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(Article begins on next page)

# Forward modelling of $D_\alpha$ camera view in ST40 informed by experimental data

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## Abstract

Embedding diagnostics in future pilot plants will be a challenging task, because of space- and irradiation-related concerns. Relying on high-fidelity synthetic diagnostics would then be valuable.

The 3D Monte-Carlo ray-tracing code CHERAB allows the development of numerous synthetic spectroscopic diagnostics. Focus of the present work is the introduction of new CHERAB models. The forward modelling of a synthetic  $D_\alpha$  camera in ST40, the privately funded, high-field spherical tokamak, owned and operated by Tokamak Energy Ltd, and the comparison against experimental data is chosen as a testbed for quality assessment. Main output of the study then consists of estimates of the neutral particle densities throughout the chamber, of crucial relevance within edge plasma studies.

Starting from simple analytical models, a 2D  $D_\alpha$  source in the poloidal plane is generated. However, the centre column limited plasmas in ST40 display an intrinsically-3D  $D_\alpha$  emission, mostly localised around the discrete poloidal limiters on the centre column, not captured by any axisymmetric source model. Hence, a novel methodology is introduced in CHERAB to approximate the 3D non-toroidally-symmetric pattern via a piece-wise emission distribution.

Irrespective of the geometry of the emission and size of the tokamak, the pronounced non-homogeneity in the edge plasma emission requires sub-millimetric ( $\sim$  power fall-off length) spatial resolution to guarantee an accurate estimate of the peak emission. Minimising the associated burden via implementation of a non-uniform source sampling algorithm, which is a modification of the standard CHERAB uniform sampling, results in a  $>10$ -fold reduction of the computational cost.

The significantly-shortened simulation time also makes the inclusion of more sophisticated models affordable. Of potential appeal in view of highly-detached divertors, the approximation of optically thin plasma is dropped, and photon-plasma interactions are accounted for. Brand-new CHERAB models able to take into account phenomena of photon absorption and scattering are so introduced.

*Keywords:* Synthetic diagnostic,  $D_\alpha$  radiation, Spherical tokamak, Raysect, Cherab

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## 1. Introduction

In the next generation of nuclear fusion devices, a significantly increased plasma performance with respect to existing tokamaks is aimed at being achieved and sustained for pilot-plant-relevant timescales. Such a mission requires plasma properties and power and particle exhaust to be reliably quantified during the design phase, and ultimately diagnosed during operations to build the basis for relevant code validation and extrapolation to pilot plants.

Pilot plants, with the aim of proving the feasibility of electricity production from nuclear fusion, will soon populate the landscape also. In such devices, the neutron irradiation would reach levels never experienced before and become of primary concern from the viewpoints of operation and servicing. The diagnostic apparatus would thus struggle to properly operate in, and even to survive *in primis*, this harsh environment [1].

It is in this framework that synthetic diagnostics become a tool of appeal to the fusion community. On the one hand, synthetic diagnostics offer support within the experimental data interpretation, of well-known peculiarly-challenging nature. On the other hand, synthetic diagnostics allow for a careful diagnostic design and integrated analyses devoted to fast capturing the essential plasma features [2]. The  $D_\alpha$  camera is one such example: by allowing to infer properties of the neutral deuterium distribution throughout the chamber, this tool becomes of interest, e.g., in diagnosing the edge plasma, especially in regimes of detached divertor.

In this work we employ the 3D Monte-Carlo inverse ray-tracing code CHERAB [3][4] to model the 2D view of a synthetic  $D_\alpha$  camera in the ST40 tokamak [5], the privately funded, high-field spherical tokamak, owned and operated by Tokamak Energy Ltd. With the ultimate goal of inferring valuable neutral parameters, focus is also put on the improvement of the existing numerical tools and methods in view of later systematic applications.

To this extent, the  $D_\alpha$  emission source is built via simplified analytical models for the core and edge plasma as consistently as possible with the experimental data, as discussed in Sec. 2. Instead, Sec. 3 highlights the new CHERAB models developed. A new CHERAB model qualitatively capturing the substantial lack of toroidal symmetry of the ST40 centre column is introduced in Sec. 3.1. Then, a novel implementation of a non-uniform sampling algorithm in CHERAB aimed at reducing the computational burden of spatially resolving the dishomogeneous edge plasma radiation emission is discussed in Sec. 3.2. Last, Sec. 3.3 details the process of replacing the approximation of optically thin plasma with the implementation of photon absorption and scattering, of potential interest in pilot-plant-relevant detached plasmas. The results of the work, with comparison of the different models and assessment of the overall sensitivity to the input parameters, are discussed in Sec. 4, while Sec. 5 summarises the conclusions and outlook of the paper.

## 2. Experimental data and analytical source

### 2.1. Experimental data

Time instant 65 ms within plasma shot #9229 in the ST40 database is selected for the present analysis because of: (i) the satisfactory plasma current of 490 kA, the central ion and electron temperature of  $\sim 5.3$  keV and  $\sim 2.5$  keV, respectively, and the central electron density approaching  $\sim 2.6 \times 10^{19} \text{ m}^{-3}$ ; (ii) the thorough investigations conducted by

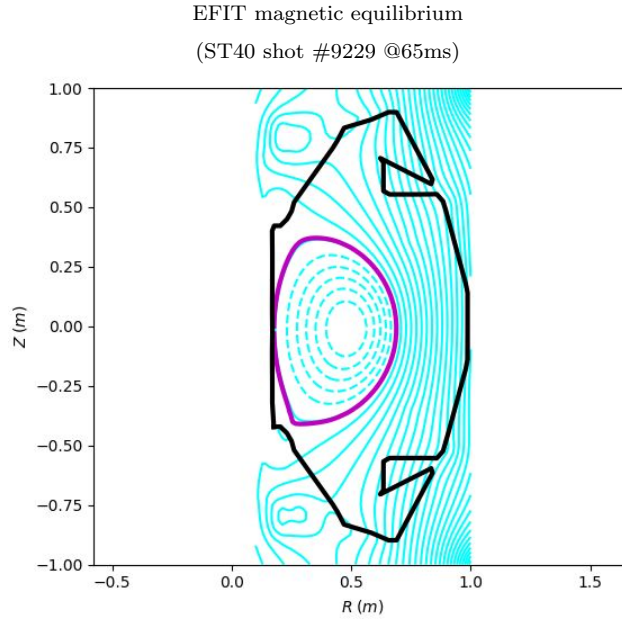
77 the ST40 Team. These make such discharge an appropriate testbed for the new CHERAB  
 78 models presented in this paper. The corresponding EFIT-reconstructed magnetic equilib-  
 79 rium [6] and experimental  $D_\alpha$  camera view are shown in Fig. 1 and Fig. 2.a, respectively,  
 80 while a schematic of the view in the toroidal plane ( $R, \phi$ ) is given in Fig. 3. The plasma is  
 81 limited to the centre column and the  $D_\alpha$  emission (lux) is hence localised on the  $N_{\text{LIM}} = 8$   
 82 discrete poloidal limiters (4 visible in Fig. 2.a), each of  $\Delta\phi_{\text{LIM}} = 0.23$  rad (= 13 degrees)  
 83 in width along the toroidal direction  $\phi$ .

84 Further noteworthy features of the experimental view are:

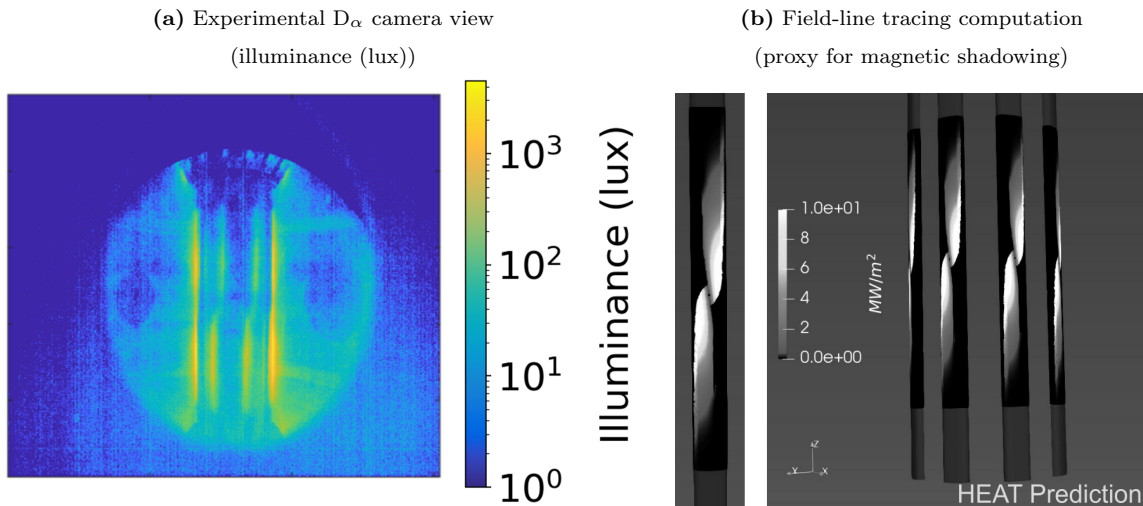
- 85 • The up-down asymmetry in the radiation emission, which is preferentially localised  
 86 in the lower half of the centre column. Reason of this is ascribed to the asymmetry  
 87 in the magnetic coil setup, reverberating in an asymmetry in the equilibrium (see,  
 88 e.g., magenta separatrix in Fig. 1).
- 89 • The sigmoid pattern along the vertical direction  $Z$ , visible upon the two central  
 90 limiters. As suggested by field-line tracing computations (Fig. 2.b), the surface  
 91 struck by the plasma (where most of the  $D_\alpha$  emission is localised) must indeed be  
 92 the left (right) half of the limiters for  $Z < 0$  ( $Z > 0$ ). This is due to the interplay  
 93 between magnetic shadowing and the direction of the magnetic field lines - impinging  
 94 on the limiters from right to left (left to right) for  $Z < 0$  ( $Z > 0$ ).
- 95 • The non-zero emission from the vicinity of the passive plates (top and bottom ex-  
 96 tremities of the centre column) caused by a fraction of power and particles actually  
 97 striking the passive plates rather than the limiters.

