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# Experimental and analytical investigation of the effect of ballasted track on the dynamic response of railway bridges under moving loads

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## ABSTRACT

Ballasted tracks are among the most widespread railway track typologies. The ballast possesses multiple functions. Among them, it significantly affects the dynamic interaction between a rail bridge and a moving load in terms of damping and load distribution. These effects entail accurate modelling of the train-track-bridge interaction (TTBI). The paper presents a finite-difference formulation of the TTBI. The governing equations of the track and the bridge, modelled as Euler-Bernoulli (EB) beams, are coupled by a distributed layer of springs representing the ballast. The two equations are solved under a moving load excitation using a Runge–Kutta family algorithm and the finite-difference method for the temporal and spatial discretization, respectively. The authors validated the mathematical model of the TTBI against the displacement response of a rail bridge with a ballasted sub-structure. In a first step, the modal parameters of the bridge, obtained from ambient vibration measurements, are used to estimate the bending stiffness of an equivalent EB beam representative of the tested bridge. In a second step, the authors estimated the coupling effect of the ballast by assessing the model sensitivity to the modelling parameters and optimizing the agreement with the experimental data. Comparing the bridge' experimental displacement responses

24 highlights the ballast's significant effect on the load distribution and damping. Additionally, opti-  
25 mizing the vertical ballast stiffness and damping provided experimental assessments for predicting  
26 TTBI phenomena with the numerical model. The considerable difference between the damping es-  
27 timated from output-only identification and that determined from the displacement response under  
28 moving load proves the dominant role of the ballast in adsorbing the vibrations transmitted to the  
29 bridge under the train passage and the different damping sources under high-amplitude excitation.  
30 The authors discuss the trade-off between model accuracy and computational effort for a reliable  
31 estimation of ballasted tracks response under moving loads.

## 32 INTRODUCTION

33 Ballast is one of the principal components of railway track structures. The components of  
34 typical ballasted track structures may be grouped into two main categories, the superstructure  
35 (rails, fastening system, sleepers), and the substructure (ballast, sub-ballast subgrade).

36 Railway ballast is a natural or crushed rock material placed underneath the track superstructure and  
37 above the sub-ballast (capping) and subgrade. Standard ballast is a coarse-sized, angular, crushed  
38 hard stone and rock uniformly graded, free from dirt and not prone to cementing action. According  
39 to (Robnett et al. 1975; Selig and Waters 1994), ballast fulfils multiple functions. The most essential  
40 are:

- 41 • Retain the track in its required position by opposing vertical, lateral and longitudinal forces  
42 applied to the sleepers;
- 43 • Provide the required degree of elasticity and dynamic resilience to track superstructure;
- 44 • Distribute stresses from the sleeper bearing area to acceptable stress levels for the underlying  
45 material;
- 46 • Facilitate maintenance surfacing and lining operations (to adjust track geometry) by an  
47 ability to rearrange ballast particles with tamping;
- 48 • Provide immediate drainage of water falling onto the track;
- 49 • Provide sufficient voids for storage of fouling material in the ballast, and to accommodate

50 the movement of particles through the ballast.

51 Field measurements and the modal identification of medium spans bridges (Rebelo et al. 2005;  
52 Rebelo et al. 2008) showed that the presence of the railway platform, particularly the ballast, sig-  
53 nificantly affects the boundary conditions and damping of the structure. The dynamic response  
54 of railway bridges subjected to moving trains is a popular topic of research in recent years (Xia  
55 and Zhang 2005; Frýba 2013; Yang et al. 2004; Zhai et al. 2013; Xia et al. 2014). Most of the  
56 studies focused on the train-bridge interaction modelling (Ribeiro et al. 2012; Majka and Hartnett  
57 2008; Ouyang 2011; Xia et al. 2018; Zhang et al. 2001; Au et al. 2001). For railway bridges,  
58 these studies may be generally divided into three categories based on the modelling techniques of  
59 trainloads, i.e., the moving load model (Liu et al. 2009; Yang et al. 1997), the moving mass model  
60 (Ichikawa et al. 2000; Mao and Lu 2013), and the moving spring-damper system model (Cheng  
61 et al. 2001). Parallely, the studies of railway bridges can be grouped in two categories based on  
62 the track modelling. In more complex models, the bridge and the track are represented by 3D finite  
63 element (FE) models (Wu et al. 2001; Salcher and Adam 2015; Zhu et al. 2018). Both the bridge  
64 and the rack are treated separately and then coupled via a coupling condition at the point of contact  
65 (Zhang et al. 2010). Some scholars pursue simplified modelling approaches based on beam-like or  
66 shell-like models of the bridge, and the track (Di Lorenzo et al. 2017; Svedholm et al. 2016). The  
67 ballast is represented in a simplified manner by spring-damper elements between the track, and the  
68 bridge (Das and Luo 2016).

