

Assessment of Blade Element Momentum Theory-based engineering models for wind turbine rotors under uniform steady inflow

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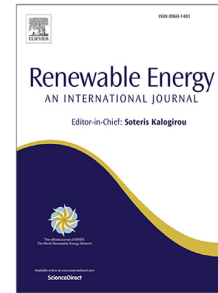
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1 Highlights

2 **Assessment of Blade Element Momentum Theory-based engineer-**  
3 **ing models for wind turbine rotors under uniform steady inflow**

4 Umberto Boatto, Paul A. Bonnet, Francesco Avallone, Daniele Ragni

- 5 • CFD-based assessment of BEMT corrections for tip-root-losses and  
6 stall delay
- 7 • Prandtl tip-loss correction is missing tip-vortex-induced lift coefficient  
8 reduction
- 9 • Zhong tip-loss model improves the Prandtl one for high tip-speed ratio  
10 and blunt tip
- 11 • CFD inboard blade loading matched by only a stall-delay-corrected lift  
12 coefficient
- 13 • Capturing the root-vortex downwash requires a modified Prandtl root-  
14 loss correction

15 Assessment of Blade Element Momentum Theory-based  
16 engineering models for wind turbine rotors under  
17 uniform steady inflow

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19 **Abstract**

Blade Element Momentum Theory (BEMT) -based approaches provide 15-20% inaccurate load predictions under uniform steady inflow. Inaccuracies were related to unclear aerodynamic mechanisms causing the Prandtl Tip-Root Loss (TRL) correction to fail at high Tip-Speed Ratio (TSR) and the ambiguous drag coefficient treatment under Stall Delay (SD). With this study, we provide an in-depth flow analysis to explain what physical mechanisms are poorly captured in TRL and SD corrections and offer a potential solution for the improvement of the Prandtl model. Corrections are assessed by comparison with RANS CFD simulations of the NREL 5MW rotor at its design TSR, TSR 4, and TSR 10. The Prandtl tip-loss correction provides 5-15% higher loading than CFD at TSR 10. The reason is a lower ( $0.5^\circ$ ) downwash angle of attack, providing a 5-10% lift coefficient overestimation. We recommend employing the tip-loss correction of Zhong, as it considers the lift coefficient reduction due to the tip-vortex downwash. The investigated Eggers SD correction predicts an incorrect drag coefficient at TSR 4, while a better agreement is found for the lift coefficient. We conclude that only the latter requires a correction for SD in the inboard blade.

20 *Keywords:* Wind turbine, Aerodynamics, Blade Element Momentum

21 *Theory, Computational Fluid Dynamics, Tip loss, Stall delay*

22 *PACS:* 0000, 1111

23 *2000 MSC:* 0000, 1111

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## 24 1. Introduction

25 The preliminary design of efficient rotor blades requires accurate, robust,  
26 and reasonably fast aerodynamic prediction methods to estimate their power  
27 output and wind-induced load distributions [1, 2, 3].

28 The preliminary design methods used by the wind turbine industry consist  
29 of low-fidelity engineering methods such as the Blade Element Momentum  
30 Theory (BEMT) [4] extended with semi-empirical corrections [5, 1, 6]. Sub-  
31 stantial efforts were made in the past to compare BEMT-based codes with 3D  
32 RANS simulations and wind tunnel experiments [7, 2, 8]. Engineering meth-  
33 ods failed to accurately predict the rotor load distributions under uniform  
34 steady inflow, notably at high speed. In the NREL "blind comparison" [7],  
35 BEMT-based codes overpredicted by 10 to 40% blade loading in the area  
36 beyond 70% of the rotor radius (from here onwards we will refer to this part  
37 of the blade as the outboard blade) due to inaccuracies in tip loss correc-  
38 tions. Moreover, the same codes provided wrong normal load estimates on  
39 the blade area between 20 and 40% of the rotor radius (from here onwards we  
40 will refer to this part of the blade as the inboard blade) at high inflow speed  
41 due to 2D airfoil data. On the other hand, CFD codes provided a more fa-  
42 vorable match under stall. In the Mexnext Phase I project [2], BEMT-based  
43 codes underpredicted by 20% the experimental and CFD normal force at  
44 above-rated conditions in spite of stall delay corrections, and overestimated  
45 the outboard blade loads at all inflow speeds by an average 15%. In Phase  
46 III [8], it was confirmed that the largest deviations between BEMT and CFD  
47 loads were in the tip region. These inaccuracies occur because 3D aerody-  
48 namic phenomena, such as tip losses and stall delay, are important at high  
49 inflow speed regimes, even under uniform steady inflow.

50 Concerning the outboard blade, most industrial BEMT codes employ  
51 variations of the Prandtl [9] Tip-Root Loss (TRL) factor to improve the ac-  
52 curacy of power and load computations [10]. The original Prandtl model was  
53 derived by assuming a constant blade bound circulation with concentrated  
54 trailed vorticity at the tip and a cylindrical non-expanding wake. Ramdin [11]  
55 compared the Prandtl factor with a free vortex wake model and showed that  
56 it tends to overpredict at 80% blade and underpredict at 95% the tip losses for  
57 Tip Speed Ratio (TSR) from 4 to 10 and five different rotor designs. These  
58 deficiencies were attributed to the inability of the model to capture the tip  
59 vortex axial and radial movements as well as strength variation with TSR  
60 and non-uniform high blade loading [11]. Barndlard [10] identified "wake

61 expansion, roll-up and distortion” as phenomena not included in Prandtl-  
62 based factors, and by free vortex wake analysis the researcher pointed out  
63 that TSR, thrust coefficient, and radial distribution of the blade circulation  
64 are the main parameters in decreasing order of importance. Micallef [12, 13]  
65 showed by stereo particle image velocimetry and 3D panel method simula-  
66 tions that the last 5-10% blade depending on the tip bluntness is affected  
67 by radial flow induced by the chordwise bound circulation and recommended  
68 the inclusion of these effects in the Prandtl model. A characterization of  
69 the near-wake and blade tip aerodynamic properties for varying TSR and  
70 under non-uniform blade loading is necessary to evaluate the Prandtl model  
71 limitations and its impact on the load distributions and power.

72 Regarding the inboard blade, the loading increase due to the rotating  
73 blade - the so called Himmleskamp effect [14] - is modelled by Stall Delay  
74 (SD) corrections. Most of them [15, 16, 17] modify the 2D airfoil polars ac-  
75 cording to a correction function obtained from 3D boundary layer analysis.  
76 Models differ in the specific definition of this function, but the correction  
77 term depends in general on the local chord to radius ratio  $c/r$  and empir-  
78 ically derived coefficients. The importance of the  $c/r$  ratio was found by  
79 Snel [15] and is related to the radial flow acceleration. Some SD correc-  
80 tions [15, 17] are applied only to the lift coefficient while others correct both  
81 lift and drag coefficients [16, 18, 19]. Moreover, there is no unique approach  
82 to correcting the drag coefficient, as in some models [19, 18] this quantity is  
83 increased, while in [16] it is decreased compared to its 2D value. Therefore,  
84 the drag coefficient correction under delay of stall still remains an unsolved  
85 issue [20]. A review of SD corrections was performed by Breton [3] who re-  
86 ported an overall overprediction of the experimental load distributions and  
87 deviations between models for increasing inflow speed. These observations  
88 were related to limitations of boundary layer theory on which SD corrections  
89 are based, such as the lack of transition and its loss of accuracy under sepa-  
90 ration, which may lead to an incorrect description of the flow [3]. Therefore,  
91 CFD investigations were performed to understand the rotational augmenta-  
92 tion effects responsible for the delay of stall [21, 22]. Guntur [21] performed  
93 RANS simulations and found that the radial flow develops also outside the  
94 separated region due to the action of the centrifugal force on slow moving  
95 chordwise flow. The researcher also observed that pressure distributions in  
96 the separated region have a constant non-zero gradient differently from the  
97 non-rotating case due to radial pumping [21]. Bangga [22] showed that 3D  
98 flow effects are responsible for the delay of stall and observed that this last

99 occurs in the presence of separated radial flow only when  $c/r > 0.1$ . Further-  
100 more, the role of Coriolis and centrifugal forces was investigated by Detached  
101 Eddy Simulations (DES), and it was observed that the first has its largest  
102 value near the wall and at the beginning of the separation region where the  
103 chordwise convective flow acceleration is also the largest, while the second  
104 dominates the rest of the flow around the blade inboard profiles [22].

