Modeling and Parametric Analysis of Quasi-Translational Parallel Continuum Manipulators

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Abstract	Translational Parallel Manipulators proved to be effective mechanisms in different application fields, from industries to haptic devices. The introduction of intrinsic flexibility within these mechanisms looks promising at increasing the safety of robots that are adopted in collaborative work-spaces. This paper focuses on the analysis of different types of Parallel Continuum Manipulator to find the better geometric structure to achieve quasi-translational motions. Therefore, the goal is to look for new flexible architectures that could be used instead of mechanisms composed by rigid links to improve safety in factories, in alignment with the United Nations Sustainable Development Goal 9: build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation.

Keywords (separated by '-')	SDG9 - Parallel Continuum Manipulators - Transitional Motion - Parametric Analysis

Author Proof

Modeling and Parametric Analysis of Quasi-Translational Parallel Continuum Manipulators

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Abstract. Translational Parallel Manipulators proved to be effective mechanisms in different application fields, from industries to haptic devices. The introduction of intrinsic flexibility within these mechanisms looks promising at increasing the safety of robots that are adopted in collaborative work-spaces. This paper focuses on the analysis of different types of Parallel Continuum Manipulator to find the better geometric structure to achieve quasi-translational motions. Therefore, the goal is to look for new flexible architectures that could be used instead of mechanisms composed by rigid links to improve safety in factories, in alignment with the United Nations Sustainable Development Goal 9: build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation. AQ1

Keywords: SDG9 · Parallel Continuum Manipulators · Transitional Motion · Parametric Analysis

1 Introduction

Translational Parallel Manipulators (TPMs) are closed-loop mechanisms with rigid links and kinematic joints that connect an end-effector to a fixed frame via several kinematic chains arranged in such a way that the manipulator itself is subjected to a set of permanent geometrical constraints that make the output motion a pure translational 3 DoF motion. A typical example of this class of robot is the Delta-Robot [1–3]. Classical applications for this class of robot are pick and place operation [4], adhesive dispensing, packing small products (cosmetics, pharmaceutical industry, etc.) [5], high precision assembly operation [6], 3D printers [7] and haptic controllers [8].

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Compliant mechanisms are a group of mechanisms that acquire mobility thanks to the flexibility of some of their parts [9,10]. Subsets of the aforementioned class are Parallel Continuum Mechanisms (PCMs), devices in which a rigid end-effector is connected to a fixed frame using flexible slender links whose nonlinear deformation is the cause of its mobility. The research community has already proposed methods for PCMs kinematic modeling, together with some case studies. These PCMs have a rigid end-effector connected to the base frame through six flexible rods whose length is controlled by the robot actuators. Based on this architecture, a class of lower mobility PCMs has been proposed in the study [11] to emulate the mobility of a lower mobility parallel robot, the rigid-link $3\underline{P}RS$ tripod. The main advantage of a robot with intrinsic flexibility is related to safety in collaborative workspaces. For example, if for any reason the obstacle avoidance control system fails, the compliance of the PCM could help at mitigating the effect of an impact between the robot and the operator. To model this class of parallel robots, different methods had been proposed in the past. Among them, nonlinear deformation is frequently modeled with the Cosserat rod model [12] expressed through a nonlinear system of differential equations (ODEs).

In the case of PCMs, it is not possible to reduce the full mobility of the end-effector through permanent constraints, as for their rigid counterparts. Nevertheless, some mechanical arrangements can introduce a much higher limitation of deformation in some directions, producing a similar constraining effect. The goal of this paper is to investigate the properties of a PCM in which the arrangement of the rod is conceived in a way that the expected motion is similar to the one of a Delta-Robot.

2 Mechanism Description

In literature, a six degrees of freedom flexible hexapod had already been studied in [13]. In that work, the authors devised a six-degrees of freedom PCM in analogy with the rigid parallel robot $6\underline{P}RS$, by replacing the revolute joint on each limb with the flexibility of cylindrical rod. To change the pose of the end-effector, the length of the rods is controlled by actuators. To achieve a quasi-translational motion, the flexible rods length L_i is controlled in pairs by three prismatic actuators placed under the XY plane and mounted with the axis parallel to the z direction. Referring to Fig. 1, the rods with the same color are controlled together. For this reason, the architecture showed in Fig. 1 is called $3P\underline{F}S$.