69 However, which is the trade-off between model complexity in terms of Train-Track-Bridge Interac-  
70 tion (TTBI) and the actual improvement of the experimental data fitting? For instance, Zhang et al.  
71 (Zhang et al. 2010) developed an advanced mechanical model of the rail bridge response based on  
72 the coupling between two beams representative of the bridge and the track with an elastic bedding  
73 representative of the ballasted track. Additionally, the mentioned authors modelled the train as  
74 a multidegree of freedom dynamic system. Despite the model complexity, Zhang et al. (Zhang  
75 et al. 2010) did not find a satisfactory agreement with the experimental displacement response.  
76 There is a high-frequency component in the simulated results not observed in the experimental

77 data. Additionally, the damping adopted in the calculation, taken as 2% (Xia et al. 2003; Xu et al.  
78 2004; Zhai et al. 2015), significantly underestimates the actual structural damping. Recent papers  
79 attempt to generalize the model by further increasing the model complexity. (Hirzinger et al. 2020)  
80 developed a non-classically damped beam model on viscoelastic supports crossed by a moving  
81 mass-spring system, solved using a dynamic substructuring technique. (König et al. 2021) presents  
82 for the first time a semi-analytical approach to analyze the response of a non-classically damped  
83 viscoelastic Euler-Bernoulli beam model subjected to a moving mass-spring-damper system.

84 Still, these papers do not discuss any experimental validation of the presented models, see (Majka  
85 and Hartnett 2008). Conversely, other papers on the same topic, like (Feng and Feng 2015), with  
86 experimental validation, prove that more simplified models based on the use of beam models for the  
87 track and the bridge with viscoelastic coupling, can accurately seize the experimental displacement  
88 response with a minor error, compared to more advanced models of the TTBI.

89 Likely, the growth of the model complexity, the increase of the model parameters to be calibrated  
90 can add significant uncertainty to the mathematical model and dramatically increase the computa-  
91 tional effort. The higher uncertainty of a sophisticated mathematical model could end in a worse  
92 experimental fitting than more elementary approaches. Besides, sophisticated models can only be  
93 created with great effort and do not allow parameter studies or stochastic simulations due to the  
94 high computational cost.

95 The authors believe that, at this stage, research is needed for assessing the trade-off between model  
96 complexity and model accuracy, rather than further generalizing the mathematical model of the  
97 train track bridge interaction by adding additional parameters.

98 Still, despite the overwhelming number of researches on the dynamic response of bridges under  
99 moving loads, a few pieces of research focused on estimating the ballast role using the displacement  
100 response of the bridge under moving loads using elementary analytical models and experimental  
101 data.

102 This paper attempts to understand the role of ballast modelling in predicting the bridge response  
103 under moving loads. Additionally, the authors attempt to understand whether the modelling of the

104 train as a sequence of concentrated loads can yield a satisfactory agreement with the experimental  
105 data.

106 The paper presents a finite-difference formulation of the TTBI. The governing equations of the  
107 track and the bridge, modelled as Euler-Bernoulli beams, are coupled by a distributed layer of  
108 springs representing the ballast. The two equations are solved under a moving load excitation using  
109 a Runge–Kutta family and the finite-difference method for the temporal and spatial discretization,  
110 respectively. The authors validated the mathematical model of the TBBI against the displacement  
111 response of two rail bridges with a ballasted sub-structure. The novel contributions of this article  
112 can be summarized as follows:

- 113 • Development of a finite difference to evaluate the structural response of a non-classically  
114 damped Euler-Bernoulli beam under a moving load excitation
- 115 • Validation of the model with the experimental displacement response of a bridge without  
116 the ballast under train loads.
- 117 • Optimization of the model parameters for assessing the effect of the viscoelastic coupling  
118 between the track and the bridge.
- 119 • Estimate the structural damping in the two cases and compare the damping estimation  
120 obtained from ambient vibration measurements.
- 121 • Parametric study on the effect of the train velocity on the response of bridges with and  
122 without ballasted tracks.

## 123 **MATHEMATICAL MODEL OF THE TRAIN-TRACK-BRIDGE INTERACTION (TTBI)**

124 This section describes the mathematical model of the bridge starting from the modelling of the  
125 track and the bridge up to the methods followed for the spatial and temporal discretization of the  
126 governing differential equations.

### 127 **Mathematical model of the track**

128 As is well known, the deflection  $w_r(x, t)$  of a track with constant mass per unit length  $\rho_s A_r$ ,  
129 where  $\rho_s$  is the specific mass of steel and  $A_r$  is the cross-section area of the rails, and constant

130 bending rigidity  $E_s I_r$ , where  $E_s$  is the Young's modulus of steel and  $I_r$  is the cross-section inertia  
 131 of the rails, can be described by an Euler-Bernoulli beam model. The equation of motion can be  
 132 written as: (Di Lorenzo et al. 2017; Kathnelson 1992; Valle et al. 2019)

$$133 \quad \rho_s A_r \ddot{w}_r(x, t) + E_s I_r w_{r,xxxx}(x, t) = q_r(x, t) + f_r(x, t) \quad (1)$$

134 where the two dots,  $\ddot{w}$ , indicate the second time derivative of  $w$ , and  $w_{r,xxxx}$  is the fourth derivative  
 135 of  $w$  with respect to the spatial coordinate  $x$ .

