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Fully Distributed Quantized Secure Bipartite Consensus Control of Nonlinear Multiagent Systems Subject to Denial-of-Service Attacks

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Abstract

This paper is intended to solve the distributed secure bipartite consensus problem of nonlinear multi-agent systems (MASs) with quantized information under Denial-of-Service (DoS) attacks. The attacks, which constrained on attack frequency and duration are studied. Based on the structurally balanced signed graph, we propose a novel secure output feedback control protocol integrated of the logarithmic quantizer and relative output measurements of neighboring agents, which can realize secure control under DoS attacks by choosing correctly the design parameters. We also develop an adaptive control protocol that includes dynamic coupling strengths into the control law and the state observer function. Different from the existing dis-

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tributed control schemes, this control strategy with dynamic coupling gains is fully distributed, under which agents are not required to know a priori knowledge of any global information and the quantizer only needs to quantize the output state error information of agents. Then, theoretical guarantees on the effectiveness of the proposed controllers in steering the system to a secure bipartite consensus under quantized output measurements and intermittent DoS attacks are derived. Finally, the numerical simulation inspired by a real-world physical network system is developed to verify the usefulness of the presented controllers.
Keywords: Nonlinear multi-agent systems (MASs), secure bipartite consensus, fully distributed control, observer-based control, quantized communication, DoS attacks.

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1. Introduction

In recent years, the cooperative control of multi-agent systems (MASs) has attracted much attention in the scientific community, as many real-world problems require multiple agents to cooperate with each other to collectively perform a task. Therefore, cooperative control of MASs has been extensively studied by many scholars [1, 2]. These theoretical advances have been widely used in several practical applications, including networked control systems [3], unmanned air vehicles [4], neural networks [5], and networked cyber-physical systems [6]. The main objective of the cooperative control for MASs is to drive a crowd of agents to consensus in a distributed fashion [7], and goes under the name of consensus problem. Depending on the presence of further requirements and specific characteristics of the scenario considered, different consensus problems have been formulated. In particular, we mention the leader-follower consensus, in which a flock of (follower) agents has to coordinate with the state of a leader by using distributed algorithms [8, 9].

However, in many practical situations, competitive and antagonistic interactions are also proposed [10]. For instance, in [11], it was observed that, in the industrial market, companies not only collaborate, but also compete for market resources. The co-existence of cooperative and competitive interactions has been observed as a key feature in government formation process in parliamentary democracies [12]. In a leader-follower framework, the authors of [13] investigated the cooperation and competition between employer and employees in management control systems. Signed graphs, originally proposed in [10], became a universal tool, widely used to describe and study networks with both cooperative and antagonistic relationships. The study on the dynamic behaviour of MASs over signed graphs can be traced back to the seminal work on linear systems, in which the authors investigated how the network structure determines whether the system converges to a collective agreement or a polarized scenario, termed bipartite consensus [14]. Concerning leader-follower dynamics, we mention leader-follower bipartite consensus under fuzzy sliding mode control [15], nonlinear dynamics [16], adaptive control [17], and finite-time consensus [18].

In the classical literature, authors often concentrated on the implementation for MASs and the design of control algorithms to achieve consensus in idealized scenarios, that is, ignoring important characteristics and limitations of the real environment in which the algorithms are implemented, such as limited communication resources [19], computer storage resources [20] and the actual malicious attacks [21]. These limitations hinder the possibility to apply classical consensus algorithms to achieve secure consensus against malicious cyber attacks in many real-world settings. Typically, there exist many different kinds of cyber attacks in MASs, including attacks on the dynamics of agents (for which we refer to [22, 23]) and on their communication [24, 25]. Within the second category, DoS attacks have received considerable attention in the past few years as it can be realized efficiently, thereby constituting a serious threat to the well-functioning of MASs [26, 27]. In [28], the authors presented the sufficient condition for the synchronization of MASs, in which the attacks can destroy the sensor-to-controller channels. In [29], the authors resolved the secure consensus problem of MASs under the strategic cyber attacks. In [30], the authors developed an efficient sampling control approach to realize the stability for networked control systems subject to DoS attacks. Although many studies have been performed on secure consensus control of MASs, it is usually supposed that only cooperative interactions are present and that the dynamic of each agent is linear, while competitive interactions and nonlinear systems are often present in the real world, calling for the development of new tools to deal with them. These motivated us to study secure bipartite consensus for nonlinear MASs under DoS attacks.

With the development of digital communication technology, communication constraints are becoming increasingly important in many practical situations. Quantized communication is a successful strategy to deal with these problems [31, 32]. Specifically, it has been shown that logarithmic quantizer can solve the consensus problem effectively in many different settings [33, 34]. Nevertheless, DoS attacks are commonly encountered in practical applications and it is more significant to analyze the application environment of MASs and consider the situation that the system faces both quantitative communication environment and DoS attacks. Then, the secure consensus control problem of MASs can be resolved by combining quantized information and DoS attacks [35, 36, 37]. As mentioned in above researches, global nonzero eigenvalues of Laplacian matrix were always needed, which would consume a lot of energy to process especially for a large-scale network. Therefore, developing a fully distributed control strategy without knowing a priori knowledge of any global information is in great demand. Then to avoid the utilization of global information, several attempts also have been made to investigate the fully distributed control strategy based on the event-triggered control strategy [38, 39, 40]. However, the event-triggered condition could cause unnecessary triggered instants and a few of efforts have been taken to study quantized secure consensus and fully distributed strategy under DoS attacks. This is another motivation for us to consider the fully distributed quantized secure bipartite consensus of MASs.

In addition, the relative state information of agents can not always be obtained in practical engineering. Therefore, the output feedback control played an important role in achieving asymptotic tracking by constructing a distributed controller [41, 42, 43]. However, the above mentioned literatures only study secure consensus problem under DoS attacks without quantized communication, and the limited relative state information of agents makes it difficult to consider the secure consensus, involving how to construct the output feedback control strategy without using any state information, how to combine the nonlinear control condition, how to construct the dynamic parameter without utilizing any global information and how to deal with the effects of competitive relationship between agents. These problems are challenging for realizing quantized secure bipartite consensus under DoS attacks.

Motivated by these works, we fill in this gap by considering the fully distributed secure bipartite consensus for nonlinear MASs with quantized communication subject to DoS attacks. After having formally defined the two controllers and illustrated the theorems to set the gain matrices, we performed a theoretical analysis of the proposed approaches. Through a Lyapunov-based argument, we prove that the two controllers are able to guarantee convergence of the system to a leader-follower bipartite consensus. Our theoretical findings are then illustrated via a numerical simulation on some case studies based on a real-world physical network system inspired by [21] and formed by Chua's circuit. The numerical findings show the good performances of the proposed controllers under the adaptive coupling gains, corroborating our theoretical results. The following fundamental issues are listed:

- Compared with some results using the full relative states of neighboring agents [8], [31], a novel distributed bipartite consensus control law based on quantized relative state measurements is proposed, in which only the relative output information of neighboring agents is utilized. Also different from [1], [17] that the dynamic of system state is linear, we consider a broad class of Lipschitz nonlinear dynamics, which are reflective of many real-world scenarios.

- Inspired by [28] and [33], both quantized communication and aperiodic DoS attacks are studied in the context of secure bipartite consensus over signed graph. The observer-based control strategy, based on the leader-follower framework, can guarantee that the consensus and observer errors go to zero by selecting the control parameters properly.

- Contrast to the traditional control protocols in related researches [16], [20], we also develop a new control law depending on quantized output state measurements, which is fully distributed and do not need to know a priori knowledge of any global information. Afterwards, some criteria are presented to guarantee the secure bipartite consensus under DoS attacks.

- The elements, including fully distributed control, logarithmic quantizer, DoS attacks, observer based control approach, and antagonistic interactions are investigated simultaneously for the first time to consider secure control. The derived results are more general.

The rest of the article is summarized as follows. In Section 2, we develop the notation and some preliminary results. In Section 3, we formulate the problem. In Section 4, we present our main findings, with proofs reported in the appendices. In Section 5, we formulate the numerical simulations. In Section 6, we conclude the paper and outline avenues for future research.

2. Notation and Preliminaries

2.1. Notation

We gather here the notation used throughout this paper. We denote by $\mathbb{R}^n$ the n-dimensional Euclidean space and by $\mathbb{N}_+$ the set of strictly positive integers. The $N \times N$ identity matrix is denoted by I_N. The Euclidean norm is denoted as $\|\cdot\|$. The symbol $\otimes$ is the Kronecker product, $\operatorname{sgn}(\cdot)$ is the sign function, and $\operatorname{diag}(\cdot)$ is the diagonalization operator. Given a matrix M, $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ represent its minimum and maximum eigenvalues, respectively. We also let $\mathcal{S}=\{S = \operatorname{diag}(s_1, s_2, \cdots, s_N), s_i \in \{-1, 1\}\}$.

2.2. Graph Theory

Consider a set $\mathcal{V} = \{1, \ldots, N\}$ of N agents (also referred to as *followers*) and one leader, labeled as $\{0\}$. Followers are connected through a (signed di-)graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. Specifically, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges, with $(j, i) \in \mathcal{E}$ if i can access information from j; and $\mathcal{A} \in \mathbb{R}^{N\times N}$ is the (signed) weighted adjacency matrix, whose generic entry a_{ij} measures the information that i receives from j; $a_{ij} \neq 0$ if and only if $(j, i) \in \mathcal{E}, i \neq j$, and $a_{ij} = 0$ otherwise. In addition, assume $a_{ii} = 0, i = 1, 2, \cdots, N$. Given the (signed) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, we define its (signed) Laplacian matrix $L = (l_{ij}) \in \mathbb{R}^{N\times N}$ entry-wise as follows:

$$l_{ij} := \begin{cases} \sum\limits_{j\in\mathcal{V}\setminus\{i\}} |a_{ij}|, & \text{if } i = j, \\ -a_{ij}, & \text{if } i \neq j. \end{cases} \tag{1}$$

Definition 1 (Structural balance). *A signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is structurally balanced if there is a partition of the agent set $\mathcal{V}_1$, $\mathcal{V}_2$ satisfying i) $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, ii) $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, iii) $a_{ij} \geq 0, \forall\, v_i, v_j \in \mathcal{V}_k\,(k \in \{1, 2\})$, and iv) $a_{ij} \leq 0, \forall\, v_i \in \mathcal{V}_k, v_j \notin \mathcal{V}_k\,(k \in \{1, 2\})$. if not, it is said to be structurally unbalanced.*

Lemma 1. [16] For the graph $\mathcal{G}$, there is a diagonal matrix $S \in \mathcal{S}$ such that the diagonal entries of SLS are positive, and the off-diagonal entries of SLS

are negative. In addition, S produces a division, i.e., $\mathcal{V}_1 = \{i \,|s_i > 0\}$ and $\mathcal{V}_2 = \{i \,|s_i < 0\}$ that satisfies properties i)–iv) in Definition 1.

In this paper, we consider an augmented graph $\mathcal{G}_R$ formed by the set of N followers and the leader. The augmented graph has thus agent set $\mathcal{V}_R = \mathcal{V} \cup \{0\}$ and edge set $\mathcal{E}_R = \mathcal{E} \cup \{(j, 0) : j \in \mathcal{N}_0\}$, where N_0 is the set of followers that can access the information on the leader's state. We define a nonnegative $N \times N$-dimensional diagonal matrix $R = \text{diag}([a_{10}, \dots a_{N0}])$, whose entry $a_{i0} \geq 0$ measures how much follower i interacts with the leader, with the understanding that $a_{i0} > 0$ if and only if $(i, 0)$ in $\mathcal{E}_R$. We can finally define $\bar{L} = SLS + R$, $L_R = L + R$. Based on Definition 1, one obtains $\bar{L}$ is positive definite, i.e., $\bar{L} > 0$.

2.3. Cyber Attack: Aperiodic DoS Attack Model

In this paper, assume the DoS attacks could damage temporarily both the communication and the control channel. Each DoS attack occurs over a finite time window, termed *attack interval*, after which the MAS could recover to the initial communication and control channels. Hence, DoS attacks constitute a sequence of attack intervals parametrized by a positive integer $k \in \mathbb{N}_+$. Specifically, the kth attack interval is defined as $\mathrm{T}_k := [t_k, t_k + \tau_k)$, where t_k is the time instant at which the kth attack begins, and τ_k is the duration of the kth attack. Considered a generic time interval $[t_1, t_2)$, when attack exists, a sequence of time intervals can be denoted as

$$\mathrm{T}_d\,(t_1, t_2) = \bigcup_{k \in \mathbb{N}_+} \{\mathrm{T}_k\} \cap [t_1, t_2], \tag{2}$$

and its complement $\mathrm{T}_f\,(t_1, t_2) := [t_1, t_2) \setminus \mathrm{T}_d\,(t_1, t_2)$ denotes a sequence of time intervals that no attacks occur.

