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Experimental and Theoretical Studies on Friction Contact of Bolted Joint Interfaces / Li, Dongwu; Botto, Daniele; Li, Ruozhang; Xu, Chao; Zhang, Wenming. - In: INTERNATIONAL JOURNAL OF MECHANICAL SCIENCES. - ISSN 0020-7403. - ELETTRONICO. - 236:(2022), p. 107773. [10.1016/j.ijmecsci.2022.107773]

*Availability:*

This version is available at: 11583/2973413 since: 2022-11-27T16:56:22Z

*Publisher:*

Elsevier

*Published*

DOI:10.1016/j.ijmecsci.2022.107773

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<http://dx.doi.org/10.1016/j.ijmecsci.2022.107773>

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# Experimental and Theoretical Studies on Friction Contact of Bolted Joint Interfaces

Dongwu Li<sup>1</sup>, Daniele Botto<sup>2</sup>, Ruozhang Li<sup>1</sup>, Chao Xu<sup>3</sup>, Wenming Zhang<sup>1\*</sup>

<sup>1</sup>State Key Laboratory of Mechanical System and Vibration, School of Mechanical Engineering, Shanghai Jiao Tong University Shanghai, 200240, China

<sup>2</sup>Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Turin 10129, Italy

<sup>3</sup>School of Astronautics, Northwestern Polytechnical University, Xi'an 710072, China

**Abstract:** This paper presents experimental and theoretical modeling studies on the tangential contact stiffness and friction hysteresis of bolted joint interfaces under transversal vibration. A new test rig was developed to measure the interface fretting response of bolted joints. This rig is based on a servo-hydraulic fatigue testing system in which the contact surfaces are arranged symmetrically to avoid bending of the specimen that could affect the friction measurement. Force-displacement curves were measured under different tangential loads and compared with those found in the literature. A multi-scale contact modeling method was proposed to calculate the tangential contact stiffness and reproduce the friction hysteresis of bolted joint interfaces. This method overcomes the complexity of identifying contact stiffness, which usually requires a model of the dynamic behavior of the joint and frequency responses measured with dedicated fretting test stands. Specifically, this method comprehensively considers the multi-scale features of randomly rough surfaces, the non-uniform contact pressure distribution, and the convenience of phenomenological contact models in simulating frictional hysteresis, relying on fractal contact theory and discretized Iwan model. The effectiveness of the proposed method was verified by comparing the simulation results with the experimental counterparts in different cases of tangential loading. The successful validation demonstrates the potential of the proposed modeling method for application to structures with bolted joint.

**Keywords:** Bolted joint; friction hysteresis; contact stiffness; fractal theory; Iwan model.

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\* Corresponding author: [wenmingz@sjtu.edu.cn](mailto:wenmingz@sjtu.edu.cn) (wenming Zhang)

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# 1 Introduction

1 Bolted joints are widely used in various mechanical systems, such as turbine engines, spacecraft,  
2 wind turbines, etc., to make two or more components assembled as a whole and transmit loads.  
3 Subjected to transversal vibration, the mechanical behavior induced by the relative motion of the bolted  
4 joint interface exhibits nonlinear stiffness and energy dissipation characteristics (i.e. frictional  
5 hysteresis) [1-5]. The contact condition of the interface (possibly stick, partial slip, and gross slip) is  
6 related to the level of transversal vibration. This inherent nonlinearity of friction can cause the  
7 ‘frequency shift’ and instability in the dynamic response of assembled structures, which may further  
8 lead to undesired resonance and increase the difficulty of structural control [6-11]. Therefore, an  
9 accurate description of interface frictional hysteresis behavior is of great importance for dynamic  
10 modeling and vibration control of assembled structures.

12 Extensive experimental and theoretical studies have been carried out to understand the contact  
13 mechanisms of bolted joints and to characterize the interface friction hysteresis. The work by Gaul et  
14 al. [12] is considered one of the milestones in the field of joint structural mechanics. They investigated  
15 the hysteresis response of the bolted interface through a mixed experimental and numerical strategy  
16 and discussed the effects of excitation amplitude and bolt preload on the hysteresis behavior. Eriten et  
17 al. [13] compared hysteresis curves and energy dissipation between steel and aluminum jointed  
18 samples and obtained the same exponential relationship between energy dissipation per cycle and the  
19 maximum tangential force as obtained in Ref. [14][15]. Subsequently, they studied the effects of  
20 surface roughness and lubrication on the hysteresis behavior of bolted joints, revealing the underlying  
21 mechanism affecting the friction force, contact stiffness, and damping [16]. Li et al. [17][18] studied  
22 the effect of load level and fretting wear on the friction hysteresis curve and contact parameters (contact  
23 stiffness and friction coefficient). Moreover, a theoretical contact analysis was performed to  
24 demonstrate that the residual stiffness phenomenon (found in the gross slip regime) is due to the  
25 influence of other contact surfaces in bolted joints. Recently, Zhao et al. [19] emphasized, by  
26 experiments and finite element analysis, the role of contact pressure distribution in the nonlinear  
27 softening of the tangential contact stiffness of bolted joint. From the above experimental studies, it can  
28 be found that there are many factors affecting the interface hysteresis of bolted joints. Therefore, it is

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1 still a challenge to comprehensively consider these factors in modeling the friction hysteresis of bolted  
2 joints.

3 Generally, there are mainly two families of modeling methods applied to simulate the friction  
4 contact behavior: models that account for rough contact (physics-based models) and phenomenological  
5 contact models (data-driven models). Models with rough contact characterize a rough surface as a  
6 collection of many asperities (in single-scale [20-22] or multi-scale [23-27]). The overall contact  
7 behavior of the interface is obtained by combining the deformation features of a single asperity with  
8 statistical methods. These methods consider the rough surface characteristics and physics-based  
9 constitutive relation. However, since the force-displacement expression obtained is not explicit, it is  
10 unlikely to be compatible with a structural dynamic analysis code.

11 In comparison, the phenomenological models, such as Iwan model [28-32], Valanis model  
12 [12][33], and LuGre model [34], can be easily integrated into the joint structural dynamic analysis  
13 method due to good compatibility. Especially, due to the generality of the model parameters (contact  
14 stiffness and friction coefficient) and the simple expression of the force-displacement nonlinear  
15 relation, the Iwan model has wide application in dynamic modeling of jointed structures. In its first  
16 iterations, the Iwan density function - the core of the Iwan model - is assumed to be a truncated power-  
17 law distribution with a Dirac delta function [28][30][35]. These models have 4-6 parameters that must  
18 be obtained by fitting the results of appropriate experiments, which partially limits the application of  
19 the model. Li et al. [36][37] proposed a mapping relationship from the contact pressure distribution to  
20 the Iwan density function without introducing new parameters. This model requires only two  
21 parameters, contact stiffness and friction coefficient. Li et al. [38] improved the Iwan model using  
22 similar ideas and developed a parameter identification method. Although these improved Iwan models  
23 show good accuracy in the simulation of friction hysteresis and have good compatibility with structural  
24 dynamic analysis, the model parameters estimation procedure is inconvenient for predicting dynamic  
25 response of jointed structures. Actually, this procedure requires dedicated fretting test rigs or  
26 measured frequency response curves [39-44].

27 The objective of this work is to develop a method for modeling friction contact to facilitate the  
28 dynamic analysis of jointed structures without resorting to dedicated friction tests or dynamic tests.

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1 This method comprehensively considers the multi-scale features of randomly rough surfaces, the non-  
2 uniform distribution of contact pressure, and the convenience of phenomenological contact models in  
3 friction hysteresis simulation. At the same time, to verify the effectiveness of the proposed method, a  
4 new fretting test rig was developed. This test rig was specially designed to avoid the effects of residual  
5 stiffness and to eliminate as much as possible the effect of additional moments during reaching the  
6 contact conditions on the jointed interfaces.

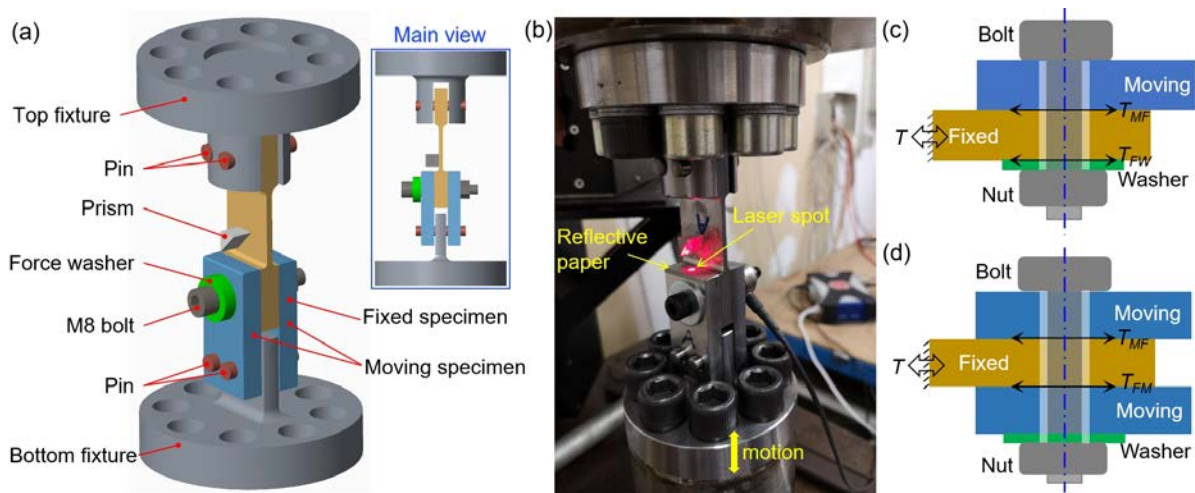
7 The remainder of this paper is organized as follows. [Section 2](#) describes the newly developed  
8 fretting test apparatus and shows the experimental hysteresis curves under different loads. [Section 3](#)  
9 presents the multi-scale modeling approach to calculate the tangential contact stiffness of jointed  
10 interfaces and reproduce friction hysteresis behavior. [Section 4](#) verifies the effectiveness of the  
11 proposed modeling approach by comparison of the simulation results with the experimental  
12 counterparts, and discusses the effect of the interface discretization on the model performance. [Section](#)  
13 [5](#) closes the paper by highlighting the novelty of the proposed method and presenting the main  
14 conclusions.