136 The distributed force  $q_r(x, t)$  results from the viscoelastic bedding counteracting the displacement  
 137 of the track:

$$138 \quad q_r(x, t) = q_b(x, t) = k_f [w_r(x, t) - w_b(x, t)] + c_f [\dot{w}_r(x, t) - \dot{w}_b(x, t)] \quad (2)$$

139 where  $k_f$  and  $c_f$  represent the stiffness and damping of the viscoelastic Winkler bedding, while  $w_b$   
 140 is the deflection of the beam representing the bridge substructure. The excitation function  $f_r(x, t)$   
 141 captures the effect of the interaction forces between the rails and the vehicles.

142 The train can be modelled by a series of moving concentrated forces with identical intervals, and  
 143 each car is modeled by a single concentrated force, as shown in Fig.1. The authors assume the  
 144 train loads are equally spaced instead of considering the exact wheel locations. The legitimacy  
 145 of this assumption has been confirmed by the satisfactory agreement between experimental and  
 146 simulated responses. Additionally, the ballast increases the load redistribution, thus endorsing the  
 147 above assumption. Thus, a train composed of  $N_v$  cars can be considered as  $N_v$  moving forces,  
 148 which are numbered as  $P_k(1, 2, \dots, N_v)$ . Assuming the first force enters the bridge at the initial  
 149 time, the time of the  $k$ -th load entering the bridge can be expressed as:

$$150 \quad t_k = (k - 1)L_v/c \quad (3)$$

151 where  $L_t$  is full length of the train and  $c$  is the speed of the train.

$$152 \quad f_r(x, t) = \sum_{k=1}^{N_v} P_k \delta [x - c(t - t_k)] \quad (4)$$

$$153 \quad P = \left\{ \frac{P_l}{2}, \left( \frac{P_l}{2} + \frac{P_c}{2} \right), P_c, \dots, P_c, \dots, P_c, \frac{P_c}{2} \right\} \quad (5)$$

155 where  $L_l$  is the length of the locomotive,  $P_k$  is the concentrated force related to the  $k$ -th car,  $P$  is  
 156 the vector collecting all values of  $P_k$ .  $P_c$  and  $P_l$  are loads of the cars and the locomotives. The  
 157 boundary conditions for a pinned-pinned track can be written as:

$$158 \quad \text{Left boundary: } w_r(0, t) = 0 \quad w_{r,xx}(0, t) = 0 \quad (6)$$

$$159 \quad \text{Right boundary: } w_r(L, t) = 0 \quad w_{r,xx}(L, t) = 0 \quad (7)$$

161 where  $L$  is the bridge length.

### 162 **Mathematical model of the bridge**

163 The bridge can be described by Euler–Bernoulli beam. The EB has a constant mass per unit  
 164 length  $(\rho_c A_c + \rho_b A_b)$ , where  $\rho_c$  is the specific mass of concrete,  $A_c$  is the cross-section area of  
 165 the beam,  $\rho_b$  is the specific mass of the ballast and  $A_b$  is the cross-section area of the ballasted  
 166 track, and constant flexural rigidity  $E_c I_c$ , where  $E_c$  is the Young's modulus of concrete and  $I_c$  the  
 167 cross-section inertia of the beam. The vertical displacement  $w_b(x, t)$  of the bridge is governed by  
 168 the following partial differential equation (Frÿba 2013):

$$169 \quad (\rho_c A_c + \rho_b A_b) \ddot{w}_b(x, t) + E_c I_c w_{r,xxxx}(x, t) = q_b(x, t) \quad (8)$$

170 The boundary conditions for a pinned-pinned track can be written as:

$$171 \quad \text{Left boundary: } w_r(0, t) = 0 \quad w_{r,xx}(0, t) = 0 \quad (9)$$

172

173

$$\text{Right boundary: } w_r(L, t) = 0 \quad w_{r,xx}(L, t) = 0 \quad (10)$$

### 174 Spatial discretization

175 The equations of motion of the bridge–soil and the track subsystems can be written in matrix  
176 form as:

$$\begin{aligned} & \begin{bmatrix} (\rho_c A_c + \rho_b A_b) & 0 \\ 0 & \rho_s A_r \end{bmatrix} \begin{Bmatrix} \ddot{w}_b(x, t) \\ \ddot{w}_r(x, t) \end{Bmatrix} + \begin{bmatrix} E_c I_c & 0 \\ 0 & E_s I_r \end{bmatrix} \begin{Bmatrix} w_{b,xxxx}(x, t) \\ \ddot{w}_{r,xxxx}(x, t) \end{Bmatrix} + \\ & + \begin{bmatrix} -k_f & k_f \\ k_f & -k_f \end{bmatrix} \begin{Bmatrix} w_b(x, t) \\ w_r(x, t) \end{Bmatrix} + \begin{bmatrix} -c_f & c_f \\ c_f & -c_f \end{bmatrix} \begin{Bmatrix} \dot{w}_b(x, t) \\ \dot{w}_r(x, t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ f_r \end{Bmatrix} \end{aligned} \quad (11)$$