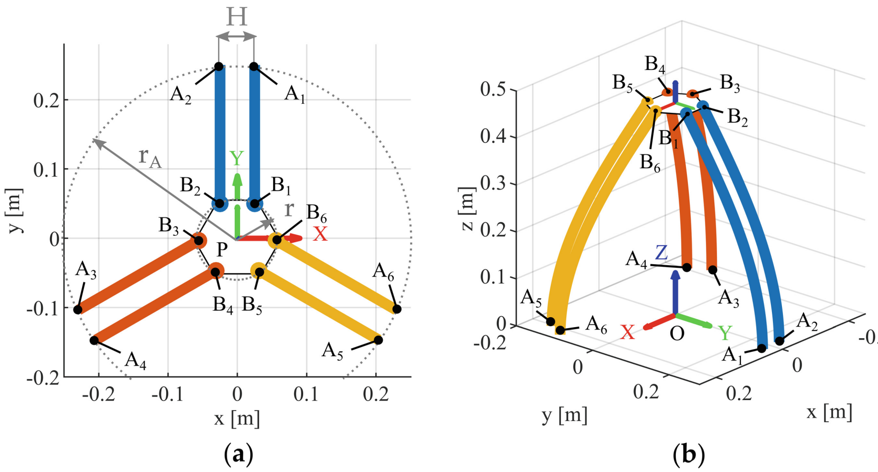

Fig. 1. Architecture 1: Flexible Tripod $3P\underline{F}S$ with parallel rods: (**a**) top view and geometric parameters definition, (**b**) 3D representation.

The end of rods $(2j-1)$ and $2j$ $(j = 1, 2, 3)$ are joined to the end-effector at points $B_{(2j-1)}$ and B_{2j} with spherical joints that introduce no restriction of rod's self rotation. The same rods $(2j-1)$ and $2j$ are connected to the base at fixed points $A_{(2j-1)}$ and $A_{(2j)}$ distributed symmetrically and with a fixed vertical orientation, again with no restriction of intrinsic rotation so that torsion effects on the rods are avoided. At the connection to the fixed base, rods are conducted through said guiding holes A_i $(i = 1, \ldots, 6)$ to the three linear actuators j below the base that control the length of the rods $(2j-1)$ and $2j$ together $(j = 1, 2, 3)$.

An alternative flexible mechanism could be devised by keeping fixed rod lengths $L_i = L$ and using the z coordinate of points $A_{(2j-1)}$ and $A_{(2j)}$ as inputs for the mechanism, as shown in Fig. 2. As for architecture 1, the rods $(2j-1)$ and $2j$ are clamped at points $A_{(2j-1)}$ and $A_{(2j)}$ with a fixed vertical orientation, while spherical joints are used in the connection with the end-effector at points $B_{(2j-1)}$ and B_{2j}.

The geometry of the mechanism is described by the following parameters (Fig. 1 and 2): r_A distance between the attachment points A_i and the fixed frame origin O, r distance between the attachment points B_i and the end-effector frame origin P, H distance between the attachment points $A_{(2j-1)}$ and $A_{(2j)}$ equal to the distance between $B_{(2j-1)}$ and $B_{(2j)}$. These geometry parameters, and in particular the parameter H and the ratio r_A/r, have a strong influence on the type of motion of the end-effector as it is presented in Sect. 4.

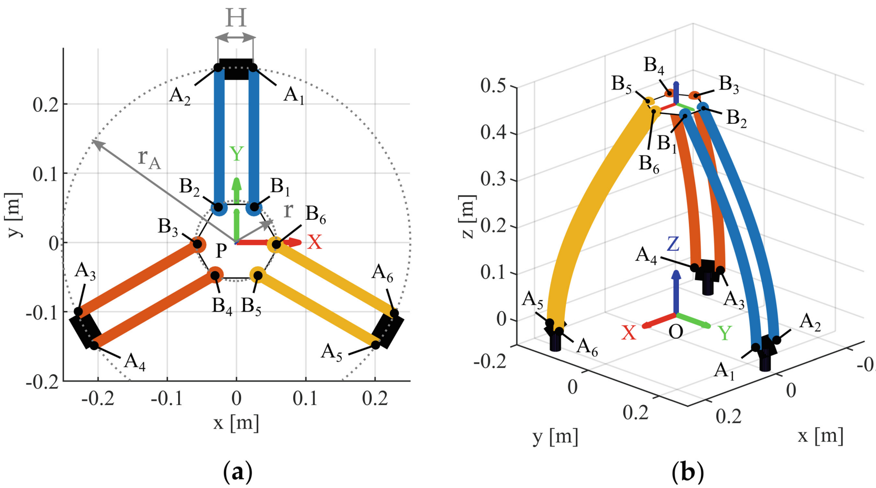

Fig. 2. Architecture 2: Flexible Tripod $3\underline{P}FFSS$ with parallel rods and fixed rod length: (**a**) top view and geometric parameters definition, (**b**) 3D representation.