105 The goal of this study is to contribute to the explanation of the physical  
106 mechanisms responsible for the reported Prandtl factor limitations under  
107 high loading and expanding wake as well as the drag coefficient treatment  
108 under rotational augmentation effects. Furthermore, based on the findings  
109 of the study, a potential solution is proposed for the improvement of the  
110 Prandtl TRL correction for high TSR. No study available investigated in  
111 detail the physical limitations of the Prandtl model under this condition  
112 and the solution proposed could help researchers in developing an improved  
113 correction for tip-losses. The analysis is carried out by comparing one BEMT  
114 code expanded by the Prandtl TRL model and the Eggers SD correction with  
115 unsteady RANS CFD simulations. The contribution from the individual and  
116 combined application of the SD and TRL corrections is considered, and CFD  
117 simulations are employed to confirm previous findings concerning the physical  
118 mechanisms of tip losses and rotational augmentation. In addition to a design  
119 condition, a high TSR case is accounted for to address the Prandtl factor  
120 inaccuracies, and a low TSR one is considered for the rotational augmentation  
121 effect on drag. In all conditions simulated the inflow is assumed steady and  
122 uniform. The NREL 5MW rotor [23] is the selected wind turbine and the  
123 blades are considered rigid to focus on the rotor aerodynamics. Finally, only  
124 one specific BEMT code is taken into account.

125 The paper follows with section 2, where the details of the applied method-  
126 ology are provided in terms of: rotor specifics and simulation conditions,  
127 modeling aspects and setups of the BEMT and CFD solvers, post-processing  
128 techniques for the extraction of lifting-line variables from the CFD solution.  
129 Section 3 reports the verification of the CFD solution. Next, a characteri-  
130 zation of the rotor aerodynamics is done in section 4 by means of the CFD  
131 results, and the comparison between BEMT and CFD predictions is treated  
132 in section 5. Finally, the main findings are summarized and guidelines for  
133 modeling improvement are given in the conclusions.

## 134 2. Methodology

### 135 2.1. Turbine and simulation condition

136 The NREL 5MW wind turbine [23] was chosen for this benchmark activ-  
 137 ity. The turbine was simulated as an isolated rotor with rigid blades and no  
 138 cone and tilt angles. The 3D blade geometry for the CFD simulations was  
 139 generated by a python script [24] that translates a list of points describing the  
 140 design provided in [23] from the Simcenter Samcef language [25] to the STL  
 mesh format. The resulting blade is shown in fig. 1, where it features a blunt

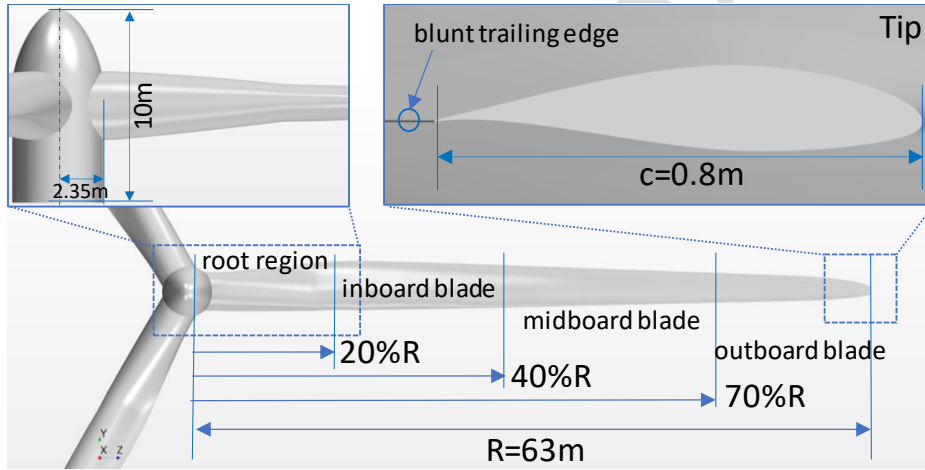


Figure 1: Planform view of the generated 3D blade and hub (with sizes) for the CFD simulations. The inboard blade is defined as the blade portion from 20 to 40% of the rotor radius and the outboard blade as the portion extending from 70% radius up to the tip. For the sake of completeness, the midboard blade goes from 40 to 70% radius and the root region extends from the blade root up to 20%, where the first aerodynamically relevant DU40 airfoil is located. Moreover, the blunt trailing edge for the NACA64 profile towards the tip is also shown.

141 trailing edge along its whole span, and the NACA64 profiles in the outboard  
 142 blade were modified by introducing a trailing edge thickness of 0.3% chord  
 143 to facilitate the generation of the CFD prism layer mesh. Furthermore, the  
 144 blade has a 0.8m tip chord and  $N=200$  profiles were defined according to a  
 145 hyperbolic tangent distribution:  
 146

$$s_i = (R - R_h) \left( \frac{\tanh\left(\frac{i}{N-1}A\right)}{\tanh(A)} \right) \quad (1)$$

147 to reconstruct the blade surface. Equation (1) defines the radial location  $s_i$  of  
 148 a generic profile  $i$  starting from the blade root located at the hub radius  $R_h =$   
 149 1.5m, and the value of the amplitude  $A$  was set to 1.5 to achieve a more refined  
 150 aerodynamic discretization towards the tip. The 200 profiles were obtained  
 151 by linear interpolation of the airfoil shapes in [23] including the profiles in  
 152 between the root cylinder and the DU40 airfoil at 20% radius to achieve a  
 153 representative geometry of wind turbine blades. Finally, a cylindrical shape  
 154 rounded on the upstream side was designed for the hub geometry (see fig. 1),  
 155 as no reference geometry could be found.

156 The NREL 5MW rotor was simulated under uniform steady inflow for  
 157 three values of the TSR  $\lambda = (\Omega R)/U$ :  $\lambda = 7$ ,  $\lambda = 4$ , and  $\lambda = 10$ . The only  
 158 parameter changing is the inflow speed  $U$ :  $U = 11.4\text{m/s}$ ,  $U = 19.95\text{m/s}$ , and  
 159  $U = 7.98\text{m/s}$ , whereas the rotor speed  $\Omega$ , blade pitch angle, and radius are  
 160 set to 12.1rpm,  $0^\circ$ , and 63m. The first condition is the design TSR of the  
 161 turbine [23], while the second and third ones are not representative of steady-  
 162 state power production operations and are selected to show the limitations  
 163 of SD and TRL corrections respectively. Finally, the air properties are the  
 164 following: density  $\rho = 1.225\text{kg/m}^3$ , dynamic viscosity  $\mu = 1.82 \cdot 10^{-5}\text{Pa}\cdot\text{s}$ , and  
 165 pressure  $P = 101325\text{Pa}$ .

## 166 2.2. BEMT model and setup

167 The employed BEMT solver is part of the Simcenter Samcef solver (ver-  
 168 sion 2022.2) [25] and is based on the classic Glauert-Prandtl theory [4]  
 169 equipped with the Burton version [26] of the Prandtl [9] TRL correction  
 170 and the the Buhl's AeroDyn [27] Turbulent Wake State correction (TWS).

171 BEMT simulations were set-up in the engineering interface Samcef for  
 172 Wind Turbines (SWT) [28] by prescribing the blade design properties and  
 173 airfoil polars for the cylindrical and DU profiles from [23], but not for the  
 174 sections with the NACA64 profile. As the latter was modified with a blunt  
 175 trailing edge, new polars were computed with the viscous Xfoil [29] solver  
 176 by using 300 panels, Reynolds number 6.6 million, 500 as maximum number  
 177 of iterations,  $N_{crit} = 9$ , and transition location at 1% chord on both profile  
 178 sides. Two types of polars were considered for the DU profiles: those with the  
 179 SD correction of Du-Selig [16] extended by the Eggers' drag treatment [19]  
 180 and those without. All polars employed in the BEMT solver are for a single  
 181 Reynolds number despite the range of TSR values simulated. SWT performs  
 182 a linear interpolation of the input polar data sets according to a prescribed

183 relative thickness distribution, which was obtained from the 3D blade geom-  
 184 etry. Finally, the blade was discretized into 30 stations with non-uniform  
 185 radial spacing according to eq. (1) with  $N=30$  and  $A=1.3$ .

186 The individual and combined effects of the SD and TRL corrections on  
 187 the BEMT solver predictions were considered by employing four BEMT ver-  
 188 sions: "BEMT", "BEMT+TRL", "BEMT+SD", and "BEMT+SD+TRL".  
 189 The "BEMT" version consisted of the classic BEMT model corrected only  
 190 for TWS (labeled "BEMT" for simplicity) and polars uncorrected for SD  
 191 were employed. The "BEMT+TRL" version added to the previous the TRL  
 192 correction by activating this option in the solver. The "BEMT+SD" ver-  
 193 sion was obtained by providing polars with the SD correction. Finally, the  
 194 "BEMT+SD+TRL" version contained all corrections.