177

178 Fig.2 illustrates the mathematical model of the TTBI. The spatial discretization is obtained using  
179 the finite difference method, by approximating the fourth derivative with the approximate fourth  
180 derivative matrix. The beam is divided into  $n$  elements with a  $\Delta x$  length. The two coupled partial  
181 derivative equations in Eq.11 can be discretized into the following:

$$\begin{aligned} & \begin{bmatrix} (\rho_c A_c + \rho_b A_b) \Delta x \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \rho_s A_r \Delta x \mathbf{I} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{w}}_b(t) \\ \ddot{\mathbf{w}}_r(t) \end{Bmatrix} + \begin{bmatrix} E_c I_c \mathbf{D}_4 - k_f \Delta x \mathbf{I} & k_f \Delta x \mathbf{I} \\ k_f \Delta x \mathbf{I} & E_s I_r \mathbf{D}_4 - k_f \Delta x \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{w}_b(t) \\ \mathbf{w}_r(t) \end{Bmatrix} + \\ & + \begin{bmatrix} -c_f \Delta x \mathbf{I} & c_f \Delta x \mathbf{I} \\ c_f \Delta x \mathbf{I} & -c_f \Delta x \mathbf{I} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{w}}_b(t) \\ \dot{\mathbf{w}}_r(t) \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_r \end{Bmatrix} = \mathbf{0} \end{aligned} \quad (12)$$

182

183 where  $\mathbf{I}^{\{n \times n\}}$ ,  $\mathbf{0}^{\{n \times n\}}$  are the identity and null matrices,  $\mathbf{D}_4^{\{n \times n\}}$  is the approximate fourth matrix  
184 derivative defined in Eq.30,  $\mathbf{w}_b(t)^{\{n \times 1\}}$  and  $\mathbf{w}_r(t)^{\{n \times 1\}}$  collect the vertical deflection of the bridge  
185 and track models discretized in  $N$  segments,  $\mathbf{f}_r^{\{n \times 1\}}$  discretizes the moving force vector described  
186 in Eq.1.

187 Matrix  $\mathbf{D}_4^{\{n \times n\}}$  must satisfy the boundary conditions. Appendix A details all the algebraic passages  
188 needed for the assemblage of the fourth derivative matrix, which satisfies the boundary conditions  
189 of a simply supported beam.

190 Eq.12 can be re-written using the conventional notation for multi-degrees of freedom dynamic

191 systems:

$$192 \quad \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (13)$$

193 where  $\mathbf{M}^{\{2n \times 2n\}}$ ,  $\mathbf{C}^{\{2n \times 2n\}}$  and  $\mathbf{K}^{\{2n \times 2n\}}$  are the mass, damping and stiffness matrices, while  
194  $\mathbf{f}(t)$  is the forcing term. The displacement vector has the following definition:  $\mathbf{x}^{\{2n \times 1\}} =$   
195  $\{\mathbf{w}_b(t)^{\{n \times 1\}}, \mathbf{w}_r(t)^{\{n \times 1\}}\}$ .

### 196 **Temporal discretization**

197 The temporal discretization requires the formulation of Eq.13 into the state space. The  
198 continuous-time state space model of Eq.13 can be written in the classical form:

$$199 \quad \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (14)$$

200 where  $\mathbf{x}(t)$ ,  $\mathbf{A}(t)$  and  $\mathbf{B}$  and  $\mathbf{u}(t)$  are defined after (Craig Jr and Kurdila 2006) using the mass  
201  $\mathbf{M}^{\{2n \times 2n\}}$ , damping  $\mathbf{C}^{\{2n \times 2n\}}$  and stiffness  $\mathbf{K}^{\{2n \times 2n\}}$  matrices, and the forcing term  $\mathbf{f}(t)^{\{2n \times 1\}}$ .

202 Eq.14 is then transformed in the following discrete form using Tustin Approximation method  
203 (Åström and Wittenmark 2013):

$$204 \quad \dot{\mathbf{x}}_k = \mathbf{A}_k\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \quad (15)$$

205 where  $k$  indicates the time step. Eq.15 is solved using the Dormand-Prince method based on an  
206 explicit Runge-Kutta temporal discretization (Dormand and Prince 1980).

207 As supplementary material of this research paper, the authors provided the Matlab code written used  
208 for estimating the approximate solution of Eq.15.

### 209 **CASE STUDY: BRIDGE WITH A BALLASTED TRACK**

210 The considered bridge is in the Orte-Falconara railway line, in the municipality of Trevi (Italy).

#### 211 **Bridge description**

212 The viaduct consists of 46 spans of about 20 m lengths, see Fig.3. Each span is a structure  
213 of 8 pre-tensioned beams equipped with four crosspieces with rectangular cross-sections. The  
214 planimetric route of the piers identifies a curve with a radius equal to about 2232 m.

