadzibabic Z 2013 Bose–Einstein condensation of atoms in a uniform potential *Phys. Rev. Lett.* **110** 200406

[33] Pasienski M and DeMarco B 2008 A high-accuracy algorithm for designing arbitrary holographic atom traps *Opt. Express* **16** 2176–90

[34] Harte T, Bruce G D, Keeling J and Cassettari D 2014 Conjugate gradient minimisation approach to generating holographic traps for ultracold atoms *Opt. Express* **22** 26548

[35] Bruce G D, Mayoh J, Smirne G, Torralbo-Campo L and Cassettari D 2011 A smooth, holographically generated ring trap for the investigation of superfluidity in ultracold atoms *Phys. Scr.* **2011** 014008

[36] Gaunt A L and Hadzibabic Z 2012 Robust digital holography for ultracold atom trapping *Sci. Rep.* **2** 721

[37] Bruce G D, Johnson M Y H, Cormack E, Mayoh R D A W J and Cassettari D 2015 Feedback-enhanced algorithm for aberration correction of holographic atom traps *J. Phys. B: At. Mol. Opt. Phys.* **48** 115303

[38] Bowman D, Ireland P, Bruce G D and Cassettari D 2015 Multi-wavelength holography with a single spatial light modulator for ultracold atom experiments *Opt. Express* **23** 8365–72

[39] Altland A, Béri B, Egger R and Tsvelik A M 2014 Multichannel Kondo impurity dynamics in a Majorana device *Phys. Rev. Lett.* **113** 076401

[40] Altland A, Béri B, Egger R and Tsvelik A M 2014 Bethe ansatz solution of the topological Kondo model *J. Phys. A: Math. Theor.* **47** 265001

[41] Grimm R, Weidemüller M and Ovchinnikov Y B 2000 Optical dipole traps for neutral atoms *Advances In Atomic, Molecular, and Optical Physics* vol 42 ed B Bederson and H Walther (New York: Academic) pp 95– 170

[42] Yurovsky V A, Olshanii M and Weiss D S 2008 Collisions, correlations, and integrability in atom waveguides *Adv. At. Mol. Opt. Phys.* vol 55 ed E Arimondo, P R Berman and C C Lin (San Diego, CA: Academic) pp 61–138

[43] Bouchoule I, van Druten N J and Westbrook C I 2011 Atom Chips and One-Dimensional Bose Gases *Atom Chips* ed J Reichel and V Vuletic (New York: Wiley) pp 331–63

[44] Cazalilla M A, Citro R, Giamarchi T, Orignac E and Rigol M 2011 One dimensional bosons: from condensed matter systems to ultracold gases *Rev. Mod. Phys.* **83** 1405–66

[45] Paredes B, Widera A, Murg V, Mandel O, Fölling S, Cirac I, Shlyapnikov G V, Hänsch T W and Bloch I 2004 Tonks–Girardeau gas of ultracold atoms in an optical lattice *Nature* **429** 277

[46] Kinoshita T, Wenger T and Weiss D 2004 Observation of a one-dimensional Tonks–Girardeau gas *Science* **305** 1125–8

[47] Tsvelik A M 2014 Topological Kondo effect in star junctions of Ising magnetic chains: exact solution *New J. Phys.* **16** 033003