### 195 2.3. CFD model and setup

196 The incompressible RANS equations were simulated by Simcenter STAR-  
 197 CCM+ (version 2021.2) [30] by both steady-state and unsteady approaches.  
 198 The  $k-\omega$  SST turbulence model [31] was employed, as in [32, 33, 34], to-  
 199 gether with the Reichardt [35] hybrid velocity wall function. The blade  
 200 boundary layer was set to fully turbulent without transition modeling. The  
 201 segregated flow solver was selected, with a constant density gas model, and  
 202 second-order space and time discretizations. In the unsteady simulations,  
 203 a time step corresponding to a  $4^\circ$  rotation angle was used with 20 internal  
 204 iterations and 10, 12, and 15 revolutions were simulated for the  $\lambda=4, 7,$  and  
 205 10 cases respectively. The rotor motion was modelled by the Multiple Refer-  
 206 ence Frame (MRF) approach in steady-state and rigid-body rotation under  
 207 unsteady conditions. The MRF approach was implemented by assigning the  
 208 rotor cylinder region in fig. 2a to a rotating frame and the remaining domain  
 209 to a stationary one with an "in-plane" conformal interface in between the  
 210 two regions.

211 To set up the CFD simulations, a similar approach as Dose [32] was  
 212 followed. As shown in fig. 2b, the far-field boundaries were located at 10 and  
 213 20 (outlet) diameters from the rotor center with assigned velocity inlets and  
 214 pressure outlet boundary conditions. Moreover, a wake refinement region  
 215 up to 3 diameters downstream was defined. The generated volume mesh  
 216 was unstructured, as depicted in fig. 3, and consisted of polyhedral cells  
 217 and 8 prismatic layers for the blade boundary layer. The latter had a total  
 218  $3.17 \cdot 10^{-4}D$  thickness and a geometric growth ratio of 1.3. The  $y^+$  values  
 219 achieved are in range 10-300 growing from the inboard to the outboard blade

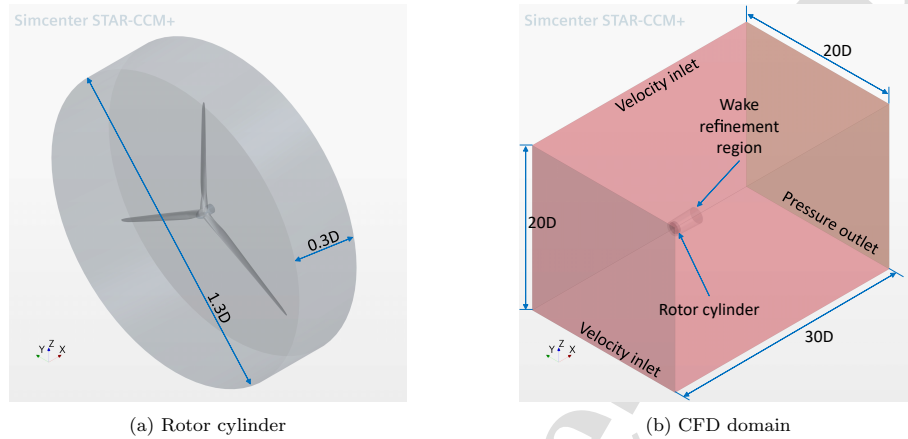


Figure 2: Regions, sizes, and boundary conditions of the CFD setup.  $D$  refers to the rotor diameter  $D=126\text{m}$  and the reference system displayed is the inertial one.

220 and with a larger variation for the  $\lambda=4$  case, as shown in fig. 4. The cell size  
 221 was set to  $0.008D$  inside the wake refinement region and at the interface for  
 222 the "fine" mesh shown in fig. 3 and defined in section 3. Finally, the volume  
 223 growth rate of mesh cells was set to 1.05 and the cell size on the far-field  
 surface was set to  $D$  in the "fine" mesh.

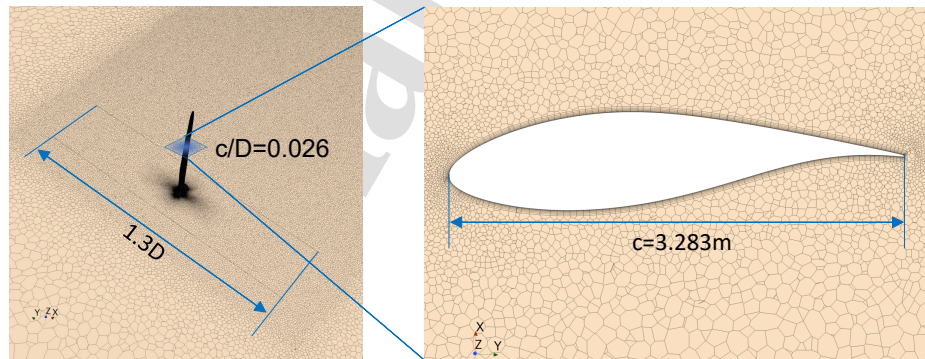


Figure 3: The "fine" CFD mesh (whose definition is in section 3) with views of the hub, blade, rotor cylinder region, and wake refinement on the left and of the blade section at 63% radius on the right.

224

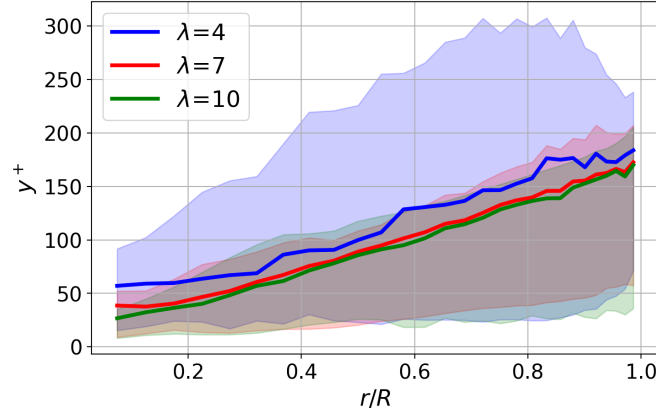


Figure 4: Range of  $y^+$  values of the CFD simulations. The solid lines show the mean  $y^+$  values over the profiles while the shed areas show the range in between the minimum and maximum values. [color in print]

225 *2.4. Post-processing of the CFD solution for the extraction of lifting line*  
 226 *variables*

227 The extraction of the induction velocity components, angle of attack,  
 228 lift and drag coefficients from the CFD simulations was carried out by the  
 229 Azimuthal-Averaging Technique (AAT) and the inverse BEMT method [36,  
 230 33, 37, 38].

231 The AAT method computes the annular-averaged axial  $\bar{a}$  and tangential  
 232  $\bar{a}'$  induction velocity components by sampling the near-wake axial  $U_a$  and  
 233 tangential  $U_t$  flow velocity components. The method was implemented by  
 234 using 6 streamwise (three upstream and three downstream) arrays of probes  
 235 sampling the rotor flow each time step during the last revolution. Probes  
 236 in each array were located in 10 annuli at 20, 40, 60, 70, 80, 85, 90, 93, 96,  
 237 and 98% radius. In each annulus there were 90 probes according to [36] and  
 238 these were axially spaced from the rotor center by the local blade chord. The  
 239 sampled  $U_a$  and  $U_t$  were processed as shown in fig. 5: (1) annular-averaging,  
 240 (2) interpolation at the rotor plane by Lagrange polynomials as in [33], (3)  
 241 time-averaging at each radial location, and (4) conversion into induction  
 242 velocity components as:

$$\bar{a} = 1 - \frac{\overline{U_a}}{U} \quad \bar{a}' = -\frac{\overline{U_t}}{2\Omega r} \quad (2)$$

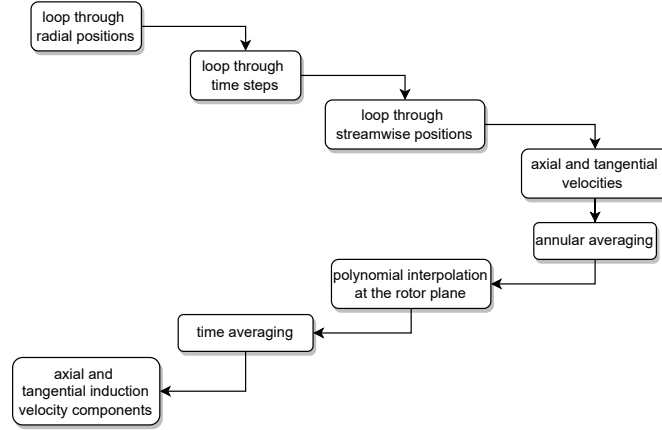


Figure 5: Flowchart showing the applied method to obtain the axial and tangential induction velocity components from the near-wake axial and tangential flow velocity components according to the AAT method.