215 Fig.4 details the cross-section of each span. The beams are 1.40 m high. The upper and lower  
216 wings are 1.20 m and 0.70 m wide, respectively. The eight beams have a shear reinforcement  
217 by the supports. Therefore the thickness of the core of the beam varies from 16 to 33cm. The  
218 prestressing reinforcement is arranged in the lower wing, and, according to the design drawings  
219 of the time, it consists of 29 cables arranged in 3 rows, sheathed at the support. The crosspieces  
220 are also prefabricated and are therefore born integral with the beam. They have a rectangular  
221 cross-section with a 40cm width and a height equal to the beams. There is a 20 cm thick reinforced  
222 concrete slab with 1.40 cantilevered elements, which support the side walkways to the railway  
223 line and the parapets. The total width of the deck is about 12.40 m and bears two running tracks.  
224 Tab.3 lists the geometrical characteristics of the bridge cross-section. The typical pile consists of  
225 a pseudo-rectangular reinforced concrete wall (with maximum dimensions equal to 11.0 x 1.50  
226 m). The pulvinus has the same shape as the pile, with plan dimensions higher than 30 cm and an  
227 average height equal to 50 cm. The foundation of the piles consists of a 2 m high reinforced concrete  
228 plinth, with plant dimensions of 5.60 x 12.80 m, sustained by 8 piles with a 1.20m diameter. The  
229 abutments are made up of reinforced concrete cantilever walls 1.10 m thick.

230

### 231 **Dynamic identification**

232 The experimental layout consists in two rows of seven equally-spaced Force Balance Accelerom-  
233 eters (FBA) (Aloisio et al. 2020b; Aloisio et al. 2020a). The two extreme accelerometers were  
234 placed by the supports with a mutual spacing equal to 3.3m. The accelerometers were arranged  
235 into two measurement chains, each one driven by a master recording unit connected to a Wi-Fi  
236 access point and synchronized by GPS receivers, see Fig.5. The dynamic tests were carried out  
237 under ambient excitation. The time series are about 20 minutes long. The modal parameters are  
238 estimated using the covariance-driven Stochastic Subspace Identification (SSI) method (Peeters  
239 and De Roeck 2001).

240 The data were sampled at a rate of 200 Hz. The cut-off frequency of the anti-aliasing filter was  
241 set to 40 Hz. The preprocessed data were used for SSI and subsequent modal analysis, resulting

242 in eigenfrequencies, damping ratios, mode shapes and covariances of these modal parameters for  
 243 each setup. The parameters used for the identification are  $i = 7$ ,  $n = 20$  and  $n_b = 70$  (Reynders  
 244 et al. 2008). The first bending and torsional modes are at approximately 8 and 9Hz, see Fig.6.  
 245 The higher modes mainly involve the deformation of the wings and are not discussed in the this  
 246 research.

### 247 **Deterministic updating of the beam model**

248 The outcomes from dynamic identification proved that the bridge exhibits both bending and  
 249 torsional modes. However, the bending modes are not coupled with the torsional ones. Specifi-  
 250 cally, the first mode closely resembles that of a pinned-pinned beam. The modal components by  
 251 the supports are almost null, proving that the bearing deformation can be considered negligible  
 252 in the current case study. Additionally, the modal components associated with the two rows of  
 253 accelerometers are almost coincident, proving prevalent bending rather than torsional modal de-  
 254 formation (Aloisio et al. 2021c). Therefore, the first bending mode in terms of mode shapes and  
 255 natural frequencies can be reasonably used to estimate the bending stiffness  $EI_b$  of an equivalent  
 256 beam model. The natural frequencies and mode shapes of a pinned-pinned EB beam are:

$$257 \quad f_n = \left( \frac{n^2 \pi}{2L^2} \right) \sqrt{\frac{EI_b}{\rho A_b}} \quad \phi_n = \sin \left( \frac{n\pi}{L} x \right) \quad (16)$$

258 where  $n$  id the mode number,  $f_n$  the  $n$ -th natural frequency,  $\phi_n$  the  $n$ -th mode shape.

259 The authors solved the following nonlinear least-squares problem: (Mottershead et al. 2011; He  
 260 et al. 2021):

$$261 \quad \hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \arg \min_{\boldsymbol{\theta}} \sum_i w_{\epsilon,i} (\epsilon_{z,i}(\boldsymbol{\theta}))^2 \quad (17)$$

262 where  $\epsilon_{z,i}$  denotes the residuals between the experimental and numerical modal data  $z$ . Herein,  
 263 only the undamped eigenvalue  $z = \lambda$  is involved by considering  $\lambda_i = (2\pi f_i)^2$

$$264 \quad \epsilon_{\lambda_i}(\boldsymbol{\theta}) = \frac{\lambda_i(\boldsymbol{\theta}) - \tilde{\lambda}_i}{\tilde{\lambda}_i} | i \in \{1 - n\} \quad (18)$$

265 where the upper tilde denotes the experimental values and  $f$  stands for the natural frequency. In  
266 Eq.(25),  $w_{\epsilon,i}$  is the weighting factor. Herein, we assumed  $w_{\epsilon,i} = 1$ .  
267 Fig.7 shows that the global minimum of the objective function corresponds to an  $EI_b \approx 126000\text{kN/mm}$ ,  
268 using the parameters in Tab.3 to estimate the mass per unit of length.

