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Validation and Practical Application of a Data Reduction Software for the Analysis of Data from Stress Relief Tests / Saltarin, Simone. - In: GEOTECHNICAL AND GEOLOGICAL ENGINEERING. - ISSN 0960-3182. - 40:7(2022), pp. 3779-3797. [10.1007/s10706-022-02115-8]

*Availability:*

This version is available at: 11583/2977457 since: 2023-03-27T07:06:59Z

*Publisher:*

Springer

*Published*

DOI:10.1007/s10706-022-02115-8

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# Validation and Practical Application of a Data Reduction Software for the Analysis of Data from Stress Relief Tests

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Received: 18 June 2021 / Accepted: 5 March 2022 / Published online: 25 April 2022  
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**Abstract** In his work "Rock Anisotropy and the Theory of Stress Measurements", Amadei (1983) highlights the importance of the correct geological-technical characterisation of the rocks involved in open pit or underground excavation works. At the laboratory of Geomechanics and Geotechnology (IGG-CNR, Polytechnic of Turin), a data reduction software has been developed which, based on the theory developed by Amadei, allows the estimation of the main stresses acting in selected volumes of rock, on the basis of strains resulting from over-coring, possibly taking into account the transverse isotropy of the material. This paper, after listing the most popular techniques for assessing the state of stress in on-site rocks, focuses on the stress relief method. In particular, the validation tests of the software, called BM2000, are presented, comparing the calculated results with those published by Amadei and with those provided by two software packages from the literature (Smith82, STRESSsOUT). After having outlined the input data and the necessary adjustments for their application in the aforementioned software, the

results obtained showed excellent correspondence, for the cases of both isotropic and transversely isotropic rock.

**Keywords** Isotropy · Transverse isotropy · Rock · Stress relief method · Over-coring · Hollow inclusion cell

## 1 Introduction

Carrying out safe rock excavations (open-pit or underground), for civil or military purposes, must always deal with the nature of the materials crossed and with their ability to self-sustain. An essential condition for carrying out rock excavations is therefore the knowledge of the strength and deformability characteristics of the materials being excavated, as well as their on-site stress conditions.

The methods for assessing the state of stress in rocks on-site, based on the physical reference principle adopted, are generally classified into four groups: stress release, hydraulic fracturing, compression of rock samples, and propagation of fractures induced by drilling. Compared to the methods of hydraulic fracturing and fracture propagation that require decidedly more relevant equipment and financial resources for their execution, the first method is certainly the most widespread, both for its adaptability to different case studies and for the relative simplicity of the experimental determination. In fact, the technique is

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based on sensors of different shapes, which are able to evaluate, according to different directions, the endured strains from small rock surfaces, consequent to their separation from the boundary rock. Together with the development of procedures and equipment for the experimental determination of the natural and/or induced stresses acting on site, methods for interpreting experimental data have been developed by different authors, through the application of mathematical techniques. In this context, specific analytical and numerical solutions have been proposed for the data collected thanks to different geotechnical instruments, such as: the "*USBM borehole deformation gage*" (Leeman 1959; Obert et al. 1962; Panek 1966; Crouch & Fairhurst 1967; Merril 1967; Niwa et al. 1971; Suzuki 1966, 1971); the "*biaxial confinement device*" (Fitzpatrick 1962); the "*flat jack*" (Merril et al. 1964; Rocha et al. 1966); the "*CSIR-doorstopper*" and "*triaxial strain cell*" or "*hollow inclusion cell*" (Mohr 1956; Leeman & Hayes 1966; Leeman 1964, 1968, 1969, 1971; Hiramatsu & Oka 1968; Oka 1979; Rocha et al. 1974; Worotnicki & Walton 1976; "*borehole slotting*" (Becker & Werner 1994; Bock & Foruria 1983); the "*strains at the hemi-spherical borehole end*" (Sugawara et al. 1984; Sugawara & Obara 1986); and the "*strains at the conical end*" (Kobayashi et al. 1991). This innovative drive has been followed, in the last twenty years, exclusively by the evolution of existing measurement methods which are the result of technology transfer, such as: automatic data acquisition by means of miniaturised PCs directly connected to the measurement device and transmission of the data acquired and stored during the measurement via Wi-Fi (Gulli et al. 2006; Iabichino & Cravero 2010).

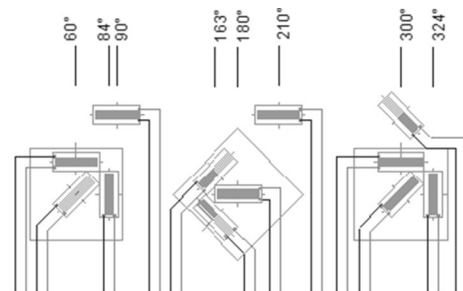
## 2 The Hollow Inclusion Stress-Relief Method

In the context of excavations, both for mining purposes and for the construction of infrastructures, the most widespread techniques for determining the state of natural and/or induced stress in the rocks on site, are those which, based on "stress relief", allow the determination of the complete stress tensor, with a single set of measurements taken from a single borehole. Among these techniques, due to the relative simplicity of execution and, above all, to the wide availability and consistency of studies on data

interpretation methods, the one most used is the technique proposed by the Commonwealth Scientific and Industrial Research Organisation (CSIRO), also called the Hollow Inclusion cell, or HI cell.

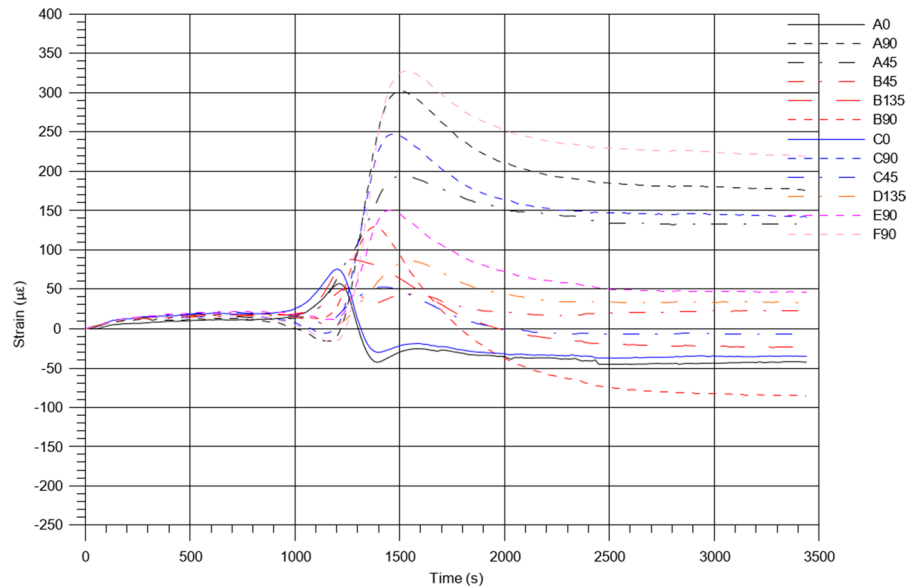
The measuring cell (HI cell) is a two-component epoxy resin cylinder with a length of approximately 100 mm, an internal diameter of approximately 32 mm and a thickness of approximately 3 mm, equipped with three (3) rectangular strain gauge rosettes, each consisting of three (3) or four (4) resistive electrical strain gauges, arranged at 120° along its inner median circumference (Fig. 1). For the execution of the experimental test, the external surface of the HI cell is made integral, by means of a gluing operation, to the internal wall of a pilot hole with a nominal diameter of 38 mm, drilled into the rock to reach the desired depth. Once the polymerisation of the glue is completed, the stresses are released by making a circular notch (over-coring) coaxially to the pilot hole equipped with the HI cell, using a thin-walled core barrel or a double-core barrel with a minimum internal diameter of about 120 mm.