15

## 2 Experimental tests

### 2.1 Experimental method

Figure 1(a and b) shows an overview and a photograph of the test apparatus used in the experiments. To capture the friction force of bolted joint interface as purely as possible, the joint sample with two moving specimens and a fixed specimen is arranged in a symmetric layout. As a comparison, the friction force measured in an asymmetric layout is mixed with that at other interfaces (between the screw head, the washer, the nut and the target interface) and the bending deformation of the bolt shank, which introduces errors into friction measurement [17], as shown in Fig. 1(c and d). Ref. [17] developed a theoretical model to analyze these factors and attributed the origin of the residual stiffness (i.e. the slope of the displacement-force curve during the gross slip regime) to them. In this experiment, to avoid the influence of the above factors on friction measurement, a symmetric specimen layout is adopted. Additionally, the experimental results are available for indirectly verifying the theoretical analyses in Ref. [17] where some ideal assumptions were employed.



**Fig. 1** (a) Schematic diagram of the fretting test apparatus and main components, (b) photograph of the servo-hydraulic fatigue testing system and the assembled bolted joint specimens. The joint specimens are periodically displaced long the piston shaft. (c) Asymmetric layout of bolt joints used in Ref. [17] in which the measured friction force includes that at other surfaces, not only that at target surface. (d) Symmetric layout in this work where the measured friction force is equal to the resultant of those at two target surfaces.

The specimens are made of ASTM 304 stainless steel and fastened together with an M8 bolt. There are two measured contact regions in parallel. Each contact region is a  $30 \times 30 \text{ mm}^2$  square with a

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1 10 mm diameter hole at the center. The moving specimens are connected to the bottom fixture by two  
2 pins with a diameter of 8 mm. There is a 1 mm gap (see the main view in [Fig. 1\(a\)](#)) between both sides  
3 of the protruding part in the middle of the bottom fixture and the moving specimen to avoid the effect  
4 of their contact on the measurements. The fixed specimen is specially designed to have greater  
5 flexibility along the normal direction of the contact surface, thereby avoiding as much as possible the  
6 tilt contact (or local contact which is found from wear scars in some tests [\[18\]](#)) between the moving  
7 and fixed surfaces caused by the extra moment during installation. The middle of the fixed specimen  
8 is made as a leaf spring, and the whole is like a dumbbell. This design idea is similar with the ‘floating  
9 body approach’ developed in Ref. [\[45\]](#). From this point of view, the thinner the leaf spring, the more  
10 favorable the formation of plane contact. However, if the leaf spring is too thin, there is a risk of  
11 buckling instability in the test apparatus under large compressive loads. Therefore, considering the  
12 combined effects of the flexibility and structural stability, the thickness of the leaf spring is set to 3  
13 mm. The other end of the fixed specimen is connected to the top fixture by two 8 mm diameter pins.  
14 Likewise, a 1 mm interval is reserved between the top fixture and the fixed specimen.

15 The top and bottom fixtures are mounted on a servo-hydraulic fatigue testing system (capacity  
16 250 kN, Instron Inc.). There is a cylindrical depression at the upper end of the top fixture and the lower  
17 end of the bottom fixture, which is used to match with the boss of the piston shaft so that the axis of  
18 the fixtures coincides with the piston axis as much as possible. The piston shaft drives the moving  
19 specimens to reciprocate and causes relative motion of the bolted joint interfaces. The friction force  
20 induced by the relative motion is measured by a built-in load cell of the testing system. Note that the  
21 measured force is the resultant of the tangential contact forces on the two surfaces, as shown in [Fig.](#)  
22 [1\(d\)](#). The relative displacement between the moving and fixed specimens is measured using a  
23 combination of a laser vibrometer (OFV-525, Polytec Inc.) and a small prism (size 10×10×10 mm)  
24 with a inclination of 45°. The prism is attached to the fixed specimen close to the contact area and is  
25 used to reflect the laser beam onto the end of the moving specimen where a reflective paper is pasted,  
26 as shown in [Fig. 1\(b\)](#). The laser spot is focused as close as possible to the contact surface. It is worth  
27 noting that since the bolted joint specimen is not strictly constrained along the normal direction of the  
28 contact surface, the piston oscillation excitation may cause the specimen to slightly vibrate along the

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1 normal direction. Therefore, to eliminate the influence of this factor on the displacement measurement  
2 accuracy, the path of the laser beam is set parallel to the contact surface, see Fig. 1(b). Another point  
3 to note is that due to the compliance of the leaf spring, the relative movement between the prism and  
4 the ideal measurement point (on the same side as the reflective paper and close to the contact surface)  
5 on the fixed specimen should not be ignored when measuring the relative displacement of the target  
6 interface. This issue will be discussed in Section 2.2. In addition, the bolt preload is recorded by an  
7 annular force washer (KMR-40 kN for M8 bolt, HBM Inc.). It should be pointed out that the present  
8 test apparatus is semi-closed loop controlled. The controlled variable is the piston displacement, not  
9 the measured interface relative displacement.

10 Before each test, the target surfaces are cleaned with acetone and dried. At the same time, the  
11 topography of all contact surfaces is measured using an Alicona Infinite Focus instrument and will be  
12 employed to estimate the parameters of the multi-scale contact model in Section 3. All tests were  
13 carried at room temperature. The test procedure is given as follows:

14 (1) Mount the fixtures on the servo-hydraulic testing system, and then assemble the fixed and  
15 moving specimens on the top and bottom fixture using two pins, respectively;

16 (2) Drive the moving part upward by the piston shaft until the bolt holes on the fixed and moving  
17 specimens are aligned, then assemble the fixed and moving specimens using an M8 bolt, a square nut,  
18 and the force washer;

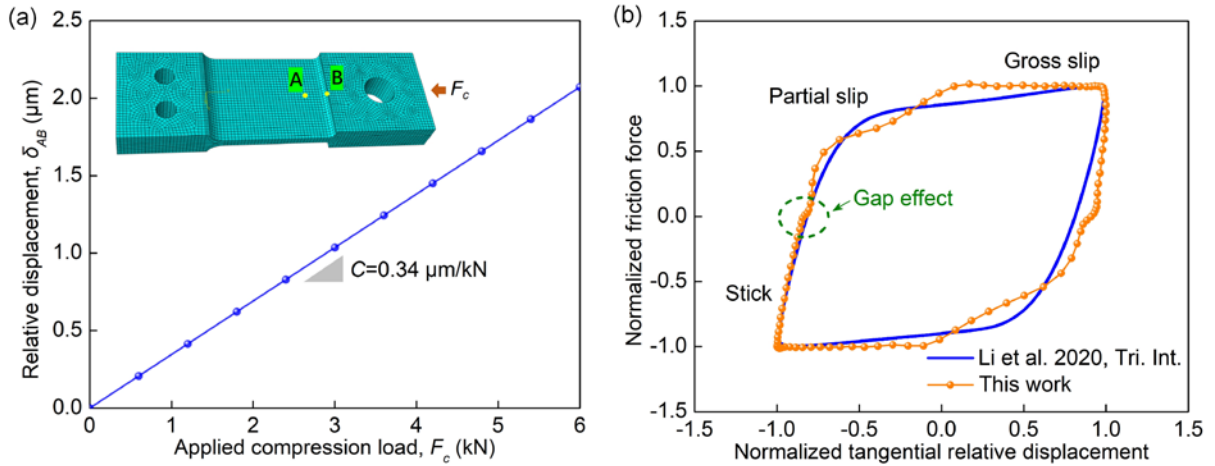
19 (3) Tighten the bolt and monitor the preload by the force washer until reaching up to a target value  
20 (the applied preload deviates from the target by  $\pm 5\%$  due to hand tightening), and then adjust the  
21 position of the laser vibrometer so that the laser spot is focused on the set measurement point;

22 (4) apply a harmonic displacement with a frequency of 5 Hz along the piston axis to the moving  
23 specimens, at the same time, record the friction force, the relative displacement between the fixed and  
24 moving specimens, and the bolt preload.