## 269 EXPERIMENTAL TESTS

270 The experimental equipment consisted of two easels supporting a laser sensor. The laser sensors  
271 are Micro-epsilon optoNCDT 1420. The sampling rate is 1000Hz. The C-Box/2A controller  
272 (Micro-epsilon) synchronizes and digitizes the two signals, which are acquired by a personal  
273 computer from an Ethernet cable. A lead-acid battery provides power to both the laser sensors, the  
274 controller and the personal computer. The two lasers measured the displacement response of 3rd  
275 and 6th beam intrados. Fig.8 shows the experimental setup for the Orte-Falconara bridge.

276 Fig.8 shows the experimental displacement response of the bridge under four different moving  
277 trains. In the following sections, the measurement No 4 has been selected for the sensitivity analysis  
278 due to knowledge of the train weight, which can be assumed. In the other train passages, the authors  
279 have no precise knowledge of the weight and geometric characteristics of the train. The inspection  
280 of the experimental data may suggest the following comments:

- 281 • **Damping.** There are no free oscillations as the train exits the bridge. Theoretical and  
282 experimental evidence proved that free oscillations can occur after the train passage, see  
283 (Frýba 2013). Likely, the lack of free oscillations depends on the high-damping of the  
284 ballasted track.
- 285 • **Load distribution.** The displacement response is the superposition of two components.  
286 One is almost quasistatic, the other oscillatory. This effect is evident in Train No 1, when  
287 the spacing between the train axes is lower than the beam length. However, this effect is  
288 also manifest in the other plots, where the spacing between the train axes equal to 22m is  
289 larger than the beam length. Theoretically, if there is no load redistribution and the train  
290 behaves like a concentrated load, the beam should return to the non-deformed configuration

291 after the train passage. Nevertheless, there is a minor beam deflection when the train has  
292 exited the beam, possibly due to the load redistribution effect of the ballast, which spreads  
293 the concentrated load to a broader influence area.

294 • **Rotational response.** The train loads are eccentric and cause an evident torsional response.  
295 The authors are modelling the bridge like an EB beam. Therefore, they purged the response  
296 from the torsional response by extracting the mean value, also shown in Fig.9.

297 • **Displacement Peaks** The difference between the peak displacement values, especially  
298 between the first or last and the central ones, depend on the weight difference between the  
299 locomotives and the cars.

## 300 SENSITIVITY ANALYSIS

301 Before estimating the values of the modelling parameters associated with the optimal agree-  
302 ment with the experimental data, a covariance-based sensitivity analysis provided a quantitative  
303 assessment of their effect on the rank correlation coefficient and the peak displacement response.  
304 Following (Gibbons 1985; Aloisio et al. 2021a), the rank correlation coefficient can be used to  
305 estimate the degree of similarity between the experimental  $\mathbf{x}_m$  and simulated  $\mathbf{x}_s$  displacement  
306 response. The correlation is defined as:

$$307 \text{corr}(\mathbf{x}_s, \mathbf{x}_m) = \frac{\mathbf{x}_s \cdot \mathbf{x}_m}{|\mathbf{x}_s| \cdot |\mathbf{x}_m|} \quad (19)$$

308 where  $(\cdot)$  is the inner product and  $||$  the norm operator. The authors chose the following parameters  
309 for the sensitivity analysis: the parameters of the ballast, the bending stiffness of the beam and the  
310 train velocity. Tab.2 lists the parameters and their range of variation.

311 The vertical stiffness of the ballast ranges between 6 and 600 MPa. The lower bound is obtained  
312 by assuming an estimate of the Winkler coefficient for compacted gravel ( $c_{\text{gravel}}$ ):

$$313 k_{f,\text{lower}} = c_{\text{gravel}} \cdot \frac{B}{2} = 10^6 [\text{kN/m}^3] \cdot 6.2 \approx 6 \cdot 10^6 = 6\text{MPa} \quad (20)$$

314 where  $B$  is the span width. The upper bound is obtained by assuming a hundred times the lower

315 bound. The ballast can be very stiff, and there is no consensus on the possible ranges of variation  
316 of its vertical deformability.

$$317 \quad k_{f,upper} = k_{f,lower} \cdot 100 = 600\text{MPa} \quad (21)$$

318 Therefore, the authors chose to consider a broad but consistent interval.

319 It is also doubting to provide a reasonable range of variation for the nonproportional damping  
320 coefficient ( $c_f$ ). Some trial tests compared the response associated with a nonproportional damping  
321 coefficient and an equivalent damping ratio, assuming that the damping matrix is proportional to  
322 the mass and stiffness matrices. Eqs.22-23 show the correspondence between the nonproportional  
323 damping coefficient  $c_f$  and the equivalent damping ratio  $\xi_{eq}$ :

$$324 \quad c_{f,lower} \approx 0.1\text{Mpa}/s \rightarrow \xi_{eq} \approx 10\% \quad (22)$$

$$325 \quad c_{f,upper} \approx 20\text{Mpa}/s \rightarrow \xi_{eq} \approx 200\% \quad (23)$$