The strain intensities, evaluated for each direction of measurement, are obtained by the difference between those acquired at the end of the over-coring operation and those acquired at the start (Fig. 2) and are related to the stress tensor through the elastic properties of the rock, which must be known. Given the wide choice of both the methods to be adopted for the determination of the pseudo-elastic constants of the rock being excavated, and the number of samples to be tested for each rock involved in the project, it follows that the choice of adopting greater/lesser



**Fig. 1** Scheme of a generic CSIRO HI cell. Both the position of the strain gauge rosettes along the median circumference of the measuring instrument inside the pilot hole and the distribution of the electrical strain gauges in each individual rosette are highlighted

**Fig. 2** Continuous strain vs time diagram obtained during the over-coring of a generic CSIRO HI cell. In the diagram, it is possible to identify the beginning of the over-coring, the moment of the passage of the diamond crown of the core barrel on the strain gauge rosettes, as well as the end part of the over-coring itself



accuracy in the characterisation of the rock mass is directly linked to the importance of the project. In any case, the best approximation that can be reached in determining the pseudo-elastic characteristics of the rock has a direct reflection in the best estimate of the stress state of the rock itself. In this regard, it should be noted that according to Hakala (2006), when performing experimental tests with the HI cell, dispersions between 1.0 MPa and 3.8 MPa are acceptable, due to the intensity of the main stresses; and between  $8^\circ$  and  $15^\circ$  for the direction of the stresses.

### 3 Interpretation of Stress Relief Data

As previously shown, different mathematical solutions are proposed in the literature, all quite complex, to determine the state of stress acting in rocks for which a continuous, homogeneous, linear elastic behaviour, both isotropic and anisotropic, has been hypothesised. The objectives of this study do not refer to the analytical developments to determine the stresses acting in a rock, as 9 or 12 strains are known that are independent of each other, so the fundamentals of the most widely used data reduction software for the purpose, those of the program to be validated, as well as the difference between the calculation schemes themselves will be summarised.

The most commonly used softwares for the interpretation of data coming from stress relief tests

(Smith 1982; Larson 1992; Worotnicki 1993) evaluate the acting stress tensor, using displacements and strains determined at the boundary of a circular hole of infinite length, made in a material considered to be continuous, linearly elastic and isotropic, with known geotechnical characteristics. A further hypothesis adopted in the numerical calculations is that of "plane deformation", which considers the strains detected, developed exclusively in the survey plane and in the plane orthogonal to the axis of the hole. The stresses acting around the hole are obtained using the classical Kirsch elastic solution (1898) or its generalised version for non-linearly dependent (non-parallel) holes provided by Hiramatsu and Oka (1962, 1968), Fairhurst (1968a, ba, b, 1968a, ba, b), Leeman (1968), and Fama and Pender (1980). The quoted solutions, albeit with the limitations implicit in their basic hypotheses (the component of the axial stress exclusively dependent on the Poisson's ratio, strength and deformability characteristics of the sensor adopted for the strain/displacement measurements, considered negligible), are widely used in construction site practice. This is largely attributable to the simplicity of the geomechanical characterisation required by the numerical calculations for the rock under study. In fact, the determination of the pseudo-elastic modules characterising the rock under excavation, although conducted on the basis of different assumptions made in the design field (Amadei 1983; Hirashima & Hamano 1987; Amadei & Stephansson 1997), is

generally carried out by hypothesising, for the on-site rock, an isotropic, linearly elastic behaviour, and adopting, for each of the three necessary characteristics  $E$  (Young's modulus),  $\nu$  (Poisson's ratio), and  $G$  (transverse stiffness), an average value derived from specifically normed tests. These modules/coefficients can be evaluated in greater detail both in the laboratory and on site using established procedures, such as those listed by Fitzpatrick (1962) or the ISRM (1979).

The software for the interpretation of data from stress release tests (Cravero et al. 2005, 2016) is called "Berni1". It was originally developed by Amadei (1983), then modified to be used on computers operating at 32/64 bit and updated with different numerical methods and mathematical routines (taken from the Fortran IMSL libraries), and equipped with specific routines for the evaluation of the main stress tensor. This software is aimed at determining the state of stress in the boundary of an elastic inclusion, and has an annular section, integral to the walls of a circular hole made in a continuous, elastic, linear and anisotropic medium. The analytical formulation of the state of stress at the boundary of holes made in an anisotropic medium was given by Lekhnitskii (1963), while Amadei (1983) contributed the generalisation applicable to the rock mass, both of the Lekhnitskii formulations and of the hypothesis of "plane deformation" adopted, to calculate the distribution of stresses around arbitrarily oriented holes in equally arbitrarily arranged anisotropic rock masses. In greater detail, the Berni1/BM2000 software addresses and provides analytical solutions to two problems: the calculation of the deformations induced around a hole however oriented in an anisotropic medium, since the stresses acting at infinity are known (direct problem); the calculation of the stresses acting around a hole in any orientation, made in an anisotropic medium, known as the induced strains (inverse problem). Although the analytical formulations proposed by Amadei (1997) for the interpretation of stress release data are widely recognised as "rigorous", as they take into account the influence both of the stiffness of the measuring instrument and of the adhesive used for making the sensor itself integral with the rock, their application to practical cases is rather rare. One of the reasons for this lack of application is certainly the difficulty in the determination of the geotechnical characteristics of the rock, when considered transversely isotropic (the case of metamorphic rocks). In fact, to calculate the

stresses acting around a hole made in a transversely isotropic medium using the strains resulting from the stress release, knowledge of five independent parameters is required, as already pointed out:  $E_1$ ,  $E_2$ ,  $\nu_1$ ,  $\nu_2$  and  $G_{1,2}$ , which cannot simply be assessed through laboratory or on-site tests.

A simple method for determining the pseudo-elastic modules is offered by Nunes (1997, 2002) who, on the basis of the transformations of the elastic and anisotropic constants developed by Lekhnitskii (1963) during his studies on the distribution of stresses in superimposed flat plates crossed by holes of different shapes, developed an original analytical method: through specific formulations, this allows the data acquired to be used by carrying out simple confinement tests on hollow cylindrical specimens equipped with a "Hollow Inclusion cell", obtaining the 5 pseudo-elastic deformability modules ( $E_{1,2}$ ,  $\nu_{1,2}$ ,  $G_{1,2}$ ), together with the orientation parameters of the rock's isotropy planes ( $\alpha$ ,  $\beta$ ). The proposed method is valid for transversely isotropic rocks with an anisotropy ratio between the elastic modules ( $E_1/E_2$ ) in the range of 1~2.

#### 4 Validation of the BM2000 Software

The validation of the BM2000 software was first required to build two sets of data relating to the geomechanical characterisation of the rocks to be examined. The results of the stress state analysis in the case of rock with linear elastic behaviour, both isotropic and anisotropic (transversely isotropic), were evaluated separately.

In the first case, with the same data of "relieving" from over-coring to be evaluated, the results of the analyses carried out through BM2000 were compared with those from Berni1, and from the most common software available in the literature and widely used in the sector (Smith82, STRESSOUT).

In the second case, again with the same data to analyse, the results obtained and shown by Amadei (1983) through Berni1 were compared with the similar results from BM2000.

In order to make the data required by the various softwares homogeneous, a first analysis was carried out regarding the sign conventions and the reference systems adopted by them, as well as the geometry of the measuring device (Hollow Inclusion cell).

In this regard, it has to be pointed out that the Smith82 and STRESsOUT software represent the strain and the compressive stress with the positive sign, whereas Berni1 and BM2000 adopt the negative sign for the same parameters. In any case, it has to be kept in mind that the over-coring of a portion of rock on-site has the local consequence of tension "relief", so that the stresses resulting from the analyses have a sign opposite to the strains used at the beginning of the calculation (positive input strains correspond to negative output stresses).