98 The plasma wetting affecting only the half-width of the limiters is taken into account in  
 99 Sec. 3.1. On the contrary, believed to give minor corrections to the overall result, the  
 100 remaining features are not included in the present analysis.

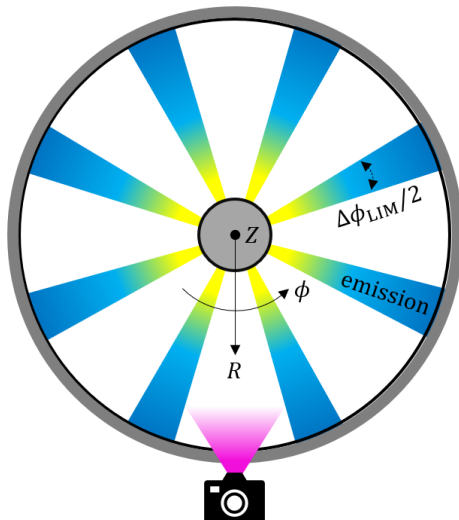
101



**Figure 1:** EFIT-reconstructed magnetic equilibrium [6] (cyan) of the plasma shot #9229 at 65 ms and separatrix (magenta). Dashed field lines belong to the plasma core region, while solid ones to the edge.



**Figure 2:** (a)  $D_\alpha$  camera experimental view of the plasma shot #9229 at 65 ms. (b) Pattern of advective and conductive plasma heat flux (proxy for magnetic shadowing) on the ST40 limiters (right) computed via field-line tracing with the HEAT code [7]. Detail of one only limiter on the left. Black regions are magnetically shadowed in the optical approximation. Courtesy of [8].



**Figure 3:** Cartoon of the top-view (toroidal plane) of the emission distribution (Sec. 3.1) enclosed by the ST40 wall (black and grey), with the reference frame adopted,  $N_{\text{LIM}} = 8$  and  $\Delta\phi_{\text{LIM}} \sim 13$  degrees (as per ST40 specs). While no emission is assumed to occur within the white volumes,  $D_\alpha$  photons isotropically originate from the complementary emitting regions (yellow = high emission, blue = low emission) and propagate throughout the chamber. At the bottom, the position of the actual camera and its field of view (magenta).

## 102 2.2. Analytical source

103 Plasma profiles of density and temperature (Sec. 2.2.1), and neutral density (Sec.  
 104 2.2.2) are required to compute the corresponding  $D_\alpha$  radiation emission (Sec. 2.2.3).  
 105 Relying upon profiles obtained via core and edge plasma simulations as in [9] would hinder  
 106 the possibility of several subsequent adjustments of the distribution itself, because of the  
 107 substantial computational cost of the underlying plasma simulations. On the other hand,  
 108 core plasma profiles computed via hierarchical data analysis (HDA) are available [2]. Non-  
 109 etheless, both the associated uncertainty in the near-separatrix region and the sensitivity of  
 110 the outcome to very near-separatrix figures (Sec. 4.2) make using HDA profiles unsuitable  
 111 in the present context. For these reasons, analytical models are employed instead.

### 112 2.2.1. Plasma profiles

113 For plasma density  $n$  and temperature  $T$  merely entering the computation of the  
 114 Amjuel-based fractional abundances of excited atoms (Sec. 2.2.3), which are a function of  
 115 the electron species only [10],  $n = n_e$  and  $T = T_e$  hold throughout the work.  
 116 The normalised poloidal flux coordinate  $\psi_N = (\psi - \psi_0)/(\psi_{\text{sep}} - \psi_0)$  is employed, with  $\psi$   
 117 being the magnetic flux,  $\psi_0$  and  $\psi_{\text{sep}}$  its value on the magnetic axis (sitting at  $(R_0; Z_0) =$   
 118  $(+0.49; -0.01)$  m) and separatrix, respectively.  
 119 A Gaussian shape<sup>1</sup> centred on the magnetic axis ( $\psi_N = 0$ ) is assumed for the core plasma  
 120 profiles [2], with a peak of  $n_e^{\psi_N=0} = 2.6 \times 10^{19} \text{ m}^{-3}$  and  $T_e^{\psi_N=0} = 2.5 \text{ keV}$  for density and

<sup>1</sup>Due to the insensitivity of the result to the core plasma parameters (Sec. 4.2), the actual shape of the core profiles is of minor relevance.

121 temperature, respectively, as informed by the available experimental measurements. The  
 122 resulting mathematical formulation hence reads:

$$\begin{aligned} n_e^{\text{core}}(\psi_N) &= n_e^{\psi_N=0} \cdot \exp\{-\psi_N^2/2\sigma_n^2\} \\ T_e^{\text{core}}(\psi_N) &= T_e^{\psi_N=0} \cdot \exp\{-\psi_N^2/2\sigma_T^2\} \end{aligned} \quad (1)$$

123 with  $\sigma_n = [2 \ln(n_e^{\psi_N=0}/n_e^{\psi_N=1})]^{-1/2}$  and  $\sigma_T = [2 \ln(T_e^{\psi_N=0}/T_e^{\psi_N=1})]^{-1/2}$  to force the pro-  
 124 files to attain the separatrix values of  $n_e^{\psi_N=1} = 1.0 \times 10^{19} \text{ m}^{-3}$  and  $T_e^{\psi_N=1} = 100 \text{ eV}$  (at  
 125  $\psi_N = 1$ ), reasonable figures obtained from [11].

126 Consistently with [11], density and temperature in the edge plasma are assumed to expo-  
 127 nentially decay along the radial direction<sup>2</sup>, as measured from the plasma magnetic axis.

128 Then:

$$\begin{aligned} n_e^{\text{edge}}(\psi_N, \theta) &= \max[n_e^{\psi_N \rightarrow \infty}; n_e^{\psi_N=1} \cdot \exp\{-(\psi_N - 1)/\lambda_n^{\psi_N}\} \cdot F(\theta)] \\ T_e^{\text{edge}}(\psi_N) &= \max[T_e^{\psi_N \rightarrow \infty}; T_e^{\psi_N=1} \cdot \exp\{-(\psi_N - 1)/\lambda_T^{\psi_N}\}] \end{aligned} \quad (2)$$

129 where the decay lengths in the  $\psi_N$ -space  $\lambda_n^{\psi_N}$  and  $\lambda_T^{\psi_N}$  are such that  $\lambda_n = 7.8 \text{ cm}$  and  $\lambda_T =$   
 130  $1.5 \text{ cm}$  in the physical space, coherently with [11]-based estimates. The resulting power  
 131 decay length for this scenario would thus be  $\lambda_q \sim (1/\lambda_n + 3/2\lambda_T)^{-1} = 0.9 \text{ cm}$  [11], of the  
 132 expected order of magnitude for a limited plasma [12]. The values of  $n_e^{\psi_N \rightarrow \infty} = 10^{14} \text{ m}^{-3}$   
 133 and  $T_e^{\psi_N \rightarrow \infty} = 0.1 \text{ eV}$  ( $\sim$  room temperature) instead serve the purpose of preventing  
 134  $n_e^{\text{edge}}$  and  $T_e^{\text{edge}}$  from dropping substantially below the minimum density and temperature  
 135 allowed by Amjuel fits [10], hence possibly leading to unreasonable extrapolations [13].  
 136 The form factor  $F(\theta) = 1 + |\theta|/\pi$  confers to  $n_e^{\text{edge}}$  the dependence on the poloidal angle  
 137  $\theta \in [-\pi; +\pi]$  expected from the 2-point model for an attached plasma [11], such that  $n_e^{\text{edge}}$   
 138 linearly doubles when moving from outer mid-plane (OMP,  $\theta = 0$ ) to the strike point on  
 139 the wall (SP,  $\theta = \pm\pi$ ). The edge plasma is instead assumed isothermal along the poloidal  
 140 direction [11].