3 Simulation Method

In this section the adopted modeling approach for Forward Kinematics (FK) resolution of PCMs is briefly presented. The description is kept concise for space limitation, the interested reader is addressed to the studies [11,13] for further details.

The FK Problem consists of determining the pose of the end-effector, i.e. the position vector of reference point P, $\mathbf{p}$, and the orientation given by the rotation matrix $\mathbf{R}_{EE}$, when the inputs are known and a given load $\mathbf{F}_{ext}; \mathbf{M}_{ext}$ is imposed.

To model the non-linear deformation of rods, the Kirchhoff model is adopted. Each section i of the rod is described with a centroid position $\mathbf{p}_i(s)$ and orientation defined with unit-quaternion $\tilde{\mathbf{q}}_i(s)$ with $s \in [0, L_i]$. Rods' internal moments $\mathbf{m}_i(s)$ are related to the curvature $\mathbf{u}(s)$ through the relation $\mathbf{u} = \mathbf{K}_{BT}^{-1}\mathbf{R}^T\mathbf{m}$, where $\mathbf{K}_{BT}$ is a stiffness matrix for bending and torsion. Extension-compression or shear effects are neglected in the evaluation of internal forces $\mathbf{n}(s)$. The change of shape of the flexible rod and the equilibrium of internal forces and moments with the load along s are related through a system of differential equations $\frac{d\mathbf{y}}{ds} = \mathbf{f}$, that can be stated with the vector $\mathbf{y}$ of variables and the vector of functions $\mathbf{f}$:

$$\mathbf{y} = \begin{Bmatrix} \mathbf{p}(s) \\ \tilde{\mathbf{q}}(s) \\ \mathbf{n}(s) \\ \mathbf{u}(s) \end{Bmatrix}; \qquad \mathbf{f} = \begin{Bmatrix} \mathbf{R}(s)\mathbf{e}_3 \\ \frac{1}{2}\tilde{\mathbf{q}}(s)\tilde{\mathbf{u}}(s) \\ \mathbf{0} \\ -\mathbf{K}_{BT}^{-1}\left(\left(\widehat{\mathbf{u}(s)}\mathbf{K}_{BT}\right)\mathbf{u}(s) + \widehat{\mathbf{e}}_3\mathbf{R}^T(s)\mathbf{n}(s)\right) \end{Bmatrix} \tag{1}$$

where $\mathbf{R}(s)$ the rotation matrix of section i associated to the unit-quaternion $\tilde{\mathbf{q}}(s)$, and $\widehat{\ }$ the operation that converts a three-dimension vector into its skew-matrix representation.

Evolution of $\mathbf{p}_i(s)$, $\hat{\mathbf{q}}_i(s)$, and $u_{xi}(s)$, $u_{yi}(s)$ can be obtained upon integration of the system of equations using Runge-Kutta method, from $s = 0$ to $s = L_i$. At $s = 0$, the position of base-tip of the rod $\mathbf{p}_i(s = 0)$ and the orientation of base-tip $\tilde{\mathbf{q}}_i(s = 0)$ are known given the inputs, while some guess values are used for $u_{xi}(s = 0)$, $u_{yi}(s = 0)$, $\mathbf{n}_i(s = 0)$. Given the inputs, each rod evolution is evaluated and an error function is calculated. This error function $\mathbf{e}_{fun}$ is made of the mechanism assembly constraints, static equilibrium on the end-effector and unit-quaternion normalization:

$$\mathbf{e}_{fun} = \left\{ \begin{array}{c} \mathbf{p}_1(L_1) - \mathbf{p} - \mathbf{R}_{EE}\mathbf{r}_1 + \mathbf{a}_1 \\ u_{x1}(L_1) \\ u_{y1}(L_1) \\ \hline \vdots \\ \hline \mathbf{p}_6(L_6) - \mathbf{p} - \mathbf{R}_{EE}\mathbf{r}_6 + \mathbf{a}_6 \\ u_{x6}(L_6) \\ u_{y6}(L_6) \\ \hline \sum_{i=1}^{6}[\mathbf{n}_i(L_i)] - \mathbf{F}_{ext} \\ \sum_{i=1}^{6}[(\mathbf{a}_i + \mathbf{p}_i(L_i)) \times \mathbf{n}_i(L_i) + \mathbf{m}_i(L_i)] - \mathbf{p} \times \mathbf{F}_{ext} - \mathbf{M}_{ext} \\ \hline |\tilde{\mathbf{q}}|^2 - 1 \end{array} \right\} \tag{2}$$

where $\mathbf{a}_i$ is the position vector of point A_i with respect to the fixed frame and $\mathbf{r}_i$ is the position vector of point B_i with respect to the end-effector frame.

To find the FK Problem solution, this residual have to be minimized following an iterative procedure, in our case through a Newton-Raphson scheme. To ensure convergence, a home position configuration is used so that contiguous configurations are solved with a boundary value problem upon slight changes of either input or output variables.

In the following sections, some simulations are presented to describe the type of motion of the mechanisms relative to the position of the end-effector within the workspace. To do so, an Inverse Kinematic algorithm would be needed, but since the resolution of the IK is still under study, an approximated IK method as been adopted. The basic assumption of this approximated method is the fact that an increment of the actuation variable j causes a displacement of the end-effector along the direction perpendicular to the segment $B_{2j-1}B_{2j}$ and an equal increment of all actuation variables causes a displacement of the end-effector along the z direction. Therefore, the approximate IK relationship can be written as:

$$\begin{Bmatrix} L_1 \\ L_2 \\ L_3 \end{Bmatrix} = \frac{1}{3} \begin{bmatrix} 0 & -2 & 1 \\ \sqrt{3} & 1 & 1 \\ -\sqrt{3} & 1 & 1 \end{bmatrix} \mathbf{p} \tag{3}$$

By using Eq. (3), the trajectories planned in the operative space are converted into joint space variables used as inputs of the FK algorithm presented before. For this reason, in the following sections the trajectories used to analyze the robot mobility are called "quasi-circular motions at a quasi-constant end-effector height".

4 Parametric Analysis and Motion Evaluation

In this chapter, a parametric analysis is presented to select the geometry for the flexible robot to obtain translational-like motions. Later, the output motions of the robot with the selected geometry is evaluated.

Since the goal is to achieve translational-like motion in space, in the following analysis the output rotations of the end effector will be called parasitic rotations, in analogy with the parasitic motions of lower mobility rigid parallel manipulators. As previously mentioned, the geometric parameters r_A/r and H affect the magnitude of the parasitic motions of the end effector and, therefore, the type of output motion. As case study, the parasitic motions of a $3P\underline{F}S$ mechanism had been studied during the execution of a quasi-circular motion at a quasi-constant end-effector height of 450 mm. The motion can be described by the polar coordinate radius $d \simeq 40\%\, r_A$ and angle θ (the same nomenclature, presented in Fig. 4, is used later to study the output motion of the mechanism). This simulation had been performed with different values of the ratio r_A/r at constant H and with different value of H at constant ratio r_A/r. From the results, it is clear that if r_A/r or H increase, lower parasitic rotations are obtained, as showed in Fig. 3 for the flexible tripod $3P\underline{F}S$ (similar values are obtained also for Architecture 2). It should be underlined, that the results are dependent on stiffness of the system (material, rods diameter and lengths). The simulations presented in this study had been performed with an elastic modulus $E = 83$GPa (similar to the one of Nitinol), a rod diameter of 2.5 mm and an initial rod length of 500 m. Nevertheless, the influence of the geometric parameters on the type of the output motion is the same.

Regarding the size ratio r_A/r a trade-off value must be selected in order to limit the total encumbrance of the manipulator, while preserving a minimum dimension of the end-effector. From these preliminary results, it seems like the beneficial effect of reducing the parasitic rotations decreases as the ratio increases. Therefore, even if no optimal evaluation has been done yet, a good starting point is the ratio $r_A/r = 5.0$. The possible values for parameter H lie in the range $[0; \sqrt{3}r]$. Therefore, the maximum value $\sqrt{3}r$ is selected, which corresponds to the configuration where $B_2 \equiv B_3$, $B_4 \equiv B_5$ and $B_6 \equiv B_1$.