During the attach intervals, every $\lambda\!\!\!\bar{} > 0$ time units, starting from the time instant in which the attach has occurred. The MAS does not resume communication immediately after the DoS attack, but only after a (successful) attempt of communication. Hence, the *effective attack interval* of the kth DoS attack may be longer than the attack interval, and it is equal to $\bar{\mathrm{T}}_k = [t_k, t_k + \bar{\tau}_k)$, where $\bar{\tau}_k = \min\{t \geq \tau_k : t/\lambda\!\!\!\bar{} \in \mathbb{N}_+\}$. The effective DoS attack time interval set and its complement can be denoted as

$$\begin{aligned}\bar{\mathrm{T}}_d\left(t_1, t_2\right) &= \bigcup_{k \in \mathbb{N}_+} \{\bar{\mathrm{T}}_k\} \cap [t_1, t_2], \\ \bar{\mathrm{T}}_f\left(t_1, t_2\right) &= [t_1, t_2) \backslash \bar{\mathrm{T}}_d\left(t_1, t_2\right).\end{aligned}$$

Definition 2. *Denote $N\left(t_1, t_2\right)$ as the number of DoS attacks in the interval $[t_1, t_2)$, and the attack frequency can be concluded as*

$$\Lambda\left(t_1, t_2\right) = \frac{N\left(t_1, t_2\right)}{t_2 - t_1}.$$

Assumption 1. *Define $\left|\mathrm{T}_d\left(t_1, t_2\right)\right|$ as the total duration of the DoS attacks in the time interval $[t_1, t_2)$. And there exist $T_0 \geq 0$, $\Lambda_0 \geq 0$, $T_1 > 1$, $\Lambda_1 > 1$ such that*

$$\begin{aligned}\left|\mathrm{T}_d\left(t_1, t_2\right)\right| &\leq T_0 + \frac{t_2 - t_1}{T_1}, \\ N\left(t_1, t_2\right) &\leq \Lambda_0 + \frac{t_2 - t_1}{\Lambda_1}.\end{aligned}$$

2.4. Logarithmic Quantizer

The quantizer $q : \mathbb{R} \to \mathbb{R}$ is assumed to be logarithmic and can be described by

$$q(r) = \begin{cases} \Im_i, & \text{if } \frac{1}{1+\xi}\Im_i < r \leq \frac{1}{1-\xi}\Im_i, r > 0, \\ 0, & \text{if } r = 0, \\ -q\left(-r\right), & \text{if } r < 0, \end{cases} \tag{3}$$

Then the accuracy constant $\xi \in (0, 1)$. The set of quantized levels can be denoted as

$$\bar{\Im} = \left\{ \pm\Im_i, \Im_i = \left(\frac{1-\xi}{1+\xi}\right)^i \Im_0, i = \pm1, \pm2, ... \right\}$$
$$\cup \{\pm\Im_0\} \cup \{0\}.$$

According to the conception of the quantizer, one has $|q(a) - a| \leq \xi |a|, \forall a \in \mathbb{R}$. For $\aleph = [\aleph_1, \aleph_2, \cdots \aleph_n]^{\mathrm{T}} \in \mathbb{R}^n$, and $q(\aleph) = [q(\aleph_1), q(\aleph_2), \cdots, q(\aleph_n)]^{\mathrm{T}}$, one has $q(\aleph) - \aleph = \mathrm{H}\aleph$, in which $\mathrm{H} = \mathrm{diag}\{\mathrm{H}_1, \mathrm{H}_2, \cdots, \mathrm{H}_n\}$ and $\mathrm{H}_i \in [-\xi, +\xi]$.

3. Problem Formulation

In this paper, consider the MAS made of a group of N followers and a leader. Each follower $i \in \mathcal{V}$ is characterized by a *state vector* $x_i(t) \in \mathbb{R}^n$, an *input vector* $u_i(t) \in \mathbb{R}^m$, and an *output measurement vector* $z_i(t) \in \mathbb{R}^r$, and its dynamic is described by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + f(x_i(t), t) + Bu_i(t), \\ z_i(t) &= Cx_i(t), \end{aligned} \tag{4}$$

in which $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{r\times n}$, and $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a function continuous and differentiable in t. Note that all the followers have the same dynamics. The dynamic of the leader can be instead given by the following equation

$$\dot{x}_0(t) = Ax_0(t) + f(x_0(t), t), \tag{5}$$

in which $x_0(t) \in \mathbb{R}^n$ stands for the leader's state. Note that, the state of the leader evolves as an autonomous nonlinear system, that is, $u_0(t) = 0$, while the states of the followers are influenced by the external input. In this paper, we will study whether the MAS made by the leader and the N followers converges to a bipartite consensus. Specifically, we introduce the following definition.

Definition 3 (Leader-follower bipartite consensus). *The leader-follower bipartite consensus problem of MAS (5) and (6) can be resolved for some* $k \in \{1, 2\}$ *if*

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, \forall\ i \in \mathcal{V}_k,$$
$$\lim_{t\to\infty} \|x_i(t) + x_0(t)\| = 0, \forall\ i \in \mathcal{V}_{3-k},$$

which can be further written by

$$\lim_{t\to\infty} \|x_i(t) - s_i x_0(t)\| = 0, i = 1, 2, \cdots, N.$$

Note that, differently from other notions of consensus [8], in Definition 3 we are not requiring that the leader converges to a fixed point, but we say that a leader-follower bipartite consensus problem can be solved if the entire system synchronizes toward a trajectory in which a set of followers has the same state of the leader, and the remaining followers have the opposite state.

Assumption 2. *The pair* (A, B, C) *is stabilizable and detectable.*

Assumption 3. *There exists a non-negative constant* $\rho > 0$ *such that*

$$\|f(a_1, t) - s_i f(a_2, t)\| \leq \rho \|a_1 - s_i a_2\|, \ \ \forall\, a_1, a_2 \in \mathbb{R}^n. \tag{6}$$

Lemma 2. [27] Consider a MAS with dynamics from (4) and (5) that satisfy Assumption 3, with a sequence of DoS attacks. If the system-related piecewise Lyapunov function satisfies: 1) when there is no DoS attacks, that is $t \in \bar{\mathrm{T}}_f$,

$$V(t) = \tilde{V}(t), \dot{V}(t) \leq -a_1 \tilde{V}(t) + a_2,$$

2) When there exist DoS attacks, that is $t \in \bar{\mathrm{T}}_d$,

$$V(t) = \hat{V}(t), \dot{V}(t) \leq a_3 \hat{V}(t) + a_4,$$

in which $a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0$, T_1 and Λ_1 satisfy the conditions:

$$\begin{aligned} \frac{1}{T_1} &< \frac{a_1 - \tau}{a_1 + a_3}, \\ \frac{1}{\Lambda_1} &< \frac{\tau}{2\ln\mu + (a_1 + a_3)\bar{\lambda}}, \end{aligned}$$

where $0 < \tau < a_1$, $\bar{\lambda} > 0$, $\mu \geq 1$, and the following inequalities hold:

$$\begin{cases} \mu \hat{V}\left((t_k + \bar{\tau}_k)^-\right) - \tilde{V}(t_k + \bar{\tau}_k) \geq 0, \\ \mu \tilde{V}\left(t_{k+1}^-\right) - \hat{V}(t_{k+1}) \geq 0, \end{cases}$$

in which $k \in N$. Therefore, $V(t)$ is bounded.

Remark 1. *Assumption 1 is concerned with the attack strength and attack frequency for the DoS attacks and it is a standard assumption made in the literature on consensus problems of MASs subjects to DoS attacks [26, 41], which bounds the maximum attack frequency. However, different from many other works in the literature [35, 40], we do not rely on the more restrictive assumption that DoS attacks are periodic.*

Remark 2. *Assumption 3 restricts the set of nonlinear functions that our controller is able to deal with those that verifies the Lipschitz condition. Note* *that all linear and piece-wise linear time-invariant continuous functions satisfy this condition, as well as many nonlinear functions often used in cyber-physical systems* [5, 8]. It is still an open problem to extend these theoretical findings to ensure secure bipartite consensus for MASs under DoS attacks and nonlinear dynamics which do not satisfy the Lipschitz condition. The design approaches presented in [6, 26] might be useful for investigating this this direction in the future research.

4. Main Results

In this section, secure bipartite consensus control of nonlinear MASs subject to DoS attacks and quantized communication is solved by both the static protocol and the adaptive protocol, respectively.

4.1. Secure bipartite consensus with static protocol under DoS attacks

In this subsection, consider the bipartite consensus of the MASs in (4)-(5) over a static control protocol. An observer-based controller based on the output measurements is developed by defining the following input functions for the followers:

$$u_i(t) = \begin{cases} cK\varphi_i(t), & \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ 0, & \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases} \tag{7}$$

in which $c > 0$ is a coupling strength, K is the feedback gain matrix, and the combining measurement $\varphi_i(t)$ satisfying

$$\varphi_i\left(t\right) = \Big[\sum_{j=1}^{N} |a_{ij}| \left(\hat{x}_i\left(t\right) - \mathrm{sgn}\left(a_{ij}\right) \hat{x}_j\left(t\right)\right) + a_{i0}\left(\hat{x}_i\left(t\right) - s_i x_0\left(t\right)\right)\Big], \tag{8}$$

and $\hat{x}_i(t)$ denotes the state observer. Then, one obtains

$$\dot{x}_i(t) = \begin{cases} Ax_i(t) + f\left(x_i(t), t\right) + cBK\Big[\sum\limits_{j=1}^{N} |a_{ij}|(\hat{x}_i(t) - \operatorname{sgn}\left(a_{ij}\right)\hat{x}_j(t)) \\ \quad + a_{i0}\left(\hat{x}_i(t) - s_i x_0\left(t\right)\right)\Big], \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ Ax_i(t) + f\left(x_i(t), t\right), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

and

$$\dot{\hat{x}}_i(t) = \begin{cases} A\hat{x}_i(t) + f\left(\hat{x}_i\left(t\right), t\right) + \varepsilon Fq[\sum\limits_{j=1}^{N} |a_{ij}|\left(\tilde{e}_i\left(t\right) - \operatorname{sgn}\left(a_{ij}\right)\tilde{e}_j\left(t\right)\right) \\ \quad + a_{i0}\left(\tilde{e}_i\left(t\right) - \operatorname{sgn}\left(a_{i0}\right)\tilde{e}_0\left(t\right)\right)] + Bu_i(t), \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ A\hat{x}_i(t) + f\left(\hat{x}_i\left(t\right), t\right), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

in which $\varepsilon > 0$ and F is the feedback matrix to be designed. After that, for any follower $i \in \mathcal{V}$, define $\tilde{e}_i\left(t\right) = z_i\left(t\right) - C\hat{x}_i\left(t\right)$ as the error between the measurement output, $z_i(t)$, and the corresponding quantity computed from the state observer, $C\hat{x}_i\left(t\right)$, for all $i \in \mathcal{V}$. Since the leader acts as a reference signal generator, it is supposed that $\hat{x}_0\left(t\right) = x_0\left(t\right)$, i.e., the leader does not need to observe its own state, and it holds $\tilde{e}_0\left(t\right) = z_0\left(t\right) - C\hat{x}_0\left(t\right) = z_0\left(t\right) - Cx_0\left(t\right) = 0$. Define the following two errors:

$$\bar{e}_i(t) = x_i(t) - \hat{x}_i(t), \quad \hat{e}_i(t) = \hat{x}_i(t) - s_i x_0\left(t\right), \tag{9}$$

in which $\bar{e}_i(t)$ represents the observer error between the agent i and its observer and $\hat{e}_i(t)$ denotes consensus tracking error between the observer of agent i and the leader or its opposite side, respectively. Based on the forgoing analysis, since $s_i s_j a_{ij} \geq 0, i, j = 1, \cdots N$, one obtains $a_{ij} s_i = |a_{ij}|\, s_j$ and