## 25 **2.2 Experimental results**

26 As mentioned above, although the prism and the ideal measurement position on the fixed  
27 specimen are in close proximity (about 7 mm), the high compliance of the leaf spring in the middle of  
28 the fixed specimen may have a significant effect on the displacement measurement. Therefore, to

1 quantify this compliance and further correct the measured displacements, a finite element model of the  
 2 fixed specimen was established, as shown in the inset in Fig. 2(a). The model takes into accounts the  
 3 fillets and structural elastic deformation. A mechanical analysis was carried out and the relative  
 4 displacement between the prism and the ideal measurement position as a function of compression load  
 5 is depicted in Fig. 2(a). It shows that the compliance of the leaf spring between the prism and the ideal  
 6 measurement position is  $0.34 \mu\text{m}/\text{kN}$ . All measured relative displacement data are corrected by  
 7 subtracting the product of the compliance and measured force.



8  
 9 **Fig. 2** (a) Relative displacement between points A and B as a function of the applied compression load.  
 10 Points A and B represent laser spots on the prism and the ideal measurement position on the fixed  
 11 specimen, respectively. (b) Comparison of the normalized hysteresis loops measured in this work with  
 12 that in Ref. [17]. Normalization is implemented by dividing the original measurement data by the  
 13 corresponding amplitude.

14  
 15 Figure 2(b) compares a typical hysteresis loop measured by the present test apparatus with that  
 16 obtained in Ref. [17] where an asymmetric specimen with an M6 bolt was tested. For a more intuitive  
 17 comparison, the original hysteresis loops were normalized by dividing by the amplitude of  
 18 force/displacement. Taking the loading stage as an example, it can be seen that the contact surface in  
 19 Ref. [17] experiences stick, partial slip, and gross slip successively. The force-displacement curve  
 20 looks smooth and the tangential contact stiffness decreases monotonically with increasing relative  
 21 displacement.

22  
 23 By contrast, the force-displacement curve measured in this work exhibits a complex shape after  
 24 the initial stick regime, and there are three distinct reductions in the tangential contact stiffness. The

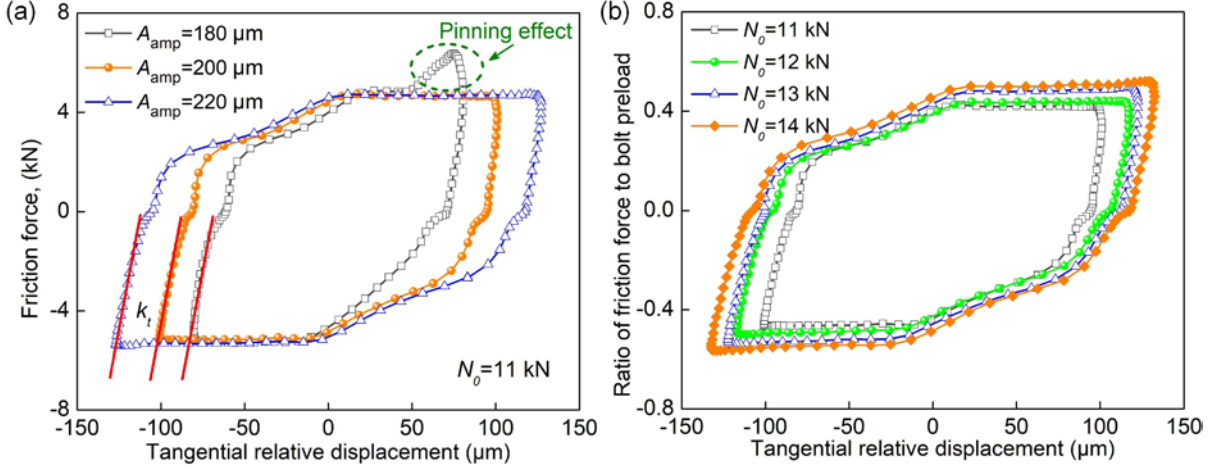
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1 first reduction occurs at zero friction force, where the friction force hardly changes with the increase  
2 of the interface relative displacement. It means that there are gaps in the load transmission path of the  
3 test apparatus. These gaps are most likely to exist at the pin connections between the fixture and the  
4 specimen. Although the gap effect results in a temporary loss of the ‘measured contact stiffness’, it  
5 does not affect the energy dissipation per cycle. The second noticeable reduction occurs during the  
6 partial slip regime. The reason for this may be that one of the contact surfaces grossly slips while the  
7 other is still in the partial slip regime. Due to the difference in microscopic surface topography, even  
8 if the two contact surfaces are distributed symmetrically and synchronously loaded, it is impossible to  
9 maintain the same motion state. The third noticeable reduction occurs in the final stage of loading,  
10 which is caused by the gross slip of both contact surfaces. It can be found that during the gross slip  
11 regime, the tangential contact stiffness is zero (no residual stiffness), different from that in Ref. [17].  
12 This result also corroborates our design idea for the test apparatus.

13 **Figure 3(a)** depicts the measured hysteresis loops under the same bolt preload of 11 kN but with  
14 different excitation amplitudes ( $A_{\text{amp}} = 185 \mu\text{m}$ ,  $205 \mu\text{m}$ , and  $225 \mu\text{m}$ ). The excitation amplitude is  
15 obtained from the piston displacement. Each test was run for 50 cycles, during which the testing system  
16 stabilized after 20 cycles. Although the tangential relative motion of the bolted joint specimen leads to  
17 the preload oscillation, this change does not exceed  $\pm 5\%$  of the initial value throughout test period. It  
18 can be found that under varying excitation amplitudes, the initial tangential contact stiffness (the slope  
19 of the red line in **Fig. 3(a)**) hardly changes, and so does the friction coefficient (which equals the ratio  
20 of friction force in the gross slip regime to the bolt preload). In addition, at the end of the gross slip  
21 phase, an ‘uplift’ (the friction force suddenly increases) in the hysteresis curve for the case  $A_{\text{amp}} = 185$   
22  $\mu\text{m}$  can be observed. The cause for this phenomenon is the contact between the screw and the bolt hole.  
23 Specifically speaking, when tightening the bolt, it is hard to control the position of the screw relative  
24 to the bolt hole. In some cases, the screw is too close to the inner wall of the bolt hole after tightening.  
25 Once the sliding distance of the contact surface exceeds the gap between the screw and the hole, a new  
26 contact will be formed between them and result in the ‘uplift’ phenomenon. This phenomenon is also  
27 observed in Refs. [17][35].

28 **Figure 3(b)** plots the ratio of friction force to bolt preload as a function of the tangential relative

1 displacement under varying bolt preloads and excitation amplitudes. The shapes of the hysteresis  
 2 curves for the given four scenarios are similar. The ratio of friction force in gross slip regime to the  
 3 bolt preload (i.e. twice the friction coefficient of contact surface) varies with the bolt preload. This  
 4 change may be caused by the evolution of the surface topography due to multiple tests.



5  
 6 **Fig. 3** (a) Friction force as a function of tangential relative displacement for varying excitation  
 7 amplitudes and the same bolt preload of 11 kN. The slope of the red lines in the stick regime represents  
 8 the tangential contact stiffness. (b) The ratio of friction force to bolt preload versus tangential relative  
 9 displacement for varying loading cases:  $N_0=11$  kN,  $A_{amp}=200$   $\mu\text{m}$ ;  $N_0=12$  kN,  $A_{amp}=225$   $\mu\text{m}$ ;  
 10  $N_0=13$  kN,  $A_{amp}=240$   $\mu\text{m}$ ;  $N_0=14$  kN,  $A_{amp}=260$   $\mu\text{m}$ .

11  
 12 The test rig developed in this paper can well avoid the influence of other contact surfaces other  
 13 than the target surface on friction force measurement, which makes it highly accurate and  
 14 unconventional among the fretting test rigs found in the literature so far. In the next section, a multi-  
 15 scale modeling method will be developed to reproduce the initial tangential contact stiffness and the  
 16 friction hysteresis behavior. Then in [Section 4](#), the above experimental results will be used as a  
 17 benchmark to verify the multi-scale contact model.

18  
 19

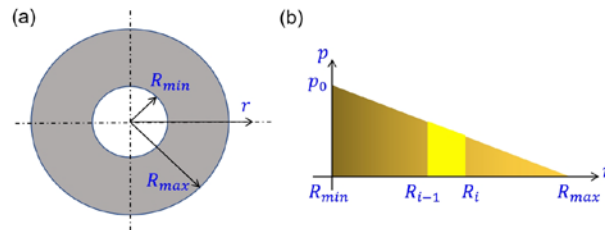
### 3 Modeling method

In this section, we propose a multi-scale modeling method based on fractal contact theory [46][47] and a discretized Iwan model to characterize the friction contact behavior of bolted joint interfaces. Fractal contact theory is used to consider the roughness characteristics of contact surfaces and to model the initial tangential contact stiffness, while the discretized Iwan model is used to reproduce the friction hysteresis behavior. The former provides necessary model parameters for the latter.

The fractal contact theory implicitly assumes that the contact pressure in flat-on-flat contact problems is uniformly distributed, which cannot be directly applied to bolted joint interface in which contact pressure monotonically decreases from a maximum value near bolt hole to zero at contact edge. Clearly, the area near the contact edge is easier to slide than the area near the contact center. Therefore, to improve accuracy, the non-uniform distribution of contact pressure is considered in the proposed modeling method. we divides the contact area into several parts and assumes a constant contact pressure in the divided local area, in which the fractal contact theory can be used.