141 Core and edge profiles are ultimately merged to give:

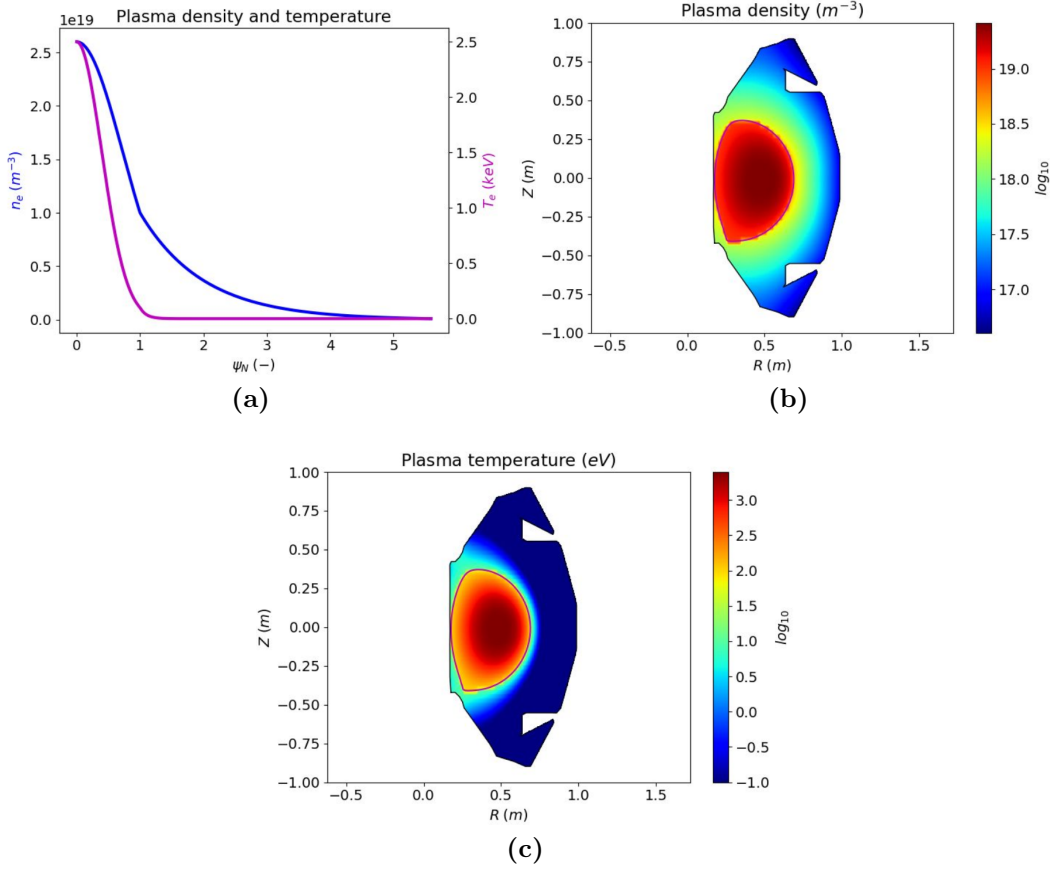
$$\begin{aligned} n_e(\psi_N, \theta) &= \begin{cases} n_e^{\text{core}}(\psi_N) & \text{if } 0 \leq \psi_N < 1 \\ n_e^{\text{edge}}(\psi_N, \theta) & \text{if } \psi_N \geq 1 \end{cases} \\ T_e(\psi_N) &= \begin{cases} T_e^{\text{core}}(\psi_N) & \text{if } 0 \leq \psi_N < 1 \\ T_e^{\text{edge}}(\psi_N) & \text{if } \psi_N \geq 1 \end{cases} \end{aligned} \quad (3)$$

142 and are depicted in Fig. 4.

143

---

<sup>2</sup>In the present scenario,  $\psi_N$  is chosen as a suitable proxy for the radial direction.



**Figure 4:** (a) Plasma density  $n_e$  (blue, left vertical axis) and temperature  $T_e$  (magenta, right vertical axis) as a function of the normalised flux coordinate  $\psi_N$ . Resulting 2D maps in the poloidal plane for density (b) and temperature (c) in logarithmic scale.

### 144 2.2.2. Neutral profiles

145 An effective neutral density  $n_g = n_D + 2n_{D_2}$  is considered [13], hence not discriminating  
 146 between atoms D and molecules  $D_2$ . Separately accounting for the two species would imply  
 147 additional degrees of freedom and complexity, and is therefore avoided in the present work.  
 148 Moreover, ST40 plasmas have been of attached (not detached) nature thus far. In such a  
 149 regime, discriminating between atomic and molecular mechanisms would presumably give  
 150 a marginal correction.

151 The overall neutral density profile is given by the superposition of two contributions:

$$\begin{aligned}
 n_g^{\text{BG}}(\psi_N) &= n_g^{\psi_N=0} + (n_g^{\psi_N \rightarrow \infty} - n_g^{\psi_N=0}) \cdot [1 + \exp\{-(\psi_N - 1)/\mu_g\}]^{-1} \\
 n_g^{\text{SP}}(R, Z) &= n_{g,0}^{\text{SP}} \cdot \sum_{i=1}^2 \exp\{-(R - R_i)^2/2\sigma_{g,R}^2\} \cdot \exp\{-(Z - Z_i)^2/2\sigma_{g,Z}^2\}
 \end{aligned} \tag{4}$$

152

$$\Rightarrow n_g(\mathbf{r}) = n_g(R, Z) = n_g^{\text{BG}}(\psi_N(R, Z)) + n_g^{\text{SP}}(R, Z)$$

153

154

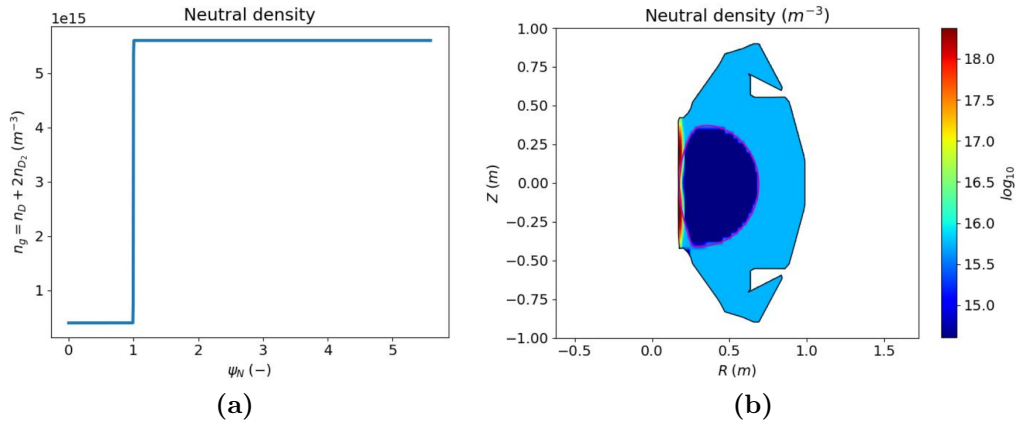
- Depicted in Fig. 5.a, a background (BG) density  $n_g^{\text{BG}}$  monotonously increasing with  $\psi_N$ , and such that  $\lim_{\psi_N \rightarrow \infty} n_g^{\text{BG}}(\psi_N) = n_g^{\psi_N \rightarrow \infty}$  away from the separatrix (wall)

vicinity), where neutral density gradients vanish. The parameter  $\mu_g$  quantifies the smoothness of the transition from core to edge plasma around the separatrix.

- Two identical anisotropic Gaussian distributions localised at the two strike points (Sec. 2.1)  $(R_1; Z_1) = (+0.177; +0.2)$  m and  $(R_2; Z_2) = (+0.177; -0.2)$  m, to mimic the recycling-driven enhanced neutral density, and featuring a peak value of  $n_{g,0}^{\text{SP}}$ . The anisotropic nature is conferred by the standard deviations  $\sigma_{g,R} = 0.5$  cm  $\ll$   $\sigma_{g,Z} = 5.0$  cm, a reminder of the neutral ionisation mean free path being substantially longer along the  $Z$  direction (towards the low-density cold edge plasma) than along the  $R$  direction (towards the high-density hot core plasma).

The actual values of the afore-mentioned free parameters are iteratively worked out by subsequent adjustments, and embody the main result of the work (Sec. 4.1). Fig. 5 pictures the resulting profiles.

167



**Figure 5:** (a) Background neutral density  $n_g^{\text{BG}}$  as a function of the normalised flux coordinate  $\psi_N$ . (b) Resulting 2D map in the poloidal plane for  $n_g(R, Z) = n_g^{\text{BG}}(\psi_N(R, Z)) + n_g^{\text{SP}}(R, Z)$  in logarithmic scale, with the peaking on the limiter conferred by  $n_g^{\text{SP}}(R, Z)$ .