$|a_{ij}|\, s_i = a_{ij}\mathrm{sgn}\,(a_{ij})\, s_i = |a_{ij}|\, s_i\mathrm{sgn}\,(a_{ij})$. Hence, one obtains

$$\dot{\bar{e}}_i\,(t) = \begin{cases} A\bar{e}_i\,(t) + f\,(x_i\,(t)\,, t) - f\,(\hat{x}_i\,(t)\,, t) - \varepsilon Fq\Big[\sum\limits_{j=1}^{N} |a_{ij}| \cdot \\ \quad \cdot\,(\tilde{e}_i\,(t) - \mathrm{sgn}\,(a_{ij})\,\tilde{e}_j\,(t)) + a_{i0}\tilde{e}_i\,(t)\Big], \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ A\bar{e}_i\,(t) + f\,(x_i\,(t)\,, t) - f\,(\hat{x}_i\,(t)\,, t)\,, \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t). \end{cases}$$

Similarly, we compute

$$\dot{\hat{e}}_i\,(t) = \begin{cases} A\hat{e}_i\,(t) + f\,(\hat{x}_i\,(t)\,, t) - s_i f\,(x_0\,(t)\,, t) + \varepsilon Fq\Big[\sum\limits_{j=1}^{N} |a_{ij}| \cdot \\ \cdot\,(\tilde{e}_i\,(t) - \mathrm{sgn}\,(a_{ij})\,\tilde{e}_j\,(t)) + a_{i0}\tilde{e}_i\,(t)\Big] + Bu_i(t), \quad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ A\hat{e}_i\,(t) + f\,(\hat{x}_i\,(t)\,, t) - s_i f\,(x_0\,(t)\,, t)\,, \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t). \end{cases}$$

According to the concepts of the Krasovskii solution and logarithmic quantizer [20], one choose $\mathrm{Z}_i\,(t) \in \mathcal{K}\,(\mathrm{H}_i\bar{e}_i\,(t))$ and by utilizing Kronecker products, one can write the equations for the errors into a compact matrix form as

$$\dot{\bar{e}}\,(t) = \begin{cases} [(I_N \otimes A) - \varepsilon\,(L_R \otimes FC)]\,\bar{e}\,(t) - \varepsilon\,(L_R \otimes FC)\,\mathrm{Z}\,(\mathrm{t}) \\ +I_N \otimes (f\,(x\,(t)\,, t) - f\,(\hat{x}\,(t)\,, t))\,, \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ (I_N \otimes A)\,\bar{e}\,(t) + I_N \otimes (f\,(x\,(t)\,, t) - f\,(\hat{x}\,(t)\,, t))\,, \quad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

and

$$\dot{\hat{e}}\,(t) = \begin{cases} [I_N \otimes A + c\,(L_R \otimes BK)]\,\hat{e}\,(t) + \varepsilon\,(L_R \otimes FC)\,\mathrm{Z}\,(\mathrm{t}) + \varepsilon\,(L_R \otimes FC)\,\bar{e}\,(t) \\ \quad + (f\,(\hat{x}\,(t)\,, t) - (SI_N \otimes f\,(x_0\,(t)\,, t)))\,, \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ (I_N \otimes A)\,\hat{e}\,(t) + (f\,(\hat{x}\,(t)\,, t) \\ \quad - (SI_N \otimes f\,(x_0\,(t)\,, t)))\,, \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

in which $f\,(x(t), t) := \left[f^\top\,(x_1(t), t)\,, \cdots, f^\top\,(x_N(t), t)\right]^\top$, $f\,(\hat{x}(t), t) := \left[f^\top\,(\hat{x}_1(t), t)\,, \cdots, f^\top\,(\hat{x}_N(t), t)\right]^\top$, $\bar{e}\,(t) := \left[\bar{e}_1^\top\,(t)\,, \bar{e}_2^\top\,(t)\,, \cdots, \bar{e}_N^\top\,(t)\right]^\top$, $\hat{e}\,(t) := \left[\hat{e}_1^\top\,(t)\,, \hat{e}_2^\top\,(t)\,, \cdots, \hat{e}_N^\top\,(t)\right]^\top$.

Theorem 1. *Assume the MAS in (4)-(5) satisfying Assumptions 1-3. Under the controller form (7), the MAS achieves leader-follower bipartite consensus (Definition 3) with feedback matrices* $K = -B^{\mathrm{T}}P, F = P^{-1}C^{\mathrm{T}}, \Gamma = PBB^{\mathrm{T}}P$*, and* $\tilde{\Gamma} = C^{\mathrm{T}}C$*, if there are two positive definite matrices* P *and* Q *and positive constants* $m_1, m_2, n_1, n_2, \rho > 0$ *and* $\tau \in (0, m_0)$ *such that the following conditions are satisfied:*

$$\begin{bmatrix} \Theta_1 & \rho P & C \\ \rho P & -I & 0 \\ * & 0 & -I \end{bmatrix} < 0, \begin{bmatrix} \Theta_2 & \rho P \\ \rho P & -I \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} \Theta_3 & \rho Q \\ \rho Q & -I \end{bmatrix} < 0, \begin{bmatrix} \Theta_4 & \rho Q \\ \rho Q & -I \end{bmatrix} < 0, \tag{11}$$

$$\mu = \max\left\{\frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}, \frac{\lambda_{\max}(Q)}{\lambda_{\min}(P)}\right\}, \tag{12}$$

$$\Lambda(t_0, t) \leq \frac{\tau}{2\ln\mu + (m_0 + n_0)\bar{\lambda}}, \tau \in (0, m_0), \tag{13}$$

$$\frac{1}{T_1} \leq \frac{m_0 - \tau}{m_0 + n_0}, \tag{14}$$

in which

$$\begin{aligned}
\Theta_1 =& A^T P + PA - \left(PB + B^T P\right) + I_N + m_1 P, \\
\Theta_2 =& A^T P + PA - C^{\mathrm{T}}C + I_N + m_2 P, \\
\Theta_3 =& A^T Q + QA + I_N - n_1 Q, \\
\Theta_4 =& A^T Q + QA + I_N - n_2 Q,
\end{aligned}$$

with constant T_1 defined in Assumption 1, $m_0 = \min_{i\in\{1,2\}} \{m_i\}, n_0 = \max_{i\in\{1,2\}} \{n_i\}, c \geq \frac{1}{2\lambda_1}, \varsigma = \varepsilon (k_1 + k_2) \leq \frac{1}{\lambda_N}, \bar{\varsigma} = \left(2\varepsilon - \frac{\xi^2}{k_1} - \frac{1}{k_2} - k_3 - \frac{\xi^2}{k_3}\right) \geq \frac{1}{\lambda_1}$, *and* λ_1 *and* λ_N *are the smallest nonzero eigenvalue and largest eigenvalue of* $\bar{L}$.

Proof: At this stage, based on a structurally balanced communication network, we can analytically prove that the observer-based control law (7) solves the quantized secure bipartite consensus for nonlinear MAS in (4)-(5) subject to DoS attacks. The following result formally guarantees our claim. The proof, which is based on a Lyapunov argument to show convergence to 0 for the two quantities in (9), is quite cumbersome and is thus reported in Appendix A, for the sake of readability.

Remark 3. *In contrast to the related references* [6, 17, 20], a static observer-based control law based on the output information is developed to ensure the secure bipartite consensus of MASs subject to DoS attacks. The result is more relevant because the state information of agents are not always available. That is to say, our results are not limited by the state information of agents. Inspired by [31, 35], a controller based on logarithmic quantizer has been poposed, which just need to quantize the output information of agents, and it can solve the secure bipartite consensus problem with communication constraints and load more effectively. As partly depicted in the proof of Theorem 1, the influences of non-uniform quantitative information, DoS attacks, the limitation of state information of agents, nonlinear term and competitive relationships between agents make it more challenging to achieve secure bipartite consensus.

4.2. Secure bipartite consensus with fully adaptive protocol under DoS attacks

In this subsection, consider the bipartite consensus of the MASs in (4)-(5) over a fully distributed control protocol. Then, the fully adaptive control law can be designed as follows

$$u_i(t) = \begin{cases} \hat{\mathrm{c}}_{ij}(t) K\varphi_i(t), & \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ 0, & \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases} \tag{15}$$

where $\hat{\mathrm{c}}_{ij}(t)$ denotes the adaptive coupling strength, and it satisfies

$$\dot{\hat{\mathrm{c}}}_{ij}(t) = -\zeta \sum_{j=1}^{N} a_{ij}\hat{\mathrm{c}}_{ij}(t) + \zeta\varphi_i^{\mathrm{T}}(t)\Gamma\varphi_i(t), \tag{16}$$

in which $\hat{\mathrm{c}}_{ij}(0) > 0$, ζ is an arbitrarily chosen positive constant. Then, one has

$$\dot{x}_i(t) = \begin{cases} Ax_i(t) + f(x_i(t), t) + \hat{\mathrm{c}}_{ij}(t) BK\Big[\sum\limits_{j=1}^{N} |a_{ij}|(\hat{x}_i(t) - \mathrm{sgn}(a_{ij})\hat{x}_j(t)) \\ \quad + a_{i0}(\hat{x}_i(t) - s_i x_0(t))\Big], \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ Ax_i(t) + f(x_i(t), t), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

and

$$\dot{\hat{x}}_i(t) = \begin{cases} A\hat{x}_i(t) + f(\hat{x}_i(t), t) + \hat{\varepsilon}_{ij}(t) Fq[\sum\limits_{j=1}^{N} |a_{ij}|(\tilde{e}_i(t) - \mathrm{sgn}(a_{ij})\tilde{e}_j(t)) \\ \quad +a_{i0}(\tilde{e}_i(t) - \mathrm{sgn}(a_{i0})\tilde{e}_0(t))] + Bu_i(t), \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ A\hat{x}_i(t) + f(\hat{x}_i(t), t), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

where the $\hat{\varepsilon}_{ij}(t)$ is an another adaptive coupling strength, and it satisfies

$$\dot{\hat{\varepsilon}}_{ij}(t) = -\tilde{\zeta} \sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}(t) - \tilde{\zeta}\varphi_i^{\mathrm{T}}(t)\tilde{\Gamma}\varphi_i(t) + \tilde{\zeta}\tilde{\varphi}_i^{\mathrm{T}}(t)\tilde{\Gamma}\tilde{\varphi}_i(t), \tag{17}$$

in which the initial condition $\hat{\varepsilon}_{ij}(0) > 0$, the constant $\tilde{\zeta} > 0$, and

$$\tilde{\varphi}_i(t) = q\Big[\sum_{j=1}^{N} |a_{ij}| \left(\tilde{e}_i(t) - \operatorname{sgn}(a_{ij})\, \tilde{e}_j(t)\right) + a_{i0}\tilde{e}_i(t)\Big]. \tag{18}$$

According to (9), one has

$$\dot{\bar{e}}_i(t) = \begin{cases} A\bar{e}_i(t) + f(x_i(t), t) - f(\hat{x}_i(t), t) - \hat{\varepsilon}_{ij}(t) Fq\Big[\sum_{j=1}^{N} |a_{ij}| \cdot \\ \quad \cdot \left(\tilde{e}_i(t) - \operatorname{sgn}(a_{ij})\, \tilde{e}_j(t)\right) + a_{i0}\tilde{e}_i(t)\Big], \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ A\bar{e}_i(t) + f(x_i(t), t) - f(\hat{x}_i(t), t), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

and

$$\dot{\hat{e}}_i(t) = \begin{cases} A\hat{e}_i(t) + f(\hat{x}_i(t), t) - s_i f(x_0(t), t) + \hat{\varepsilon}_{ij}(t) Fq\Big[\sum_{j=1}^{N} |a_{ij}| \cdot \\ \quad \cdot \left(\tilde{e}_i(t) - \operatorname{sgn}(a_{ij})\, \tilde{e}_j(t)\right) + a_{i0}\tilde{e}_i(t)\Big] + Bu_i(t), \quad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ A\hat{e}_i(t) + f(\hat{x}_i(t), t) - s_i f(x_0(t), t), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t). \end{cases}$$

Similarly, by utilizing Kronecker products, rewrite the above equations into a compact matrix form as

$$\dot{\bar{e}}(t) = \begin{cases} \left[(I_N \otimes A) - (L_\varepsilon \otimes FC)\right] \bar{e}(t) - (L_\varepsilon \otimes FC)\, \mathrm{Z}(\mathrm{t}) \\ \quad + I_N \otimes \left(f(x(t), t) - f(\hat{x}(t), t)\right), \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ (I_N \otimes A)\, \bar{e}(t) + I_N \otimes \left(f(x(t), t) - f(\hat{x}(t), t)\right), \quad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

and

$$\dot{\hat{e}}(t) = \begin{cases} \left[I_N \otimes A + (L_c \otimes BK)\right] \hat{e}(t) + (L_\varepsilon \otimes FC)\, \mathrm{Z}(\mathrm{t}) + (L_\varepsilon \otimes FC)\, \bar{e}(t) \\ \quad + \left(f(\hat{x}(t), t) - (SI_N \otimes f(x_0(t), t))\right), \qquad \text{if } t \in \bar{\mathrm{T}}_f(t_0, t), \\ (I_N \otimes A)\, \hat{e}(t) + \left(f(\hat{x}(t), t)\right. \\ \quad \left. - (SI_N \otimes f(x_0(t), t))\right), \qquad \text{if } t \in \bar{\mathrm{T}}_d(t_0, t), \end{cases}$$

in which L_c and L_ε can be defined as $L_c = \bar{L} + \bar{R}$, $L_\varepsilon = \tilde{L} + \tilde{R}$, $\bar{L} = (\hat{c}_{ij} l_{ij}) \in \mathbb{R}^{N \times N}$, $\tilde{L} = (\hat{\varepsilon}_{ij} l_{ij}) \in \mathbb{R}^{N \times N}$, $\bar{R} = \operatorname{diag}(\hat{c}_{1j} a_{10}, \ldots \hat{c}_{Nj} a_{N0})$, and $\tilde{R} = \operatorname{diag}(\hat{\varepsilon}_{1j} a_{10}, \ldots \hat{\varepsilon}_{Nj} a_{N0})$.