243 where  $\overline{U}_a$  and  $\overline{U}_t$  are the time-azimuth-averaged flow velocities interpolated  
 244 at the rotor plane.

245 The inverse BEMT method was implemented by following the conventions  
 246 and layout of the Simcenter Samcef BEMT code. The input are the blade out-  
 247 of-plane  $F_A$  and in-plane  $F_T$  forces per unit of spanwise length time-averaged  
 248 over the last revolution. The usage of the TRL correction allowed to compute  
 249 the local axial  $a_B$  and tangential  $a'_B$  induction velocity components, while  
 250 not using the TRL correction led to the annular-averaged induction velocity  
 251 components like in the AAT method. Finally, both inverse BEMT method  
 252 variations (with and without TRL) included the TWS Buhl's correction [27].

253 With the annular-averaged  $\bar{a}$   $\bar{a}'$  and local  $a_B$   $a'_B$  induction velocity compo-  
 254 nents, the corresponding annular-averaged  $\bar{\alpha}$  and local  $\alpha_B$  angles of attack  
 255 were computed as:

$$\bar{\alpha} = \arctan \left( \frac{U(1 - \bar{a})}{\Omega r(1 + \bar{a}')} \right) - \theta \quad \alpha_B = \arctan \left( \frac{U(1 - a_B)}{\Omega r(1 + a'_B)} \right) - \theta \quad (3)$$

256 where  $\theta$  is the blade station twist angle.

257 The local angle of attack  $\alpha_B$  was employed to define the downwash angle  
 258 of attack in the outboard blade as a measure of the tip vortex induction.

259 Following [38], the latter was defined as:

$$\alpha_w = \bar{\alpha} - \alpha_B \quad (4)$$

260 with  $\alpha_B$  provided by the inverse BEMT method with TRL only.

261 By using the angle of attack, normal  $c_a$  and tangential  $c_t$  force coefficients  
 262 (in the rotor plane reference system), the lift and drag coefficients  
 263 were extracted from the CFD solution:

$$c_l = c_a \cos \phi + c_t \sin \phi \quad c_d = c_a \sin \phi - c_t \cos \phi \quad (5)$$

264 with  $c_a = F_A/(q_{loc}c)$ ,  $c_t = F_T/(q_{loc}c)$ ,  $q_{loc} = 0.5\rho(U^2 + (\Omega r)^2)$ , and inflow  
 265 angle  $\phi = \bar{\alpha} + \theta$ . The annular-averaged angle of attack considered is the one  
 266 provided by the AAT method only.

### 267 3. Verification of the CFD solution

268 The CFD solution was assessed by a mesh convergence analysis. Four  
 269 different grids were simulated by systematically varying a base size  $b$  for the  
 270 polyhedral cells in the wake refinement region and on the interface. The  
 271 four grids were defined as: "coarse" ( $b/R=0.032$ ), "medium" ( $b/R=0.024$ ),  
 272 "fine", ( $b/R=0.016$ ), and "very fine" ( $b/R=0.008$ ), with  $R=63\text{m}$ . The  
 273 thickness and growth ratio of the prismatic layers were left unmodified to  
 274 achieve a constant boundary layer resolution, as recommended in [39]. Fol-  
 275 lowing [33, 32], only steady-state simulations were performed and only the  
 276 rotor thrust and power were considered to assess the mesh convergence.

277 Figure 6a shows the values of the rotor thrust and power coefficients given  
 278 by the four meshes plotted against the number of mesh cells. Convergence  
 279 towards a rotor power coefficient of 0.476 was achieved by the "fine" mesh  
 280 represented by a star in fig. 6a. This resolution provides a value for the rotor  
 281 thrust coefficient of 0.756, which is less than 0.1% different from the "very  
 282 fine" mesh value. Therefore, the "fine" resolution was deemed converged  
 283 also for thrust and selected for the subsequent unsteady simulations because  
 284 computationally cheaper than the "very fine" mesh.

285 In fig. 6a, the rotor thrust and power coefficients computed by Dose [32]  
 286 are shown. The "fine" mesh provides 2.1% thrust and 3.4% power underpre-  
 287 dictions with respect to these reference values. The radial distributions of  
 288 the annular thrust  $C_T$  and annular power  $C_P$  coefficients are shown in fig. 6b

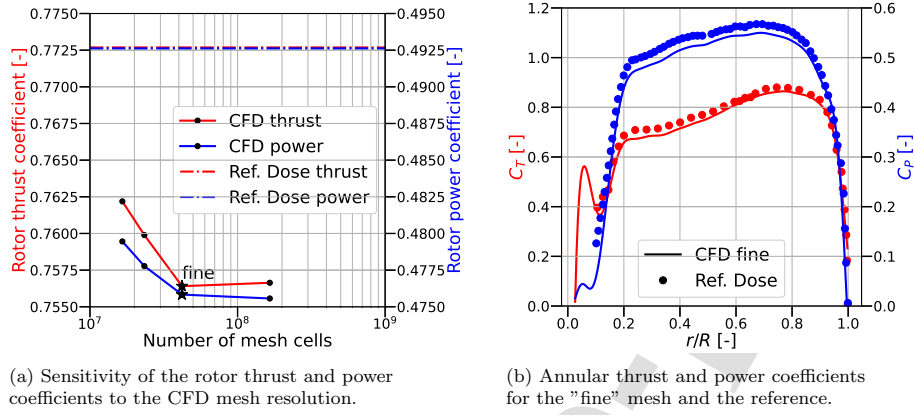


Figure 6: Results of the CFD mesh convergence study. [color in print]

289 for the selected "fine" resolution and the reference available in [32]. There  
 290 quantities are defined as:

$$C_T = \frac{3F_A}{\pi r \rho U^2} \quad C_P = \frac{3F_T \Omega}{\pi r \rho U^3} \quad (6)$$

291 and were obtained by using the Accumulated Force Table functionality in  
 292 STAR-CCM+ with 200 bins for an accurate integration and by applying a  
 293 third-order Butterworth filter with 0.1 cut-off frequency to get rid of the  
 294 nonphysical fluctuations provided by the tool. An overall good match is  
 295 shown with minor deviations in the inboard blade for  $C_T$  and in the mid-  
 296 inboard blade for  $C_P$ .

#### 297 4. Characterization of the blade and rotor near-wake aerodynamics

298 The main features of the flow over the blades are displayed by the near-  
 299 wall velocity pathlines shown in fig. 7 on the blade suction surface and for the  
 300 three conditions studied. Furthermore, areas of flow separation are visualized  
 301 by a purple threshold of zero or negative chordwise skin friction coefficient  
 302 as in [21]. The flow in the blade inboard region and near the tip is char-  
 303 acterized by a 3D radial component in all the three cases and the extent of  
 304 such radial flow increases with reducing  $\lambda$ . Near the tip beyond 90% blade,  
 305 the flow pathlines bend radially towards the inboard blade. Such a radial  
 306 flow is related to the presence of the tip vortices and is due to the radial

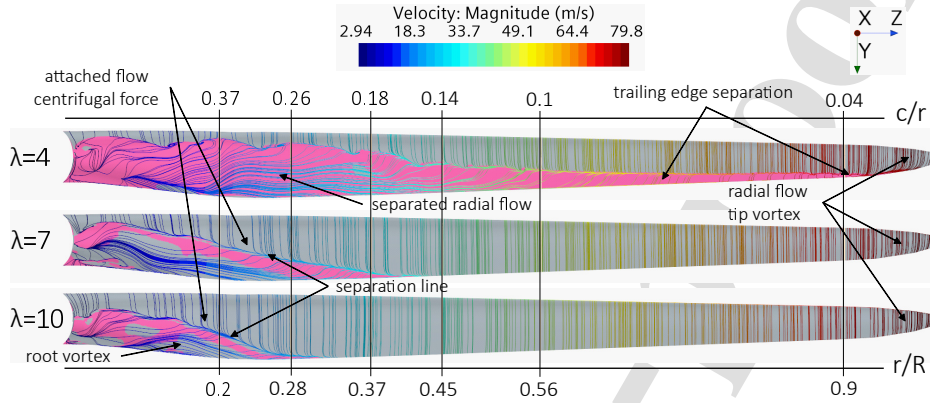


Figure 7: Near-wall pathlines on the blade suction side colored by velocity magnitude in the inertial frame as they are seen from a downwind observer, purple threshold of zero or negative chordwise skin friction coefficient, and five radial locations  $r/R$  with the corresponding value of the  $c/r$  parameter used in stall delay corrections. [color in print]

307 pressure gradient driving the pressure equalization between the two sides of  
 308 the blade surface [40, 41]. At  $\lambda = 4$ , the chordwise skin-friction threshold  
 309 reveals that the flow is separated close to the trailing edge of the outboard  
 310 blade profiles, but not in the two other cases. In the inboard blade at 28%  
 311 radius, the flow is massively separated with a dominant spanwise direction  
 312 at  $\lambda = 4$ . When the TSR increases, the flow is attached at this radial location  
 313 with separation confined to the trailing edge at  $\lambda = 7$  and almost negligible at  
 314  $\lambda = 10$ . Moreover, due to the steering action of the centrifugal force, the at-  
 315 tached flow at these two conditions increases its spanwise component before  
 316 separating, as described in [21]. Attached flow with a clear radial orienta-  
 317 tion is visible at  $\lambda = 10$  beyond the separation line and towards the trailing  
 318 edge around  $r/R = 0.20$  where profiles have the maximum chord. Previous  
 319 studies [21, 42, 3] related this phenomenon to the root vortex induction at  
 320 low inflow speed (here high  $\lambda$ ). Finally, for  $r/R > 0.56$ , the flow is separated  
 321 only near the trailing edge for  $\lambda = 4$  and with a limited radial component,  
 322 consistently with the alleviation of SD corrections for  $c/r < 0.1$  [22].