### 168 2.2.3. $D_\alpha$ emission source

169 The  $D_\alpha$  photons ( $\nu_{32}$ , Balmer line) are emitted by neutral deuterium atoms D when  
 170 radiatively de-exciting from the 3rd to the 2nd quantum level<sup>3</sup> (3-2 transition, with 1  
 171 being the ground state), according to:



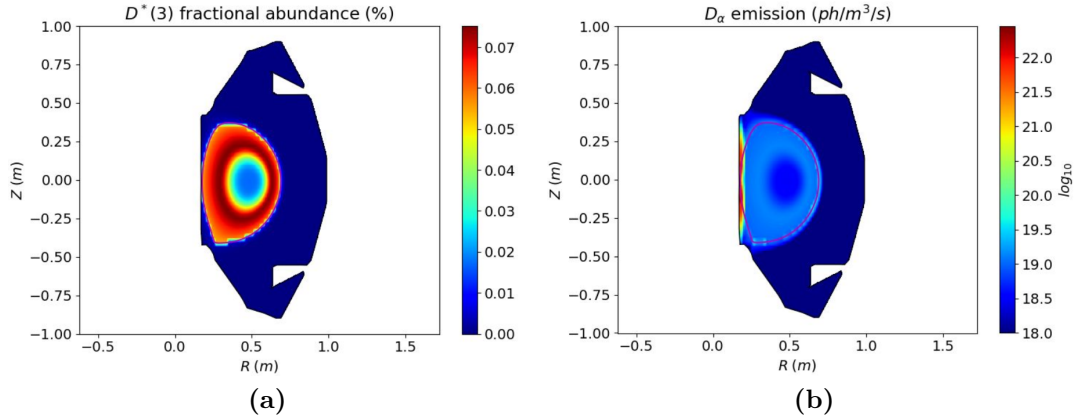
172 With the plasma profiles commented in Sec. 2.2.1, the Amjuel database<sup>4</sup> [10] allows  
 173 the computation of the fractional abundance  $f_{D^*(3)}(-)$  (Fig. 6.a) such that  $n_{D^*(3)}(\mathbf{r}) =$   
 174  $n_g(\mathbf{r}) \cdot f_{D^*(3)}(\mathbf{r})$  is the density of neutrals excited in the 3rd quantum level, with the implicit

<sup>3</sup>Splitting of the degenerate quantum levels due to the presence of a strong magnetic field (Paschen-Back effect) can be shown to be a negligible correction in this context.

<sup>4</sup>Specifically, the ratios between the fractional abundances H.12-2.1.5a and H.12-2.1.5tot.

175 assumptions of  $n_g$  comprising all the possible excited states, and of the excitation being  
 176 driven by neutral-electron interactions<sup>5</sup>.  
 177 Then, by relying on the transition probability  $A_{32} = 4.41 \times 10^7 \text{ s}^{-1}$  for the 3-2 transition  
 178 [14], the 2D  $D_\alpha$  isotropic source  $S_{D_\alpha}$  ( $\text{ph} \cdot \text{m}^{-3} \cdot \text{s}^{-1}$ ) follows (Fig. 6.b):

$$S_\alpha^{2D}(\mathbf{r}) = A_{32} \cdot n_{D^{*(3)}}(\mathbf{r}) \quad (6)$$



**Figure 6:** 2D maps in the poloidal plane of (a) fractional abundance  $f_{D^{*(3)}}(R, Z)$  (%) and (b) isotropic emission source  $S_\alpha^{2D}(R, Z)$ , in logarithmic scale.

179 in a similar manner to the approach adopted by [15] (Eq. 5 therein). The radiation  
 180 source in ( $\text{ph} \cdot \text{m}^{-3} \cdot \text{s}^{-1}$ ) is then sampled in CHERAB providing a result expressed in  
 181 ( $\text{ph} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1}$ ), and converted to (lux) for experimental comparison according to [16].

### 182 3. Novel CHERAB models and algorithms

#### 183 3.1. 3D periodic piece-wise emission distribution

184 By default, CHERAB allows the selection among two methods to build 3D emission  
 185 sources: revolving a 2D source defined in the poloidal plane around the tokamak  $Z$ -axis,  
 186 or considering a source intrinsically 3D.

187 Via the introduction of a dichotomic function  $\mathbb{D}(\phi)$  periodic along the toroidal direction  
 188  $\phi$ , a novel 3D emission source model of piece-wise nature is implemented in CHERAB.  
 189 Because of the experimental emission pattern (Fig. 2.a), such new model is of vital  
 190 relevance in the present context, but would similarly be of appeal in circumstances as  
 191 those found in [17] (Fig. 3 therein).

192 With  $N_{\text{LIM}} = 8$  and  $\Delta\phi_{\text{LIM}} = 0.23 \text{ rad}$  ( $= 13 \text{ degrees}$ ) as per Sec. 2.1, and by defining  
 193  $\phi_k = 2k\pi/N_{\text{LIM}}$  with  $k \in [0; N_{\text{LIM}} - 1]$ , the mathematical expression of the dichotomic

<sup>5</sup>Because of the small neutral density of the present limited scenario (Sec. 4.1), excitation driven by neutral-neutral collisions is neglected.

194 function and of the corresponding 3D (isotropic) source (schematically represented in Fig.  
195 3) read:

$$\mathbb{D}(\phi) = \begin{cases} 1 & \text{if } \phi_k < \phi < \phi_k + \Delta\phi_{\text{LIM}}/2 \\ 0 & \text{if } \phi_k + \Delta\phi_{\text{LIM}}/2 < \phi < \phi_{k+1} \end{cases} \quad (7)$$

196

$$\Rightarrow S_{\alpha}^{3\text{D}}(\mathbf{r}, \phi) = \mathbb{D}(\phi) \cdot S_{\alpha}^{2\text{D}}(\mathbf{r})$$

197 Particularly, considering the isotropic emission being non-zero only within a region of  
198 width  $\Delta\phi_{\text{LIM}}/2$  (yellow-blue in Fig. 3) comes from the magnetic shadowing, as discussed  
199 in Sec. 2.1. Instead, assuming exactly zero emission in the complementary areas (white  
200 in Fig. 3) could be relaxed and some non-zero emission included, but is outside the scope  
201 of the present work.

202 The dichotomic function is then also applied to all the plasma and neutral variables (Sec.  
203 2.2.1 and Sec. 2.2.2, respectively).

### 204 3.2. Non-uniform source sampling

205 Whenever dishomogeneous emission sources are taken into account, sampling (and  
206 sample integration afterwards) is carried out along the directions identified by the rays  
207 (lines of observation) fired by the camera within the computational scene (reverse ray-  
208 tracing) [9][18]. By default and for the sake of simplicity, CHERAB relies upon a uniform  
209 sampling (exemplified by the white arrow and dots in Fig. 7.a), i.e. adjacent samples are  
210 spaced of a constant quantity  $\Delta s$  (m) - sampling step. Given the strong dishomogeneities  
211 featured by the edge plasma and developing over a length in the order of  $\lambda_q$ , the most  
212 appropriate sampling step would be such that  $\Delta s < \lambda_q$ , ideally of a factor  $\geq 2$ . Neverthe-  
213 less,  $\lambda_q$  attaining values in the millimetres range [19][20] implies a prohibitively small  $\Delta s$ .  
214 A new non-uniform sampling algorithm is therefore developed, and schematically shown in  
215 magenta in Fig. 7.a. Driving idea is acknowledging the need of fine-sampling exclusively  
216 in those regions of space characterised either (1) by elevated emission or (2) by elevated  
217 emission gradients, leaving coarse-sampling elsewhere. Mathematically, emphasising such  
218 specific areas can be translated in the following criterion:

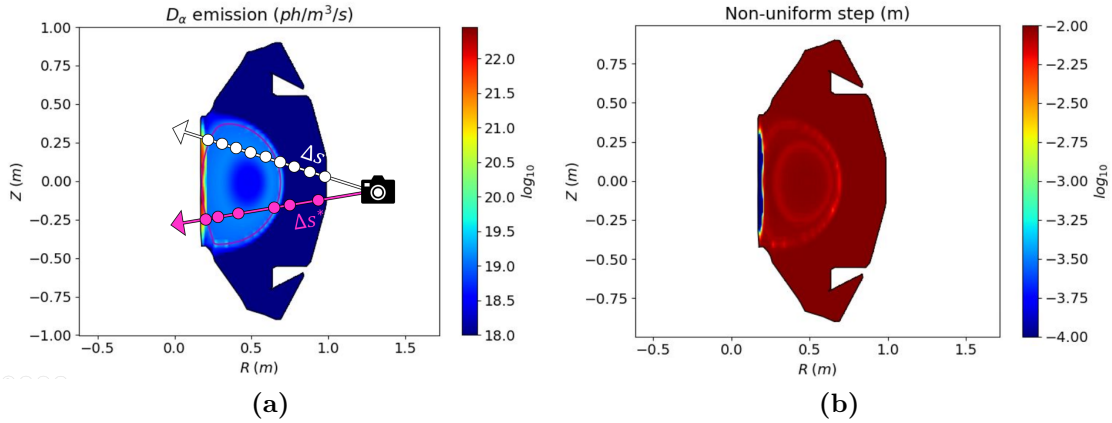
$$\Delta s^*(\mathbf{r}) = \Delta s_{\text{max}} \cdot \left( \frac{\Delta s_L(\mathbf{r})}{\Delta s_{\text{max}}} \right)^{\frac{w_1 S_{\alpha}^{2\text{D}}(\mathbf{r})}{\max_{\mathbf{r}} [S_{\alpha}^{2\text{D}}(\mathbf{r})]}} \quad (8)$$