For future comparison with commercial linear Delta Robot, the following value for the base radius has been selected $r_A = 210$ mm, which leads to the following dimensions: $r = 42$ mm and $H = 73$ mm. Moreover, for the flexible tripod $3\underline{P}FFSS$ a constant length of $L = 500$ mm has been defined. In Fig. 4 the two architectures of flexible tripods are represented with these geometric parameters.

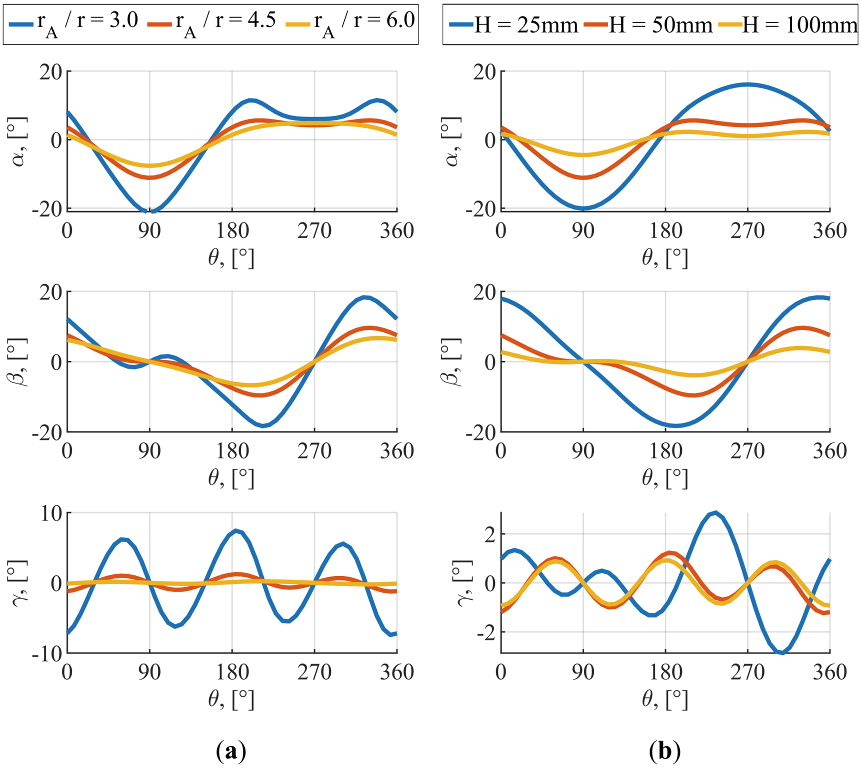

Fig. 3. Parasitic rotations during a quasi-circular motion, defined by a polar radius $d \simeq 40\%\, r_A$ and the angle θ, expressed in Euler angles (XYZ): (**a**) influence of the size ratio r_A/r with constant H, (**b**) influence of H with constant size ratio $r_A/r = 4.5$ and $r_A = 250$ mm.

To test the effectiveness of these mechanisms at generating pure translational motions in space, quasi-circular trajectories have been planned for the two manipulators at increasing radius d on a plane at a quasi-constant height. The resulting parasitic rotations, expressed in Euler angles (XYZ) are presented in Fig. 5.

From the results, it can be said that during motions on a plane at a constant height the flexible mechanisms $3P\underline{F}S$ and $3\underline{P}FFSS$ are characterized by parasitic rotations that increase in magnitude as the distance from the symmetric home position increases. Quantitatively, the absolute value of the rotation in below 5° is the distance d is below the value $r_A/2$. While the intensity of the parasitic rotation increases a lot in this range. A physical explanation of this result is the fact that when the projection on the XY plane of the attachment point B_i gets close to its homologous A_i the rod flexion is close to zero, and therefore the rod stiffness makes the end-effector pivoting around the point B_i. Nevertheless, it can be said that quasi-translational motions are possible in a limited range of