#### 3.1 Contact pressure distribution

The contact pressure distribution of bolted joint interfaces is related to many factors, such as the thickness of the connected parts, surface waviness, assembly process, etc. It is hard to obtain an analytical formation of the contact pressure distribution. Therefore, a widely accepted approximation function of the contact pressure distribution is used, which assumes that the contact pressure decreases linearly in the radial direction and is unchanged in the circumferential direction [48][49], as shown in Fig. 4.



**Fig. 4** (a) Nominal contact area of bolted joint interface, and (b) approximated contact pressure distributed along the contact radius.

The contact pressure  $p(r)$  distributed along the contact radius can be expressed as

$$p(r) = p_0 \left( \frac{R_{max} - r}{R_{max} - R_{min}} \right), \quad (1)$$

1 where  $R_{min}$  and  $R_{max}$  represent the radius of bolt hole and contact radius, respectively,  $r$  the  
 2 distance to the hole center,  $p_0$  the maximum pressure. The relationship between the bolt preload  $N_0$   
 3 and the contact pressure function satisfies,

$$N_0 = \int_{R_{min}}^{R_{max}} 2\pi r p(r) dr. \quad (2)$$

4 Substituting Eq. (1) into Eq.(2) yields the maximum contact pressure,

$$p_0 = \frac{3N_0}{\pi(R_{max}^2 + R_{max}R_{min} - 2R_{min}^2)}. \quad (3)$$

5 The distribution of contact pressure determines the frictional state (sticking or sliding) of the local  
 6 contact region. To consider the effect of contact pressure distribution on the friction contact, we divide  
 7 the contact area equally into  $h$  annular sub-regions along the contact radius. The radial length of each  
 8 annular region equals  $(R_{max} - R_{min})/h$ . The normal load  $N_{0i}$  of the  $i^{\text{th}}$  annular region can be  
 9 expressed as

$$N_{0i} = \int_{R_{i-1}}^{R_i} 2\pi r p(r) dr = \frac{2\pi p_0}{R_{max} - R_{min}} \left[ \frac{R_{max}(R_i^2 - R_{i-1}^2)}{2} - \frac{(R_i^3 - R_{i-1}^3)}{3} \right], \quad (4)$$

10 where the boundaries of the  $i^{\text{th}}$  annular region are (see Fig. 4(b)),

$$R_{i-1} = R_{min} + \frac{i-1}{h} (R_{max} - R_{min}), \quad (5)$$

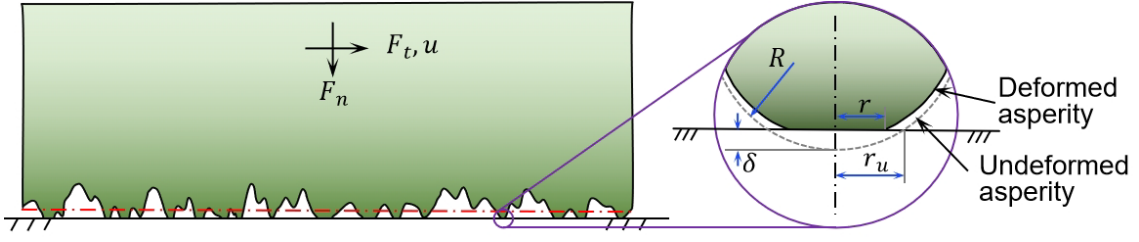
$$R_i = R_{min} + \frac{i}{h} (R_{max} - R_{min}). \quad (6)$$

11 In each divided annular region, the contact pressure is assumed to be uniformly distributed.  
 12 Therefore, the fractal contact theory can be applied to analyze the contact area of the bolted joint  
 13 interface and further calculate the tangential contact stiffness.

### 14 **3.2 Multi-scale contact model**

15 When two rough flat surfaces are in contact, it can be simplified to a contact between a rough  
 16 surface and a rigid smooth surface, as shown in Fig. 5, where a graphical representation of the  
 17 simplified contact and the deformation geometry of an individual asperity are given. Considering the  
 18 multi-scale nature of surface topography of rough surfaces, the fractal theory is used to describe the  
 19 random rough features of connected surfaces. For completeness, the essential steps will be briefly  
 20 reviewed below. Then it is combined with the contact mechanics model of individual asperity to

1 simulate the normal and tangential contact.



2

3 **Fig. 5** Schematic diagram of the contact between a random rough surface and a rigid smooth surface,  
4 and the deformation geometry of an individual asperity.

5

6 The modified Weierstrass-Mandelbrot (W-M) function [47] is widely used to characterize a three-  
7 dimensional surface topography of random rough surfaces. For simplicity, we employ a simplified W-  
8 M function to describe the rough surface height  $z(x)$ ,

$$z(x) = L \left( \frac{G}{L} \right)^{D-2} (\ln \gamma)^{0.5} \sum_{n=0}^{n_{max}} \gamma^{(D-3)n} \left[ \cos \phi_{1,n} - \cos \left( \frac{2\pi \gamma^n x}{L} - \phi_{1,n} \right) \right], \quad (7)$$

9 where  $L$  denotes the sampling length of surface profile,  $G$  the fractal roughness,  $D$  the fractal dimension  
10 ( $2 < D < 3$ ),  $\gamma$  the scaling factor that is related to the frequency spectrum of surface profile (typically  
11  $\gamma = 1.5$ ),  $n$  the frequency index and  $n_{max}$  corresponding to the high cutoff frequency of the profile,  
12  $\phi_{1,n}$  the random phase, and  $x$  the coordinate along the surface. The deformation of an individual  
13 asperity can be expressed as

$$\delta = 2G^{D-2} (\ln \gamma)^{0.5} (2r_u)^{3-D}, \quad (8)$$

14 where  $r_u$  is the truncated radius of the undeformed asperity (see the local enlarged view in Fig. 5).

15 According to the deformation geometry of an individual asperity,  $R^2 - (R - \delta)^2 = r_u^2$  and  
16  $R \gg \delta$ , the curvature radius  $R$  of the asperity can be derived as

$$R = 2^{D-5} G^{2-D} (\ln \gamma)^{-0.5} \left( \frac{a_u}{\pi} \right)^{0.5D-0.5}, \quad (9)$$

17 where  $a_u$  is the truncated area of the undeformed asperity and equals  $a_u = \pi r_u^2$ .

### 18 3.2.1 Normal contact

19 Under a certain normal load, the asperities of the rough surface with different heights will  
20 generally be in three deformation states, including purely elastic, elastic-plastic, and fully plastic  
21 deformations, denoted by the subscripts  $e$ ,  $ep$ , and  $p$ , respectively. Therefore, the asperities undergo  
22 different normal loads, deformations, and contact areas, from each other.

---

1 (1) Purely elastic contact

2 When the height of the asperity is low (namely, the deformation of the asperity is relatively small),  
3 the asperity is in a purely elastic deformation state. According to Hertz contact theory [50], the contact  
4 force  $F_e$  can be expressed as

$$F_e = \frac{4}{3} E^* R^{0.5} \delta^{1.5}, \quad (10)$$

5 where  $E^*$  is the reduced elastic modulus,  $E^* = [(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]^{-1}$  with  $\nu_1$ ,  $E_1$ ,  $\nu_2$ ,  
6 and  $E_2$  being the Poisson's ratios and elastic moduli of two contacting bodies. Substituting Eqs. (8)  
7 and (9) into Eq. (10) yields the relationship between the contact force  $F_e$  and the truncated contact  
8 area  $a_u$ ,

$$F_e(a_u) = \frac{2^{5.5-D}}{3\pi^{2-0.5D}} E^* G^{D-2} (\ln\gamma)^{0.5} a_u^{2-0.5D}. \quad (11)$$

9 (2) Fully plastic contact

10 When the asperity is relatively high, the fully plastic deformation of the asperity occurs. Based  
11 on Ref. [22], the contact force  $F_p$  can be expressed as

$$F_p(a_u) = 2\pi R H \delta = \eta \sigma_y a_u, \quad (12)$$

12 where  $H$  is the hardness of softer material and equals  $H = \eta \sigma_y$  (typically  $\eta=2.79$ ) with  $\sigma_y$  being the  
13 yield strength.