219

$$\begin{aligned} & \text{with } \Delta s_L(\mathbf{r}) = \max [\Delta s_{\text{min}} ; \min [\Delta s_{\text{max}} ; L_{1\text{st}}(\mathbf{r})/w_2 ; L_{2\text{nd}}(\mathbf{r})/w_2]] \\ & \text{and } L_{1\text{st}}(\mathbf{r}) = S_{\alpha}^{2\text{D}}(\mathbf{r})/|\nabla S_{\alpha}^{2\text{D}}(\mathbf{r})| \\ & \text{and } L_{2\text{nd}}(\mathbf{r}) = |\nabla S_{\alpha}^{2\text{D}}(\mathbf{r})|/\|\mathbf{H} [S_{\alpha}^{2\text{D}}(\mathbf{r})]\|_2 \end{aligned} \quad (9)$$

220 In other words, the non-uniform sampling step  $\Delta s^*(\mathbf{r})$  depends on the actual location in  
221 space  $\mathbf{r}$  via the power law of Eq. 8, graphically represented in Fig. 7.b. If  $S_{\alpha}^{2\text{D}}(\mathbf{r}) = 0$  then  
222  $\Delta s^*(\mathbf{r}) = \Delta s_{\text{max}}$  (under user-selection, likewise  $\Delta s_{\text{min}}$ ) and the sampling is the most coarse  
223 allowed. Conversely, if  $S_{\alpha}^{2\text{D}}(\mathbf{r}) > 0$  then  $\Delta s^*(\mathbf{r})$  would depend on  $\Delta s_L(\mathbf{r}) \in [\Delta s_{\text{min}} ; \Delta s_{\text{max}}]$ ,  
224 with  $L_{1\text{st}}$  and  $L_{2\text{nd}}$  (involving the 2-norm  $\|\cdot\|_2$  of the Hessian matrix  $\mathbf{H}$  [21]) being the  
225 characteristic length scales (m) over which the emission  $S_{\alpha}^{2\text{D}}(\mathbf{r})$  and its gradient  $\nabla S_{\alpha}^{2\text{D}}(\mathbf{r})$   
226 vary, respectively, and, hence, must be spatially resolved. The weights  $w_1$  and  $w_2$  (-)

227 are under user-control and put more emphasis (proportionally smaller  $\Delta s^*$ ) on the regions  
 228 where the emission is closest to the maximum and where it varies the most, respectively.  
 229 The values employed in this work are  $w_1 = 45$  and  $w_2 = 30$ .  
 230 Once a satisfactory 2D function  $\Delta s^*(\mathbf{r})$  is built in the pre-processing phase by choosing  
 231  $\Delta s_{\min}$  and  $\Delta s_{\max}$  and adjusting  $w_1$  and  $w_2$  (Fig. 7.b), the corresponding 3D version is ob-  
 232 tained by a mere revolution around the  $Z$  axis (no dependence on  $\phi$ ). Whether employing  
 233 a method likewise Sec. 3.1's would give an advantage despite the enhanced complexity  
 234 (especially around the periodic discontinuities) is currently dubious and left for future  
 235 investigation.  
 236



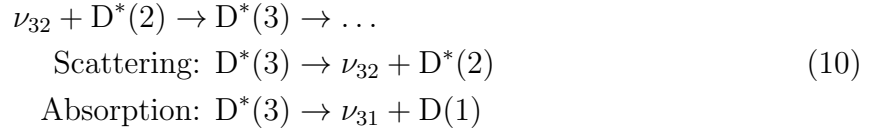
**Figure 7:** (a) Cartoon of the uniform (white, default in CHERAB) and non-uniform (magenta, newly-implemented) source sampling along the direction of two representative rays fired by the camera (reverse ray-tracing), overlaid to the emission distribution. The arrows identify the ray direction (observation line), the dots are the samples collected. (b) Map of the newly-implemented non-uniform sampling step  $\Delta s^*(R, Z)$  with  $\Delta s_{\min} = 10^{-4}$  m,  $\Delta s_{\max} = 10^{-2}$  m,  $w_1 = 45$ , and  $w_2 = 30$ , as used in the present work.

### 237 3.3. Opaque plasma: photon absorption and scattering

238 Customarily relied upon is the approximation of optically thin plasma [22], which as-  
 239 sumes the photons emitted by the plasma to leave the plasma itself unperturbed. In  
 240 CHERAB this translates into rectilinear ray trajectories the source samples are collected  
 241 along (Fig. 7.a). However, with neutral deuterium densities foreseen to exceed  $10^{21}$  m<sup>-3</sup> in  
 242 partially- and fully-detached regimes [13][23], plasma opaqueness to photon propagation  
 243 might be non-negligible. Although such densities are far above those achieved in the ST40  
 244 shot under consideration, a  $\sim 12\%$  variation is already implied when considering plasma  
 245 opaqueness (Sec. 4.1), which is therefore meritorious of implementation in view of future  
 246 applications. This is here carried out according to two strategies, differing in terms of  
 247 complexity and computational cost.

248 First, the propagation of rays in an opaque environment is directly simulated via a Monte  
 249 Carlo procedure, and the emission samples collected along the resulting non-rectilinear  
 250 trajectory (absorption & scattering). At a fundamental level lies the microscopic cross-  
 251 section  $\sigma_{32}^{\text{abs}} \sim 1.2 \times 10^{-14}$  m<sup>2</sup>, which regulates the process of absorption of a  $\nu_{32}$  photon

252 by the hand of a  $D^*(2)$  excited atom. Such parameter can be computed from [24], where  
 253 (i) spontaneous emission ( $A$ -factors of Sec. 2.2.3), (ii) elastic, and (iii) inelastic contribu-  
 254 tions to the transition are accounted for, with (i) being the dominant and henceforth the  
 255 only one actually considered here<sup>6</sup>. Downstream the photon absorption, if the resulting  
 256  $D^*(3)$  atom subsequently meets a radiative 3-2 de-excitation via emission of a  $\nu_{32}$  photon  
 257 (44% probability<sup>7</sup>), the original  $\nu_{32}$  is considered as (isotropically) scattered. Instead, if a  
 258 radiative 3-1 de-excitation is met by  $D^*(3)$  (56% probability), the original  $\nu_{32}$  is assumed  
 259 as effectively absorbed<sup>8</sup>.  
 260 Such processes are summarised below:



261 and implemented in CHERAB via the Delta-tracking method [27] regulated by the ab-  
 262 sorption macroscopic cross section  $\Sigma_{32}^{\text{abs}}(\mathbf{r}, \phi) = \sigma_{32}^{\text{abs}} \cdot n_{D^*(2)}(\mathbf{r}, \phi)$  ( $\text{m}^{-1}$ ) of Fig. 8 (with  
 263  $n_{D^*(2)}$  computed from the  $D^*(2)$  fractional abundance - Sec. 2.2.3).  
 264 Second, in addition to the above-mentioned strategy, a simpler purely-absorbing dynamics  
 265 in the fashion of [28] is implemented by properly weighting the emission samples collected  
 266 along rectilinear ray trajectories (linear absorption). By relying on the linear attenuation  
 267 law along the rectilinear direction of each ray, the intensity  $I_\alpha$  ( $\text{ph} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$ ) integrated  
 268 between two consecutive emission samples ( $S_\alpha^{3D}(\mathbf{r}_i, \phi_i)$  and  $S_\alpha^{3D}(\mathbf{r}_{i+1}, \phi_{i+1})$ ) at a distance  
 269  $\Delta s^*(\mathbf{r}_i)$  (Sec. 3.2) reads<sup>9</sup>:

$$\begin{aligned}
 I_\alpha^{i \rightarrow i+1} = & 1/2 \cdot [S_\alpha^{3D}(\mathbf{r}_i, \phi_i) + S_\alpha^{3D}(\mathbf{r}_{i+1}, \phi_{i+1})] \cdot \Delta s^*(\mathbf{r}_i) \\
 & \cdot \exp \left\{ -\Sigma_{32}^{\text{abs}}(\mathbf{r}_i, \phi_i) \cdot \Delta s^*(\mathbf{r}_i) \right\}
 \end{aligned}
 \tag{11}$$

270 where the exponential factor embodies the attenuation. The default CHERAB integration  
 271 algorithm is recovered by setting  $\Sigma_{32}^{\text{abs}}(\mathbf{r}, \phi) = 0$ .