The reference systems adopted by the different software used for validation are generally three (3): two sets of axes orthogonal to each other and three Cartesian two-dimensional coordinates lying in a known position on planes tangential to the measurement hole. The first orthogonal triad generally coincides with the NS and EW axes of the geographical reference, completed by the Zenith axis. The second one has an axis coinciding with the axis of the hole used for the execution of the experimental test and the other two axes lying in the plane normal to the axis of the hole. The third reference system identifies the position of the measuring elements (strain gauge) by means of two angles: the first (circumferential angle) identifies the position of the centre of the single strain gauge along the circumference of the section of the hole; the second (rotational angle) identifies the inclination of the axis of the single strain gauge with respect to the horizontal axis of the two-dimensional reference system associated with the plane tangential to the measurement hole at the point of application of the strain gauge rosette. For example, this latter angle is evaluated in an anticlockwise direction according to Berni1 and BM2000, thus assuming a value of  $90^\circ$  if the axis of the strain gauge considered has the same direction as the axis of the measuring hole, and a value of  $0^\circ$  if the axis of the strain gauge is arranged according to the measurement section. For the different software used, Table 1 presents the individual reference systems adopted to identify the arrangement of the strain gauges in the hole in order to highlight the differences. Table 2 shows the orientations of the electrical strain gauges along the circumference of the measuring instrument, depending on the specific software analysed. Again, with the aim of highlighting the differences among the software used, Table 3 shows the arrangement of the orthogonal reference triad associated with the measurement hole.

Table 4 presents the "transformations" necessary to make the inputs of a generic data reduction software "compatible" with those of another data reduction software selected for the validation of BM2000.

Further preliminary adaptations that were necessary to compare the results of the software chosen to validate BM2000 on the basis of "homogeneous" data were: the adaptation of the data relating to the geometry of the instrument used and the choice of the four (4) " $K_i$ " stress concentration factors related to the calculation conditions (plane deformation), as well as the dimensions and deformability values of the HI cell used. In detail, respectively  $E=2000$  MPa and  $\nu=0.3$  (-) for the Young's modulus and the Poisson's ratio of inclusion were adopted, as well as the symbols (" $a$ ") for the internal radius of the measuring cell; (" $b$ ") for the radius of the pilot hole used to install the cell and (" $r$ ") for the radius of the circumference where the electrical strain gauges are positioned. The data on the radii necessary for the calculation are expressed as a ratio, where  $a/b=2$  (-),  $r/a=0.75$  (-). The  $K_i$  coefficients used in the Smith82 and STRESsOUT software are calculated based on the solution developed by Savin (1961) in the case of a ring integral with a hole made in an isotropic and linear elastic material (Fama & Pender 1980; Jalkanen 1982).

#### 4.1 Isotropic case

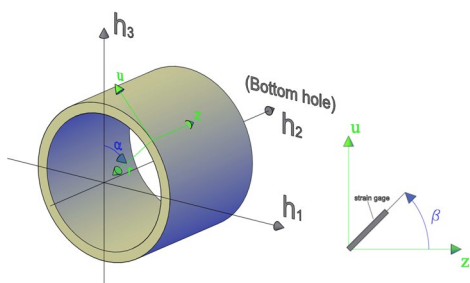
In a rock with isotropic, homogeneous, elastic, linear behaviour, the first assessment for the validation of BM2000 was carried out using the input data proposed by Amadei (1983) (p. 234). Tables 5, 6 and 9 show the strain values, the orientation of the hole and the characteristics of the rock under study.

The results obtained from the stress state analyses performed in the case of homogeneous rock with linear elastic behaviour using the software Berni1, BM2000, Smith82, STRESsOUT are shown in Tables 8 and 11. The tables also show the results of the comparison carried out both in terms of the components of the acting stresses referred to the global reference system XYZ adopted by Amadei (1983), and in terms of the maximum principal stresses. These latter stresses have also been graphically reported in equi-angle stereographic projections. In this regard, it seems appropriate to highlight that the directions and inclinations of the results obtained by Amadei were deduced from the stereographic projections drawn up

**Table 1** "Circumferential" and "rotational" reference angles associated with the position of the sensitive elements (electrical strain gauges) distributed along the circumference of the

hole section chosen for the stress state measurement performed with the over-coring stress relief method

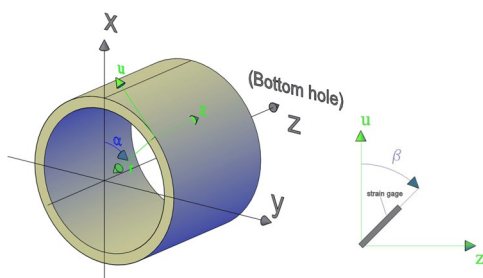
### Software convention



#### Smith82

$h_2$  axis positive—direction entering into the hole

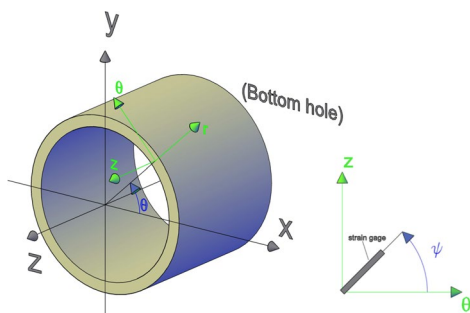
- $\alpha$ —circumferential angle: from the top to the bottom of the hole, positive clockwise from the axis lying on the vertical plane normal to the axis of the hole and perpendicular to it (from  $h_3$  to  $h_1$  axis)
- $\beta$ —rotational angle: positive counterclockwise from  $z$  axis to  $u$  axis until the axis passing through the centreline of the strain gauge of interest ( $z$  axis in the same direction as  $h_2$  axis)



#### STRESsOUT

$z$  axis positive—direction entering into the hole

- $\alpha$ —circumferential angle: from the top to the bottom of the hole, positive clockwise from the axis lying on the vertical plane normal to the axis of the hole and perpendicular to it (from  $x$  to  $y$  axis)
- $\beta$ —rotational angle: positive clockwise from  $z$  axis to  $u$  until the axis passing through the centreline of the strain gauge of interest



#### Berni1—BM2000

$z$  axis positive—direction coming out of the hole

- $\theta$ —circumferential angle: positive counterclockwise from  $x$  (from the horizontal to  $y$  axis);
- $\Psi$ —rotational angle: positive counterclockwise starting from the  $u$  axis lying on a plane locally tangential to the strain gauge considered and orthogonal to the axis of the hole

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Table 8 shows, for the different components of the stresses, the substantial equivalence of the results obtained from the analyses conducted with the different selected software. Assuming Berni1 as the reference software, as it is the application of an original theory capable of taking into account the peculiar characteristics of the instrument adopted for the experimental determinations, it can be noticed that, in absolute value, the results identified by the other softwares deviate from it by a maximum of a few hundredths of MPa (0.034 MPa). As regards the

differences in sign that can be detected in the individual results of the analyses, reference is made to Tables 3, 9, and 10. The first, as previously pointed out, shows the differences between the reference systems associated with the measurement hole adopted by each single software used; the second shows how to "transform" the output data obtained by each software, in order to obtain uniformity of orientation with respect to a measurement hole having a positive direction angle clockwise from North (towards East) and a positive angle of inclination towards up; the third shows the different sign conventions to be considered in the analysis of the results, in terms of stress

**Table 2** Detail of the angular position of the electrical strain gauges along the circumference of the instrument, according to the specific software analysed, as reported in the diagrams in Table 1 (the arrangement proposed by Amadei, 1983 is also used). The first column shows the position of the centre of the single strain gauge along the circumference of the section of the hole involved in the measurement ( $\theta$  or  $\alpha$ , depending on the software); the other columns show the inclination of the axis of the single strain gauge with respect to the horizontal axis of the two-dimensional reference system associated with the plane tangential to the measurement hole at the point of application of the strain gauge rosette ( $\psi$  or  $\beta$ , depending on the software)

$\theta, \alpha$ (°)	$\epsilon_A$ $\psi, \beta$ (°)	$\epsilon_B$ $\psi, \beta$ (°)	$\epsilon_C$ $\psi, \beta$ (°)
<i>Berni1—BM2000</i>			
0	0	90	225
90	0	90	225
225	0	90	225
<i>Smith82</i>			
90	90	0	45
0	90	0	45
225	90	0	45
<i>STRESSsOUT</i>			
90	0	90	45
0	0	90	45
225	0	90	45

components, again in order to make the output data homogeneous and comparable. To pursue the aforementioned purpose, it is also useful to highlight the appropriate "precautions" to be adopted for the graphing of the results by means of polar stereographic projections (lower hemisphere). The use of this hemisphere requires careful evaluation of the value of the inclination returned in the output compared, with particular attention to its sign. For example, if the sign of the inclination is positive, in order to obtain a comparable polar stereographic projection (lower hemisphere) it is necessary to either change the sign by making it negative, or to rotate its direction by 180°. Table 7 shows, for each software, any changes to be applied to the direction and inclination data of the results obtained by the individual software used for verifying the BM2000 software.