323 The profile distributions of the pressure coefficient  $c_p$  and radial skin-  
 324 friction coefficient  $c_{f,r}$  are shown in fig. 8 at  $r/R = 0.28$ . In fig. 8-left, the  
 325 pressure distribution for  $\lambda = 4$  differentiates from the other two in terms of  
 326 shape of the suction peak and the almost constant-slope from 20% chord up

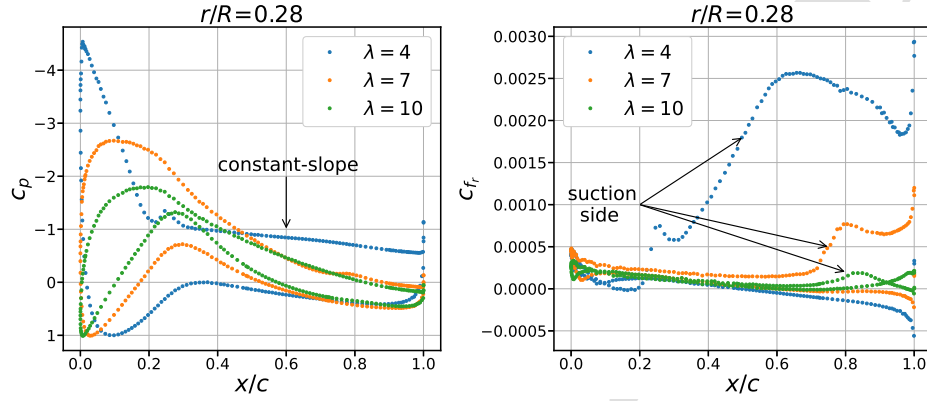


Figure 8: Pressure  $c_p$ , and radial  $c_{fr}$  skin friction coefficient components on the inboard blade profiles. The velocity used in the normalization dynamic pressure includes both the blade peripheral velocity  $\Omega r$  and inflow velocity  $U$ . [color in print]

327 to the trailing edge where the flow is largely separated (see fig. 7). Small  
 328 variations in  $c_p$  are visible at  $x/c = 0.25$  for  $\lambda = 4$  and  $x/c = 0.8$  for  $\lambda = 7$   
 329 corresponding to separation. Next, the radial  $c_{fr}$  plots show that there is  
 330 a substantial radial outboard flow (positive  $c_{fr}$ ) at  $\lambda = 4$  and a limited or  
 331 negligible one at  $\lambda = 7$  and  $10$ . Furthermore, at  $\lambda = 4$  the portion of the profile  
 332 over which the radial flow is present coincides with the constant pressure  
 333 gradient under flow separation in fig. 8-left. Such a relation between constant  
 334 pressure gradient, separation, and radial flow characterizes the rotational  
 335 augmentation phenomenon [21, 22]. Therefore, it could be argued that at  
 336 28% blade the delay of stall happens more likely at  $\lambda = 4$  compared to  $\lambda = 7$   
 337 and  $10$ .

338 The curved surface pathlines on the outboard blade in fig. 7 are the  
 339 footprint of the tip vortices. These and the root vortices are visualized in fig. 9  
 340 for the rotor near-wake up to one diameter downstream. The figure shows  
 341 also areas of localized Q-criterion due to the mesh refinement at the interface.  
 342 The axial spacing along the X direction between the tip vortices increases for  
 343 reducing  $\lambda$  due to the faster downstream convection of the vortices, which is  
 344 proportional to the inflow speed ( $\lambda \sim 1/U$ ). Next, the expansion of the tip  
 345 vortices along the radial direction (Y- or Z-axis) increases for increasing  $\lambda$ .  
 346 The wake expansion is related to the rotor thrust coefficient increase with  $\lambda$   
 347 for zero blade pitch, which correlates with an increase in wake expansion [43].

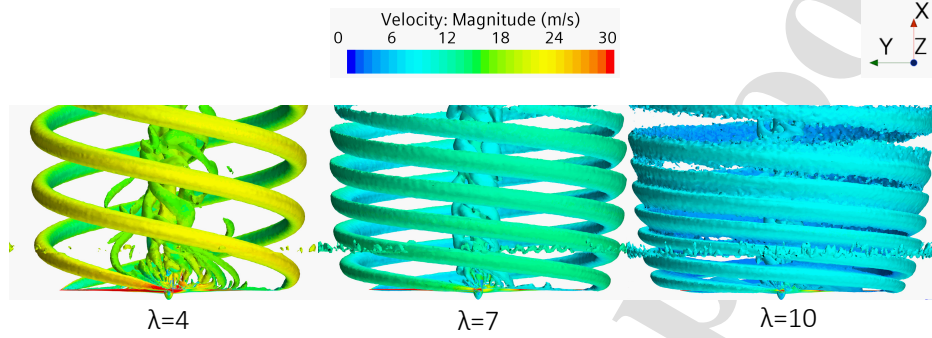


Figure 9: Rotor wake vortex structures visualized by isosurfaces of the Q-criterion colored by velocity magnitude. The rotor blades are at the bottom and the wake develops towards the top. [color in print]

348 At  $\lambda = 10$ , the mutual induction of the tip vortices is most likely to be not  
 349 negligible anymore. Finally, the root vortex is the largest at  $\lambda = 4$  due to the  
 350 extensive blade flow separation (fig. 7).

351 Both axial and tangential tip vortex motions are measured in fig. 10-left  
 352 by tracking the paths of the vortex centers on the X-Z plane with respect  
 353 to the tip location. The axial movement can be quantified by the pitch of  
 354 the tip vortex helix along the X-axis, and it increases by almost three times  
 when  $\lambda$  reduces from 10 ( $X \approx 0.5$ ) to 4 ( $X \approx 1.4$ ), similarly as in [43]. The

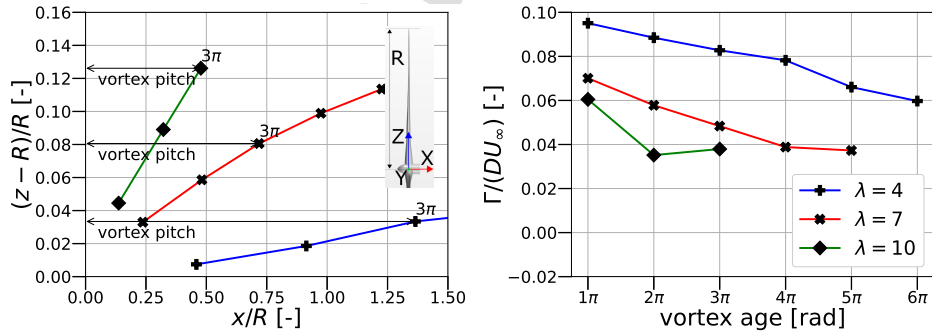


Figure 10: Centers' trajectories and normalized circulation of the tip vortices for the three CFD simulation cases. [color in print]

355 wake expansion is instead correlated with the TSR and the Z-wise radial  
 356 component of the third vortex age  $3\pi$  is around three times larger at  $\lambda = 10$   
 357

358 ( $Z \approx 0.12$ ) than at  $\lambda = 4$  ( $Z \approx 0.04$ ). This means that, as the TSR  
 359 decreases, the axial component gets much larger (factor 35) than the radial  
 360 one, while there is a smaller (factor 4) difference at high TSR. Since the  
 361 very first vortex ages  $X/R < 2$  are those that dominate the reduction of the  
 362 outboard blade loading (tip loss), these results confirm that the cylindrical  
 363 wake assumption of the Prandtl model loses its reference with the expanding  
 364 wake of the CFD unsteady RANS solution for increasing TSR.