Remark 4. *Note that in* [5, 8], the developed controllers that needed to obtain the Laplacian of the graph for designing the control gains, however, in our paper, we propose an approach for adaptively tuning the gains $\hat{\mathrm{c}}_{ij}(t)$ and $\hat{\varepsilon}_{ij}(t)$ depending on sampled relative quantized output information, and thereby the application of global information on the basis of the Laplacian is avoided. Then, a fully distributed control strategy integrating the state observer, logarithmic quantizer, adaptive parameters, and antagonistic interactions is developed for the first time. And the scope of application for (21) is wider from the practical point of view.

Theorem 2. *Assume the MAS in (4)-(5) satisfying Assumptions 1-3. Under the controller (15) with adaptive control laws (16) and (17), the MAS achieves leader-follower bipartite consensus (Definition 3) and* $\hat{\mathrm{c}}_{ij}(t), \forall (j,i) \in \mathcal{E}$ *and* $\hat{\varepsilon}_{ij}(t), \forall (j,i) \in \mathcal{E}$, *converge to some positive constants with feedback matrices* $K = -B^{\mathrm{T}}P, F = P^{-1}C^{\mathrm{T}}, \Gamma = PBB^{\mathrm{T}}P$, *and* $\tilde{\Gamma} = C^{\mathrm{T}}C$, *if there are two positive definite matrices* P, Q *and positive constants* m_1, m_2, n_1, n_2, ρ, *and* $\tau \in (0, m_0)$ *such that the following conditions are satisfied:*

$$\begin{bmatrix} \tilde{\Theta}_1 & \rho P & C \\ \rho P & -I & 0 \\ * & 0 & -I \end{bmatrix} < 0, \begin{bmatrix} \tilde{\Theta}_2 & \rho P \\ \rho P & -I \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} \tilde{\Theta}_3 & \rho Q \\ \rho Q & -I \end{bmatrix} < 0, \begin{bmatrix} \tilde{\Theta}_4 & \rho Q \\ \rho Q & -I \end{bmatrix} < 0, \tag{20}$$

$$\mu = \max\left\{ \frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}, \frac{\lambda_{\max}(Q)}{\lambda_{\min}(P)} \right\}, \tag{21}$$

$$\Lambda\left(t_0, t\right) \leq \frac{\tau}{2\ln\mu + \left(m_0 + n_0\right)\bar{\lambda}}, \tau \in \left(0, m_0\right), \tag{22}$$

$$\frac{1}{T_1} \leq \frac{m_0 - \tau}{m_0 + n_0}, \tag{23}$$

in which

$$\begin{aligned}
\tilde{\Theta}_1 =& A^T P + PA - \left(PB + B^T P\right) + I_N + m_1 P,\\
\tilde{\Theta}_2 =& A^T P + PA - C^{\mathrm{T}} C + I_N + m_2 P,\\
\tilde{\Theta}_3 =& A^T Q + QA + I_N - n_1 Q,\\
\tilde{\Theta}_4 =& A^T Q + QA + I_N - n_2 Q,
\end{aligned}$$

with $\hat{\varsigma} = k_4 + k_5 = \tilde{\zeta}(1+\xi)^2, \tilde{\varsigma} = \frac{\xi^2}{k_4} + \frac{1}{k_5} + k_6 + \frac{\xi^2}{k_3}$, $\bar{c}_0 \geq \frac{1}{\zeta\lambda_1}$, $\bar{\varepsilon}_0 \leq \frac{1}{\tilde{\zeta}(1-\xi)^2\lambda_N}$, $\ell = \zeta\bar{c}_0$, $\tilde{\ell} = \tilde{\zeta}\bar{\varepsilon}_0(1-\xi)^2$, $m_0 = \min_{i\in\{1,2\}}\left\{m_i\right\}, n_0 = \max_{i\in\{1,2\}}\left\{n_i\right\}, \Delta = \frac{m_0}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{c}_0^2 + \frac{m_0}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{\varepsilon}_0^2$, $\iota\left(z\right) = \max\left\{\Delta, 0\right\}$, $\bar{c} = e^{(m_0+n_0)T_0 + [(m_0+n_0)\bar{\lambda} + \ln\mu]\Lambda_0}$, $\nu = m_0 - \left(m_0 + n_0\right)\frac{1}{T_1} - \tau > 0$, *and* λ_1 *and* λ_N *are the smallest nonzero eigenvalue and largest eigenvalue of* $\bar{L}$.

Proof: Similar to the scenario with static protocol, we can analytically prove that the fully distributed control law (15) and observer-based control strategies with gain matrices defined via (19)-(20) can solve the leader-following bipartite consensus of nonlinear MAS in (4)-(5) under DoS attacks. The following result, whose proof is reported in Appendix B, formally guarantees our claim, under some conditions on the Lemma 2.

Remark 5. *For general nonlinear MASs, it will be challenging to design a fully distributed protocol only based on relative states of neighboring agents and quantized output information over signed networks, and the static control strategy is no longer applicable. To amend the drawback of this fact,*

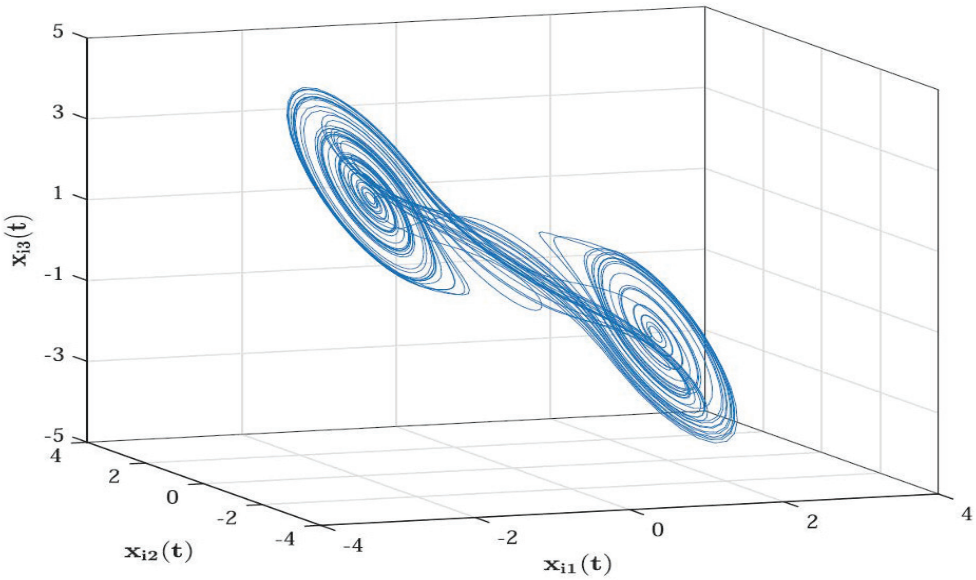

Figure 1: The chaotic behavior of Chua's circuit.

the incorporation of adaptive couplings is obviously necessary to accomplish fully distributed schemes without any global information [38], [39]. And it is theoretically proved in Theorem 2 that the leader-follower control errors and observer errors converge to zero, which reflects that the fully distributed secure bipartite consensus subject to DoS attacks and quantized communication can be realized successfully.

5. Numerical example

In this section, we conclude the paper by providing a simulation example [21], which is given to corroborate the theoretical results presented in the previous section. In our example, we consider a network made by coupled

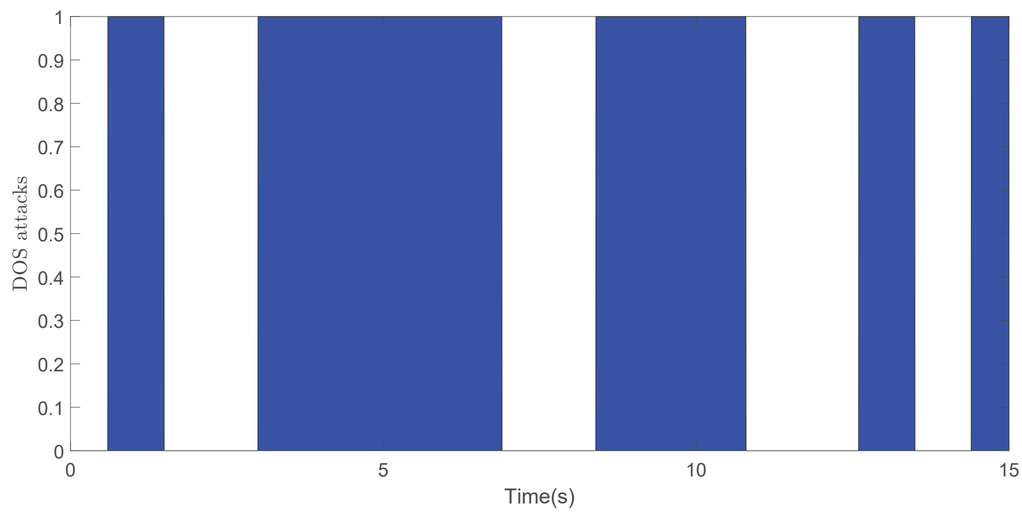

Figure 2: Time sequences of DoS attacks.

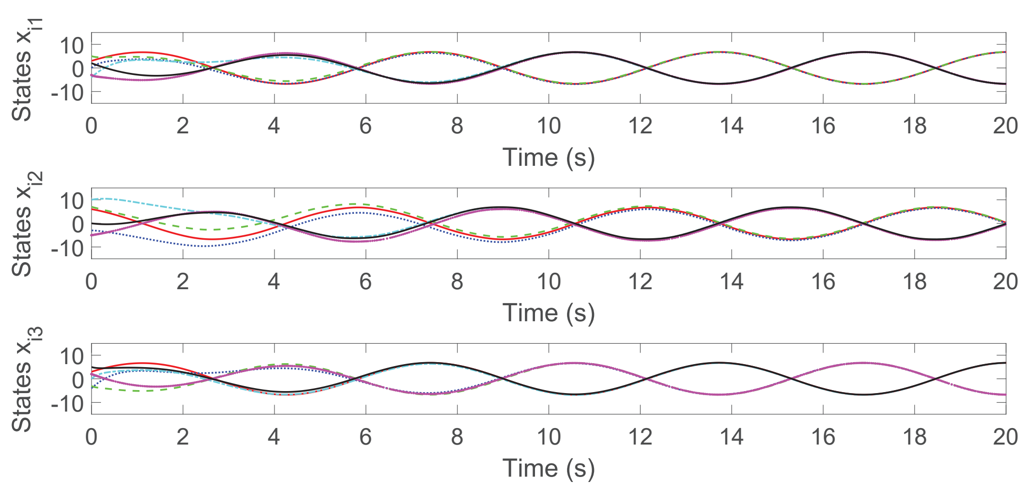

Figure 3: Time evolutions of state in Example 1.