14 (3) Elastic-plastic contact

15 When the height of the asperity is moderate, the asperity is in a transition from the purely elastic  
16 deformation to the fully plastic deformation. Following Ref. [51], the contact force  $F_{ep}$  induced by  
17 the elastic-plastic deformation of asperity is

$$F_{ep} = k_\vartheta H \pi R \delta_{ec}^{-\beta} \delta^{1+\beta}, \quad (13)$$

18 where  $k_\vartheta$  is the mean contact pressure factor,  $\delta_{ec}$  the critical deformation at which plastic yielding  
19 begins, and  $\beta = \ln(2/k_\vartheta)/\ln(\delta_{pc}/\delta_{ec})$  in which  $\delta_{pc}$  the critical deformation at the beginning of  
20 fully plastic deformation and equals  $\delta_{pc} = 110\delta_{ec}$  [51]. According to Ref. [52],  $k_\vartheta$  can be  
21 expressed as a function of Poisson's ratio,

$$k_\vartheta = 0.3097 + 0.2094\vartheta + 0.1295\vartheta^2. \quad (14)$$

22 The critical deformation  $\delta_{ec}$  at onset of plastic yielding is given, based on Ref. [53],

$$\delta_{ec} = \left( \frac{0.75\pi k_g H}{E^*} \right)^2 R. \quad (15)$$

1 Then, substituting Eqs.(8, 9, and 15) into Eq.(13), the expression of the normal force  $F_{ep}$  as a  
2 function of the truncated contact area  $a_u$  can be obtained,

$$F_{ep}(a_u) = 2^{13\beta-2\beta D-1} \left( \frac{E^*}{3} \right)^{2\beta} \pi^{\beta(D-4)} (k_g H)^{1-2\beta} G^{2\beta(D-2)} (\ln \gamma)^\beta a_u^{1+2\beta-\beta D}. \quad (16)$$

3 The reason why the contact forces  $F_e$ ,  $F_{ep}$ , and  $F_p$  are expressed as a function of the truncated  
4 area  $a_u$  is that the total contact force  $F_n$  can be obtained by integrating the product of the contact  
5 force of a single asperity and the area distribution function  $n(a_u)$ , as follows,

$$F_n = \begin{cases} \int_0^{a_{u,p}} F_p n(a_u) da_u + \int_{a_{u,p}}^{a_{u,e}} F_{ep} n(a_u) da_u + \int_{a_{u,e}}^{a_{u,l}} F_e n(a_u) da_u, & a_{u,l} > a_{u,e} \\ \int_0^{a_{u,p}} F_p n(a_u) da_u + \int_{a_{u,p}}^{a_{u,l}} F_{ep} n(a_u) da_u, & a_{u,p} < a_{u,l} < a_{u,e} \\ \int_0^{a_{u,l}} F_p n(a_u) da_u, & a_{u,l} < a_{u,p} \end{cases} \quad (17)$$

6 where the integral limits  $a_{u,p} = 2\pi R \delta_{pc}$ ,  $a_{u,e} = 2\pi R \delta_{ec}$ , and  $a_{u,l}$  represents the maximum  
7 truncated area. The area distribution function  $n(a_u)$  is given, according to Ref. [47],

$$n(a_u) = 0.5(D-1)a_{u,l}^{0.5D-0.5} a_u^{-0.5D-0.5}. \quad (18)$$

8 The maximum truncated area  $a_{u,l}$  of a single asperity can be expressed as a function of the real  
9 contact area  $A_r$  [24], as follows,

$$a_{u,l} = 2 \frac{3-D}{D-1} A_r. \quad (19)$$

10 Once the fractal and material parameters are known by measurement, the real contact area  $A_r$   
11 can be obtained setting the normal force  $F_n$  (Eq. (17)) to be equal to the applied normal load  $N_{0i}$  (Eq.  
12 (6)) in the  $i^{\text{th}}$  annular region of the bolted joint interface. A detailed expression of the normal force  $F_n$   
13 is given in **Appendix A**. Then, the real contact area  $A_{ri}$  of the  $i^{\text{th}}$  annular contact region is used as a  
14 known parameter to solve for the tangential contact.

### 15 3.2.2 Tangential contact stiffness

16 Similarly, we first consider the tangential friction contact of a single asperity. Then the tangential  
17 contact stiffness  $k_{ti}$  of the  $i^{\text{th}}$  annular contact region can be determined by integrating the individual  
18 contact stiffness with the area distribution function  $n(a_u)$ . Note that the ‘tangential contact stiffness’  
19 here refers to the initial value of the tangential contact stiffness at the onset of the stick regime of the

1 contact interface. Mindlin [54] investigated the friction behavior of the contact between two elastic  
 2 spheres (equivalent to a elastic sphere interacting with a rigid plane) and derived the elastic  
 3 deformation of the sphere under a monotonic tangential load as

$$u = \frac{3\mu F_n}{16G^*r} \left[ 1 - \left( 1 - \frac{F_t}{\mu F_n} \right)^{2/3} \right], \quad (20)$$

4 where  $\mu$ ,  $G^*$ ,  $r$ , and  $F_t$  denote the friction coefficient, reduced shear modulus, contact radius, and  
 5 tangential load, respectively, and  $G^* = [(2 - \vartheta_1)/G_1 + (2 - \vartheta_2)/G_2]^{-1}$ . The tangential contact  
 6 stiffness of a single asperity can be obtained by finding the limit of the derivative  $\partial F_t / \partial u$  at  $u \rightarrow 0$ ,

$$k_{t\_asp} = \lim_{u \rightarrow 0} \frac{\partial F_t}{\partial u} = \lim_{u \rightarrow 0} 8G^*r \left( 1 - \frac{16G^*ru}{3\mu F_n} \right)^{1/2} = 8G^*r. \quad (21)$$

7 According to Hertz contact theory and the deformation geometry of a single asperity, the contact  
 8 radius can be determined by  $r = \sqrt{a_u/2\pi}$ . Substituting the contact radius into Eq. (21) yields the  
 9 tangential contact stiffness of a single asperity as a function of the truncated area  $a_u$ ,

$$k_{t\_asp} = 4 \sqrt{\frac{2}{\pi}} G^* a_u^{0.5}. \quad (22)$$

10 Finally, the tangential contact stiffness  $k_{ti}$  of the  $i^{\text{th}}$  annular contact region is determined by  
 11 integrating the product of  $k_{t\_asp}$  and the area distribution function,

$$k_{ti} = \int_{a_{u,e}}^{a_{u,l}} k_{t\_asp} n(a_u) da_u = \frac{4\sqrt{2}G^*(D-1)\psi^{1.5-0.5D}}{\sqrt{\pi}(2-D)a_{u,l}^{0.5-0.5D}} (a_{u,l}^{1-0.5D} - a_{u,e}^{1-0.5D}). \quad (23)$$

### 12 3.3.3 Friction hysteresis

13 In dynamic modeling of joint structures such as turbine blade systems, the phenomenological  
 14 models are frequently used to reproduce the friction hysteresis behavior of joint interfaces, due to their  
 15 good compatibility with nonlinear dynamics analysis methods. On the other hand, when the local  
 16 contact pressure is uniformly distributed, the contact surface is in the stick or slip regime under  
 17 tangential load. This behavior can be well captured by the Jenkins element (a basic component of the  
 18 Iwan model). Therefore, in this paper, the Iwan model is discretized to simulate the friction behavior  
 19 by comprehensively considering the non-uniform contact pressure distribution and the surface rough  
 20 characteristics, as shown in Fig. 6(a).

1 Different from the original Iwan model, the discretized Iwan model is composed of several  
 2 Jenkins elements in parallel and each Jenkins element has the different stiffness  $k_{ti}$  and critical sliding  
 3 force  $f_i = \mu N_{0i}$ , see Fig. 6(b and c). The stiffness  $k_{ti}$  is determined by the fractal contact theory (Eq.  
 4 (23)), while the critical sliding force  $f_i$  depends on the normal load distribution function (Eq. (6)).

5 For a monotonic tangential loading case, the force-displacement relation of Jenkins element can  
 6 be expressed as

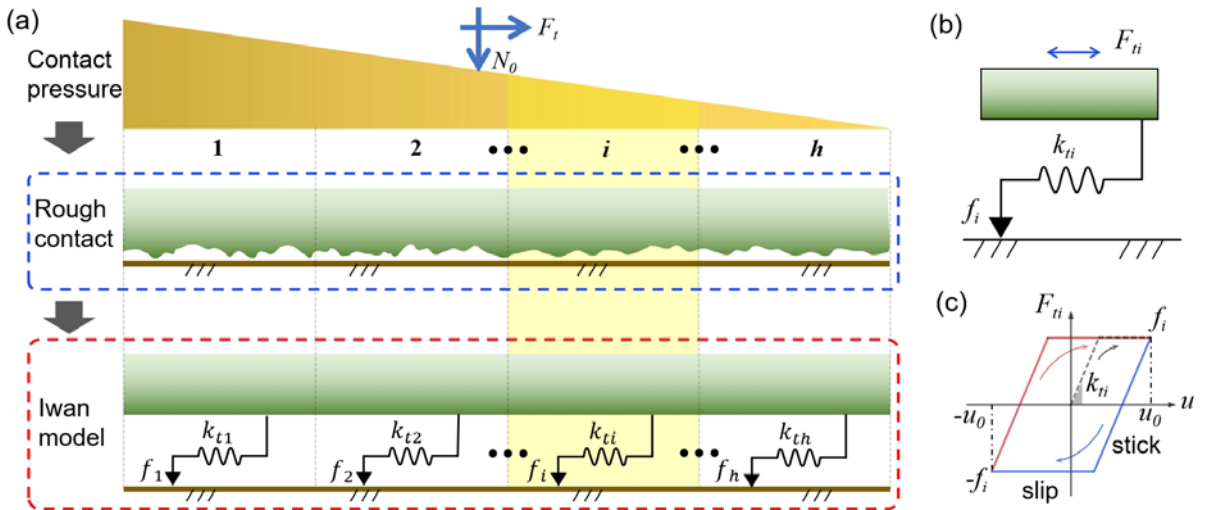
$$F_{ti} = \begin{cases} k_{ti}u, & 0 < u \leq \frac{f_i}{k_{ti}} \\ f_i, & \text{else} \end{cases} \quad (24)$$