Similarly to what was observed for Tables 11 and 12 and the related stereographic projection in the lower hemisphere (Fig. 3) show that the intensities of the maximum main stresses calculated with the software used for validation of BM2000

deviate from the results obtained through Berni1, once again, by a maximum of a few hundredths of MPa (0.043 MPa). Even with regard to the directions of the main stresses, it can be highlighted that they differ from those chosen as a reference by very small values, equal to a few sexagesimal degrees (max  $\beta = 1.68^\circ$  and max  $\delta = 1.2^\circ$ ).

To consolidate the results obtained from the stress state analyses in an isotropic, homogeneous medium with linear elastic behaviour, a further analysis (second verification) was carried out, using the same data relating to the strains recorded during the over-coring operations, and modifying the data on the geometry and deformability characteristics of the instrument. In this comparison, the values of 1.19 (–) and 0.919 (–) have been adopted for the a/b and r/a ratios respectively, while for the Young’s modulus and Poisson’s ratio the following values have been adopted:  $E = 2.647,68$  (MPa) and  $\nu = 0.40$  (–). The results obtained from the use of the selected software are shown in Tables 13 and 14, which give the comparison between the six stress components referred to the global reference system XYZ and between the 3 main maximum stresses acting.

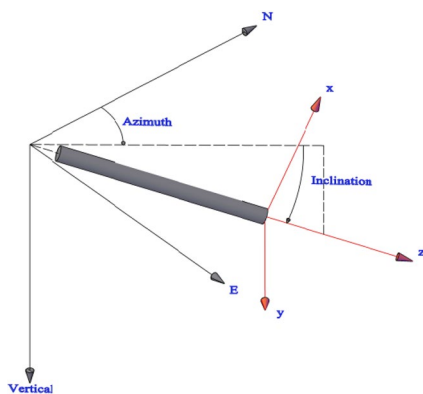
Once again, Table 14 shows, for the different components of the stresses, the substantial equivalence of the results obtained from the analyses carried out with the different software. In fact, assuming Berni1 as the reference software, it can be noticed that, in absolute value, the results identified by the other software deviate from the first by a maximum of 0.143 MPa. By analogy to what was found in Tables 13 and 14 and the stereographic projection (Fig. 4) show that the maximum principal stresses calculated with the software used for the validation of BM2000 differ from the results obtained with Berni1 by a maximum of 0.071 MPa. Even as far as the directions of the main stresses are concerned, it is useful to highlight that they differ from those chosen as a reference by a few sexagesimal degrees ( $\delta = 2.96^\circ$  and  $\beta = 4.16^\circ$ ).

#### 4.2 Transverse isotropic case

The scheme adopted for the validation of the BM2000 software, based on the comparison of the results obtained in determining the state of natural and/or induced stress performed (with comparable input data) with the Smith82 and STRESSsOUT software, cannot be extended to the case of rock

**Table 3** Orthogonal reference triad associated with the measurement hole (in red) adopted by the individual software chosen for the validation of BM2000. In detail, the direction and

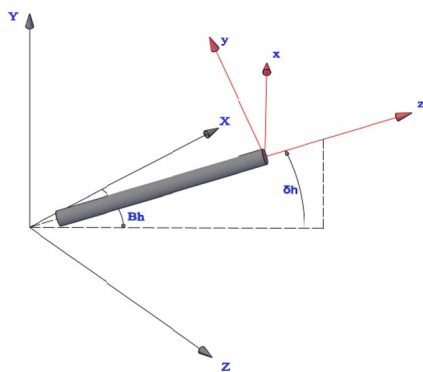
inclination of the angles that relate the local triads to the global reference system of the software used for the validation are highlighted



#### Smith82

Orthogonal triad  $x$ - $y$ - $z$  positioned at the top of the hole, where  $y$  axis is oriented towards North and  $x$  axis towards East

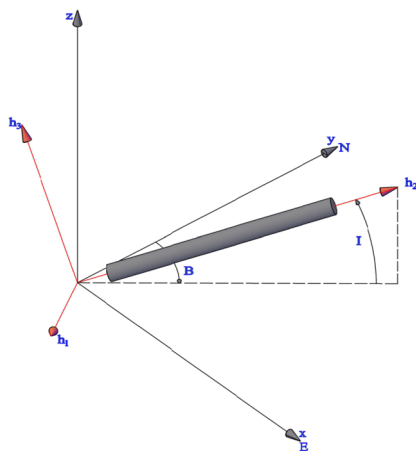
- Bearing (angle of direction  $B$ ): positive starting from the positive  $y$  axis (from North to East)
- Inclination (angle of inclination  $I$ ): positive counterclockwise (upward) from  $xy$  (horizontal plane)



#### STRESsOUT

N-E-V triad located at the top of the hole, with the vertical axis downwards

- Azimuth (angle of direction): positive clockwise from North to East
- Inclination (angle of inclination): positive clockwise (downwards) from the horizontal plane



#### Berni1—BM2000

X-Z-Y triad located at the bottom of the hole arbitrarily. For convenience, the X axis is placed towards the East and the Z axis towards the South)

- $\beta$  (angle of direction): positive clockwise from X (from East to South)
- $\delta$  (angle of inclination): positive counterclockwise (upward) from  $xy$  (horizontal plane)

with anisotropic behaviour (transversely isotropic) as the theories underlying the software are valid only in the case of isotropic rock with linear elastic behaviour. In the particular case of anisotropic

rock (transversely isotropic), the validation of the BM2000 software can only take place through the comparison, with the same input data, of the results of the software under examination with those

**Table 4** Adjustments to adapt inclination and bearing data used with BM2000 to the Smith82 and STRESsOUT software. Notice that the transformation for the bearing angle is the same for both softwares, while the sign must be changed for the angle of inclination, as shown in the diagrams in Table 3

Software	Inclination (positive)	Bearing
Smith82	Up	− 90°
STRESsOUT	Down	− 90°

**Table 5** Strains and relative rotational angles adopted in the calculation of the state of natural stress by Amadei (1983). Note both the value of the angle  $\theta$  (evaluated counterclockwise from the reference axis normal to the axis of the hole), and the sign of the strain, which indicates a "stress relief" in almost all components

Strain measurements			
$\theta$ (°)	$\epsilon_A$ ( $\mu\epsilon$ )	$\epsilon_B$ ( $\mu\epsilon$ )	$\epsilon_C$ ( $\mu\epsilon$ )
0	− 79	− 86	44
90	− 430	− 85	− 187
225	− 52	− 87	− 210

**Table 6** Angle of direction and inclination of any transverse isotropy planes and of the measurement hole adopted in the calculation of the state of natural stress by Amadei (1983)

Angle			
Strike $\beta$ (°)	Dip $\psi$ (°)	$\beta_h$ (°)	$\delta_h$ (°)
0	30	90	0

obtained by Amadei. In fact, only the theory proposed by Amadei and implemented in the Bernil software is based on the generalisation of the formulas of the plane deformation case, placing no limits on the symmetry planes of the rock, or on the orientation of the main stresses. This formulation requires, for its application to the case of transverse

**Table 7** Characteristic properties of the isotropic rock adopted in the calculation of the state of natural stress by Amadei (1983).  $E_{1,2,3}$ =modulus of elasticity evaluated in the direction of the orthogonal reference axes associated with the isotropic

Rock properties						
$E_1$ (MPa)	$E_2$ (MPa)	$E_3$ (MPa)	$G_{12}, G_{13}$ (MPa)	$G_{23}$ (MPa)	$\nu_{21}, \nu_{31}$ (-)	$\nu_{23}$ (-)
40,000	40,000	40,000	16,000	16,000	0.25	0.25

isotropy, in addition to the strain measurements and their arrangement in the measurement hole, the knowledge of six (6) deformability characteristics of the rock ( $E_1, E_2, \nu_1, \nu_2, G_1, G_2$ ). In detail, the input data necessary for this last assessment are collected in Table 15 and refer to 4 cases where the deformability characteristics of the rock ( $E, \nu$ ) vary, while the strains ( $\epsilon_{A, B, C}$ ) evaluated around the measurement hole and the rotational angles ( $\theta$ ) of the problem are the same as those used for the validation of the results in the case of a rock with isotropic behaviour, shown in Table 5.