272

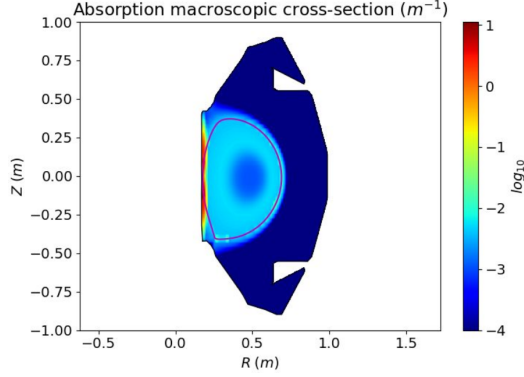
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<sup>6</sup>Particularly, (iii) can be evaluated via experimentally measured values of [25] (2-1 transition) and conservatively according to Drawin formula of [26] (Eq. 1 and 2 therein for 3-2 and 3-1 transitions).

<sup>7</sup>As dictated by the ratio  $A_{32}/(A_{32} + A_{31})$  as a function of the transition probabilities for the 3-2 and 3-1 radiative de-excitation ( $A_{31} = 5.58 \times 10^7 \text{ s}^{-1}$ ) [14].

<sup>8</sup>The circumstance of  $\nu_{31}$  being further absorbed by a  $D(1)$  atom eventually leading to a re-emission of  $\nu_{32}$  is outside the scope of the present work, and therefore neglected. Codes capable of self-consistently evolving the photon transport in a plasma background nonetheless exist.

<sup>9</sup>Assuming a proper selection of  $\Delta s^*(\mathbf{r})$ , the plasma can be considered as homogeneous over the spatial scale defined by the sampling step.



**Figure 8:** Poloidal distribution of the absorption macroscopic cross section  $\Sigma_{32}^{\text{abs}}(R, Z)$ .

## 273 4. Results

274 The location, field of view, observation direction, and pixel arrangement ( $1520 \times 1000$   
 275 uniformly spaced along the horizontal and vertical directions, respectively) of the camera  
 276 are extracted via the CalCam software [29]. Matching the actual camera pixel distribution  
 277 is imperative for the synthetic view, to ensure a spatial resolution consistent with the  
 278 experimental view.

279 Common to all the simulations, highlights of the settings adopted to build the non-uniform  
 280 step map are those of Fig. 7.b, with a number of rays (Monte Carlo samples [18][30])  
 281 which guarantees a statistical error below 1%. While achieving the same overall accuracy,  
 282 employing the non-uniform sampling in the present scenario guarantees a  $>10$ -fold decrease  
 283 in the computational cost if compared to equivalent simulations with uniform sampling  
 284 (with  $\Delta s = \Delta s_{\text{min}}$ ). This advantageously leads to the possibility of relying on PC-class  
 285 devices, rather than on HPCs. By employing  $\Delta s_{\text{max}} = 1 \times 10^{-2}$  m and  $\Delta s_{\text{min}} = 1 \times 10^{-4}$   
 286 m as in the present study, the computational time with non-uniform sampling approaches  
 287  $\sim 0.8$  hours, whilst  $\sim 15$  hours are required with uniform sampling of step  $\Delta s = 1 \times 10^{-4}$   
 288 m (8 CPUs in both instances).

### 289 4.1. Effect of the different models

290 A two-fold strategy is employed.

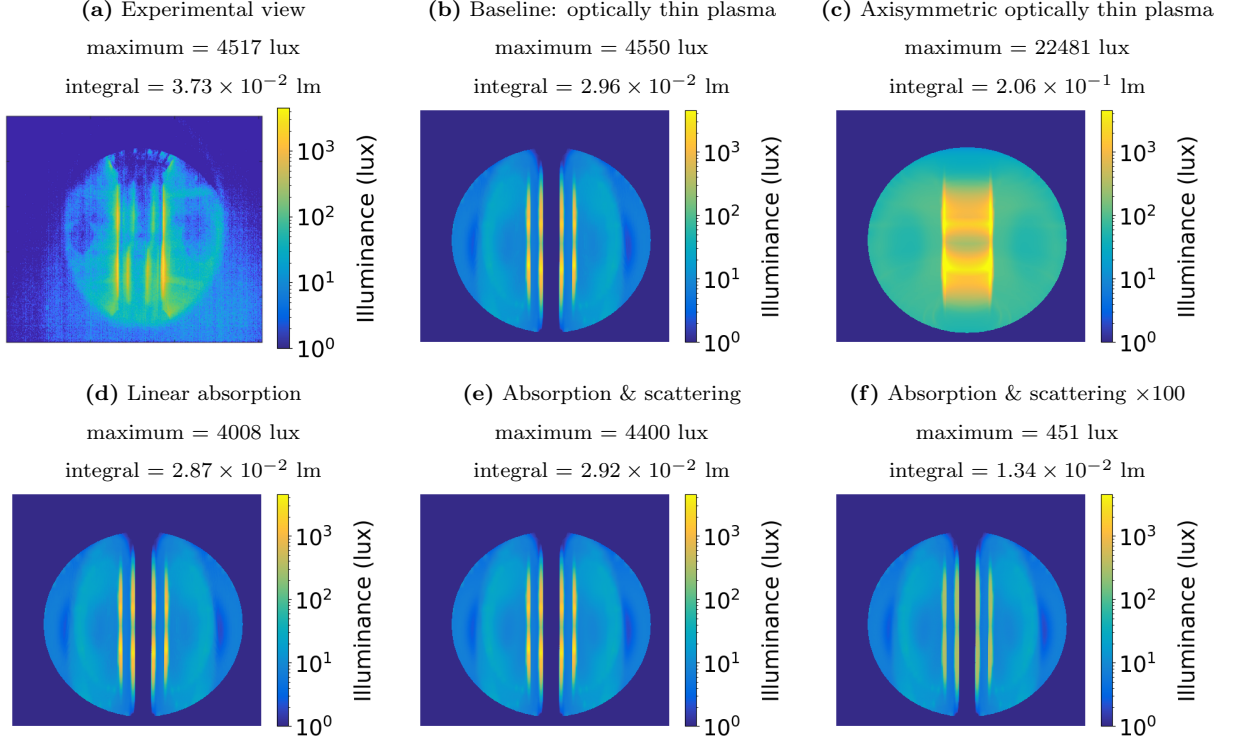
291 First, a best match with experimental data in the case of optically thin plasma model  
 292 (baseline) is achieved, both in terms of maximum illuminance (lux) and of integral lu-  
 293 minous flux (lm) - computed by integrating the illuminance over the pixel areas. Then,  
 294 further simulations with the different models are run, with the same plasma and neutral  
 295 input parameters of the baseline. This allows to gain insight into how different models  
 296 impact, if at all, the resulting synthetic emission. The results are pictured in Fig. 9.  
 297 The optically thin plasma baseline case (Fig. 9.b) well matches the experimental view  
 298 (Fig. 9.a), both from a qualitative and quantitative standpoint. The strong qualitative  
 299 similarity crucially follows from the exploitation of the novel 3D periodic piece-wise model  
 300 of Sec. 3.1: the inappropriateness of an axisymmetric source model is clarified by Fig.  
 301 9.c. The quantitative agreement is instead justified by the synthetic maximum illuminance  
 302 (maximum) and integral luminous flux (integral) satisfactorily falling within 2% and 21%

303 of the experimental data's, respectively. Neutral densities of  $n_g^{\psi_N=0} \sim 4.0 \times 10^{14} \text{ m}^{-3}$  in  
 304 the core plasma,  $n_g^{\psi_N \rightarrow \infty} \sim 5.6 \times 10^{15} \text{ m}^{-3}$  in the far-edge plasma (outer wall vicinity), and  
 305  $n_{g,0}^{\text{SP}} \sim 3.8 \times 10^{18} \text{ m}^{-3}$  around the strike points are estimated. The other results (from Fig.  
 306 9.c to .f) departing from the baseline is therefore to be entirely ascribed to the different  
 307 models considered. In the respect of Sec. 3.3, the maximum emission obtained in Fig.  
 308 9.d by considering rectilinear ray trajectories and attenuating the weight of the emission  
 309 samples (linear absorption) leads to a  $\sim 12\%$  and  $\sim 3\%$  reduction in the maximum and  
 310 integral, respectively. Instead, a mere  $\sim 3\%$  and  $\sim 1\%$  drop of the maximum and integral  
 311 can be appreciated in Fig. 9.e when both absorption and scattering are activated and  
 312 non-rectilinear ray trajectories simulated (absorption & scattering). The mild effects are  
 313 presumably due to the reduced neutral density ( $< 10^{19} \text{ m}^{-3}$ ) characteristic of this limited  
 314 plasma, in contrast to predictions estimating neutral densities 2 orders of magnitude larger  
 315 in detached regimes [13][23]. With absorption and scattering heavily depending on such  
 316 parameter, a marginal importance in this low-neutral-density case is reasonable. For the  
 317 sake of illustration, Fig. 9.f shows a synthetic view obtained with a 100-fold increase in  
 318 the microscopic cross-section<sup>10</sup> (absorption & scattering  $\times 100$ ): the corresponding 10-fold  
 319 reduction in the maximum stresses how plasma opaqueness could substantially impact the  
 320 predictions.