365 As the strength of the tip vortices was reported to be an important as-  
 366 pect in the modeling of tip losses [11], this is measured by computing the  
 367 circulation of the first vortex ages. The circulation was calculated by sur-  
 368 face integration of the vorticity component  $\omega_j$  sampled on rectangular areas  
 369 around the tip vortex centers on the X-Z plane shown in fig. 9. A threshold  
 370 of  $\omega_j < 0.8\text{Hz}$  was considered to eliminate unwanted contributions and cap-  
 371 ture only the vortex cores. The normalized circulation is shown on the right  
 372 plot of fig. 10. It is observed that the lower  $\lambda$  the larger the circulation of  
 373 the first tip vortex, as reported in [44], with an increase around 40% from  
 374 TSR 4 to 10. This can be explained by the larger extent of the spanwise  
 375 flow on the blade tip near-wall pathlines in fig. 7 at  $\lambda = 4$ , as it determines  
 376 increased trailing vorticity release and so circulation in the tip vortex [41, 8].  
 377 Figure 10-right shows an up to 40% circulation decrease for increasing vortex  
 378 age, which is not expected for the coherent tip vortices before their break-  
 379 down. As reported in [8], this can be associated to the numerical diffusion of  
 380 second-order discretization scheme in the unsteady RANS simulations and  
 381 the possibly too large ( $0.016R$ ) cell size in the wake refinement region of the  
 382 "fine" mesh, despite the convergence of the rotor loads for this resolution.

### 383 5. Comparison and discussion of BEMT and CFD predictions

384 Figure 11 shows the error percentage difference of the rotor thrust and  
 385 power coefficients between the four BEMT versions introduced in section 2  
 386 and the unsteady RANS CFD solution for the three conditions. At  $\lambda = 4$ , all  
 387 BEMT versions significantly underpredict the CFD thrust coefficient up to  
 388 25% and power coefficient by 30%. A better match is found at the rated  $\lambda = 7$   
 389 case with an error difference below 5%, and at  $\lambda = 10$  the error difference  
 390 is below 10%. Including the SD correction at  $\lambda = 4$  improves the thrust  
 391 coefficient prediction by 10% and the power coefficient one by 5%. The TRL  
 392 contribution is around 5% at  $\lambda = 10$  where it leads to better predictions of  
 393 both quantities, while at  $\lambda = 7$  it leads to slightly worse results by 2-3%.

Furthermore, the fully-corrected version "BEMT+TRL+SD" is not the one

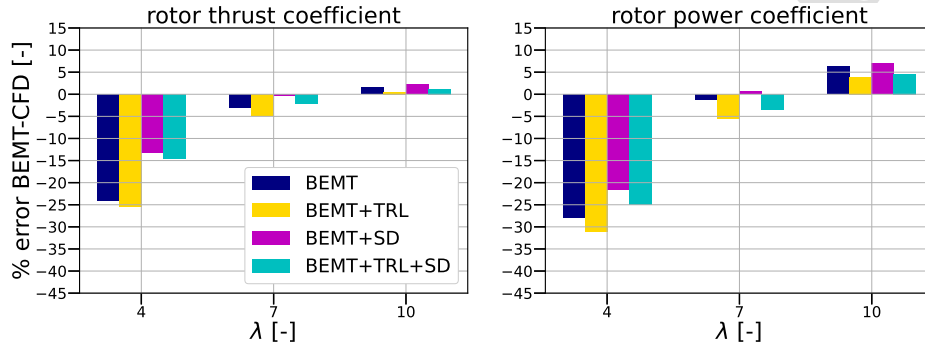


Figure 11: Error percentages between BEMT versions and the CFD of the rotor thrust and power coefficients. The CFD values are obtained by applying their definition to the spanwise integrated thrust  $F_A$  and tangential  $F_T$  blade force densities averaged over the last revolution period simulated. [color in print]

394  
 395 providing the lowest error at all conditions. The best performing version is  
 396 the "BEMT+SD" one at  $\lambda = 4$  and 7, whereas the "BEMT+TRL" version  
 397 has the most favorable match with CFD at  $\lambda = 10$ . However, the latter is  
 398 also the worst performing at low TSR. Overall, BEMT predictions are below  
 399 5% only at the design TSR, while at off-design conditions they are not as  
 400 satisfactory. Especially at low  $\lambda$ , the error difference in rotor power and  
 401 thrust gets up to 30% when root effects are dominant over a large part of  
 402 the blade span, and this result is in line with those in [2, 8].

403 To elaborate on the effect of the individual TRL and SD corrections with  
 404 respect to the reference CFD solution, the rotor regions where the aerody-  
 405 namic models diverge are considered. This is done by plotting in fig. 12 the ra-  
 406 dial distributions of the annular thrust  $C_T$  and power  $C_P$  coefficients defined  
 407 in eq. (6) for the overall best "BEMT+SD" and overall worst "BEMT+TRL"  
 408 versions based on fig. 11. Substantial differences can be noticed over the  
 409 whole span depending on the operational condition and these are analyzed  
 410 in the remainder of this section.

411 From 20% to 40% of the radius, the effect of the SD correction is dominant  
 412 at  $\lambda = 4$ . This is consistent with the strong spanwise separated flow in fig. 7  
 413 and the different  $c_p$  distribution compared to the other conditions at  $r/R =$   
 414 0.28 in fig. 8. Figure 12 reveals a good match with the CFD  $C_T$  distribution  
 415 at  $\lambda = 4$  for the "BEMT+SD" version, but not an equally satisfactory one for

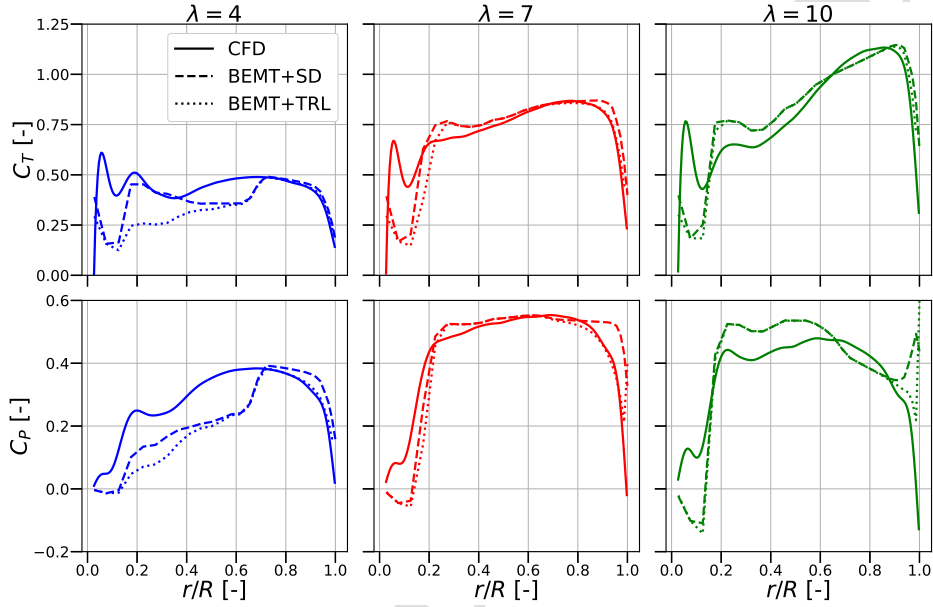


Figure 12: Annular thrust  $C_T$  and power  $C_P$  coefficient distributions for the CFD solution, the "BEMT+SD" and "BEMT+TRL" versions. [color in print]