The results obtained from the stress state analyses performed in the 4 cases examined led to the results shown in Table 16 and Fig. 5. The table highlights the results of the comparison in terms of the maximum principal stresses. The maximum principal stresses were also plotted in equi-angle stereographic projections. In this regard, it has to be pointed out that the results of Amadei’s analyses were obtained from the graphs published in his PhD thesis, appropriately corrected with photographic items, to remove the distortions produced by the reproduction process.

Table 16 shows that the intensities of the maximum main stresses calculated with BM2000 differ from those obtained with the Bernil software by a few hundredths of MPa (maximum value of 0.027 MPa). Even with regard to the directions of the main stresses, it has to be highlighted that they differ from those chosen as a reference by a few sexagesimal degrees (maximum value of  $\delta = 2.36^\circ$  and  $\beta = 3.51^\circ$ ). Considering that the differences in the results obtained by the two types of software are repeated in all the analyses performed, it can be deduced that these can be attributed to the possible graphic reproduction errors indicated above.

planes; Poisson’s ratio  $\nu_{21}, \nu_{31}, \nu_{23}$ , evaluated in the direction of Young’s modulus; tangential (shear) modulus of elasticity  $G_{1,2}; G_{1,3}; G_{2,3}$ , evaluated in the isotropy plans of the material under study

**Table 8** Transformations to be made to the results obtained, in terms of direction of the reference system associated with the measurement hole, by the software used for the validation of BM2000 for their standardisation. It should be noted that the direction of the reference hole obtained by using the Berni1 and BM2000 software must be increased by  $90^\circ$  and that the only modification to be made on the angle of inclination concerns STRESsOUT, where the sign of the output value must be changed

	Berni1	BM2000	Smith82	STRESsOUT
Bearing	$+90^\circ$	$+90^\circ$	N/A	N/A
Inclination	N/A	N/A	N/A	$-I$

**Table 9** Transformations to be made on the stress values obtained by the software used for the validation of BM2000, in terms of sign, in order to standardise them for a correct comparison

Berni1	BM2000	Smith82	STRESsOUT
$\sigma_x$	$\sigma_x$	$\sigma_{EW}$	$\sigma_{EW}$
$\sigma_y$	$\sigma_y$	$\sigma_{Ver}$	$\sigma_{Ver}$
$\sigma_z$	$\sigma_z$	$\sigma_{NS}$	$\sigma_{NS}$
$\tau_{yz}$	$\tau_{yz}$	$-\tau_{NS/V}$	$\tau_{V/NS}$
$\tau_{xz}$	$\tau_{xz}$	$-\tau_{Hor}$	$-\tau_{Hor}$
$\tau_{xy}$	$\tau_{xy}$	$\tau_{EW/V}$	$-\tau_{EW/V}$

**Table 10** Transformations to be made on the values of the direction angles of the maximum main stresses obtained from the calculations carried out through the software used for the validation of the BM2000 to standardise them for a correct comparison and stereographic representation

Software	Inclination sign	Bearing
Berni1 / BM2000	+	=
	-	$+180^\circ$
Smith82	+	$+180^\circ$
	-	=
STRESsOUT	+	=
	-	$+180^\circ$

**Table 11** Stress components obtained by the different software used for the validation of BM2000. Data are referred to the isotropic case proposed by Amadei (1983)

	Berni1 (MPa)	BM2000 (MPa)	Smith82 (MPa)	STRESsOUT (MPa)
$\sigma_x$	7.000	7.030	7.033	7.034
$\sigma_y$	2.500	2.503	2.506	2.507
$\sigma_z$	5.800	5.823	5.832	5.832
$\tau_{yz}$	$-1.800$	$-1.808$	1.809	$-1.809$
$\tau_{xz}$	1.000	1.010	$-1.018$	$-1.018$
$\tau_{xy}$	2.600	2.572	2.611	$-2.611$

#### 4.3 Further validation of the transverse isotropy case

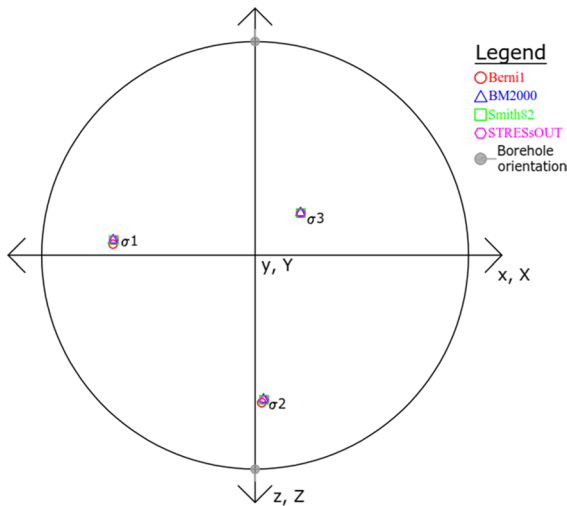
To further validate the BM2000 software, a comparison was also made between the results obtained by Amadei in the other examples proposed by him where, using the characteristics of the rock assumed for Case 1 and shown in Table 15, the characteristic angles of the transverse isotropy plane shown in Fig. 6 vary: the inclination angle  $\Psi$  was considered equal to  $30^\circ$  for the first case and  $90^\circ$  for the second, while the transverse isotropy plane angle  $\beta$  varies with intervals of  $10^\circ$ , from  $0^\circ$  up to  $90^\circ$  for both cases analysed.

The results obtained by Amadei through the BM2000 software are presented in Table 17 and in Fig. 7 in the case where  $\Psi$  is  $30^\circ$ , and in Table 18 and in Fig. 8 when  $\Psi$  is  $90^\circ$ .

To better compare the results obtained in the two cases developed, Figs. 9 and 10 show the trends of the three main stresses for each of the two transverse isotropy configurations, analysed as a function of the variation of  $\beta$ . The results are superimposed on those plotted by Amadei. As in the previous paragraphs, it can be noticed that the slightest differences seem

**Table 12** Intensity, inclinations  $\delta$  and directions  $\beta$  of the main maximum stresses obtained by the different software used for the validation of BM2000. Data are referred to the isotropic case proposed by Amadei (1983)

	Berni1			BM2000			Smith82			STRESSsOUT		
	(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)
$\sigma_1$	8.200	- 29.97	274.42	8.210	- 29.88	276.06	8.240	- 30.15	276.05	8.243	- 30.20	276.10
$\sigma_2$	6.580	- 27.97	177.38	6.610	- 29.17	176.61	6.620	- 29.02	176.41	6.620	- 29.07	176.40
$\sigma_3$	0.520	- 63.76	47.51	0.535	- 63.96	47.16	0.510	- 63.85	47.42	0.510	- 63.85	47.40



**Fig. 3** Stereogram of the maximum principal stresses calculated with the different software used for the validation of BM2000 in the isotropic case. The symbols  $\Delta$ ,  $\square$ ,  $\circ$ ,  $\diamond$ , respectively, identify the maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  (MPa), for the software Berni1, BM2000, Smith82 and STRESSsOUT

**Table 13** Intensity of the stress components obtained by the different software. The data refer to the isotropic case proposed by Amadei (1983), where the geometry data and deformability characteristics of the instrument adopted have been modified

	BM2000	Smith82	STRESSsOUT
	(MPa)	(MPa)	(MPa)
$\sigma_x$	6.914	6.914	6.914
$\sigma_y$	3.523	3.524	3.524
$\sigma_z$	6.049	6.057	6.057
$\tau_{yz}$	- 1.906	1.907	- 1.907
$\tau_{xz}$	1.065	- 1.073	- 1.073
$\tau_{xy}$	2.098	1.955	- 1.955

essentially due to the errors in acquiring the comparison data from its graphic representation.