Chua's circuits, which is described as follows:

$$\begin{cases} \dot{x}_1(t) = a\,(x_2(t) - x_1(t) - f(x_1(t))), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) = -bx_2(t), \end{cases} \tag{24}$$

in which $a = 10, b = 14.87$. A simulation of the trajectory of the uncoupled dynamical systems is illustrated in Figure 1. The system matrices are designed by

$$A = \begin{bmatrix} -c & c & 0 \\ 1 & -1 & 1 \\ 0 & -d & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

The description for chaotic behavior of Chua's circuit can be expressed by $x_0 = [5, 5, 5]^T$, $x_1 = [0.5, 0.7, .2]^T, x_2 = [1.5, 1.2, 1.6]^T$,$x_3 = [5, 5, 5]^T, x_4 = [3.2, 2.4, 2.3]^T, x_5 = [0.1, 0.1, 0.1]^T, x_6 = [0.8, 0.8, 0.8]^T$. In addition, $f(x_i(t)) = [0.333 \sin\left(x_{i3}\left(t\right)\right), 0, 0]^T$ is the nonlinear function of system. Select parameters as $\mu = 0.04, \tilde{\varsigma} = 12, \bar{c}_0 = 0.04, \hat{\varsigma} = 0.1, k_4 = 0.08, k_5 = 1, k_6 = 0.01, \tilde{\ell} = 0.9, \bar{\varepsilon}_0 = 2.3, \bar{c} = 0.08, \nu = 0.043, \ell = 0.071$. The quantized parameter is $\xi = 0.005$. Solving the LMI, one choose $t_{\min} = -0.0146$, $n_0 = 3$, $m_0 = 4$. Based on (19) and (20), we compute the following matrices P and Q

$$P = \begin{bmatrix} 1.0770 & 0.0386 & -1.0067 \\ 0.0386 & 0.4641 & -0.2941 \\ -1.0067 & -0.2941 & 6.4827 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.3126 & 0.1631 & -2.1389 \\ 0.1631 & 0.6582 & -0.4017 \\ -2.1389 & -0.4017 & 1.5214 \end{bmatrix}.$$

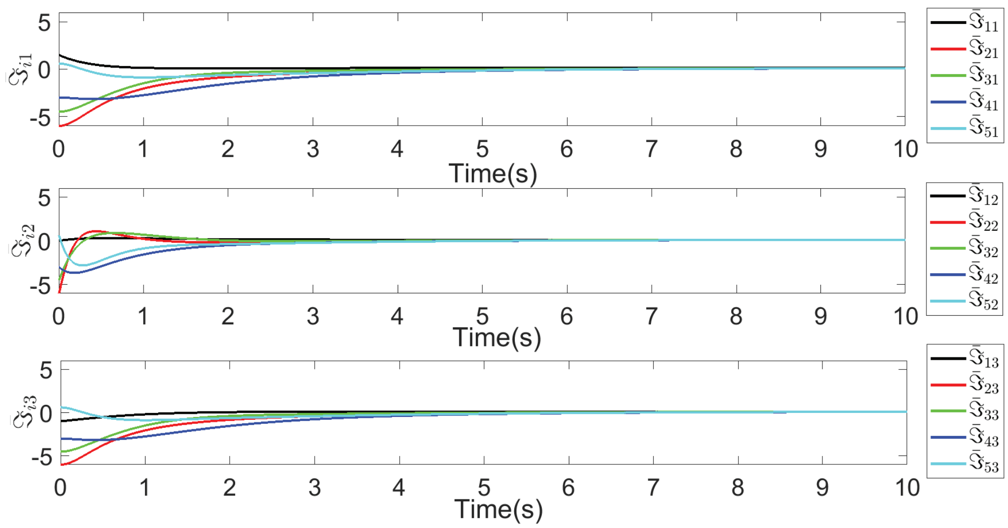

Figure 4: Time evolutions of leader-follower errors.

Then, one can further get

$$K = \begin{bmatrix} -1.0770 & -0.0386 & 1.0067 \\ -0.0386 & -0.4641 & 0.2941 \\ 1.0067 & 0.2941 & -6.4827 \end{bmatrix},$$

$$F = \begin{bmatrix} 2.1896 & 1.2558 & 1.2728 \\ 2.2528 & 0.1203 & 2.3391 \\ 0.4422 & 0.3547 & 0.4580 \end{bmatrix}.$$

It can be seen that after attacks destroy the control channels, the system errors turn from convergence to divergence, and the destroyed controller can no longer guarantee the normal quantization communication. In the attack's sleeping interval, the communication has a complete recovery, the system can continue to converge, and finally the system error will converge to 0. Then, the sequences of DoS attacks are shown in Figure 2. The state trajectories of five followers and one leader are shown in Figure 3. The Figure 4 depicts

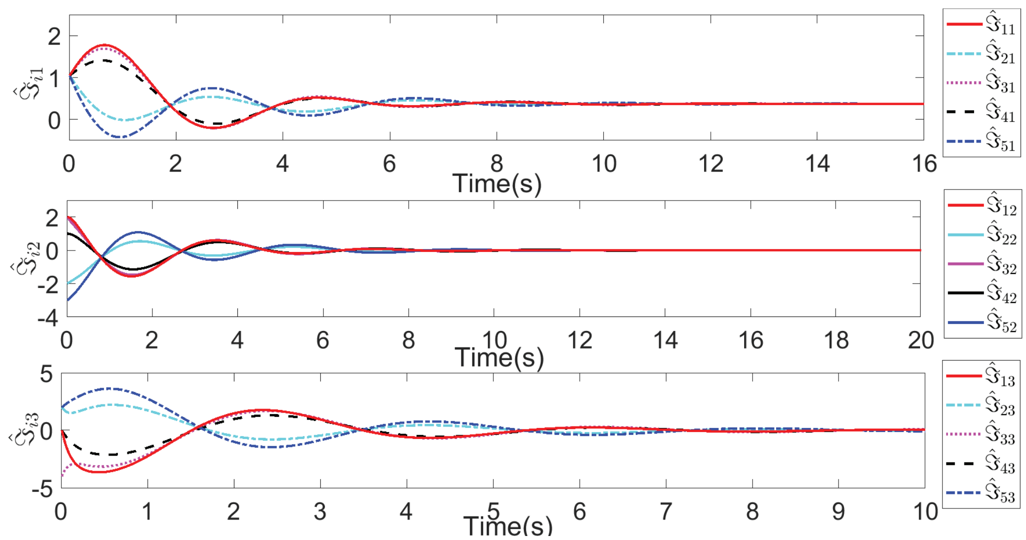

Figure 5: Time evolutions of observer errors.

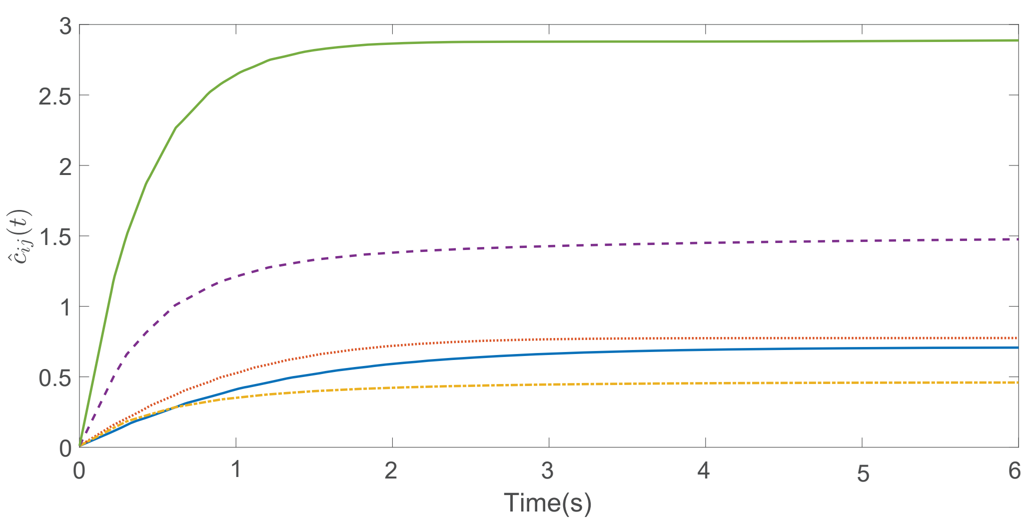

Figure 6: Time evolutions of $\hat{c}_{ij}(t)$.

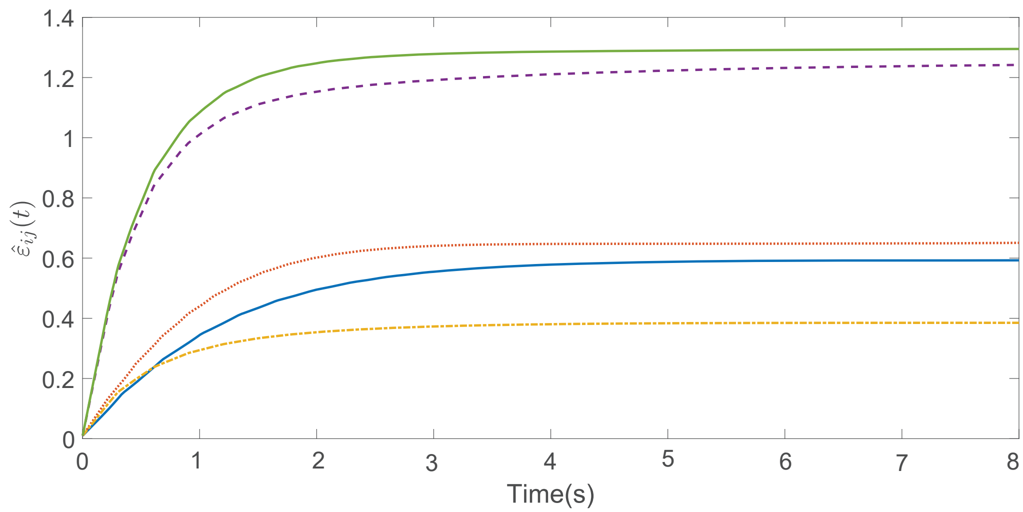

Figure 7: Time evolutions of $\hat{\varepsilon}_{ij}(t)$.

the error between the (signed) leader's state and each follower agent on a logarithmic quantizer under the proposed control protocol (15). Figure 5 reports the temporal evolution of the state of the agents, when the observer-based controller is enacted, showing that the state of the followers converges to a leader-follower bipartite consensus. Note that many existing controllers proposed in the literature cannot handle this scenario [16, 20]. The Figures 6 and 7 depicted that the evolutions of adaptive parameters $\hat{c}_{ij}(t)$ and $\hat{\varepsilon}_{ij}(t)$. Under the energy constraints and the designed distributed controller (21) based on relative quantitation information, MASs (4)-(5) can finally achieve the secure bipartite consensus.

6. Conclusions

In this paper, we have solved the secure bipartite consensus problem for nonlinear MASs with quantized information subject to DoS attacks. Based on a connected structurally balanced signed graph, a new secure output feedback control protocol integrated of logarithmic quantizer and relative output measurements of neighboring agents is proposed to realize secure control under DoS attacks. Furthermore, we also develop a control strategy with dynamic coupling gains, which is fully distributed and agents are not required to know a priori knowledge of any global information and the quantizer only need to quantize the output state error information of agents. Then, theoretical guarantees on the effectiveness of the proposed controller in steering the system to a secure bipartite leader-follower consensus under quantized output measurements and intermittent DoS attacks are derived. Finally, numerical simulations inspired by a real-world physical MAS are provided to verify the usefulness of the presented controllers.