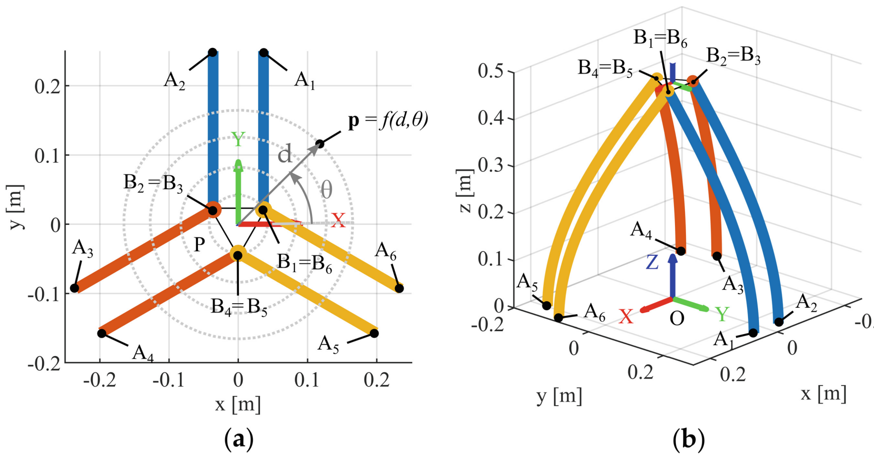

Fig. 4. Selected geometry for the flexible mechanism: (**a**) top view, (**b**) 3D representation. With the dotted gray line, the quasi-circular path, that it is used afterwards for the motion evaluation, is represented.

displacement. Moreover, from the results presented in Fig. 5, Architecture 1 is characterized by lower parasitic rotations with respect to Architecture 2. This fact is due to the different stiffness of the flexible rods during motions. In Architecture 1, as the end-effect moves away from points $A_{(2j-1)}$ and $A_{(2j)}$, the rod length L_j increases with a subsequent lower stiffness, while in Architecture 2 the rod length is constant and, therefore, the increase in stiffness is greater than the one in Architecture 1.

Finally, it is worth mentioning that the analysis had been performed with no load applied. Contrary to rigid parallel manipulators, the kinematic position problem solution in flexible PCMs is not independent of the load because the end-effector equilibrium is a fundamental component of the kinematic model. Future studies will investigate the influence of loads over position analysis. Nevertheless, it can be stated that this dependence is not always a drawback in these systems. In fact, as compliant grippers are used to compensate for positioning errors in certain tasks execution, such as peg-in-hole [14], the intrinsic flexibility of PCMs could be exploited to achieve the same result, while at the same time, providing improved safety in case of collisions.

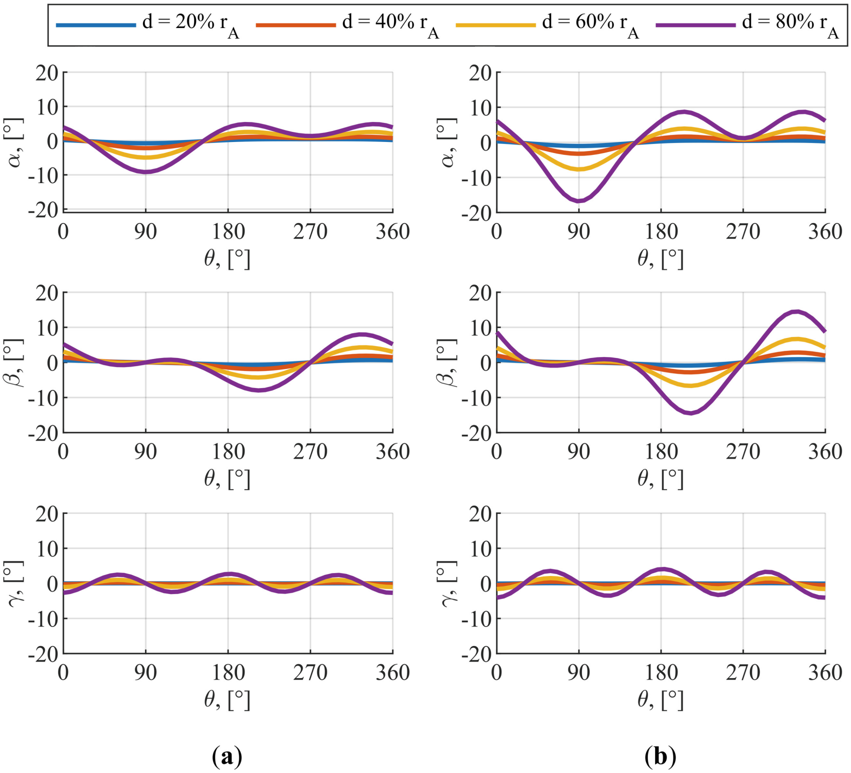

Fig. 5. Parasitic rotations of the flexible mechanism $3P\underline{F}S$ (**a**) and $3\underline{P}FFSS$ (**b**) expressed in Euler angles (XYZ) during the execution of circular-like trajectories in the XY plane at increasing radius.