327 The equivalent damping ratio is estimated by minimizing the squared error between the maximum  
328 displacement response modelled as non proportional damping and using an equivalent viscous  
329 damping coefficient following (Aloisio et al. 2021b). The upper bound might seem an exaggerated  
330 overestimation of damping. However, as shown in the following paragraphs, the damping in  
331 ballasted tracks under the train transit is considerable and may result in super-critical damping.  
332 The lower and upper bounds of the bending stiffness are obtained by lowering or increasing the  
333 optimum bending stiffness found from deterministic model updating using the first experimental  
334 natural frequency. The train velocity ranged between 30 and 200 km/h, which are velocity limits  
335 in specific railway trails.

336 The sensitivity analysis is limited to the parameters in Tab.2 because the goal is assessing the effect  
337 of the modelling choices. The authors seek the following answers: Is the ballast's contribution  
338 significant?; is it possible to achieve an engineering model by neglecting the ballast and considering

339 the sole beam bending stiffness?; Is the load velocity a high-sensitive parameter?. In conclusion,  
340 the analysis would declare the error in adopting the simplest mechanical model, a beam with a static  
341 load representative of the moving train axis, neglecting the effect of velocity and ballast. Tab.3 lists  
342 the input parameters, assumed as known in the sensitivity analysis. Fig.10 shows qualitative plots  
343 indicating the effect of the chosen parameters on the displacement response.

- 344 • **Effect of  $k_f$ :** The vertical stiffness of the ballast affects both the amplitude of the response  
345 and the delay between train transit and beam deflection. If the vertical stiffness is lower,  
346 the train behaves like a concentrated load, while a higher vertical stiffness determines a  
347 significant load distribution and lower displacement values.
- 348 • **Effect of  $c_f$ :** The authors show the displacement response using the equivalent viscous  
349 damping to assess the damping effect qualitatively. If the damping is very low,  $\xi_{eq} \approx 20\%$ ,  
350 there is a significant growth in the amplitude of the response. Additionally, the oscillations  
351 tend to be symmetric to the undeformed configuration. Higher damping is associated with  
352 a higher reduction of the displacement response after the first oscillation. Therefore, the  
353 succession of the different train loads does not allow the beam to oscillate, whose response  
354 almost corresponds to a quasi-static loading. Besides, it is also manifest that high damping  
355 is not associated with significant free-oscillations occurring in the experimental data after  
356 the end of the trainloads.
- 357 • **Effect of  $EI$ :** The bending stiffness has the sole effect of amplifying or reducing the  
358 displacement response by a specific scaling factor. It has no significant impact on the delay  
359 between the load application and the beam deflection.
- 360 • **Effect of  $c$ :** The train velocity mainly affects the displacement amplitude and, obviously,  
361 the duration of the time series. Therefore, the velocity variation has been eliminated in the  
362 sensitivity analysis of the rank correlation coefficient.

363 Fig.10 proved that the chosen parameters have a significant effect on the displacement response.  
364 Therefore, the authors carried out a systematic covariance-based sensitivity analysis. The multi-

365 variate sensitivity analysis allows decomposing the variance of the output (objective function, and  
366 peak displacement) of the model into fractions which can be attributed to the chosen parameters  
367 (Pasca et al. 2021). The first step is setting the inputs sampling range (Tab.2) and generate the  
368 model inputs according to the Saltelli's sampling scheme (Saltelli and Sobol' 1995). ( $N \cdot (2D + 2)$   
369 model inputs were generated, where  $N = 100$  is the number of samples, and  $D = 4$  is the number  
370 of input parameters).

371 After running all the model inputs the first-order ( $S_1$ ) and total-order ( $S_T$ ) sensitivity indices were  
372 calculated.  $S_1$  and  $S_T$  measure respectively, the effect of varying a single parameter alone and the  
373 contribution to the output variance of the selected parameter including all variance caused by its  
374 interactions with the other parameters.

375  
376 Tab.4 lists the values of the sensitivity indicators in two separate analyses. The first focused on  
377 the sensitivity to the peak displacement of the  $k_f$ ,  $c_f$ ,  $EI$  and  $c$ . The latter evaluated the sensitivity  
378 of the rank correlation coefficient to  $k_f$ ,  $c_f$ ,  $EI$ .

379  $k_f$  and  $EI$  manifest the most significant influences. The  $S_1$  and  $S_T$  coefficients attain approximately  
380 40% in both analyses. The damping coefficient of the ballast is also determinant, with sensitivity  
381 indicators close to 20%. The train velocity has a minor effect compared to the other parameters.  
382 This fact is confirmed by several literature findings, as remarked by (Frýba 2013).