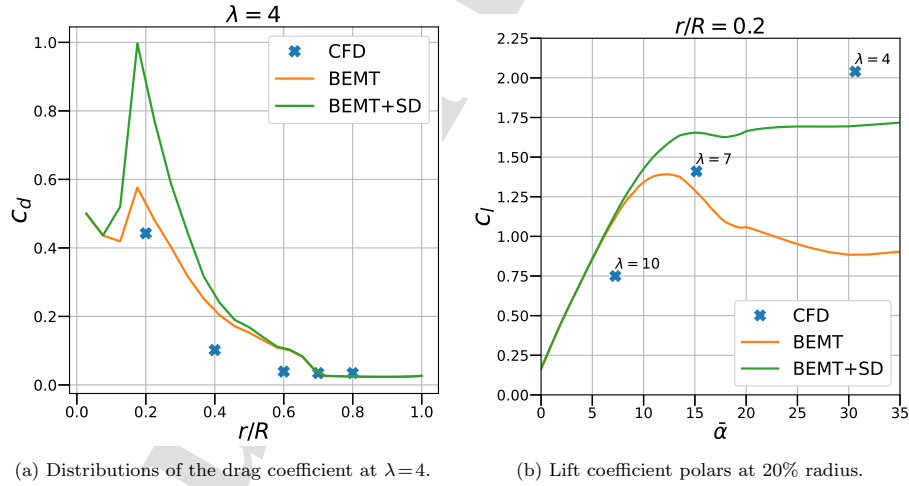
416 the  $C_P$  by both BEMT versions. Such a  $C_P$  underprediction can be explained  
 417 by an overestimation up to a factor of 2 of the drag coefficient  $c_d$  provided  
 418 by the Eggers [19] SD correction compared to the CFD simulation and the  
 419 uncorrected "BEMT", as shown in fig. 13a. The drag overestimation is related  
 420 to the assumptions of the Eggers model [19], where only the first-order effect  
 421 due to the Coriolis force under deep stalled flow is considered. Therefore,  
 422 the larger drag leads to a reduction of the tangential force per unit span  $F_T$   
 423 on the profile the due to the minus sign preceding  $c_d$ :

$$F_T = \frac{1}{2} \rho U_{rel}^2 c (c_l \sin \phi - c_d \cos \phi) \quad (7)$$

424 with  $U_{rel}$  the relative profile inflow speed. The lower  $F_T$  leads finally to an  
 425 annular power coefficient  $C_P$  underestimation, despite the large inflow angle  
 426  $\phi$  in the inboard blade at high inflow speeds. As already discussed, the  $c_d$   
 427 treatment is an unsolved problem in SD modeling. Moreover, this was shown  
 428 to be blade dependent: the NREL Phase VI turbine showed an increase in 3D

429  $c_d$  [3], Bangga [22] predicted a decrease in the inboard blade for the AVATAR  
 430 turbine by DES simulations, and Boorsma computed a slight decrease in  $c_d$   
 431 for the Mexico rotor [8]. The CFD solution in this study provides lower  $c_d$   
 432 compared to the corrected and uncorrected BEMT versions for SD in fig. 13a.  
 433 However, as shown in fig. 13b, the CFD predicts a larger lift coefficient at 20%  
 434 blade and  $\lambda=4$ , and the corrected lift polar provides a better match in this  
 435 case compared to the corrected drag polar. Therefore, a more accurate  $C_P$   
 436 prediction could be obtained from a lift coefficient corrected for SD and an  
 437 uncorrected drag coefficient. This confirms the modeling approach in [16, 15],  
 438 at least at  $r/R=0.2$ , and shows that the drag coefficient increase in [19] seems  
 439 to be unjustified for the NREL 5MW turbine.

440 Coming back to the  $C_T$  and  $C_P$  distributions in fig. 12, at  $\lambda=7$  and 10 and  
 441 in the inboard blade both the "BEMT+SD" and "BEMT+TRL" versions  
 442 overpredict the reference CFD distributions by 20%. At the rated TSR the  
 443 role of the SD correction can be seen up to 30% of the radius, while at the  
 444  $\lambda=10$  condition it has a negligible contribution. It was observed in section 4  
 445 that the blade aerodynamics at  $\lambda=10$  is characterized by negligible separated  
 446 spanwise flow and delay of stall. Furthermore, most of the radial flow is  
 attached around 20% blade due to the root vortex induction. The root



(a) Distributions of the drag coefficient at  $\lambda=4$ . (b) Lift coefficient polars at 20% radius.

Figure 13: Drag and lift coefficients for the BEMT versions corrected and uncorrected for SD and for the CFD solution obtained as described in section 2. The polars used in the BEMT simulations are for a single Reynolds number value of  $7 \cdot 10^6$  [23]. [color in print]

448 vortex leads to the downwash effect visible in fig. 13b as a smaller CFD  $c_l$   
 449 for  $\lambda = 10$  compared with the BEMT polars, which are unaffected by SD  
 450 due to the low angle of attack. Such an effect was observed in previous CFD  
 451 simulations and wind tunnel experiments [21, 42, 3] and an important finding  
 452 of this study is that it is responsible for a 20% load reduction not adequately  
 453 captured by the "BEMT" and "BEMT+TRL" versions. Figure 13b shows  
 454 that at the root, there is a tiny  $C_T$  reduction due to the Prandtl root-loss  
 455 correction, but this is insufficient because the hub radius ( $R_h = 1.5\text{m}$ ) is used  
 456 - as common practice - to define the blade root radial distance. Therefore,  
 457 the radial location of maximum chord could be used in the model to enforce  
 458 the downwash where the CFD results indicate it occurs. This solution was  
 459 already tested by Schepers [45] obtaining a favorable match with the root-  
 460 losses predicted by free vortex wake simulations, but without comparing load  
 461 distributions.

462 The  $C_T$  and  $C_P$  distributions in fig. 12 show that for  $r/R > 0.7$  the effect  
 463 of the TRL correction is dominant, in particular for the  $C_P$  distribution.  
 464 Moreover, the Prandtl factor positively affects the accuracy of the BEMT  
 465 load distributions at  $\lambda = 4$  and 7 for both quantities. However, at high  $\lambda$ , the  
 466 TRL correction overestimates the  $C_T$  and  $C_P$  over the last 10% blade by 5 to  
 467 15%. The degradation of the Prandtl factor at high TSR can be explained  
 468 by considering the tip vortex properties of the CFD solution in fig. 10 and  
 469 the downwash angle of attack  $\alpha_w$ . The latter is defined in eq. (4) and shown  
 470 in fig. 14 for the BEMT model and CFD solution by means of the inverse  
 471 BEMT ("invBEMT") approach. Concerning the tip vortex, the CFD solu-  
 472 tion predicts at  $\lambda = 4$  an almost cylindrical helix with the largest pitch and  
 473 circulation, while at  $\lambda = 10$  the tip vortex expands the most with the smallest  
 474 circulation. This means that at a TSR of 4 the strongest tip vortex produces  
 475 a downwash covering 40% of the outboard blade and reaching  $\alpha_w \approx 2.5^\circ$  at  
 476 the tip. Moreover, the almost cylindrical tip vortex helix predicted by CFD  
 477 and assumed by the Prandtl TRL correction leads to the smallest deviation  
 478 in downwash angle of attack between the two models. Different is what hap-  
 479 pens at  $\lambda = 10$  where the expanding tip vortex induction is concentrated on  
 480 the last 10% blade and its weakest circulation leads to  $\alpha_w \approx 1.5^\circ$  at the tip.  
 481 Furthermore, the deviation between BEMT and CFD  $\alpha_w$  distributions at  
 482 this condition is the largest, with the latter providing a  $0.5^\circ$  higher down-  
 483 wash and so lower  $\alpha_B$ . Given the attached flow regime, this translates into  
 484 a 5-10% difference in  $c_l$  and so the 5-15% "BEMT+TRL" loading overpre-  
 485 dictions. Therefore, another important result of this study is to relate the

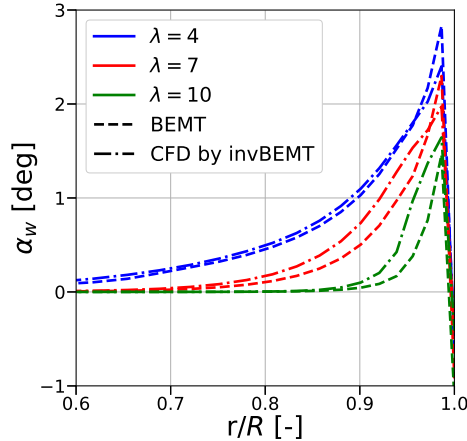


Figure 14: Downwash angle of attack in the outboard blade according to the BEMT and CFD by the inverse BEMT method. In both approaches, the local angles of attack are obtained by including the TRL correction, while the annular-averaged ones do not. [color in print]

486 overprediction of annular power and thrust coefficient in the outboard blade  
 487 by BEMT to the insufficient downwash provided by the Prandtl TRL model  
 488 compared to unsteady RANS at TSR 10.

489 Based on the conclusion drawn in the previous paragraph, an improve-  
 490 ment of the Prandtl correction for  $\lambda$  values around 10 and blunt blade tips  
 491 (as the one simulated) necessitates the modeling of the tip-vortex downwash  
 492 effect on the 2D lift coefficient under attached flow conditions in order to  
 493 achieve more accurate predictions of aerodynamic forces in the outboard  
 494 blade. To capture such an effect, alternative tip-loss models are required,  
 495 such as the one recently developed by Zhong [46], where this effect is explic-  
 496 itly modeled. The Zhong tip-loss correction models the tip-vortex contribu-  
 497 tion in two parts: one part describes the effect of the 3D tip-vortex trailed  
 498 by the non-rotating blade (same as for a fixed-wing aircraft) on the blade  
 499 profiles polars by means of the lifting-line theory, and the other part accounts  
 500 for the near-wake tip-vortex rotation (modeled similarly as Prandtl [9]). The  
 501 first part introduces an induced angle of attack which is used to reduce the  
 502 lift coefficient and increase the drag coefficient in the outboard blade cross-  
 503 sections due to the tip-vortex downwash. Such an aspect is not accounted  
 504 for by the Prandtl model and the researchers showed that the Zhong model

505 can predict closer normal and tangential force distributions to CFD in the  
 506 outboard blade than the Prandtl one for an almost TSR 10 condition and  
 507 the NREL 5 MW rotor [23]. Therefore, we claim that the Zhong correction  
 508 could be a replacement for the Prandtl one, especially at high TSR and with  
 509 blunt tip designs.