321

---

<sup>10</sup>Increasing the neutral density instead of the cross section would nullify the validity of the assumptions of Sec. 2.2.1 and 2.2.2, as the plasma would enter a detached state and more sophisticated models would be required. The *escamotage* of artificially increasing the cross section is indeed for illustration purposes only.



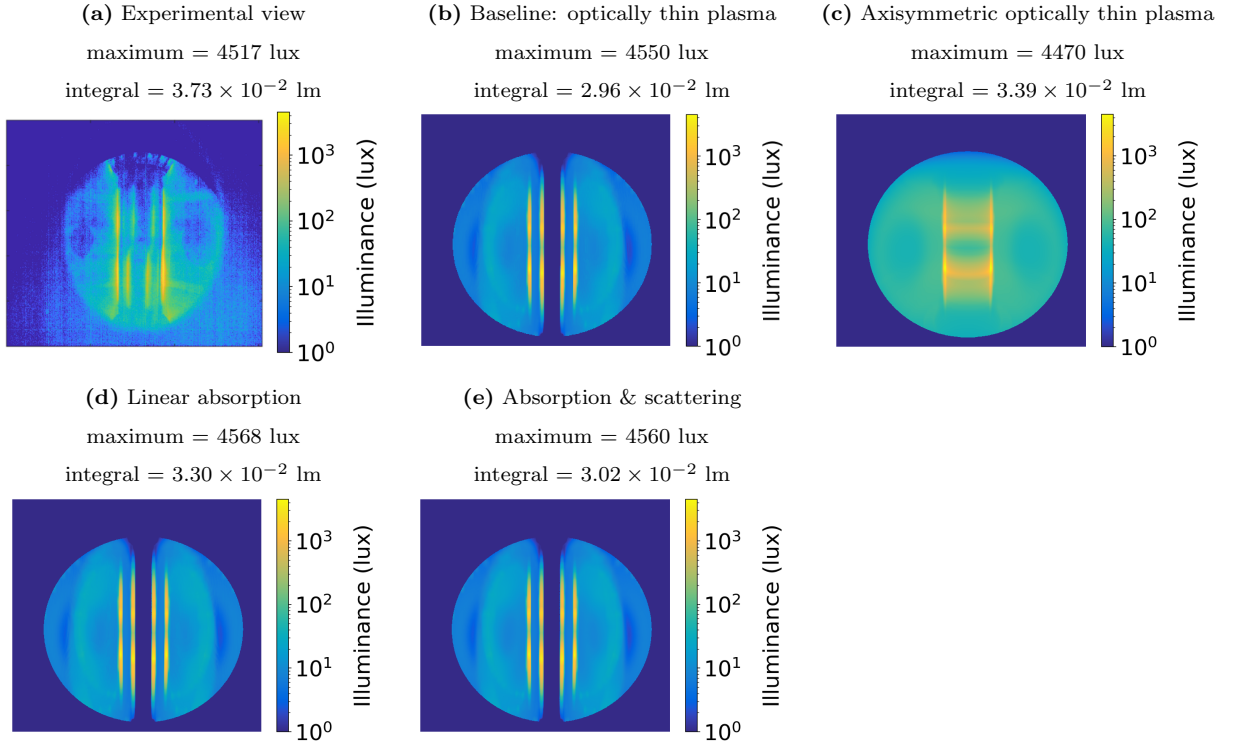
**Figure 9:** (a)  $D_\alpha$  camera experimental view (lux) (as per Fig. 1.b, repeated here for the sake of ease of comparison). (b) Baseline scenario, approximation of optically thin plasma with novel 3D periodic piece-wise emission pattern. (c) Approximation of optically thin plasma with CHERAB default axisymmetric source. (d) Newly-implemented linear absorption. (e) Newly-implemented Delta-tracking-based absorption and scattering, in (f) with  $\times 100$  magnification. Free parameters of (b) are employed in each instance. The top end of each logarithmic colorbar matches the maximum illuminance of instance (a).

322 Second, simulations with the different models are run, with case-specific neutral para-  
 323 meters so that each instance is independently best-matched against experimental data, to  
 324 understand the sensitivity of the neutral densities (main output) to the particular model  
 325 adopted. The case-specific parameters are listed in Tab. 1 and the results in Fig. 10,  
 326 with a satisfactory agreement with the data achieved in each instance: within 2% for the  
 327 maximum value of the illuminance, and within 21% for the integral luminous flux. No par-  
 328 ticular neutral parameter stands out as a most influential: by excluding the axisymmetric  
 329 instance (in qualitative spark contrast with the experiment),  $n_g^{\psi_N=0}$ ,  $n_g^{\psi_N \rightarrow \infty}$ , and  $n_{g,0}^{SP}$   
 330 are seen to vary up to 15% at most, hence suggesting the robustness of the obtained results  
 331 with respect to the neutral figures, at least within an uncertainty of 15% (not accounting  
 332 for other sources of uncertainty, e.g. model uncertainty).

333

Case	Maximum (lux)	$n_g^{\psi_N=0}$ (m <sup>-3</sup> )	$n_g^{\psi_N \rightarrow \infty}$ (m <sup>-3</sup> )	$n_{g,0}^{\text{SP}}$ (m <sup>-3</sup> )
<i>Experimental</i>	4517	—	—	—
Optically thin plasma	4550	4.0E+14	5.6E+15	3.8E+18
Axisymmetric thin plasma	4470	1.0E+12	1.0E+13	1.3E+18
Linear absorption	4568	4.6E+14	6.5E+15	4.4E+18
Absorption & scattering	4560	4.1E+14	5.8E+15	3.9E+18

**Table 1:** Summary of the maximum emission and neutral parameters adopted in the different cases, separately tuned to achieve the best match possible in terms of maximum illuminance and integral luminous flux.



**Figure 10:** (a)  $D_\alpha$  camera experimental view (lux) (as per Fig. 1.b, repeated here for the sake of ease of comparison). (b) Baseline scenario, approximation of optically thin plasma with novel 3D periodic piece-wise emission pattern. (c) Approximation of optically thin plasma with CHERAB default axisymmetric source. (d) Newly-implemented linear absorption. (e) Newly-implemented Delta-tracking-based absorption and scattering. The free parameters are separately tuned in each instance. The top end of each logarithmic colorbar matches the maximum illuminance of instance (a).

334 *4.2. Assessment of robustness to free and uncertain parameters*

335 The present section aims at quantifying the overall sensitivity of the outcome with  
336 respect to the parameters involved. By adopting the same settings as per the baseline

337 simulation of Sec. 4.1 (also in Fig. 11.a), and by changing one parameter at a time of an  
 338 order of magnitude, the impact on the synthetic view is highlighted. Primary judgement  
 339 metrics selected are the qualitative agreement to the baseline view, the maximum illuminance  
 340 (lux), and the integral luminous flux (lm) over the picture.