7 For the unloading and reloading cases, the friction force can be determined as

$$F_{ti} = \begin{cases} k_{ti}(u - u_0) + f_i, & u_0 \geq \frac{f_i}{k_{ti}}, \dot{u} < 0, u_0 - \frac{2f_i}{k_{ti}} < u \leq u_0 \\ -f_i, & u_0 \geq \frac{f_i}{k_{ti}}, \dot{u} < 0, -u_0 < u \leq u_0 - \frac{2f_i}{k_{ti}} \\ k_{ti}(u + u_0) - f_i, & u_0 \geq \frac{f_i}{k_{ti}}, \dot{u} > 0, -u_0 \leq u < \frac{2f_i}{k_{ti}} - u_0 \\ f_i, & u_0 \geq \frac{f_i}{k_{ti}}, \dot{u} > 0, \frac{2f_i}{k_{ti}} - u_0 \leq u < u_0 \\ k_{ti}u, & u_0 < \frac{f_i}{k_{ti}} \end{cases} \quad (25)$$

8 where  $\dot{u}$  denotes the velocity and  $u_0$  the amplitude of displacement. The total friction force can be  
 9 obtained by

$$F_t = \sum_{i=1}^{i=h} F_{ti}. \quad (26)$$



10 **Fig. 6** (a) Schematic diagram of modeling the friction hysteresis behavior by considering the contact  
 11 pressure distribution and rough contact in the Iwan model. (b) Jenkins element and (c) corresponding  
 12

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1 force-displacement curve under periodic tangential loads. The blue and red curves represent the  
2 unloading and reloading cases, respectively.

3

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## 4 Results and discussion

In this section, the effectiveness of the proposed multi-scale contact modeling method is validated by comparison with the experimental results in Section 2. Then, a parametric analysis is performed to discuss the effect of the number of the divided annular sub-regions on the model accuracy.

### 4.1 Model parameters estimation

To assess the capability of the developed multi-scale contact model to reproduce the friction contact of bolted joint interfaces, the first necessary step is to estimate model parameters. These parameters include contact radius  $R_{max}$ , fractal dimension  $D$ , fractal roughness  $G$ , and friction coefficient  $\mu$ .

Refs. [19][55] investigated the contact pressure distribution at the interface of bolted joints using a direct measurement method (based on pressure film) and a finite element analysis method. The contact radius obtained by these two methods shows good agreement, with a relative error of less than 5%. Therefore, in this paper we employ the finite element contact analysis method to calculate the contact radius of the bolted joint interfaces. The finite element contact analysis was performed using the software Abaqus, and all components were meshed using 8-node linear hexahedron element (type C3D8R). The model has 31521 nodes, 25936 elements, and 1088 contact elements (only for each target contact surface). Figure 7(a) shows the contact pressure distribution obtained by finite element contact analysis and plots the contact radius as a function of the bolt preload. With the increase of bolt preload, the contact radius increases monotonically and slightly.

The fractal parameters,  $D$  and  $G$ , are identified using the PSD (Power Spectral Density) function method [56]. The logarithm of the PSD of surface profile can be expressed as

$$\begin{aligned} \log P(\omega) = & (2D - 7)\log\omega + (2D - 4)\log G - \log(\ln\gamma) + (6 - 2D)\log\pi \\ & + (5 - 2D)\log 2, \end{aligned} \quad (27)$$

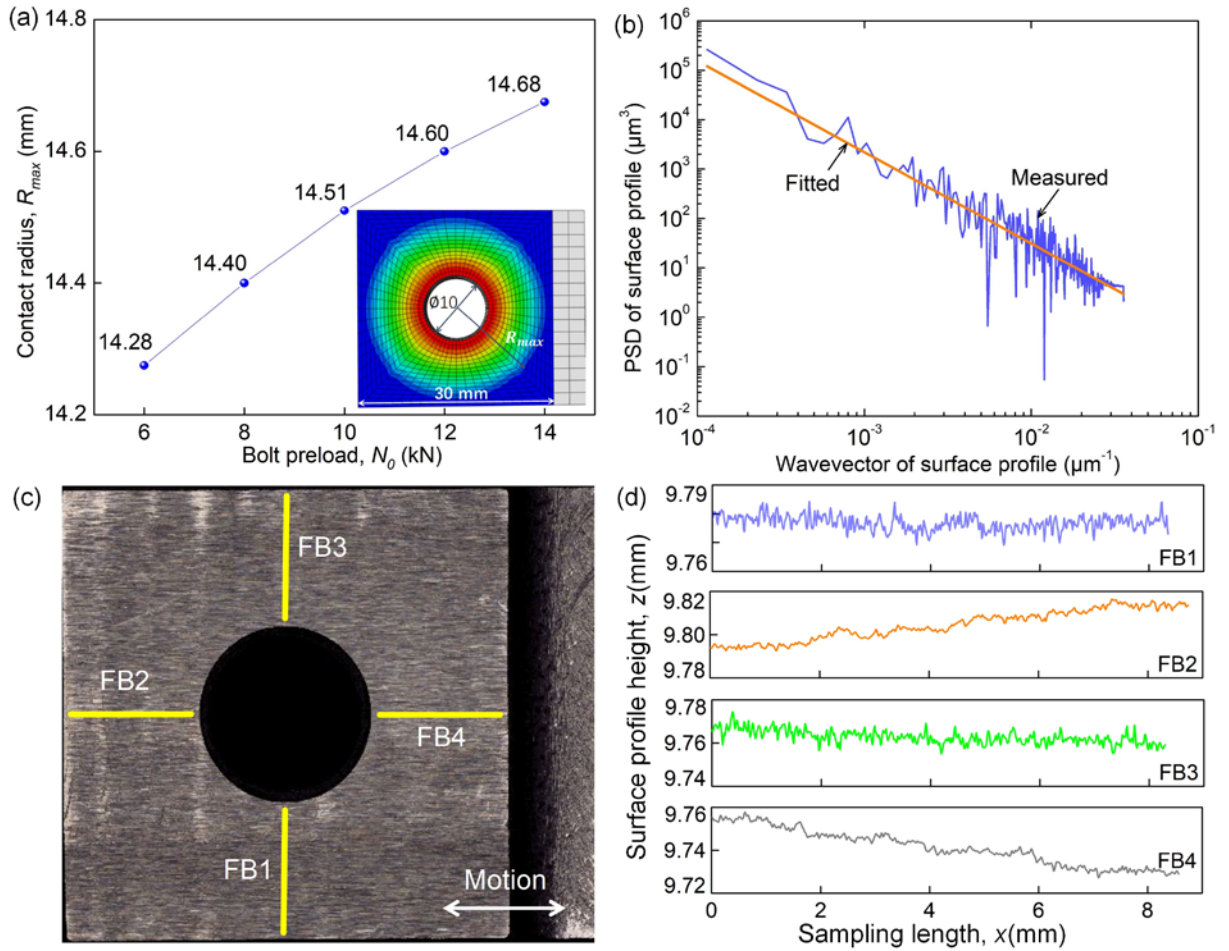
where  $P$  denotes the PSD and  $\omega$  the wavevector of surface profile. Once the PSD-wavevector curve of the surface profile height in the double logarithmic coordinate system is obtained, the fractal parameters,  $D$  and  $G$ , can be identified according to the following expression,

$$k_{PSD} = 2D - 7, \quad (28a)$$

$$b_{PSD} = (2D - 4)\log G - \log(\ln\gamma) + (6 - 2D)\log\pi + (5 - 2D)\log 2, \quad (28b)$$

1 where  $k_{PSD}$  and  $b_{PSD}$  are the slope and intercept of the PSD-wavevector curve, respectively. These  
 2 two parameters,  $k_{PSD}$  and  $b_{PSD}$ , can be directly determined from the fitted PSD-wavevector curve.

3 Before fretting tests, all contact surfaces are cleaned with acetone and then the surface topography  
 4 is measured using an optical 3D measurement system (Alicona G4 Infinite Focus). Figure 7(c) shows  
 5 the measured surface photography of the fixed specimen (set as an example), where the straight yellow  
 6 lines (FB1-4) represent the measurement paths for the surface profile height. Figure 7(d) depicts the  
 7 measured surface profile height as a function of the sampling length. The sampling length of the  
 8 measurement paths is around 9 mm.



9  
 10 **Fig. 7** (a) Contact radius  $R_{max}$  under different bolt preloads obtained from finite element contact  
 11 analysis. The inset shows the finite element model for one of the specimens and the contact pressure  
 12 distribution. (b) PSD curve of surface profile height (FB1 in Fig. 7c) in the double logarithmic  
 13 coordinate system as a function of wavevector: measurement and least square fitting. (c) Surface  
 14 photography measured by an optical 3D measurement system (Alicona G4 Infinite Focus). (d) Surface  
 15 profile height curves at different measurement positions (FB1-4).

From the measured surface topography data, the surface roughness  $R_a$  and the PSD curve can be obtained. The average value of the measured roughness is regarded as the roughness of the contact surface. The PSD curve is fitted using the least square method, as shown in Fig. 7(b), where an example (FB1 in Fig. 7(c)) is given. Substituting the slope and intercept of the fitted PSD curve into Eq.(28) yields the fractal parameters. Table 1 lists the surface roughness  $R_a$  and fractal parameters,  $D$  and  $G$ , of the contact surfaces. Since the material of the connected specimens is the same and the surface roughness is close, the equivalent fractal parameters are determined using the average slope  $k_{PSD}$  and intercept  $b_{PSD}$  of all fitted PSD curves. Note that the fractal parameters cannot be averaged directly.

