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## Maths Games without Frontiers

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### ABSTRACT

The use of games in higher education significantly benefits cognitive, motivational, affective, and sociocultural perspectives. This paper describes a mathematical challenge for first-year STEM students at the “College of Merit Camplus”, located in different cities in Italy from South to Nord, during the autumn of 2020 and 2021. Students in pairs play different puzzler games to reinforce mathematical prerequisites and basic knowledge. Due to the pandemic situation, non-digital games were forced to adjust to the remote environment. On the one hand, moving online was a challenge both from the organisational side and for the students that sometimes need to cooperate from different locations. On the other end, approaching the games remotely allowed students from colleges all around Italy to participate. The work describes and comments on each game in detail, considering the students’ performance. In general, it can be stated that students liked the playful experience, although they found themselves not wholly confident with some topics and the related time restriction. These games, first-of-all helped review and train the basic concepts; therefore, the students have a better approach to studying the first Mathematical course at the University. They found the games’ dynamics helpful in highlighting some simple tricks and common mistakes.

*Keywords:* Game based learning, Mathematics, Online challenge, Pair work, Playful environment

### Introduction

Gamification and game-based learning are well-implemented approaches that foster cognitive development by increasing students’ motivation and engagement through game elements within educational environments (Danniels & Pyle, 2018). While gamification is defined as using game features in a non-game setting, game-based learning identifies real games, usually on a digital platform, used for an educational purpose with a defined learning outcome (Plass et al., 2015). The level

of engagement and the social perspective of games makes them an ideal learning medium (Squire, 2011). In higher education, game-based learning is very effective as it increases enjoyment reducing anxiety with improvements in deep learning and higher-order thinking (Bawa, 2020; Crocco, 2016). Although the creation of digital games requires economic and time effort, the non-digital ones are cost-effective, require low administrative overheads, demand little or no prerequisite skills, and provide opportunities for enhanced social interaction (Mustafa et al., 2011; Whitton, 2012). Then, non-digital game-based learning suits mathematics' educational goals in higher education (Naik, 2017). Numerous applications are designed to be used directly in the classroom or for homework activities: card games, puzzlers, or strategic games (Lai, 2018; Lee et al., 2016). In designing a game, the five playful pillars should be considered to ensure cognitive activation (Zosh et al., 2017). Neuroscience studies show that people learn best from experiences that are joyful, meaningful, actively engaging, iteratively and socially interactive (Liu et al., 2017). Play requires an integrated approach based on creative, emotional, physical and social skills (Riley, 2013). The role of the teacher also has to be defined accordingly to the goal that wants to be achieved; it can span from free play to guided play to teacher-directed play (Weisberg et al., 2013)

Due to the COVID-19 pandemic, non-digital games have been pushed to shift to be played in a remote environment.

This study describes a mathematical competition that first occurred during the lockdown in the autumn of 2020. The games were originally the non-digital type but were adjusted to be played remotely. Thanks to this positive experience, the second edition in autumn 2021 replicated the same format. This online choice allowed the participation of students living in Camplus all over Italy.

The following section highlights the context in which the experimentation took place. The third section describes the methodology, while the fourth contains deeper details of each proposed game. Finally, a general discussion with the conclusion is given.

## Context

The Italian educational system is organised into two cycles of training. The first education cycle lasts five years and consists of primary school from 6 to 10. There is no specialised path, and the goal is to acquire fundamental knowledge and skills in all subjects. The second education cycle lasts eight years and is divided into lower secondary school (from 11 to 13) and upper secondary (from 14 to 18). Like primary schools, the lower one is common to all students and shares a nationwide curriculum. Instead, the upper secondary is classified by the curricula as Lycée (general schools), technical institutes, and vocational institutes. The Ministry of Education defines basic curricula for all schools. Upper

secondary education is further classified into sub-paths, such as Scientific Lycée, Classical Lycée, Mechanical technical institute, or ICT technical institute.

Mathematics is a fundamental subject for all schools, but how it is taught could differ in terms of time per week and general goals.

This paper describes the experience “Math games without frontiers”, which aims to support studying the first university Mathematics course for students from a Camplus college in Italy. The Ministry of Education recognises Camplus as a College of Merit offering residences and training for students. The training activities are compulsory and in addition to the university programs. It has different locations distributed throughout the Italian territory.

“Math games without frontiers” offers the opportunity to improve logic and calculation skills through playful exercises in a competition between couples of students and between the different Camplus locations. In five thematic evenings, various games are presented, and students are evaluated with an updated score.

## Methodology

This paper follows the narrative case study with the support of qualitative research. The analysis includes the data related to the games (time and score), the in-field observations, and a final survey. The Camplus board ethically approved the current research, and each student signed a consent declaration, including a factsheet.