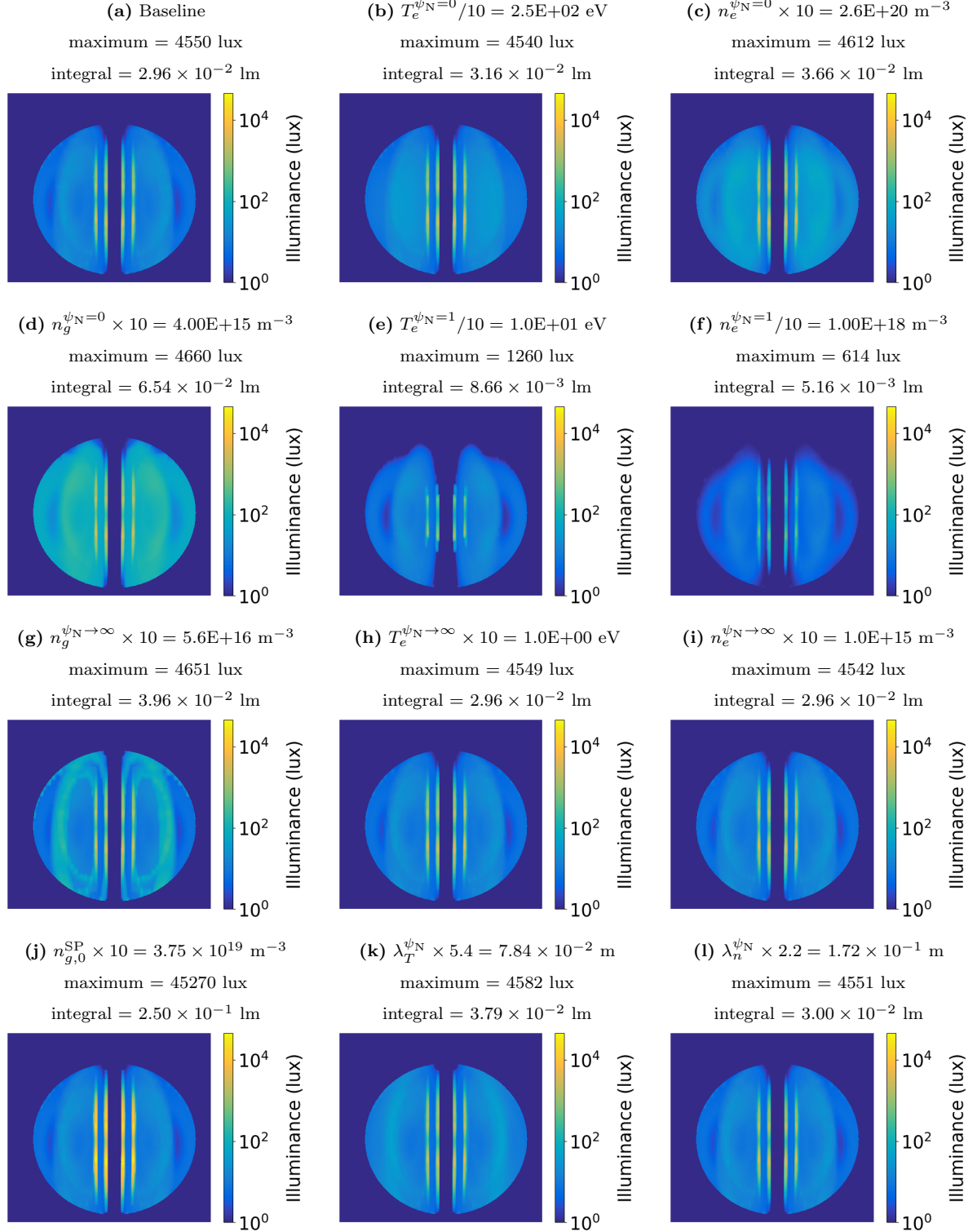
341 Fig. 11 shows the results of the study. Minor sensitivity as a function of the core plasma  
 342 parameters ( $\psi_N = 0$ , e.g. uncertainty in experimental measurements of Sec. 2.1) is ap-  
 343 parent from Fig. 11.b and 11.c, in fact implied by the mild dependence of Amjuel fits on  
 344 plasma density and temperature [10], at least for the narrow ranges spanned by a limited  
 345 plasma.

346 Conversely, edge plasma features at the separatrix ( $\psi_N = 1$ ) do play a substantial role,  
 347 presumably because of the direct impact on the  $D_\alpha$  emission distribution which peaks in  
 348 the very attached edge plasma, hence characterised by strong dependence of the down-  
 349 stream (at the limiter) quantities on their upstream (at the separatrix) counterparts [11].  
 350 Although the values of  $T_e^{\psi_N=1}/10 = 10$  eV and  $n_e^{\psi_N=1}/10 = 10^{18}$  m $^{-3}$  seemingly lie well  
 351 outside a reasonable error-bar [2], in-depth specific analyses of the ST40 edge plasma  
 352 would shed light in this respect, but are outside the scope of the present paper. Then,  
 353 the farther away from the separatrix, the milder the impact of any changes, as justified  
 354 by the absence of any appreciable effects by varying the background plasma quantities  
 355 ( $\psi_N \rightarrow \infty$ ), a welcomed result given their primary purely-numerical purpose of preventing  
 356 accidental divergence of the Amjuel fits (Sec. 2.2.1). Similarly, the actual value of the  
 357 decay lengths  $\lambda_n^{\psi_N}$  and  $\lambda_T^{\psi_N}$  (Fig. 11.k and .l, respectively) does not heavily impact the  
 358 result either.

359 Because of the direct link to the maximum  $D_\alpha$  emission, the peak neutral density on the  
 360 limiters  $n_{g,0}^{\text{SP}}$  has a strong impact on the maximum illuminance and on the integral lumin-  
 361 ous flux, both scaling almost linearly with such parameter. Sensitivity of the simulations  
 362 results to  $n_{g,0}^{\text{SP}}$ , on the one hand, warns the user on the uncertainty of the output but, on  
 363 the other hand, implies this to be a reliable method to estimate the peak neutral density,  
 364 which would be instead nullified by a marginal effect of  $n_{g,0}^{\text{SP}}$ .

365 Conversely,  $n_g^{\psi_N=0}$  and  $n_g^{\psi_N \rightarrow \infty}$ , while not appreciably affecting the maximum illuminance,  
 366 do show an effect on the integral luminous flux and on the qualitative aspect of the image.  
 367 In particular, a 10-fold increase in  $n_g^{\psi_N=0}$  implies an enhanced background emission and,  
 368 as a consequence, an increase of the same order on the integral luminous flux with respect  
 369 to the baseline scenario. In regards to  $n_g^{\psi_N \rightarrow \infty}$  being increased of a factor 10, a negligible  
 370 change in the integral luminous flux is found, but accompanied by a substantially different  
 371 image from a qualitative standpoint, hence no more in agreement with the experimental  
 372 view.

373 Recovering a good agreement with experimental data in each of the aforementioned in-  
 374 stances has been attempted by re-tuning the remaining parameters, but satisfactory ac-  
 375 curacy could not be achieved, hence suggesting the baseline scenario results to be the one  
 376 most reliable. The overall sensitivity to the neutral densities implies the welcomed feature  
 377 of this approach to firmly constraining such values, at least within the uncertainties of the  
 378 present method.



**Figure 11:** (a) Baseline scenario of Sec. 4.1, then modified by changing: (b) core plasma temperature; (c) core plasma density; (d) core neutral density; (e) separatrix plasma temperature; (f) separatrix plasma density; (g) background neutral density; (h) background plasma temperature; (i) background plasma density; (j) neutral density at strike points; (k) edge plasma temperature decay length; (l) edge plasma density decay length. The top end of each logarithmic colorbar matches the maximum illuminance of instance (a).

## 379 5. Conclusions

380 In the present work, the synthetic 2D view of the  $D_\alpha$  camera in Tokamak Energy ST40  
381 device [5] is reconstructed using the CHERAB code [3][4]. The  $D_\alpha$  emission model is  
382 based on simple analytical models informed by experimental data. A centre-column lim-  
383 ited plasma scenario is considered, and the synthetic data compared with the camera's.  
384 The emission model accounts for the 3D nature of the source in limited configuration as  
385 well as light scattering and absorption.

386 Satisfactory qualitative and quantitative agreement is achieved with the baseline simula-  
387 tion, which assumes the plasma to be optically thin (transparent) to the radiation emitted  
388 therein, by reliance on the novel piece-wise 3D emission model and by tuning the free  
389 model parameters. Corresponding values of  $\sim 4.0 \times 10^{14} \text{ m}^{-3}$ ,  $\sim 5.6 \times 10^{15} \text{ m}^{-3}$ , and  $\sim$   
390  $3.8 \times 10^{18} \text{ m}^{-3}$  are estimated for the neutral deuterium density in the core and far-edge  
391 plasma, and in the vicinity of the strike points on the centre column, respectively.

392 Inclusion of the newly-implemented models accounting for  $D_\alpha$  absorption and scattering  
393 show, on the one hand, minor modifications for the present limited plasma. On the other  
394 hand, the illustrative scenario featuring detached-plasma-like neutral characteristics pre-  
395 dicta a substantial reduction of the maximum emission, thus cautioning future analyses in  
396 such regime of divertor operation.

397 Inclusion of more sophisticated emission models and possibility of fast scoping out the  
398 parameter space in each instance are made possible by reliance on the novel algorithm for  
399 non-uniform sampling. Preferentially placing samples in regions of interest implies a corres-  
400 ponding  $>10$ -fold reduction in computational cost. Alongside clear benefits computationally-  
401 time-wise, enhanced ease in the resolution of the maximum emission and the alleviated  
402 burden of proving independence of the results on the sampling settings falling on the user  
403 [9] are also implied.

404 In perspective, the flexibility and applicability of the present method will help on improving  
405 the analytical models and overall accuracy in view of future applications. Specifically, dis-  
406 criminating between atomic and molecular deuterium, informing the choice of geometrical  
407 free parameters via estimates of the neutral mean free path, and relaxing the assumption  
408 of zero-emission in between adjacent limiters would be worth including. Moreover, the  
409 generality of the novel CHERAB models and algorithms of Sec. 3 allows their application  
410 in circumstances where a numerically-computed radiation source is relied upon, e.g. as  
411 computed via core and edge plasma transport codes [9].

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416 made possible its implementation.

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