**Table 1** Surface roughness and fractal parameters of contact surfaces. Moving\_A<sub>surf</sub>, Fixed\_A<sub>surf</sub>, Moving\_B<sub>surf</sub>, and Fixed\_B<sub>surf</sub> represent contact surfaces of joint specimens. Interface\_A and Interface\_B represent the two target interfaces.

Surface roughness, $R_a$ ( $\mu\text{m}$ )		Fractal parameters, $D$ and $G$		
Moving_A <sub>surf</sub>	1.28	Interface_A	$D$	2.523
Fixed_A <sub>surf</sub>	1.39		$G$ (m)	$8.11 \times 10^{-8}$
Moving_B <sub>surf</sub>	1.38	Interface_B	$D$	2.588
Fixed_B <sub>surf</sub>	1.47		$G$ (m)	$3.25 \times 10^{-9}$

Friction coefficient  $\mu$  is one of the parameters of the discretized Iwan model. This paper does not attempt to estimate the friction coefficient from the measured surface profile height. Therefore, to accurately predict the friction hysteresis response, the the friction coefficient is directly extracted from the measured hysteresis loops. The ratio of the friction force during the gross slip regime against the bolt preload is regarded as the friction coefficient. Moreover, the friction coefficients of two contact interfaces are assumed to be the same.

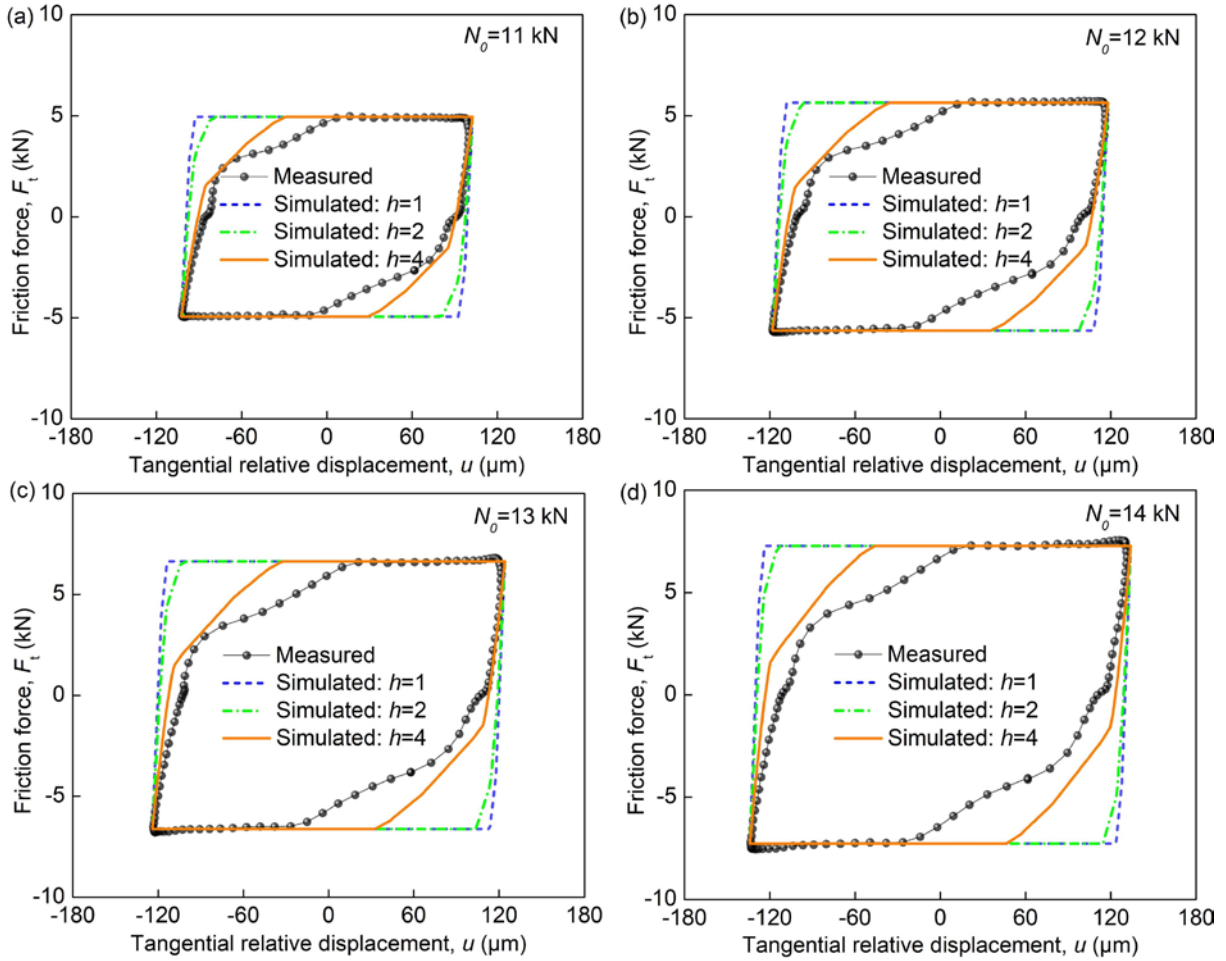
#### 4.2 Model validation

This section is meant for validation of the developed multi-scale modeling method and quantization of model performance. The material parameters of the connected components are as follows: 200 GPa elastic modulus, 0.3 Poisson's ratio, 86 GPa shear modulus, and 205 MPa yield strength. Four experimental hysteresis curves under different loading conditions were selected for

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1 comparison with the simulation results. Besides, a comparison among the discretized Iwan models  
2 consisting of different numbers of Jenkins elements (or divided sub-regions) was also involved.

3 **Figure 8** illustrates the simulated and the measured hysteresis curves for four different preloads.  
4 The approximate hexagonal hysteresis curves measured in the experiments can be well reproduced by  
5 the proposed contact model ( $h=4$ ). The difference between the simulated and the measured results  
6 mainly appears in the partial slip stage of joint interfaces. The reasons for this difference are manifold.  
7 On one hand, after several tests, the surface topography changes due to the wear effect, and the cleaning  
8 operation (before each test) can only eliminate the influence of wear particles. It may lead to the  
9 deviation of the identified fractal parameters. On the other hand, the actual contact pressure distribution  
10 of the joint interface may differ from the assumption due to surface waviness and possible additional  
11 bending moments. In addition, the gap effect observed in the experiments was not considered in the  
12 contact model, and the actual friction coefficient of the two contact surfaces is also different.  
13 Nevertheless, given that the modeling approach is physics-based and multi-scale, these differences are  
14 acceptable to some extent. Comparing with the cases ( $h=1$  and  $2$ ), the simulated results by the model  
15 ( $h=4$ ) match better with the experimental results. A detailed analysis of the effect of the number of  
16 Jenkins elements (or divided sub-regions) on the model prediction capability is given in section 4.3.

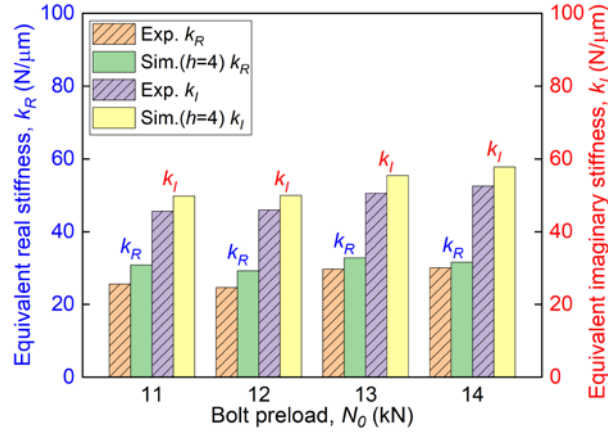


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2 **Fig. 8** Comparison of simulated hysteresis curves with experimental curves under different bolt  
3 preloads: (a)  $N_o=11$  kN, (b)  $N_o=12$  kN, (c)  $N_o=13$  kN, (d)  $N_o=14$  kN. The comparison among the models  
4 with different numbers ( $h=1, 2,$  and  $4$ ) of Jenkins elements is also involved.

5

6 To quantitatively evaluate the accuracy of the proposed modeling method, the equivalent real  
7 stiffness  $k_R$  (represents elastic stiffness) and the imaginary stiffness  $k_I$  (represents damping) are  
8 obtained according to the first-order Fourier expansion of the force-displacement curves [57]. As  
9 shown in Fig. 9, the equivalent real stiffness and imaginary stiffness obtained from the simulated  
10 curves are compared with those from the experimental curves. It can be seen that, except for the case  
11 of  $N_o=11$  kN and 12 kN, where the relative difference of the the equivalent real stiffness is large (about  
12 18%), the relative differences in the other comparisons are small (lower than 10%). Overall, the  
13 proposed multi-scale modeling method can reproduce the friction hysteresis behavior of bolted joint  
14 interfaces with acceptable accuracy.