The promising results, supported by the example illustrated in Section 5, suggest the possible extension of our methodology to different practical scenarios. In particular, following [1, 3], it would be interesting to investigate event-based secure bipartite consensus of MASs under directed switching topologies and sequential scaling attacks. In addition, following [17], a promising idea can be that of implementing dynamic event-triggered strategies to realize the fully distributed secure bipartite consensus for MASs under DoS attacks. This idea will be investigated in our future study.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proof of Theorem 1

Let us construct the Lyapunov candidate $V(t)$ as follows

$$V(t)=\begin{cases} \hat{e}^{\top}(t)(L_R\otimes P)\hat{e}(t)+\bar{e}^{\top}(t)(L_R\otimes P)\bar{e}(t), t\in\bar{\mathrm{T}}_f(t_0,t),\\ \hat{e}^{\top}(t)(L_R\otimes Q)\hat{e}(t)+\bar{e}^{\top}(t)(L_R\otimes Q)\bar{e}(t), t\in\bar{\mathrm{T}}_d(t_0,t).\end{cases}$$

Without DoS attacks on the system, that is, $t\in\bar{\mathrm{T}}_f(t_0,t)$, let $V_1(t)=\hat{e}^{\top}(t)(L_R\otimes P)\hat{e}(t)$, $V_2(t)=\bar{e}^{\top}(t)(L_R\otimes P)\bar{e}(t)$, then taking the derivative $V_1(t)$ and $V_2(t)$, one obtains

$$\begin{aligned}
\dot{V}_1(t)=&2\hat{e}^{\top}(t)(L_R\otimes P)\dot{\hat{e}}(t)\\
\leq&\hat{e}^{\top}(t)\left[L_R\otimes\left(A^{\mathrm{T}}P+PA\right)-2cL_R^2\otimes\Gamma\right]\hat{e}(t)\\
&+2\varepsilon\hat{e}^{\top}(t)\left(L_R^2\otimes PFC\right)\mathrm{Z}(t)+2\varepsilon\hat{e}^{\top}(t)\left(L_R^2\otimes PFC\right)\bar{e}(t)\\
&+2\hat{e}^{\top}(t)(L_R\otimes P)\times(f(\hat{x}(t),t)-(SI_N\otimes f(x_0(t),t))),\\
\dot{V}_2(t)=&2\bar{e}^{\top}(t)(L_R\otimes P)\dot{\bar{e}}(t)\\
\leq&\bar{e}^{\top}(t)\left[L_R\otimes\left(A^{\mathrm{T}}P+PA\right)-2\varepsilon L_R^2\otimes PFC\right]\bar{e}^{\top}(t)\\
&-2\varepsilon\bar{e}^{\top}(t)\left(L_R^2\otimes PFC\right)\mathrm{Z}(t)\\
&+2\bar{e}^{\top}(t)(L_R\otimes P)\times(f(x(t),t)-f(\hat{x}(t),t)).
\end{aligned}\tag{A.1}$$

Based on $F = P^{-1}C^{\top}$, $\mathrm{Z}(t) \in \mathcal{K}(\mathrm{H}\bar{e}(t))$, $\mathrm{H}_i \in [-\xi, +\xi]$, and according to Assumption 3 and Young's inequality, we obtain

$$\begin{aligned}
&2\varepsilon \hat{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t) \\
\leq & \varepsilon k_1 \hat{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \hat{e}(t)+\frac{1}{k_1} \mathrm{Z}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t) \\
\leq & \varepsilon k_1 \hat{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \hat{e}(t)+\frac{\xi^2}{k_1} \bar{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \bar{e}(t),
\end{aligned} \tag{A.2}$$

and

$$\begin{aligned}
&2\varepsilon \hat{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \bar{e}(t) \\
\leq & \varepsilon k_2 \hat{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \hat{e}(t)+\frac{1}{k_2} \bar{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \bar{e}(t),
\end{aligned} \tag{A.3}$$

and

$$\begin{aligned}
&2\bar{e}^{\mathrm{T}}(t)\left(L_R \otimes P\right) \times (f(x(t), t)-f(\hat{x}(t), t)) \\
\leq & 2\bar{e}^{\mathrm{T}}(t)\left(\sqrt{L_R} \otimes I_N\right)\left(\sqrt{L_R} \otimes P\right) \rho \bar{e}(t) \\
\leq & \bar{e}^{\mathrm{T}}(t)\left(L_R \otimes I_N\right) \bar{e}(t)+\rho^2 \bar{e}^{\mathrm{T}}(t)\left(L_R \otimes P^{\mathrm{T}} P\right) \bar{e}(t).
\end{aligned} \tag{A.4}$$

Similarly, one gets

$$\begin{aligned}
&-2\bar{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t) \\
\leq & k_3 \bar{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \bar{e}^{\mathrm{T}}(t)+\frac{1}{k_3} \mathrm{Z}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t) \\
\leq & k_3 \bar{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \bar{e}(t)+\frac{\xi^2}{k_3} \bar{e}^{\mathrm{T}}(t)\left(L_R^2 \otimes C^{\mathrm{T}} C\right) \bar{e}(t),
\end{aligned} \tag{A.5}$$

and

$$\begin{aligned}
&2\hat{e}^{\mathrm{T}}(t)\left(L_R \otimes P\right) \times \left(f(\hat{x}(t), t)-\left(S I_N \otimes f\left(x_0(t), t\right)\right)\right) \\
\leq & 2\hat{e}^{\mathrm{T}}(t)\left(\sqrt{L_R} \otimes I_N\right)\left(\sqrt{L_R} \otimes P\right) \rho \hat{e}(t) \\
\leq & \hat{e}^{\mathrm{T}}(t)\left(L_R \otimes I_N\right) \hat{e}(t)+\rho^2 \hat{e}(t)\left(L_R \otimes P^{\top} P\right) \hat{e}(t).
\end{aligned} \tag{A.6}$$

Then, substituting (A.2)-(A.6) into (A.1), we obtain

$$\begin{aligned}\dot{V}(t) \leq & \hat{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}P + PA\right) - 2cL_R^2 \otimes \Gamma + \varsigma L_R^2 \otimes C^{\mathrm{T}}C\right. \\ & + L_R \otimes I_N + \rho^2 L_R \otimes P^{\mathrm{T}}P\left.\right]\hat{e}(t) + \bar{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}P + PA\right)\right. \\ & - \bar{\varsigma} L_R^2 \otimes C^{\mathrm{T}}C + L_R \otimes I_N + \rho^2 L_R \otimes P^{\mathrm{T}}P\left.\right]\bar{e}(t),\end{aligned} \tag{A.7}$$

in which $\varsigma = \varepsilon(k_1 + k_2), \bar{\varsigma} = \left(2\varepsilon - \frac{\xi^2}{k_1} - \frac{1}{k_2} - k_3 - \frac{\xi^2}{k_3}\right)$. Define $\hat{\wp}(t) = (S \otimes I_n)\hat{e}(t)$ and $\bar{\wp}(t) = (S \otimes I_n)\bar{e}(t)$, we have

$$\begin{aligned}\dot{V}(t) \leq & \hat{\wp}^{\mathrm{T}}(t)\left[\left(\bar{L} \otimes I_n\right) \otimes \left(A^{\mathrm{T}}P + PA - 2c\bar{L} \otimes \Gamma + \varsigma \bar{L} \otimes C^{\mathrm{T}}C\right.\right. \\ & + I_N + \rho^2 P^{\mathrm{T}}P\left.\right]\hat{\wp}(t) + \bar{\wp}^{\mathrm{T}}(t)\left[\left(\bar{L} \otimes I_n\right) \otimes \left(A^{\mathrm{T}}P + PA\right)\right. \\ & - \bar{\varsigma}\bar{L} \otimes C^{\mathrm{T}}C + I_N + \rho^2 P^{\mathrm{T}}P\left.\right]\bar{\wp}(t).\end{aligned} \tag{A.8}$$

Depended on Lemma 1, one concludes $U^T\bar{L}U = \mathrm{diag}(\lambda_1, \ldots, \lambda_N) = \Delta$. Then we can get $\bar{L} = U^{\mathrm{T}}\Delta U$. Let $\tilde{\wp}(t) = \left(U^{\mathrm{T}} \otimes I_n\right)\hat{\wp}(t)$ and $\widehat{\wp}(t) = \left(U^{\mathrm{T}} \otimes I_n\right)\bar{\wp}(t)$, and it follows from the facts $c \geq \frac{1}{2\lambda_1}, \varsigma \leq \frac{1}{\lambda_N}, \bar{\varsigma} \geq \frac{1}{\lambda_1}$, one further obtains

$$\begin{aligned}\dot{V}(t) \leq & \tilde{\wp}^{\mathrm{T}}(t)\left[(\Delta \otimes I_n) \otimes \left(A^{\mathrm{T}}P + PA - PBB^{\mathrm{T}}P + C^{\mathrm{T}}C\right.\right. \\ & + I_N + \rho^2 P^{\mathrm{T}}P\left.\right)\left.\right]\tilde{\wp}(t) + \widehat{\wp}^{\mathrm{T}}(t)\left[(\Delta \otimes I_n) \otimes \left(A^{\mathrm{T}}P + PA\right.\right. \\ & - C^{\mathrm{T}}C + I_N + \rho^2 P^{\mathrm{T}}P\left.\right)\left.\right]\widehat{\wp}(t),\end{aligned} \tag{A.9}$$

Based on (10), one has

$$\begin{aligned}\dot{V}(t) \leq & - m_1\tilde{\wp}^{\mathrm{T}}(t)(\Delta \otimes P)\tilde{\wp}(t) - m_2\widehat{\wp}^{\mathrm{T}}(t)(\Delta \otimes P)\widehat{\wp}(t) \\ \leq & - m_1\hat{e}^{\top}(t)(L_R \otimes P)\hat{e}(t) - m_2\bar{e}^{\top}(t)(L_R \otimes P)\bar{e}(t) \\ \leq & - m_0 V(t).\end{aligned} \tag{A.10}$$

With DoS attacks on the system, that is $t \in \bar{\mathrm{T}}_d(t_0, t)$, let $\tilde{V}_1(t) = \hat{e}^{\top}(t)$ $(L_R \otimes Q)\hat{e}(t)$, $\tilde{V}_2(t) = \bar{e}^{\top}(t)(L_R \otimes Q)\bar{e}(t)$, one obtains

$$V(t) = \tilde{V}_1(t) + \tilde{V}_2(t).$$

Then, one has

$$
\begin{aligned}
\dot{\hat{V}}_1(t) =& 2\hat{e}^{\mathrm{T}}(t)(L_R \otimes Q)\dot{\hat{e}}(t) \\
\leq& \hat{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}Q + QA\right)\right]\hat{e}(t) \\
& + 2\hat{e}^{\mathrm{T}}(t)(L_R \otimes Q) \times (f(\hat{x}(t), t) - (SI_N \otimes f(x_0(t), t))), \\
\dot{\hat{V}}_2(t) =& 2\bar{e}^{\mathrm{T}}(t)(L_R \otimes Q)\dot{\bar{e}}(t) \\
\leq& \bar{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}Q + QA\right)\right]\bar{e}^{\mathrm{T}}(t) \\
& + 2\bar{e}^{\mathrm{T}}(t)(L_R \otimes Q) \times (f(x(t), t) - f(\hat{x}(t), t)).
\end{aligned}
$$

Similar to the forgoing analysis, we have

$$
\begin{aligned}
\dot{V}(t) \leq & \hat{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}Q + QA\right) + L_R \otimes I_N + \rho^2 L_R \otimes Q^{\mathrm{T}}Q\right]\hat{e}(t) \\
& + \bar{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}Q + QA\right) + L_R \otimes I_N + \rho^2 L_R \otimes Q^{\mathrm{T}}Q\right]\bar{e}^{\mathrm{T}}(t).
\end{aligned}
\tag{A.11}
$$

Then, according to (11), one obtains $\dot{V}_1(t) < n_1\bar{e}^{\mathrm{T}}(t)(L_R \otimes Q)\bar{e}(t), \dot{V}_2(t) < n_2\hat{e}^{\mathrm{T}}(t)(L_R \otimes Q)\hat{e}(t)$, therefore,

$$
\dot{V}(t) \leq n_0 V(t), \tag{A.12}
$$

where $n_0 = \max_{i\in\{1,2\}}\{n_i\}$. The derivation function of Lyapunov function satisfies the following conditions

$$
\dot{V}(t) \leq \begin{cases} -m_0 V(t), t \in [t_{2k}, t_{2k+1}), \\ n_0 V(t), t \in [t_{2k+1}, t_{2k+2}), \end{cases}
$$

in which $[t_{2k}, t_{2k+1})$ and $[t_{2k+1}, t_{2k+2})$ denote the time sequences of $\bar{\mathrm{T}}_f(t_0, t)$ and $\bar{\mathrm{T}}_d(t_0, t)$, respectively. After that, one has

$$
\dot{V}(t) \leq \begin{cases} -m_0 V(t), t \in \bar{\mathrm{T}}_f(t_0, t), \\ n_0 V(t), t \in \bar{\mathrm{T}}_d(t_0, t). \end{cases}
$$

For any time interval $[t_0, t)$, when $t \in [t_{2k}, t_{2k+1})$, by the mathematical induction, one obtains

$$\begin{aligned}
V(t) \leq & e^{-m_0(t-t_{2k})} V\left(t_{2k}\right) \\
\leq & \frac{\lambda_{\max}\left(P\right)}{\lambda_{\min}\left(Q\right)} e^{-m_0(t-t_{2k})+n_0(t_{2k}-t_{2k-1})} V(t_{2k-1}) \\
\leq & \frac{\lambda_{\max}\left(P\right)}{\lambda_{\min}\left(Q\right)} \frac{\lambda_{\max}\left(Q\right)}{\lambda_{\min}\left(P\right)} e^{-m_0(t-t_{2k})+n_0(t_{2k}-t_{2k-1})} \times \\
& e^{-m_0(t_{2k-1}-t_{2k-2})+\cdots-m_0(t_1-t_0)} V(t_0) \\
\leq & \mu^{2k} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right|+n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0) \\
\leq & \mu^{N(t_0,t)} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right|+n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0).
\end{aligned} \tag{A.13}$$