[48] Oreg Y, Refael G and von Oppen F 2010 Helical liquids and Majorana bound states in quantum wires *Phys. Rev. Lett.* **105** 177002
[49] Lutchyn R M, Sau J D and Sarma S D 2010 Majorana fermions and a topological phase transition in semiconductor-superconductor heterostructures *Phys. Rev. Lett.* **105** 077001
[50] Mourik V, Zuo K, Frolov S M, Plissard S R, Bakkers E P A M and Kouwenhoven L P 2012 Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices *Science* **336** 1003–7
[51] Altland A and Egger R 2013 Multiterminal Coulomb–Majorana junction *Phys. Rev. Lett.* **110** 196401
[52] Béri B 2013 Majorana–Klein hybridization in topological superconductor junctions *Phys. Rev. Lett.* **110** 216803
[53] Zazunov A, Altland A and Egger R 2014 Transport properties of the Coulomb–Majorana junction *New J. Phys.* **16** 015010
[54] Affleck I and Ludwig A W W 1991 universal non-integer ground-state degeneracy in critical quantum systems *Phys. Rev. Lett.* **67** 161–4
[55] Jerez A, Andrei N and Zarand G 1998 Solution of the multichannel Coqblin–Schrieffer impurity model and application to multilevel systems *Phys. Rev.* B **58** 3814–41
[56] Galpin M R, Mitchell A K, Temaismithi J, Logan D E, Béri B and Cooper N R 2014 Conductance fingerprint of Majorana fermions in the topological Kondo effect *Phys. Rev.* B **89** 045143
[57] Giamarchi T 2003 *Quantum Physics in One Dimension* (Oxford: Clarendon)
[58] Eriksson E, Nava A, Mora C and Egger R 2014 Tunneling spectroscopy of Majorana–Kondo devices *Phys. Rev.* B **90** 245417
[59] Bellazzini B, Mintchev M and Sorba P 2007 Bosonization and scale invariance on quantum wires *J. Phys. A: Math. Theor.* **40** 2485
[60] Giuliano D and Sodano P 2009 *Y*-junction of superconducting Josephson chains *Nucl. Phys.* B **811** 395–419
[61] Komnik A and Egger R 1998 Non-equilibrium transport for crossed Luttinger liquids *Phys. Rev. Lett.* **80** 2881–4
[62] Burioni R, Cassi D, Rasetti M, Sodano P and Vezzani A 2001 Bose–Einstein condensation on inhomogeneous complex networks *J. Phys. B: At. Mol. Opt. Phys.* **34** 4697
[63] Brunelli I, Giusiano G, Mancini F P, Sodano P and Trombettoni A 2004 Topology-induced spatial Bose–Einstein condensation for bosons on star-shaped optical networks *J. Phys. B: At. Mol. Opt. Phys.* **37** S275
[64] Tokuno A, Oshikawa M and Demler E 2008 Dynamics of one-dimensional Bose liquids: Andreev-like reflection at Y junctions and the absence of the Aharonov–Bohm effect *Phys. Rev. Lett.* **100** 140402
[65] Lieb E H and Liniger W 1963 Exact analysis of an interacting Bose gas: I. the general solution and the ground state *Phys. Rev.* **130** 1605–16
[66] Yang C and Yang C 1969 Thermodynamics of a one-dimensional system of bosons with repulsive delta-function interaction *J. Math. Phys.* **10** 1115–22
[67] Olshanii M 1998 Atomic scattering in the presence of an external confinement and a gas of impenetrable bosons *Phys. Rev. Lett.* **81** 938
[68] Tonks L 1936 The complete equation of state of one, two and three-dimensional gases of hard elastic spheres *Phys. Rev.* **50** 955–63
[69] Girardeau M 1960 Relationship between systems of impenetrable bosons and fermions in one dimension *J. Math. Phys.* **1** 516–23
[70] Chin C, Vuletić V, Kerman A J, Chu S, Tiesinga E, Leo P J and Williams C J 2004 Precision Feshbach spectroscopy of ultracold Cs_2 *Phys. Rev.* A **70** 032701
[71] Haller E, Gustavsson M, Mark M J, Danzl J G, Hart R, Pupillo G and Nägerl H-C 2009 Realization of an excited, strongly correlated quantum gas phase *Science* **325** 1224–7
[72] Kormos M, Mussardo G and Trombettoni A 2009 Expectation values in the Lieb–Liniger Bose gas *Phys. Rev. Lett.* **103** 210404
[73] Lewenstein M, Sanpera A and Ahufinger V 2012 *Ultracold Atoms in Optical Lattices: Simulating Quantum Many-Body Systems* (Oxford: Oxford University Press)
[74] Jaksch D, Bruder C, Cirac J I, Gardiner C W and Zoller P 1998 Cold bosonic atoms in optical lattices *Phys. Rev. Lett.* **81** 3108–11
[75] Trombettoni A and Smerzi A 2001 Discrete solitons and breathers with dilute Bose–Einstein condensates *Phys. Rev. Lett.* **86** 2353
[76] Jaksch D and Zoller P 2005 The cold atom Hubbard toolbox *Ann. Phys., NY* **315** 52–79 special issue
[77] Eckardt A, Weiss C and Holthaus M 2005 Superfluid-insulator transition in a periodically driven optical lattice *Phys. Rev. Lett.* **95** 260404
[78] Fisher M P A, Weichman P B, Grinstein G and Fisher D S 1989 Boson localization and the superfluid-insulator transition *Phys. Rev.* B **40** 546–70
[79] Friedberg R, Lee T and Ren H 1993 Equivalence between spin waves and lattice bosons with applications to the heisenberg model *Ann. Phys., NY* **228** 52–103
[80] Crampé N and Trombettoni A 2013 Quantum spins on star graphs and the Kondo model *Nucl. Phys.* B **871** 526 – 538
[81] Schlottmann P and Sacramento P 1993 Multichannel Kondo problem and some applications *Adv. Phys.* **42** 641–82
[82] Tsvelick A and Wiegmann P 1984 Solution of the-channel Kondo problem (scaling and integrability) *Z. Phys.* B **54** 201–6
[83] Tsvelick A and Wiegmann P 1985 Exact solution of the multichannel Kondo problem, scaling, and integrability *J. Stat. Phys.* **38** 125–47
[84] Andrei N and Destri C 1984 Solution of the multichannel Kondo problem *Phys. Rev. Lett.* **52** 364–7
[85] Fabrizio M and Gogolin A O 1994 Toulouse limit for the overscreened four-channel Kondo problem *Phys. Rev.* B **50** 17732–5
[86] Reyes S A and Tsvelik A M 2005 Crossed spin-1/2 Heisenberg chains as a quantum impurity problem *Phys. Rev. Lett.* **95** 186404
[87] Affleck I and Ludwig A 1991 Critical theory of overscreened Kondo fixed points *Nucl. Phys.* B **360** 641–96
[88] Bakr W S, Gillen J I, Peng A, Fölling S and Greiner M 2009 A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice *Nature* **462** 74–7
[89] Haller E, Hudson J, Kelly A, Cotta D A, Peaudecerf B, Bruce G D and Kuhr S 2015 Single-atom imaging of fermions in a quantum-gas microscope *Nat. Phys.* **11** 738–42
[90] Preiss P M, Ma R, Tai M E, Lukin A, Rispoli M, Zupancic P, Lahini Y, Islam R and Greiner M 2015 Strongly correlated quantum walks in optical lattices *Science* **347** 1229
[91] Laflamme C, Baranov M A, Zoller P and Kraus C V 2014 Hybrid topological quantum computation with Majorana fermions: a cold-atom setup *Phys. Rev.* A **89** 022319
[92] Yin S and Béri B 2016 Universality and quantized response in bosonic nonfractionalized tunneling *Phys. Rev.* B **93** 245142
[93] Albiez M, Gati R, Fölling J, Hunsmann S, Cristiani M and Oberthaler M K 2005 Direct observation of tunneling and nonlinear self-trapping in a single bosonic Josephson junction *Phys. Rev. Lett.* **95** 010402
[94] Valtolina G *et al* 2015 Josephson effect in fermionic superfluids across the bec-bcs crossover *Science* **350** 1505