### 5 Conclusions

The results obtained show that the BM2000 software developed at the IGG-CNR Geomechanics Laboratory, applied to rock with linear elastic behaviour, both isotropic and transversely isotropic, is fully satisfactory, provided that the strain characteristics of the medium and the geometry of the measuring instrument adopted are known.

In fact, observing the results in the isotropic case in the comparison software (Berni1, Smith82, STRESSsOUT), after having correctly homogenised the input data, the differences are in the order of a few hundredths of MPa for the stresses and a few degrees for the angles of the stresses with respect to the reference axes.

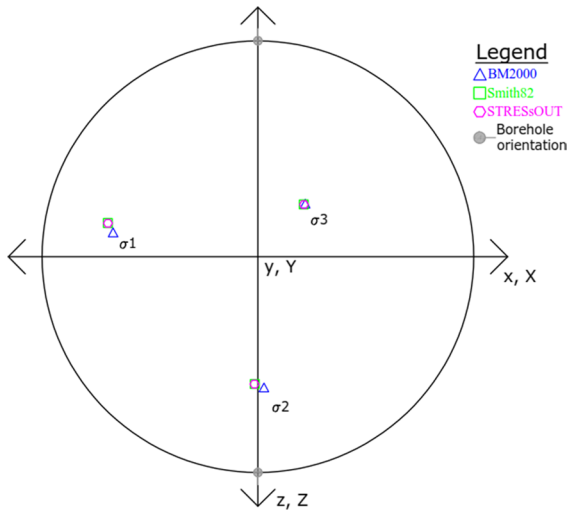
Most likely, the differences detected are due to the greater calculation precision obtained thanks to the use of different IMSL subroutines within the numerical methods and to the inaccuracies due to the positioning of the individual sensors (strain gauges) along the circumference of the measurement section of the pilot hole. The software, which has been successfully validated, allows, once the deformability characteristics of the rock are known, the complete state of stress to be determined even in the case of rocks with transversely isotropic behaviour.

In this respect, the state of natural and/or induced stress in open pit or underground activity of dimension stones with more or less evident planes of weakness, where the ratio between the elastic modules in orthogonal directions is  $> 1$ , can be examined more

**Table 14** Stress intensity, inclinations  $\delta$  and directions  $\beta$  obtained by the different software used for the validation of BM2000. The data refer to the isotropic case proposed by

Amadei (1983), where the geometry data and deformability characteristics of the instrument adopted have been modified

	BM2000			Smith82			STRESsOUT		
	(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)
$\sigma_1$	7.910	- 28.85	279.07	7.850	- 25.89	282.54	7.846	- 25.92	282.50
$\sigma_2$	7.070	- 34.93	177.44	7.080	- 36.84	181.56	7.079	- 36.88	181.60
$\sigma_3$	1.500	- 60.93	42.40	1.570	- 61.05	41.36	1.571	- 61.02	41.40



**Fig. 4** Stereogram of the maximum principal stresses calculated with the different software used for the validation of BM2000 in the modified isotropic case. The symbols  $\Delta$ ,  $\square$ ,  $\circ$ , respectively, identify the maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  (MPa), for the software BM2000, Smith82 and STRESsOUT

closely. This, in agreement with Hakala (2006), implies that the strain anisotropy ( $E_1/E_2$ ) between the intensities of 1.14 MPa and 1.33 MPa has a noticeable systematic effect on the interpretation of the stress state on-site and, for this purpose, the elastic parameters of the rock at the measuring point should be defined as accurately as possible.

In this regard, the required experimental determination can be carried out directly on-site with a circumferential press capable of containing the over-cored rock sample with the hollow inclusion cell still inside it, using the method proposed by Nunes (1997, 2002). Another experimental method, still being tested, for defining the deformability characteristics of the on-site rock with transversely isotropic linear elastic behaviour is based on the use of a circumferential press capable of containing the over-cored rock sample with the hollow inclusion cell still inside it, and a numerical process of minimisation implemented by means of the BM2000 software.

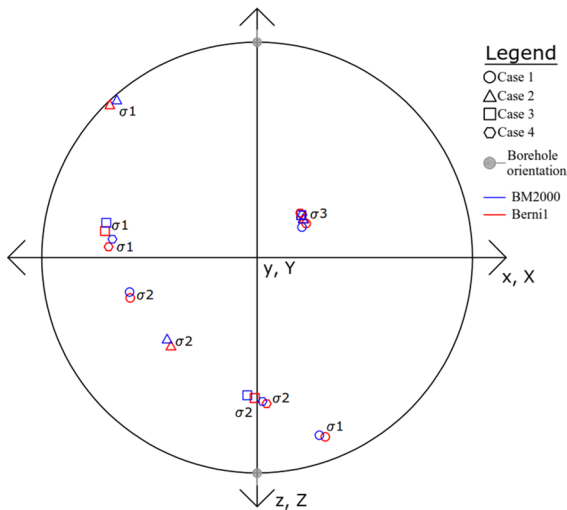
**Table 15** Values of the characteristic properties of the transverse isotropic rock adopted in the calculation of the state of natural stress by Amadei (1983) (p. 241), for the 4 different cases examined.  $E_{1, 2, 3}$ =modulus of elasticity evaluated in the direction of the orthogonal reference axes associated with the

isotropic planes; Poisson’s ratio  $\nu_{21}$ ,  $\nu_{31}$ ,  $\nu_{23}$ , evaluated in the direction of Young’s modulus; tangential (shear) modulus of elasticity  $G_{1,2}$ ;  $G_{1,3}$ ;  $G_{2,3}$ , evaluated in the isotropy plans of the material under study

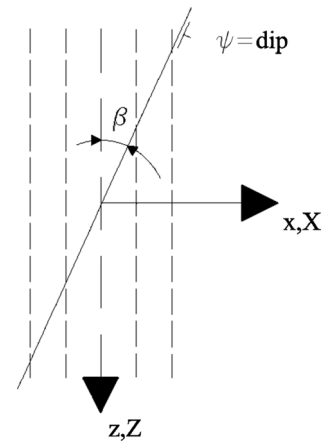
Rock properties							
	$E_1$ (MPa)	$E_2$ (MPa)	$E_3$ (MPa)	$G_{12}, G_{13}$ (MPa)	$G_{23}$ (MPa)	$\nu_{21}, \nu_{31}$ (-)	$\nu_{23}$ (-)
<b>Case 1</b>	20,000	40,000	40,000	4,000	16,000	0.40	0.25
<b>Case 2</b>	30,000	40,000	40,000	8,000	16,000	0.27	0.25
<b>Case 3</b>	35,000	40,000	40,000	14,000	16,000	0.23	0.25
<b>Case 4</b>	39,000	40,000	40,000	15,500	16,000	0.24	0.25

**Table 16** Intensity, inclinations  $\delta$  and directions  $\beta$  of the main maximum stresses obtained for the validation of BM2000 software. The data refer to the transverse isotropic cases proposed by Amadei (1983)