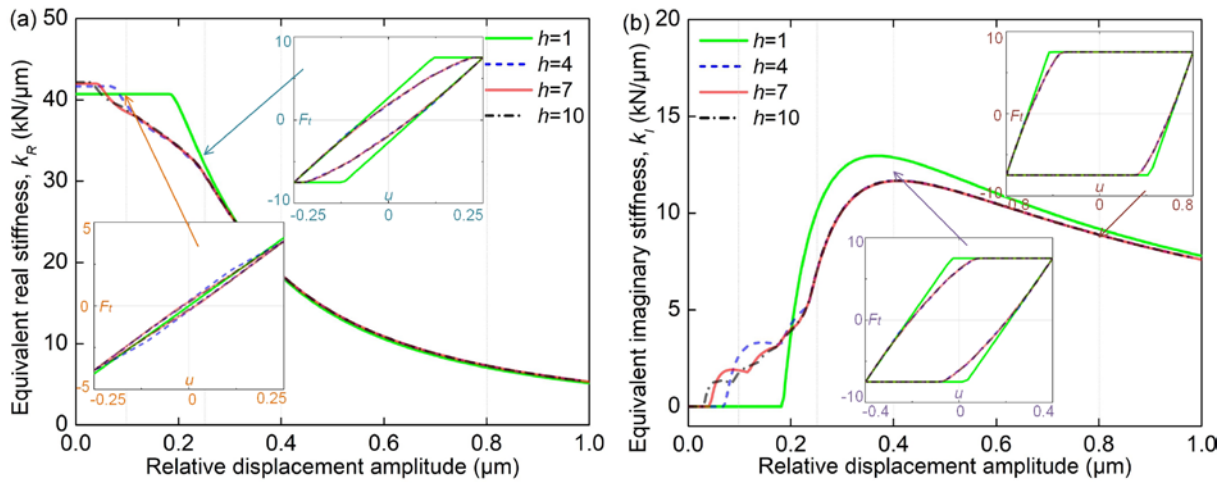


**Fig. 9** Quantitative comparison of the simulated equivalent real stiffness  $k_{eq}$  and the equivalent imaginary stiffness  $k_I$  with the experimental counterparts under different bolt preloads.

### 4.3 Effect of the interface discretization

This section evaluates the effect of the number of the divided sub-regions (or the Jenkins elements) on the model performance over a wide range of displacement amplitudes. A simple case with only one contact surface is considered. The model parameters are as follows: contact radius  $R_{max}=15$  mm, fractal dimension  $D=2.6$ , fractal roughness  $G=1\times 10^{-9}$ , friction coefficient  $\mu=0.5$ , and bolt preload  $N_0=15$  kN.

Figure 10 plots the equivalent real stiffness  $k_R$  and the imaginary stiffness  $k_I$  as a function of the relative displacement amplitudes. The comparison among the models with different numbers ( $h=1, 4, 7, \text{ and } 10$ ) of the divided sub-regions is also involved. With the increase of the number  $h$ , the difference in the equivalent stiffness predicted by different models becomes smaller and smaller. When the relative displacement amplitude is small, this difference is evident. In this case, the force-displacement curve predicted by the model with larger  $h$  is still a loop, while that obtained by the model with smaller  $h$  is a straight line (no hysteresis), see the inset in Fig.10(a). For the case of large displacement amplitude ( $>0.1 \mu\text{m}$ ), the difference in equivalent stiffness predicted by different models ( $h=4, 7, \text{ and } 10$ ) is barely visible. It can be concluded that under the comprehensive consideration of the model complexity and the prediction accuracy, the model with  $h=4$  is suitable for capturing the friction hysteresis behavior of bolted joint structures.



1

2 **Fig. 10** (a) Equivalent real stiffness  $k_R$  and (b) equivalent imaginary stiffness  $k_I$  as a function of relative  
 3 displacement amplitudes, obtained by the proposed modeling method with the different divided sub-  
 4 regions (or the Jenkins elements).

5

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## 5 Conclusions

In this paper, the tangential contact stiffness and friction hysteresis behavior of bolted joint interfaces were experimental and theoretically studied. In terms of experiments, a new test apparatus in which the joint specimens are arranged in a symmetric layout was developed to measure the friction force of bolted joint interfaces as purely as possible. In order to avoid the influence of additional moment generated during the installation process on the contact conditions of joint interfaces, the fixed specimen was designed to have high flexibility under the premise of ensuring the buckling stability of the entire assembly. The force-displacement curves under different loads were measured. The experimental friction hysteresis loops confirms the theoretical speculation about the origin of residual stiffness in Ref. [17] from the side.

A multi-scale contact modeling method was developed to reproduce the friction hysteresis behavior of bolted joint interfaces. This method comprehensively considers the multi-scale features of the randomly rough contact surface, the non-uniform distribution of contact pressure, and the convenience of the phenomenological contact model in simulating friction hysteresis, based on the fractal contact theory and the discretized Iwan model. The fractal contact theory was used to calculate the initial tangential contact stiffness of the jointed interface which is divided into several sub-regions for considering the contact pressure distribution. Then, a discretized Iwan model consisting of several Jenkins element with different contact stiffness (obtained from fractal contact analysis) and critical sliding forces was develop to describe the friction hysteresis. The effectiveness of the proposed method was verified comparing the simulation results with the experimental counterparts under different loading cases, which shows the potential for application to bolted joint structures.

It is worth emphasizing that in the fractal contact analysis the modeling of the initial tangential contact stiffness does not depend on the friction coefficient, so a physics-based model of friction coefficient could be incorporated into the method as a separate module in the future. The significance of this study is to provide structural engineers with a friction contact modeling approach for dynamic analysis of bolted joint structures without the need for complex contact experiments or dynamic testing in the initial design stage.

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## 1 Declaration of Competing Interest

2 The authors declare that they have no known competing financial interests or personal  
3 relationships that could have appeared to influence the work reported in this paper.

## 5 Acknowledgments

6 This work was financially supported by China Postdoctoral Science Foundation (grant number  
7 2022M712068), and the National Natural Science Foundation of China (grant numbers 12030215,  
8 12121002, and 12072268). The authors thanks Prof. Muzio Gola (in Politecnico di Torino) for his  
9 valuable advice on the design of the test apparatus.

## 11 Appendix A: Formula for total normal force

12 If  $a_{u,l} > a_{u,e}$ , the normal load can be expressed as

$$F_n = \int_0^{a_{u,p}} F_p n(a_u) da_u + \int_{a_{u,p}}^{a_{u,e}} F_{ep} n(a_u) da_u + \int_{a_{u,e}}^{a_{u,l}} F_e n(a_u) da_u. \quad (A1)$$

13 The first integral in Eq.(A1) represents the normal force of those asperities suffering from fully  
14 plastic deformation, which is equal to

$$F_{n,p} = \frac{0.5\eta\sigma_y(D-1)}{1.5-0.5D} a_{u,l}^{0.5D-0.5} a_{u,p}^{1.5-0.5D}. \quad (A2)$$

15 The second integral represents the normal force caused by the elastic-plastic deformation of  
16 asperities and equals,

$$F_{n,ep} = \left\{ \begin{array}{l} \frac{2^{13\beta-2\beta D-2} \pi^{\beta(D-4)}}{1.5+2\beta-\beta D-0.5D} (k_\vartheta H)^{1-2\beta} G^{2\beta(D-2)} (\ln\gamma)^\beta (D-1) a_{u,l}^{0.5D-0.5} \\ \times \left(\frac{E^*}{3}\right)^{2\beta} (a_{u,e}^{1.5+2\beta-\beta D-0.5D} - a_{u,p}^{1.5+2\beta-\beta D-0.5D}) \end{array} \right\}. \quad (A3)$$

17 The third integral represents the normal force of those asperities in the purely elastic deformation,

$$F_{n,e} = \frac{2^{4.5-D} E^* G^{D-2} (\ln\gamma)^{0.5} (D-1)}{(3\pi^{2-0.5D})(2.5-D)} a_{u,l}^{0.5D-0.5} (a_{u,l}^{2.5-D} - a_{u,e}^{2.5-D}). \quad (A4)$$

18 If  $a_{u,p} < a_{u,l} < a_{u,e}$ , the normal load is expressed as

---


$$F_n = \int_0^{a_{u,p}} F_p n(a_u) da_u + \int_{a_{u,p}}^{a_{u,l}} F_{ep} n(a_u) da_u$$

$$= \left\{ F_{n,p} + \frac{2^{13\beta-2\beta D-2} \pi^{\beta(D-4)}}{1.5 + 2\beta - \beta D - 0.5D} (k_\vartheta H)^{1-2\beta} G^{2\beta(D-2)} (\ln \gamma)^\beta (D-1) a_{u,l}^{0.5D-0.5} \left(\frac{E^*}{3}\right)^{2\beta} \right. \\ \left. \times (a_{u,e}^{1.5+2\beta-\beta D-0.5D} - a_{u,p}^{1.5+2\beta-\beta D-0.5D}) \right\} \quad (\text{A5})$$

1 If  $a_{u,l} < a_{u,p}$ , the normal load is expressed as

$$F_n = \int_0^{a_{u,l}} F_p n(a_u) da_u = \frac{\eta \sigma_y (D-1) a_{u,l}}{3-D} \quad (\text{A6})$$

2

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