5 Conclusions

In conclusion, this paper investigated PCMs as an alternative to rigid parallel manipulator, to improve safety in collaborative workspace. Different set of inputs had been proposed for an hexapod flexible mechanism to achieve quasi-translational motion. A parametric analysis had been used to find the best geometric structure at achieving quasi-translational motions. Later, the parasitic rotations during motions at constant height had been studied. From the results, it can be said that it is possible to achieve quasi-translational motions with PCMs, even if not negligible rotations are observed for large displacement from the symmetric home position. Future analysis will focus on the performance comparison between a Delta-Robot and the flexible counterpart with geometry and actuation system defined in this work. Moreover, a prototype will be developed to validate the simulation methods and to study the stability of the system.

References

1. Laribi, M.A., Romdhane, L., Zeghloul, S.: Analysis and dimensional synthesis of the delta robot for a prescribed workspace. Mech. Mach. Theory **42**(7), 859–870 (2007)
2. Stock, M., Miller, K.: Optimal kinematic design of spatial parallel manipulators: application to linear delta robot. J. Mech. Des. **125**(2), 292–301 (2003)
3. Vischer, P., Clavel, R., et al.: Kinematic calibration of the parallel delta robot. Robotica **16**(2), 207–218 (1998)
4. Cheng, H., Li, W.: Reducing the frame vibration of delta robot in pick and place application: an acceleration profile optimization approach. Shock Vibr. **2018** (2018)
5. Weber, A.: Delta robots feed need for speed. Assembly **58**(2), 28–31 (2015)
6. McClintock, H., Temel, F.Z., Doshi, N., Koh, J., Wood, R.J.: The milliDelta: a high-bandwidth, high-precision, millimeter-scale delta robot. Sci. Robot. **3**(14), eaar3018 (2018)
7. Wang, Y., Liu, J., Guo, M., Wang, L.: Research on the printing error of tilted vertical beams in delta-robot 3D printers. Rapid Prototyping J. **27**, 1633–1649 (2021)
8. Mitsantisuk, C., Stapornchaisit, S., Niramitvasu, N., Ohishi, K.: Force sensorless control with 3D workspace analysis for haptic devices based on delta robot. In: 41st Annual Conference of the IEEE Industrial Electronics Society, IECON 2015, pp. 001747–001752. IEEE (2015)
9. Howell, L.L.: Compliant mechanisms. In: McCarthy, J. (ed.) 21st century kinematics, pp. 189–216. Springer, Heidelberg (2013). https://doi.org/10.1007/978-1-4471-4510-3_7
10. McClintock, H., Temel, F.Z., Doshi, N., Koh, J., Wood, R.J.: The milliDelta: a high-bandwidth, high-precision, millimeter-scale delta robot. Sci. Robot. **3**(14) (2018)
11. Altuzarra, O., Tagliavini, L., Lei, Y., Petuya, V., Ruiz-Erezuma, J.L.: On constraints and parasitic motions of a tripod parallel continuum manipulator. Machines **11**(1) (2023)
12. Antman, S.S.: Problems in nonlinear elasticity. In: Antman, S.S. (ed.) Nonlinear Problems of Elasticity, pp. 513–584. Springer, Heidelberg (2005). https://doi.org/10.1007/0-387-27649-1_14
13. Black, C.B., Till, J., Caleb Rucker, D.: Parallel continuum robots: modeling, analysis, and actuation-based force sensing. IEEE Trans. Robot. **34**(1), 29–47 (2018)
14. Sanji, M., Nakamura, T., Suzuki, M., Aoyagi, S.: Robot task of pin insertion to a hole without chamfering and with small clearance using fuzzy control. In: Shirase, K., Aoyagi, S. (eds.) Service Robotics and Mechatronics, pp. 33–36. Springer, London (2010). https://doi.org/10.1007/978-1-84882-694-6_6

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Chapter 7

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