When $t \in [t_{2k+1}, t_{2k+2})$, there is

$$\begin{aligned}
V(t) \leq & e^{-m_0(t-t_{2k+1})} V\left(t_{2k+1}\right) \\
\leq & \mu^{2k} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right|+n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0) \\
\leq & \mu^{N(t_0,t)} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right|+n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0).
\end{aligned} \tag{A.14}$$

Thus, for any time interval $[t_0, t)$, there is

$$V(t) \leq e^{\ln\mu N(t_0,t)} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right|+n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0).$$

Then, one has

$$\begin{aligned}
-m_0 & \left|\bar{\mathrm{T}}_f(t_0,t)\right| + n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right| \\
& = - m_0\left(t - t_0 - \bar{\mathrm{T}}_d(t_0,t)\right) + n_0\left|\bar{\mathrm{T}}_d(t_0,t)\right| \\
& \leq - m_0\left(t - t_0\right) + (m_0 + n_0)\left(T_0 + \frac{t - t_0}{T_1}\right) \\
& \leq - \tau\left(t - t_0\right) + (m_0 + n_0)\, T_0.
\end{aligned} \tag{A.15}$$

According to (12), (13) and (14), there is

$$e^{\ln\mu N(t_0,t)} \leq e^{(2\ln\mu + (m_0+n_0)\lambda) N(t_0,t)} \leq e^{\tau(t-t_0)}.$$

In conclusion, the multi-agent systems (4) and (5) can achieve the secure bipartite consensus between leader and follower under the relative quantitative state control protocol (7). ■

Appendix B. Proof of Theorem 2

Construct a Lyapunov function candidate $V(t)$ as follows

$$V(t)=\begin{cases} V_1(t)+V_2(t)+\frac{1}{2}\sum_{i=1}^{N}\left(\hat{\varepsilon}_{ij}(t)-\bar{\varepsilon}_0\right)^2 \\ +\frac{1}{2}\sum_{i=1}^{N}\left(\hat{\mathrm{c}}_{ij}(t)-\bar{c}_0\right)^2, t\in\bar{\mathrm{T}}_f(t_0,t), \\ \bar{V}_1(t)+\bar{V}_2(t), t\in\bar{\mathrm{T}}_d(t_0,t), \end{cases} \tag{B.1}$$

where $V_1(t)=\hat{e}^\top(t)(L_R\otimes P)\hat{e}(t), V_2(t)=\bar{e}^\top(t)(L_R\otimes P)\bar{e}(t)$. Similar to Appendix A, when considering the case that without DoS attacks, taking the derivative $V_1(t)$ and $V_2(t)$, one obtains

$$\begin{aligned} \dot{V}_1(t)=&2\hat{e}^\top(t)(L_R\otimes P)\dot{\hat{e}}(t) \\ \leq&\hat{e}^\top(t)\left[L_R\otimes\left(A^{\mathrm{T}}P+PA\right)-2L_RL_c\otimes PBB^{\mathrm{T}}P\right]\hat{e}(t) \\ &+2\hat{e}^\top(t)(L_RL_\varepsilon\otimes PFC)\bar{\mathrm{Z}}(t)+2\hat{e}^\top(t)(L_RL_\varepsilon\otimes PFC)\bar{e}(t) \\ &+2\hat{e}^\top(t)(L_R\otimes P)\times\left(f(\hat{x}(t),t)-(SI_N\otimes f(x_0(t),t))\right), \\ \dot{V}_2(t)=&2\bar{e}^\top(t)(L_R\otimes P)\dot{\bar{e}}(t) \\ \leq&\bar{e}^\top(t)\left[L_R\otimes\left(A^{\mathrm{T}}P+PA\right)-2L_RL_\varepsilon\otimes PFC\right]\bar{e}^\top(t) \\ &-2\bar{e}^\top(t)(L_RL_\varepsilon\otimes PFC)\bar{\mathrm{Z}}(t)+2\bar{e}^\top(t)(L_R\otimes P) \\ &\times\left(f(x(t),t)-f(\hat{x}(t),t)\right). \end{aligned} \tag{B.2}$$

Then, one has

$$\begin{aligned}&2\hat{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t)\\ \leq & k_1 \hat{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \hat{e}(t)+\frac{1}{k_1} \mathrm{Z}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t)\\ \leq & k_1 \hat{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \hat{e}(t)+\frac{\xi^2}{k_1} \bar{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \bar{e}(t),\end{aligned} \tag{B.3}$$

$$\begin{aligned}&2\hat{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \bar{e}(t)\\ \leq & k_2 \hat{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \hat{e}(t)+\frac{1}{k_2} \bar{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \bar{e}(t),\end{aligned} \tag{B.4}$$

and

$$\begin{aligned}&2\bar{e}^{\mathrm{T}}(t)\left(L_R \otimes P\right) \times (f(x(t), t)-f(\hat{x}(t), t))\\ \leq & 2\bar{e}^{\mathrm{T}}(t)\left(\sqrt{L_R} \otimes I_N\right)\left(\sqrt{L_R} \otimes P\right) \rho \bar{e}(t)\\ \leq & \bar{e}^{\mathrm{T}}(t)\left(L_R \otimes I_N\right) \bar{e}(t)+\rho^2 \bar{e}^{\mathrm{T}}(t)\left(L_R \otimes P^{\mathrm{T}} P\right) \bar{e}(t).\end{aligned} \tag{B.5}$$

Similarly, one gets

$$\begin{aligned}&2\bar{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t)\\ \leq & k_3 \bar{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \bar{e}^{\mathrm{T}}(t)+\frac{1}{k_3} \mathrm{Z}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \mathrm{Z}(t)\\ \leq & k_3 \bar{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \bar{e}(t)+\frac{\xi^2}{k_3} \bar{e}^{\mathrm{T}}(t)\left(L_R L_\varepsilon \otimes C^{\mathrm{T}} C\right) \bar{e}(t),\end{aligned} \tag{B.6}$$

and

$$\begin{aligned}&2\hat{e}^{\mathrm{T}}(t)\left(L_R \otimes P\right) \times \left(f(\hat{x}(t), t)-\left(S I_N \otimes f\left(x_0(t), t\right)\right)\right)\\ \leq & 2\hat{e}^{\mathrm{T}}(t)\left(\sqrt{L_R} \otimes I_N\right)\left(\sqrt{L_R} \otimes P\right) \rho \hat{e}(t)\\ \leq & \hat{e}^{\mathrm{T}}(t)\left(L_R \otimes I_N\right) \hat{e}(t)+\rho^2 \hat{e}(t)\left(L_R \otimes P^{\top} P\right) \hat{e}(t).\end{aligned} \tag{B.7}$$

Also according to $\Gamma = PBB^{\mathrm{T}}P$, one concludes

$$\begin{aligned}\dot{V}(t) \leq & \hat{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}P + PA\right) - 2L_RL_c \otimes \Gamma\right.\\ & + \hat{\varsigma}L_RL_\varepsilon \otimes C^{\mathrm{T}}C + L_R \otimes I_N + \rho^2 L_R \otimes P^{\mathrm{T}}P\big]\,\hat{e}(t)\\ & + \bar{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}P + PA\right) - 2L_RL_\varepsilon \otimes C^{\mathrm{T}}C\right.\\ & + \tilde{\varsigma}L_RL_\varepsilon \otimes C^{\mathrm{T}}C + L_R \otimes I_N + \rho^2 L_R \otimes P^{\mathrm{T}}P\big]\,\bar{e}^{\mathrm{T}}(t)\\ & + \sum_{i=1}^{N}\hat{\mathrm{c}}_{ij}(t)\dot{\hat{\mathrm{c}}}_{ij}(t) - \bar{c}_0\sum_{i=1}^{N}\dot{\hat{\mathrm{c}}}_{ij}(t)\\ & + \sum_{i=1}^{N}\hat{\varepsilon}_{ij}(t)\dot{\hat{\varepsilon}}_{ij}(t) - \bar{\varepsilon}_0\sum_{i=1}^{N}\dot{\hat{\varepsilon}}_{ij}(t),\end{aligned} \tag{B.8}$$

where $\hat{\varsigma} = k_1 + k_2, \tilde{\varsigma} = \frac{\xi^2}{k_1} + \frac{1}{k_2} + k_3 + \frac{\xi^2}{k_3}$. Based on the fact $2 - \tilde{\varsigma} = \hat{\varsigma}$, one has

$$\begin{aligned}\dot{V}(t) \leq & \hat{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}P + PA\right) - 2L_RL_c \otimes \Gamma + \hat{\varsigma}L_RL_\varepsilon \otimes C^{\mathrm{T}}C\right.\\ & + L_R \otimes I_N + \rho^2 L_R \otimes P^{\mathrm{T}}P\big]\,\hat{e}(t) + \bar{e}^{\mathrm{T}}(t)\left[L_R \otimes \left(A^{\mathrm{T}}P + PA\right)\right.\\ & - \tilde{\varsigma}L_RL_\varepsilon \otimes C^{\mathrm{T}}C + L_R \otimes I_N + \rho^2 L_R \otimes P^{\mathrm{T}}P\big]\,\bar{e}^{\mathrm{T}}(t)\\ & + \sum_{i=1}^{N}\hat{\mathrm{c}}_{ij}(t)\dot{\hat{\mathrm{c}}}_{ij}(t) - \bar{c}_0\sum_{i=1}^{N}\dot{\hat{\mathrm{c}}}_{ij}(t) + \sum_{i=1}^{N}\hat{\varepsilon}_{ij}(t)\dot{\hat{\varepsilon}}_{ij}(t) - \bar{\varepsilon}_0\sum_{i=1}^{N}\dot{\hat{\varepsilon}}_{ij}(t).\end{aligned}$$

On the other hand, one has

$$\begin{aligned}\sum_{i=1}^{N}\hat{\mathrm{c}}_{ij}(t)\dot{\hat{\mathrm{c}}}_{ij}(t) = & -\zeta\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\mathrm{c}}_{ij}^2(t) + \zeta\sum_{i=1}^{N}\hat{\mathrm{c}}_{ij}(t)\varphi^{\mathrm{T}}(t)\,\Gamma\varphi(t)\\ \leq & -\zeta\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\mathrm{c}}_{ij}^2(t) + \zeta\hat{e}^{\top}(t)\left(L_RL_c \otimes \Gamma\right)\hat{e}(t),\end{aligned} \tag{B.9}$$

and

$$\begin{aligned}-\sum_{i=1}^{N}\bar{c}_0(t)\,\dot{\hat{\mathrm{c}}}_{ij}(t) = & \zeta\bar{c}_0\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\mathrm{c}}_{ij}(t) - \zeta\sum_{i=1}^{N}\bar{c}_0\varphi^{\mathrm{T}}(t)\,\Gamma\varphi(t)\\ \leq & \zeta\bar{c}_0\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\mathrm{c}}_{ij}(t) - \zeta\bar{c}_0\hat{e}^{\top}(t)\left(L_R^2 \otimes \Gamma\right)\hat{e}(t),\end{aligned} \tag{B.10}$$

with

$$
\begin{aligned}
\sum_{i=1}^{N} \hat{\varepsilon}_{ij}(t)\,\dot{\hat{\varepsilon}}_{ij}(t) =& -\tilde{\zeta}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}^{2}(t) - \tilde{\zeta}\sum_{i=1}^{N}\hat{\varepsilon}_{ij}(t)\varphi^{\mathrm{T}}(t)\,\tilde{\Gamma}\varphi(t) \\
& + \tilde{\zeta}\sum_{i=1}^{N}\hat{\varepsilon}_{ij}(t)\tilde{\varphi}^{\mathrm{T}}(t)\,\Gamma\tilde{\varphi}(t) \\
\leq & -\tilde{\zeta}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}^{2}(t) - \tilde{\zeta}(1+\xi)^{2}\hat{e}^{\top}(t)\left(L_R L_\varepsilon \otimes \tilde{\Gamma}\right)\hat{e}(t) \\
& + \tilde{\zeta}(1+\xi)^{2}\bar{e}^{\top}(t)\left(L_R L_\varepsilon \otimes \tilde{\Gamma}\right)\bar{e}(t),
\end{aligned}
$$

and

$$
\begin{aligned}
-\sum_{i=1}^{N}\bar{\varepsilon}_0\dot{\hat{\varepsilon}}_{ij}(t) =& \tilde{\zeta}\bar{\varepsilon}_0\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}(t) + \tilde{\zeta}\sum_{i=1}^{N}\bar{\varepsilon}_0\varphi^{\mathrm{T}}(t)\,\tilde{\Gamma}\varphi(t) \\
& - \tilde{\zeta}\sum_{i=1}^{N}\bar{\varepsilon}_0\hat{\varepsilon}_{ij}(t)\tilde{\varphi}^{\mathrm{T}}(t)\,\Gamma\tilde{\varphi}(t) \\
\leq & \tilde{\zeta}\bar{\varepsilon}_0\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}(t) + \tilde{\zeta}\bar{\varepsilon}_0(1-\xi)^{2}\hat{e}^{\top}(t)\left(L_R^{2}\otimes\tilde{\Gamma}\right)\hat{e}(t) \\
& - \tilde{\zeta}\bar{\varepsilon}_0(1-\xi)^{2}\bar{e}^{\top}(t)\left(L_R^{2}\otimes\tilde{\Gamma}\right)\bar{e}(t).
\end{aligned}
$$