The challenge was open to all the first-year students living in one of the Italian Camplus and with at least a mathematical exam into the university's first year. The first edition involved 52 participants (15 females and 37 males) from 9 different colleges. In the second edition, there were 30 participants (14 female and 16 male) from 7 colleges. They autonomously group into couples within the same Camplus, with some exceptions in the first edition when two mixed pairs came from different Camplus. Due to the pandemic situation, the challenge was entirely run on an online platform. Each team was free to choose either to play physically or remotely together. A private instantaneous message channel between lecturers and each couple was established to collect the game's solutions.

Applying the game-based learning methodology, the games recall famous puzzlers to familiarise themselves with Mathematical basics contents. They are partially inspired by the work of Morando (2009) and further developed in order to be played remotely. The “Math games without frontier” foresees five meetings one hour long of playful scored activities. The events were one week apart and required some mathematical knowledge as well as some teamwork and strategy. In the edition 2020, the first three evenings worked on essential knowledge (elementary functions such as trigonometric, exponential, and logarithmic functions; equations and inequalities), while the last 2 reinforced the concepts



Fig. 1 shows a possible path that starts with false, true, false, etc. When the reading ends and 30 seconds have elapsed, all the statements appear on the screen for 1 minute; the game ends 30 seconds later (a total of 120 seconds from the end of the reading). Teams gain 1 point for each correct answer, two bonus points if they submit within 30 seconds from the end of the reading, and one bonus point if they submit within 90 seconds from the end of the reading. The first submitting team gains another two bonus points.

### Ordering

The game is divided into two rounds: in the first one, teams have to order eight values of trigonometric functions in ascending order, while in the second, eight values must be sorted in a decreasing manner. Each heat has a duration of 4 minutes. Each team has a “STOP” to indicate the point at which to end the sequence’s validation. Teams have one extra minute for the sequence’s delivery. Pairs gain 1 point for each correctly ordered sequence but get 0 points if the series order is wrong, even for a single value. Teams get two bonus points if the complete sequence is correct and delivered before time runs out.

### Math Twins

A table of 12 + 12 elements to be paired is given to each team. In 5 minutes, the team have to form as many pairs as possible, and within 1 minute, they have to send the complete list of pairs. Each team gains 1 point for each correct pair but loses 1 for each wrong couple. If all pairs are matched correctly, two bonus points are awarded. If the total score is negative, the team score equals zero.

### Colouring puzzle

The game requires teams to blacken spaces of a figure in which there are the solutions of 15 mathematical expressions (for example, inequalities, equations or limits), see Fig. 2. The available time is 12 minutes. Once completed, the team has to send the photo of the obtained image to tutors.



Figure 2. Solution of a colouring puzzle

Suppose the figure is coloured correctly; the couple scores ten points. The team loses 1 point for each incorrect or missing box. The team gains two bonus points if the correct figure is delivered before time runs.

### Treasure hunt

Each team has to solve a system of inequalities graphically; the goal is to identify the object constrained in the solution plane region of the system. Axes origin is assumed in the lower-left corner (see Fig. 3). The available time to complete the game is 5 minutes. Teams that deliver the correct object within 3 minutes gain ten points; couples that provide the proper object after 3 minutes earn five points. If the object is wrong, the team gets no points. Two bonus points are assigned to the first team that identifies the correct object.



Figure 3. Treasure hunt scheme with the identified object (the target) [image: freepik.com]

### Guess what?

Thirty plots of function are shown to teams. The tutor secretly chooses one between all figures. Couples can ask questions about a specific feature defined in advance, with yes or no answers, to guess the tutor's choice. Teams who think they have identified the correct function send a message to the tutors, and the game ends as soon as a pair correctly guesses. Each team has at most three attempts, and the game is played in multiple rounds. The team that identifies the correct function gains three points; the same team gains one bonus point if it identifies an admissible analytic expression within 30 seconds and two bonus points if it identifies two different expressions in  $30 + 30$  seconds.

### Target

A target made up of boxes is shown to teams. Each box contains one function. The game's goal is to reach the centre of the target by passing over each box at most once, starting from the pointed box. One can move from one box to another if the two boxes are adjacent and the functions follow a prescribed property (for example, Fig. 4 shows the case in which the rule is: the intervals of increasing monotony of two functions have a non-empty intersection).

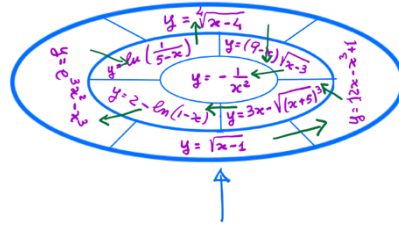


Figure 4. Solution of a target round

The available time is 7 minutes plus one extra minute for delivery. Teams gain 1 point for each correct passage between boxes. They earn one bonus point if they deliver within 7 minutes and gain another two bonus points if, for each box, they indicate the specific property (for example, the interval in which the function is increasing).