		BerniI			BM2000		
		(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)
<b>Case 1</b>	$\sigma_1$	5.46	- 9.88	69.16	5.47	- 11.54	70.66
	$\sigma_2$	3.62	- 34.38	162.34	3.62	- 34.80	164.71
	$\sigma_3$	0.67	- 65.03	325.2	0.683	- 67.39	325.86
<b>Case 2</b>	$\sigma_1$	6.02	- 1.69	225.86	6.03	- 2.23	228.16
	$\sigma_2$	5.29	- 38.11	133.93	5.28	- 38.98	137.44
	$\sigma_3$	0.59	- 65.01	321.30	0.617	- 65.13	320.61
<b>Case 3</b>	$\sigma_1$	7.31	- 25.55	189.83	7.29	- 25.38	192.92
	$\sigma_2$	6.16	- 31.24	90.98	6.15	- 32.29	94.07
	$\sigma_3$	0.42	- 64.45	316.39	0.439	- 64.48	316.08
<b>Case 4</b>	$\sigma_1$	7.99	- 27.84	184.15	7.97	- 29.10	187.25
	$\sigma_2$	6.47	- 28.87	86.25	6.47	- 29.89	87.84
	$\sigma_3$	0.48	- 64.10	314.02	0.504	- 64.01	316.80



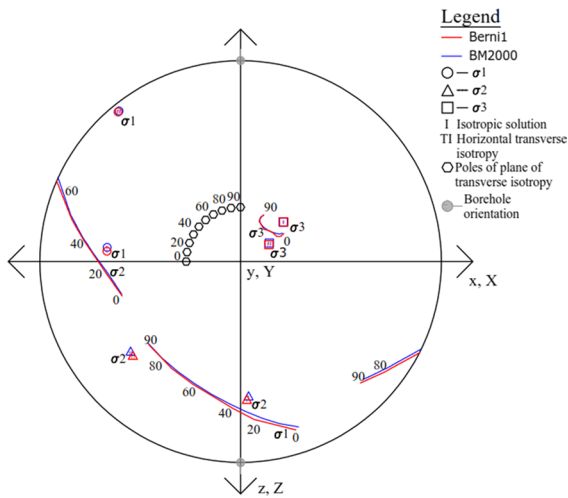
**Fig. 5** Stereogram of the maximum principal stresses calculated with BerniI and BM2000 software to validate the transverse isotropic case. The symbols  $\circ$ ,  $\Delta$ ,  $\square$ ,  $\hexagon$ , respectively, identify the maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ , in MPa, for the 4 cases analysed



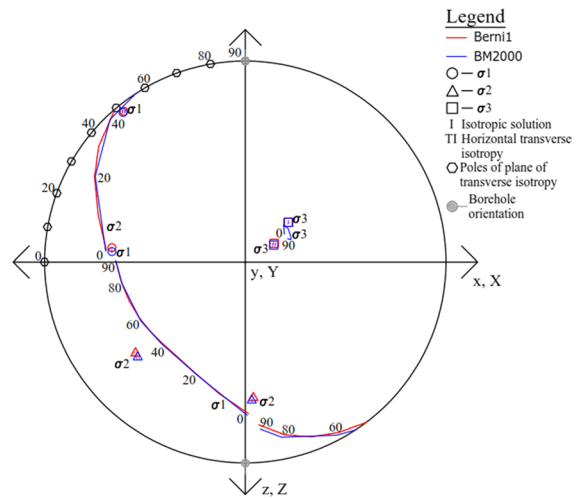
**Fig. 6** Representation of the characteristic angles of the transverse isotropy plane.  $\beta$  shows the strike (direction), positive clockwise from  $-Z$ , while  $\psi$  is the inclination angle (dip)

**Table 17** Results and differences, in terms of intensity, inclinations  $\delta$  and directions  $\beta$ , of the maximum principal stresses obtained for the validation of the BM2000 software, when  $\beta=0^\circ\sim 90^\circ$  and  $\psi=30^\circ$ . The data refer to the transversely isotropic case proposed by Amadei (1983)

	Bernil			BM2000			Difference		
	(MPa)	$\delta$ ( $^\circ$ )	$\beta$ ( $^\circ$ )	(MPa)	$\delta$ ( $^\circ$ )	$\beta$ ( $^\circ$ )	$\Delta$ (MPa)	$\Delta\delta$ ( $^\circ$ )	$\Delta\beta$ ( $^\circ$ )
BETA $0^\circ$									
$\sigma_1$	5.49	- 10.63	71.80	5.47	- 11.54	70.66	0.02	0.92	1.14
$\sigma_2$	3.59	- 34.76	163.83	3.62	- 34.80	164.71	- 0.03	0.04	- 0.88
$\sigma_3$	0.67	- 66.93	327.08	0.68	- 67.39	325.86	- 0.01	0.46	1.22
BETA $10^\circ$									
$\sigma_1$	5.59	x	x	5.58	- 15.33	76.48	0.01	x	x
$\sigma_2$	3.69	x	x	3.70	- 32.14	171.53	- 0.01	x	x
$\sigma_3$	0.71	x	x	0.74	- 68.41	324.98	- 0.04	x	x
BETA $20^\circ$									
$\sigma_1$	5.70	- 18.99	85.04	5.68	- 19.08	82.04	0.02	0.09	3.00
$\sigma_2$	3.77	- 28.43	176.93	3.79	- 28.63	177.58	- 0.02	0.20	- 0.65
$\sigma_3$	0.79	x	x	0.81	- 69.51	322.17	- 0.02	x	x
BETA $30^\circ$									
$\sigma_1$	5.77	x	x	5.76	- 22.76	88.00	0.01	x	x
$\sigma_2$	3.90	x	x	3.93	- 23.89	183.27	- 0.03	x	x
$\sigma_3$	0.88	x	x	0.88	- 70.63	317.30	0.00	x	x
BETA $40^\circ$									
$\sigma_1$	5.88	- 25.72	95.77	5.86	- 26.21	94.76	0.02	0.49	1.01
$\sigma_2$	4.10	- 19.57	187.74	4.11	- 17.94	189.23	- 0.02	- 1.63	- 1.49
$\sigma_3$	0.93	x	x	0.94	- 71.41	310.51	0	x	x
BETA $50^\circ$									
$\sigma_1$	6.02	x	x	5.97	- 29.21	102.40	0.05	x	x
$\sigma_2$	4.33	x	x	4.35	- 11.11	195.40	- 0.02	x	x
$\sigma_3$	0.95	x	x	0.95	- 71.47	303.31	0.00	x	x
BETA $60^\circ$									
$\sigma_1$	6.13	- 31.06	111.16	6.09	- 31.61	110.81	0.04	0.55	0.35
$\sigma_2$	4.60	- 3.91	201.08	4.62	- 3.90	201.90	- 0.02	- 0.01	- 0.82
$\sigma_3$	0.94	x	x	0.92	- 70.74	297.61	0.02	x	x
BETA $70^\circ$									
$\sigma_1$	6.25	x	x	6.22	- 33.32	119.80	0.03	x	x
$\sigma_2$	4.89	x	x	4.91	- 3.10	28.79	- 0.02	x	x
$\sigma_3$	0.86	x	x	0.84	- 69.51	294.64	0.03	x	x
BETA $80^\circ$									
$\sigma_1$	6.37	- 33.78	126.35	6.33	- 34.35	129.28	0.04	0.57	- 2.93
$\sigma_2$	5.21	- 8.61	37.43	5.19	- 9.01	36.21	0.02	0.40	1.22
$\sigma_3$	0.75	x	x	0.73	- 68.17	294.36	0.02	x	x
BETA $90^\circ$									
$\sigma_1$	6.47	- 34.55	137.73	6.42	- 34.72	138.87	0.05	0.17	- 1.14
$\sigma_2$	5.48	- 13.51	45.34	5.46	- 13.80	43.87	0.01	0.29	1.47
$\sigma_3$	0.61	- 66.58	296.55	0.61	- 66.96	296.18	0.00	0.38	0.37