According to $\tilde{\Gamma} = C^{\mathrm{T}}C$, one obtains

$$
\begin{aligned}
\dot{V}(t) \leq & \hat{e}^{\mathrm{T}}(t)\left[L_R\otimes\left(A^{\mathrm{T}}P + PA\right) - \ell L_R^{2}\otimes\Gamma + \tilde{\ell}L_R^{2}\otimes\tilde{\Gamma} + L_R\otimes I_N\right. \\
& \left. + \rho^{2}L_R\otimes P^{\mathrm{T}}P\right]\hat{e}(t) + \bar{e}^{\mathrm{T}}(t)\left[L_R\otimes\left(A^{\mathrm{T}}P+PA\right) - \tilde{\ell}L_R^{2}\otimes\tilde{\Gamma}\right. \\
& \left. + L_R\otimes I_N + \rho^{2}L_R\otimes P^{\mathrm{T}}P\right]\bar{e}^{\mathrm{T}}(t) - 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\mathrm{c}}_{ij}^{2}(t) \\
& + 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{c}_0\hat{\mathrm{c}}_{ij}(t) - \tilde{\zeta}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}^{2}(t) + \tilde{\zeta}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{\varepsilon}_0\hat{\varepsilon}_{ij}(t).
\end{aligned}
$$

Then, depended on the facts $\bar{c}_0 \geq \frac{1}{\zeta\lambda_1}$, $\ell = \zeta\bar{c}_0$, $\varsigma = \tilde{\zeta}(1+\xi)^{2}$, $\tilde{\ell} = \tilde{\zeta}\bar{\varepsilon}_0(1-\xi)^{2}$, $\bar{\varepsilon}_0 \leq \frac{1}{\tilde{\zeta}(1-\xi)^{2}\lambda_N}$, and similar to the proof of Appendix A, one obtains

$$
\begin{aligned}
\dot{V}(t) \leq & \tilde{\wp}^{\mathrm{T}}(t)\left[(\Delta \otimes I_n) \otimes \left(A^{\mathrm{T}}P + PA - PBB^{\mathrm{T}}P + C^{\mathrm{T}}C\right.\right. \\
& \left.\left. + I_N + \rho^2 P^{\mathrm{T}}P\right)\right] \tilde{\wp}(t) + \widehat{\wp}^{\mathrm{T}}(t)\left[(\Delta \otimes I_n) \otimes \left(A^{\mathrm{T}}P + PA\right.\right. \\
& \left.\left. - C^{\mathrm{T}}C + I_N + \rho^2 P^{\mathrm{T}}P\right)\right] \widehat{\wp}(t) - 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\mathrm{c}}_{ij}^2(t) \\
& + 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{c}_0\hat{\mathrm{c}}_{ij}(t) - \tilde{\zeta}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{\varepsilon}_{ij}^2(t) + \tilde{\zeta}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{\varepsilon}_0\hat{\varepsilon}_{ij}(t).
\end{aligned}
$$

Since (19) holds, one concludes

$$
\dot{V}(t) < -m_0 V(t) + \Delta,
$$

where $m_0 = \min_{i\in\{1,2\}}\{m_i\} = \min\{2, m_2\}$, $\Delta = \frac{m_0}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{c}_0^2 + \frac{m_0}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\bar{\varepsilon}_0^2$. With DoS attacks on the system, similarly, we have

$$
\dot{V}(t) \leq n_0 V(t), \tag{B.11}
$$

where $n_0 = \max_{i\in\{1,2\}}\{n_i\}$. The derivation function of Lyapunov function satisfies the following conditions

$$
\dot{V}(t) \leq \begin{cases} -m_0 V(t) + \Delta, t \in \bar{\mathrm{T}}_f(t_0, t) \\ n_0 V(t), t \in \bar{\mathrm{T}}_d(t_0, t) \end{cases}
$$

After that, we utilize the intervals $[t_{2k}, t_{2k+1})$ and $[t_{2k+1}, t_{2k+2})$. And $V(t)$ can be further expressed as

$$
\dot{V}(t) \leq \begin{cases} -m_0 V(t) + \Delta, t \in [t_{2k}, t_{2k+1}), \\ n_0 V(t), t \in [t_{2k+1}, t_{2k+2}). \end{cases}
$$

After that, let

$$
V(t) = \begin{cases} \tilde{V}(t), t \in [t_{2k}, t_{2k+1}), \\ \hat{V}(t), t \in [t_{2k+1}, t_{2k+2}), \end{cases} \quad \chi(t) = \begin{cases} -m_0, t \in [t_{2k}, t_{2k+1}), \\ n_0, t \in [t_{2k+1}, t_{2k+2}), \end{cases}
$$

and

$$\iota(t)=\begin{cases}\Delta, t\in[t_{2k},t_{2k+1}),\\ 0, t\in[t_{2k+1},t_{2k+2}).\end{cases}$$

Then, according to Lemma 2, one obtains

$$V(t)\leq\begin{cases} e^{\chi(t_{2k})(t-t_{2k})}\tilde{V}(t_{2k})+\int_{t_{2k}}^{t}e^{\chi(t_{2k})(t-z)}\iota(z)\,dz, t\in[t_{2k},t_{2k+1}),\\ e^{\chi(t_{2k+1})(t-t_{2k+1})}\hat{V}(t_{2k+1}), t\in[t_{2k+1},t_{2k+2}),\end{cases}$$

and

$$\begin{cases}\mu\hat{V}\left(t_{2k}^{-}\right)-\tilde{V}(t_{2k})\geq 0,\\ \mu\tilde{V}\left(t_{2k+1}^{-}\right)-\hat{V}(t_{2k+1})\geq 0.\end{cases}$$

For any time interval $[t_0,t)$, when $t\in[t_{2k},t_{2k+1})$, by the mathematical induction, we conclude

$$\begin{aligned}V(t)\leq&\mu e^{\chi(t_{2k})(t-t_{2k})}\hat{V}\left(t_{2k}^{-}\right)+\int_{t_{2k}}^{t}e^{\chi(t_{2k})(t-z)}\iota(z)\,dz\\ &\vdots\\ \leq&\mu^{2k}e^{\chi(t_{2k})\left|\bar{\mathrm{T}}_f(t_0,t)\right|+\chi(t_{2k-1})\left|\bar{\mathrm{T}}_d(t_0,t)\right|}V(t_0)\\ &+\mu^{2k}\int_{t_0}^{t_1}e^{\psi_{2k}(t,2)+\chi(t_0)(t_1-z)}\iota(z)\,dz\\ &+\mu^{2k}\int_{t_1}^{t_2}e^{\psi_{2k}(t,3)+\chi(t_1)(t_2-z)}\iota(z)\,dz\\ &+\cdots+\mu^{2}\int_{t_{2k-2}}^{t_{2k-1}}e^{\psi_{2k}(t,2k)+\chi(t_{2k-2})(t_{2k-1}-z)}\iota(z)\,dz\\ &+\mu^{2}\int_{t_{2k-1}}^{t_{2k}}e^{\psi_{2k}(t,2k+1)+\chi(t_{2k-1})(t_{2k}-z)}\iota(z)\,dz\\ &+\int_{t_{2k}}^{t}e^{\chi(t_{2k})(t-z)}\iota(z)\,dz,\end{aligned}\tag{B.12}$$

where $\psi_{2k}(t,p) = \chi(t_{2k})(t - t_{2k}) + \sum_{q=p}^{2k} \chi(t_{q-1})(t_q - t_{q-1})$. When $t \in [t_{2k+1}, t_{2k+2})$, there is

$$
\begin{aligned}
V(t) \leq & \mu^{2k+2} e^{\chi(t_{2k})\left|\bar{\mathrm{T}}_f(t_0,t)\right| + \chi(t_{2k+1})\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0) \\
& + \mu^{2k+2} \int_{t_0}^{t_1} e^{\psi_{2k+1}(t,2) + \chi(t_0)(t_1 - z)} \iota(z)\, dz \\
& + \mu^{2k+2} \int_{t_1}^{t_2} e^{\psi_{2k+1}(t,3) + \chi(t_1)(t_2 - z)} \iota(z)\, dz \\
& + \cdots + \tau^4 \int_{t_{2k-1}}^{t_{2k}} e^{\psi_{2k+1}(t,2k+1) + \chi(t_{2k-1})(t_{2k} - z)} \iota(z)\, dz \\
& + \mu^2 \int_{t_{2k}}^{t_{2k+1}} e^{\psi_{2k+1}(t,2k+2) + \chi(t_{2k})(t_{2k+1} - z)} \iota(z)\, dz \\
& + \mu^2 \int_{t_{2k+1}}^{t} e^{\chi(t_{2k+1})(t - z)} \iota(z)\, dz,
\end{aligned}
\tag{B.13}
$$

where $\psi_{2k+1}(t,p) = \chi(t_{2k+1})(t - t_{2k+1}) + \sum_{q=p}^{2k+1} \chi(t_{q-1})(t_q - t_{q-1})$. According to Definition 2, it can be found that when $t \in [t_{2k}, t_{2k+1})$, the number of DoS attacks is $N(t_0,t) = k$, and when $t \in [t_{2k+1}, t_{2k+2})$, the number of attacks is $N(t_0,t) = k+1$. Combined with (B.13) and (B.14), we can further have

$$
\begin{aligned}
V(t) \leq & \mu^{2N(t_0,t)} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right| + \chi(t_{2k+1})\left|\bar{\mathrm{T}}_d(t_0,t)\right|} V(t_0) \\
& + \int_{t_0}^{t} \mu^{2N(z,t)} e^{-m_0\left|\bar{\mathrm{T}}_f(t_0,t)\right| + \chi(t_{2k+1})\left|\bar{\mathrm{T}}_d(t_0,t)\right|} \iota(z)\, dz,
\end{aligned}
\tag{B.14}
$$

where $\iota(z) = \max\{\Delta, 0\}$. Then, one has $\left|\bar{\mathrm{T}}_f(t_0,t)\right| = t - t_0 - \left|\bar{\mathrm{T}}_d(t_0,t)\right|$, and $\left|\bar{\mathrm{T}}_d(t_0,t)\right| \leq \left|\mathrm{T}_d(t_0,t)\right| + N(t_0,t)\lambda$. Then, by utilizing Assumption 1 and Lemma 2, one concludes

$$
V(t) \leq c e^{-\nu(t-t_0)} V(t_0) + \frac{\iota c}{\nu},
$$

in which $\bar{c} = e^{(m_0+n_0)T_0 + [(m_0+n_0)\lambda + \ln\mu]\Lambda_0}$ and $\nu = m_0 - (m_0+n_0)\frac{1}{T_1} - \tau > 0$. Define $0 \leq t_f < \infty$, and it satisfies the equation $V(t_f) = V_{\max}$, in which

$V_{\max} > 0$. Furthermore, recall the conditions (21)-(23), one has

$$t_f = \frac{1}{\nu} \ln \frac{V_{\max} - \frac{\iota c}{\nu}}{\bar{c} V\left(t_0\right)} + t_0,$$

which reflects that $V\left(t\right) \leq V_{\max}$ if $t > t_f$. Therefore, $V\left(t\right)$ is bounded. ■

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