**Sudoku**

Each team has to solve a sudoku with some empty coloured dots (see Fig. 5). Each colour is associated with a definite integral. Solving them, they must place the numbers and complete the sudoku correctly. The available time to complete the game is 20 minutes plus one minute for the delivery.

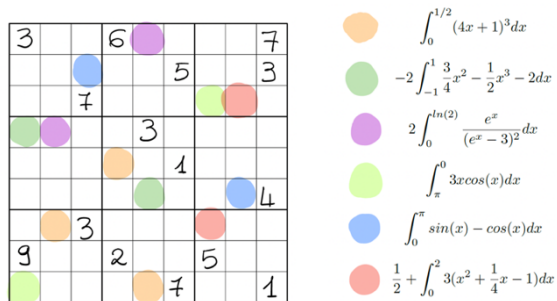


Figure 5. Sudoku example

Teams are asked to deliver the starting scheme with the integrals computed in order to obtain points: one point for each correct result within 10 minutes and 0.5 points for each proper answer delivered before the end. Six bonus points are gained by the teams that complete the sudoku within 15 minutes, five points within 20 minutes and 4 points if the solved scheme is delivered at the end of the time.’

**Connect the dots**

Each team must solve ten limits and, following the given order, use the solutions to connect the related dots (see Fig. 6).

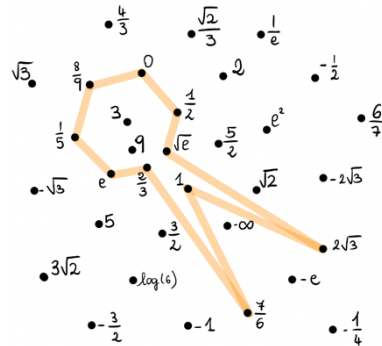


Figure 6. Solution of a “Connect the dots” game

The aim is to draw the secret image within 10 minutes. Each number correctly linked gives one point. The team receives two bonus points if it delivers the correct picture within 10 minutes.

## Discussion and conclusions

Different observations can be raised considering the pairs’ results in each game (scores and times) and the final survey results.

The Labyrinth game was played on three different evenings. The main difficulty was that answering T/Fs to a spoken sentence requires great initial concentration. Indeed, students are mainly used to solving mathematical problems by reading a written text. Therefore, in both editions, only a few teams delivered their solution on the first evening before instructions appeared on the screen. However, on the last evening, the number of couples who returned immediately after the end of the reading increased considerably. This denotes how the formalisation skills of a problem spoken aloud can be improved and reinforced.

The Ordering game was played only on the first evening but also highlighted a critical fact discovered in other games. Students are unfamiliar with trigonometric expressions and find it difficult to quantify them. In 2020, 75% of teams scored zero despite having the “stop” available. Similar results came out in the 2021 edition, where 50% of couples were unable to solve at least one of the two “ordering” games proposed. This suggests that many students still feel confident in the results, although they do not have a complete mastery of trigonometric functions. Therefore, this could explain why few have used the stop.

The Math Twins game confirmed students’ difficulty associating the algebraic expression of a function with its graphs. The game was played on three different evenings, and each evening there was a different type of twins: trigonometric expressions and numerical values, functions and derivatives, functions and graphs. In the game, teams reached higher scores when the associations were not graphical. Conversely, scores are lower when students associated the graph

with the analytic expression; notice that almost 50% of teams scored zero in the first evening on both editions.

Scores of the other games suggest the same conclusions. In particular, the Treasure hunt game highlighted again that most students are unfamiliar with the graphical exercise's solution. Furthermore, a common fact of all games is the short time to complete challenges: quickly solving exercises requires a specific mastery of mathematics. Since the goals of this challenge are to develop the ability to intuitively visualise the graph of an analytic function and support preparation in differential calculus, these games suggest that more emphasis has to be devoted to the graphical part.

We introduced two new games in the second edition: Sudoku and Connect the dots. Regarding the Sudoku, only three teams were able to solve the integrals and complete the scheme. It seems that students made mistakes in calculating elementary integrals quickly. Then, when they got stuck on the sudoku, they did not realise that the error was on the computation of the integrals, not the scheme solution. Similar considerations can be drawn for the second added game. As before, the time limit seems a significant obstacle for students.

An important fact concerns the online procedure. Indeed, working online as a team was more complicated than the same work done in person. But on the other hand, it was possible to organise this challenge on a national basis only because of the online environment.

Analysing the comments from the final survey, it is clear that many students liked the playful experience. Moreover, most of them found it helpful and educational how the exercises were carried out: many games helped students understand “tricks of the trade”. The limited time teams in a healthy competition, making mathematics more challenging. In general, answers show that the students were satisfied and would like to continue with a new edition next year.

### Author Note

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