510 The differences between BEMT and CFD predictions under uniform aligned  
 511 inflow are not only related to the modeling of SD and TRL. As shown  
 512 in fig. 11, the best performing "BEMT+SD" version provided around 15%  
 513 and 20% error differences with the CFD rotor thrust and power coefficients  
 514 at  $\lambda=4$ . Furthermore, it is shown in fig. 12 that such deviations are concentrated  
 515 below 15% radius and around 60% in the midboard blade according  
 to the annular  $C_T$  and  $C_P$  distributions. In the root region, BEMT under-

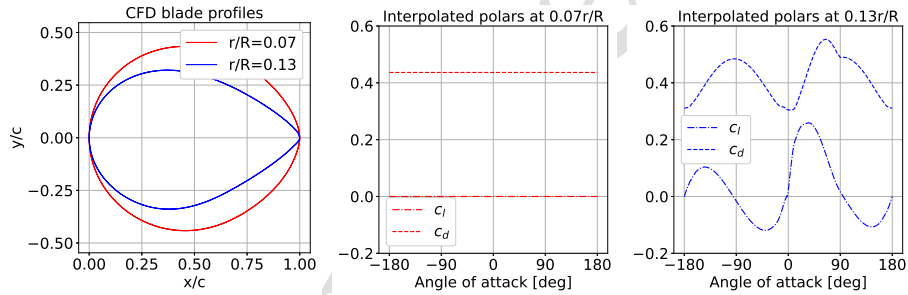


Figure 15: Profiles in the root region and the corresponding linearly interpolated lift and drag coefficient polars used by BEMT. [color in print]

516 estimates by a factor of 2-3 the  $C_T$  and  $C_P$  distributions at all TSR, not just  
 517 at  $\lambda=4$ . This is due to the polars depicted in fig. 15, which were obtained  
 518 by linear interpolation between the constant  $c_d$  polar of the cylindrical profile  
 519 and those of the DU40 airfoil. Despite this approach being consistent  
 520 with the CFD blade geometry generation described in section 2.1, such polars  
 521 do not represent the 3D, largely separated, and rotationally-augmented  
 522 flow ( $c/r \gg 0.1$ ) over these profiles discussed in section 4. In the midboard  
 523 blade, there is a clear mismatch between aerodynamic models only at  $\lambda=4$ .  
 524 The underlying reasons are the different CFD-extracted and BEMT polars  
 525 of the DU21 airfoil under separation, as shown in fig. 16. The former predict  
 526 limited separation with the lift coefficient still rising, while the latter are ex-  
 527 perimentally derived and predict a post-stall condition. Similar observations  
 528 were reported in [47] by 2D RANS, which are comparable to 3D RANS due  
 529

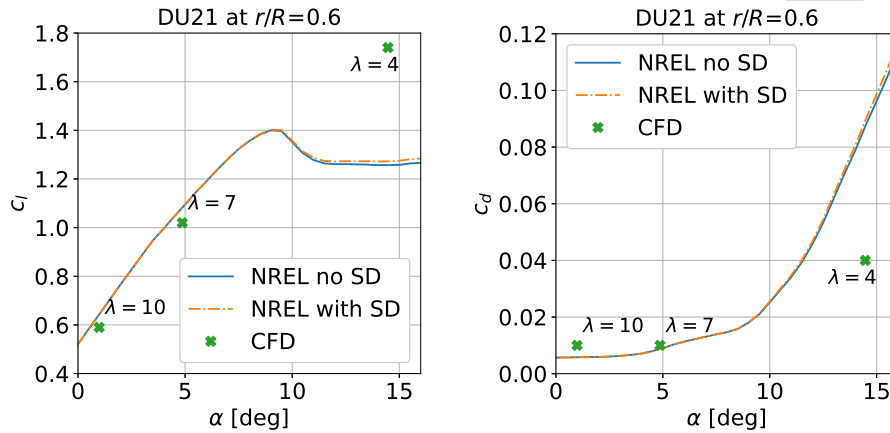


Figure 16: DU21 lift and drag coefficient polars from the NREL report with and without SD corrections (used by BEMT) and extracted from the CFD solutions as explained in section 2. [color in print]

530 to the negligible SD effects at 60% blade and  $\lambda = 4$ . Furthermore, it was  
 531 observed in [47] that transition is an important aspect to improve the match  
 532 with the experimental polars for profiles with 20% chord thickness and with  
 533 sharp suction peak and low pressure gradient, as the DU21. Therefore, both  
 534 airfoil polars and engineering corrections are critical aspects for the accuracy  
 535 of load predictions at low TSR, and it is pointless to study the accuracy  
 536 of SD corrections if poor polar data are available. The availability of reli-  
 537 able polar data for profiles with thickness greater than 40% chord could be  
 538 helpful to achieve an improvement over the simplistic linearly interpolated  
 539 polars between the cylindrical and DU40 polars before the application of SD  
 540 corrections.

## 541 6. Conclusions

542 In this study, the aerodynamic mechanisms responsible for the Prandtl  
 543 tip-loss factor inaccuracies at high TSR and the modeling of the drag coeffi-  
 544 cient by the Eggers SD correction were investigated. This was carried out by  
 545 comparing a BEMT-based solver and URANS simulations for NREL 5MW  
 546 rotor.

547 The tracking of the CFD tip-vortex trajectories confirmed that the cylin-

548 drical wake assumption of the Prandtl model is challenged at TSR 10 due to  
549 the mutual induction of the tip-vortices. As a consequence, BEMT load dis-  
550 tributions in the last 10% blade are 5-15% larger. A relevant finding is that  
551 BEMT provides insufficient ( $0.5^\circ$ ) downwash angle of attack compared with  
552 CFD at TSR 10. This result was related to the Prandtl factor which does  
553 not model the lift coefficient reduction (5-10%) provided by the tip-vortex  
554 downwash effect. Therefore, we recommend the tip-loss model of Zhong for  
555 TSR conditions of 10 and blunt blade tips as the one simulated, as it accounts  
556 for the previous effect and it was shown by its developers to provide more  
557 accurate force predictions compared to the Prandtl model. However, even  
558 if the Zhong model better captures the underlying physics of the tip-losses,  
559 it may require further parameter adjustment or tuning. It is likely that the  
560 rotating part of the model might deviate from CFD predictions under ex-  
561 panding wake conditions (high TSR). This deficiency could be mitigated by  
562 calibrating this part of the model with the annular-averaged and local axial  
563 induction factors extracted from CFD simulations over a wide set of TSR  
564 conditions and blade-tip designs.

565 Concerning the delay of stall, its presence at a TSR of 4 was shown by  
566 the relation between flow separation, radial flow, and pressure distributions  
567 at 20% radius in the CFD simulations. The only modeling of the first-order  
568 Coriolis force effect under deep stall was identified as the reason behind the  
569 incorrect prediction of the drag coefficient by the Eggers model compared  
570 with CFD. This resulted in wrong power coefficient distributions at TSR 4.  
571 At the same time, the Du-Selig SD correction for the lift coefficient provided  
572 a better match with the CFD resulting in consistent thrust coefficient distri-  
573 butions. It was concluded that a lift coefficient polar corrected for SD and  
574 an uncorrected 2D drag coefficient polar are an appropriate choice for the  
575 NREL 5MW rotor.

576 In the inboard blade and at TSR 10, the effect of the root vortex down-  
577 wash was observed on the CFD near-wall pathlines by the radial attached  
578 flow near the trailing edge of the radial location of maximum chord, simi-  
579 larly as previous studies. Another relevant result was to relate the root  
580 vortex downwash to the overestimation (20%) of the blade loading due to  
581 the use of the hub radius in the Prandtl root-loss model instead of the radial  
582 location where the phenomenon is observed.

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592 **Umberto Boatto:** Conceptualization, Methodology, Software, Validation,  
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### 599 Declaration of Competing Interest

600 The authors declare that they have no known competing financial inter-  
601 ests or personal relationships that could have appeared to influence the work  
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CRediT authorship contribution statement:

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**Declaration of interests**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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