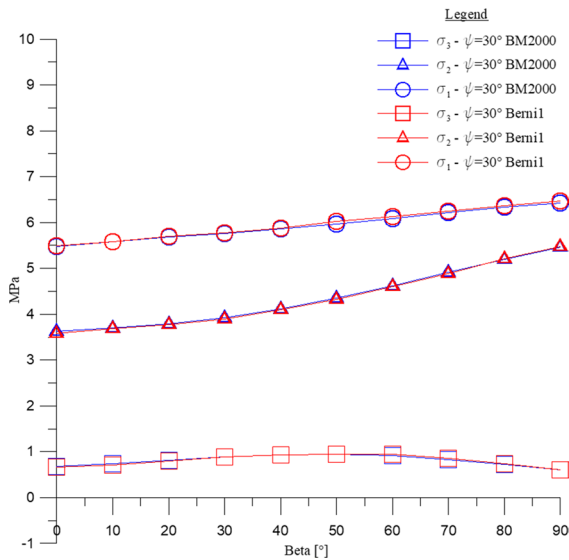
**Fig. 7** Stereogram of the trend of the maximum main stresses, calculated with Berni1 and BM2000 for the validation in the transversely isotropic case, when  $\beta=0^\circ \sim 90^\circ$  and  $\psi=30^\circ$ . The maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are respectively identified with the symbols  $\circ$ ,  $\Delta$ ,  $\square$



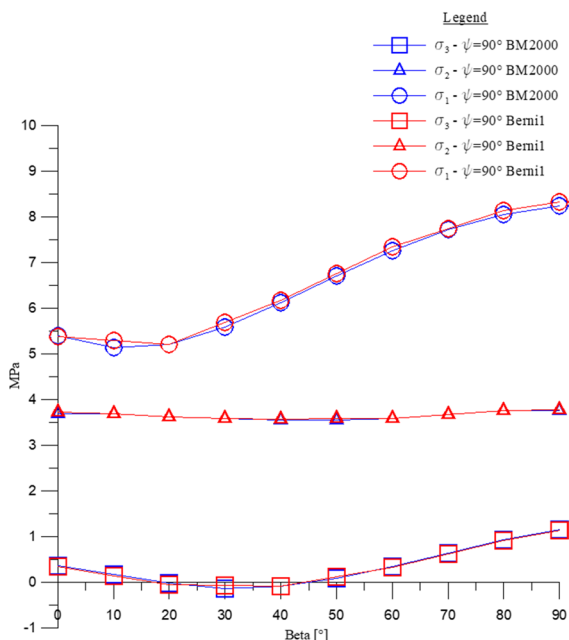
**Fig. 8** Stereogram of the trend of the maximum main stresses, calculated with Berni1 and BM2000 for the validation in the transversely isotropic case, when  $\beta=0^\circ \sim 90^\circ$  and  $\psi=90^\circ$ . The maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are respectively identified with the symbols  $\circ$ ,  $\Delta$ ,  $\square$

**Table 18** Results and differences, in terms of intensity, inclinations  $\delta$  and directions  $\beta$ , of the maximum principal stresses obtained for the validation of the BM2000 software, when  $\beta=0^\circ\sim 90^\circ$  and  $\psi=90^\circ$ . The data refer to the transversely isotropic case proposed by Amadei (1983)

	Berni1			BM2000			Difference		
	(MPa)	$\delta$ (°)	$\beta$ (°)	(MPa)	$\delta$ (°)	$\beta$ (°)	$\Delta$ (MPa)	$\Delta\delta$ (°)	$\Delta\beta$ (°)
BETA 0°									
$\sigma_1$	5.38	- 21.38	89.30	5.39	- 22.16	88.80	- 0.01	0.78	0.50
$\sigma_2$	3.73	- 27.18	184.44	3.69	- 27.45	184.86	0.04	0.27	- 0.42
$\sigma_3$	0.34	- 68.56	324.77	0.35	- 68.96	323.54	- 0.01	0.40	1.23
BETA 10°									
$\sigma_1$	5.29	x	x	5.13	- 29.89	101.26	0.16	x	x
$\sigma_2$	3.69	x	x	3.68	- 20.82	197.55	0.01	x	x
$\sigma_3$	0.14	x	x	0.18	- 68.04	318.38	- 0.03	x	x
BETA 20°									
$\sigma_1$	5.20	- 35.15	113.58	5.20	- 35.67	115.05	0.00	0.52	- 1.47
$\sigma_2$	3.62	- 12.56	209.66	3.62	- 12.37	209.48	0.00	- 0.19	0.18
$\sigma_3$	- 0.06	x	x	- 0.02	- 66.57	314.90	- 0.03	x	x
BETA 30°									
$\sigma_1$	5.68	x	x	5.58	- 37.80	125.84	0.10	x	x
$\sigma_2$	3.59	x	x	3.58	- 6.32	218.27	0.01	x	x
$\sigma_3$	- 0.07	x	x	- 0.13	- 65.82	315.54	0.06	x	x
BETA 40°									
$\sigma_3$	6.17	- 37.56	136.42	6.13	- 38.01	134.85	0.04	0.45	1.57
$\sigma_2$	3.57	- 2.02	226.37	3.55	- 2.57	225.85	0.02	0.55	0.52
$\sigma_1$	- 0.09	x	x	- 0.10	- 65.90	318.68	0.01	x	x
BETA 50°									
$\sigma_1$	6.76	x	x	6.71	- 37.37	143.59	0.05	x	x
$\sigma_2$	3.58	x	x	3.56	- 0.28	53.36	0.02	x	x
$\sigma_3$	0.12	x	x	0.07	- 66.46	323.18	0.04	x	x
BETA 60°									
$\sigma_1$	7.34	- 36.23	150.55	7.26	- 36.34	152.37	0.08	0.11	- 1.82
$\sigma_2$	3.59	- 2.35	62.07	3.59	- 3.01	61.18	0.00	0.66	0.89
$\sigma_3$	0.32	x	x	0.33	- 67.20	327.71	- 0.01	x	x
BETA 70°									
$\sigma_1$	7.74	x	x	7.72	- 35.13	161.45	0.02	x	x
$\sigma_2$	3.67	x	x	3.67	- 6.28	69.18	0.00	x	x
$\sigma_3$	0.61	x	x	0.64	- 67.91	331.36	- 0.03	x	x
BETA 80°									
$\sigma_1$	8.14	- 33.83	170.34	8.06	- 33.72	170.96	0.08	- 0.11	- 0.62
$\sigma_2$	3.76	- 9.89	78.27	3.75	- 10.77	77.26	0.01	0.88	1.01
$\sigma_3$	0.90	x	x	0.93	- 68.34	332.95	- 0.03	x	x
BETA 90°									
$\sigma_1$	8.34	- 31.92	180.63	8.25	- 31.88	180.56	0.09	- 0.04	0.07
$\sigma_2$	3.77	- 14.90	84.90	3.75	- 16.92	85.27	0.02	2.02	- 0.37
$\sigma_3$	1.13	- 68.42	331.21	1.15	- 68.12	331.31	- 0.02	- 0.30	- 0.10



**Fig. 9** Trend of the maximum principal stresses (MPa), calculated with Bemil and BM2000 for the validation in the transversely isotropic case, where  $\beta=0\sim 90^\circ$  and  $\psi=30^\circ$ . The maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are respectively identified with the symbols  $\circ$ ,  $\Delta$ ,  $\square$



**Fig. 10** Trend of the maximum principal stresses (MPa), calculated with Bemil and BM2000 for the validation in the transversely isotropic case, where  $\beta=0\sim 90^\circ$  and  $\psi=90^\circ$ . The maximum principal stresses calculated  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are respectively identified with the symbols  $\circ$ ,  $\Delta$ ,  $\square$

**Acknowledgements** A special thanks to Prof. Giorgio Iabichino and Prof. Marilena Cardu for suggestions and supervision. A further thank is due to Eng. Valentina Isaia for the graphical contribution.

**Funding** Open access funding provided by Politecnico di Torino within the CRUI-CARE Agreement. This research received no external funding.

**Availability of Data and Material** Not applicable.

**Code Availability** Not applicable.

**Declarations**

**Conflicts of Interest** The author declare no conflict of interest.

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