383 The closeness between the first-order and total-order proves that the chosen parameters are substan-  
384 tially uncorrelated. Therefore, they can be used for global optimization without the risk of solving  
385 an overdetermined problem.

## 386 **OPTIMIZATION OF THE MODEL PARAMETERS**

387 The experimental displacement data are used to calibrate the stiffness and damping parameters  
388 of the ballasted track. The calibration was performed by using a genetic optimization algorithm  
389 (Pellicciari et al. 2018; Sirotti et al. 2021). The genetic algorithm performs iteration of parameters  
390 with the goal of minimizing the following objective function: The parameters which yield the

391 maximum correlation in Eq.21 are chosen as optimum parameters:

$$392 \quad \hat{X} = \arg \min_X \text{corr}(\mathbf{d}_{s,s}, \mathbf{d}_{s,m}) \quad (24)$$

$$393 \quad \text{obj}(\mathbf{p}) = \frac{\sum_{i=1}^N | [w_{b,ei} - w_{b,si}(\mathbf{p})] \Delta t_i |}{\sum_{i=1}^N |w_{b,ei} \Delta t_i|} \quad (25)$$

395 where  $N$  is the number of data points,  $\mathbf{p}$  is the parameter vector containing the ballasted track  
 396 parameters,  $w_{b,ei}$  and  $\Delta t_i$  are the experimental deflection of the bridge, and  $w_{si}(\mathbf{p})$  is the simulated  
 397 beam deflection. Note that the objective function is defined as normalized integral of the difference  
 398 between experimental and simulated displacement. This gives a measure of discrepancy between  
 399 experimental data and model simulation. The optimization was carried out by defining the lower  
 400 and upper bounds for the model parameters displayed in Tab.2.

401 As anticipated in the previous sections, the authors assumed the value of the bending stiffness  
 402  $EI_{opt}$  in Tab.1 to further constrain the optimization problem. Differently from (Feng and Feng  
 403 2015), the train speed is not included in the updating, being estimated from the video recording of  
 404 the train passage. Tab.3 reports the additional geometric and mechanical parameters used in the  
 405 calculation. The weight of the train is known in the sole passage No 4, plotted in Fig.9. Therefore,  
 406 the optimization is limited to the mentioned displacement record.

$$407 \quad \mathbf{p} = \{k_f, c_f\} = \{490.49 \text{ Mpa}, 14.50 \text{ Mpa} \cdot s\} \quad (26)$$

408 Eq.26 shows the parameters obtained from the global optimization, while Fig.11 superposes the  
 409 experimental and simulated displacement response associated with the optimized parameters. The  
 410 comparison is very satisfactory. The displacement peaks are almost corresponding. Additionally,  
 411 the oscillations damp after each train load, as observed in the experimental data. The optimum  
 412 value of  $k_f$  is almost 80 times the expected value for compacted gravel. Additionally,  $c_f$  close to 14  
 413 Mpa·s is associated to an equivalent damping ratio close to 100%. To the authors' knowledge, there  
 414 are no experimental estimates for  $k_f$  and  $c_f$  based on the model updating of the experimental dis-

415 placement response. (Feng and Feng 2015) achieved an excellent agreement with the experimental  
 416 displacement by using an EB beam without the ballast and focused the optimization to the bending  
 417 stiffness. The mentioned research paper presents the results of a sensitivity analysis of  $k_f$  and  $c_f$   
 418 to the train velocity, but it does not provide an estimate using the experimental data. Therefore, the  
 419 authors cannot provide a comparison with experimental estimates from the scientific literature.  
 420 Fig.12 illustrates the qualitative effects of different damping values. Damping mainly affects the  
 421 amplitude of oscillation after the exit of the train load. Damping does not affect the dominant  
 422 frequency  $f_{\text{dominant}}$  of the response arises which depends from repeated trainloads (Ju et al. 2009):

$$423 \quad f_{\text{dominant}} = \frac{nc}{L_v} \quad (27)$$

424 where  $n$  is the order of the dominant frequency.

## 425 **EFFECT OF THE TRAIN VELOCITY ON THE BRIDGE RESPONSE**

426 The authors carried out a parametric analysis to verify the effect of velocity on the displacement  
 427 amplification. The dynamic amplification factor has the following definition:

$$428 \quad A = \frac{v_{\text{max}}}{v_{\text{max,static}}} \quad (28)$$

429 where  $A$  is the dynamic amplification factor,  $v_{\text{max}}$  is the maximum value of the absolute displacement  
 430 and  $v_{\text{max,static}}$  is the maximum displacement under an almost static load ( $c = 30\text{km/h}$ ). The velocity  
 431 is varied in the range 5-1500 km/h to observe the all trend of the dynamic amplification curve.  
 432 Fig.13 plots the dynamic amplification factor versus the adimensional velocity  $\alpha$  and the equivalent  
 433 viscous damping. The adimensional velocity has the following definition:

$$434 \quad \alpha = \frac{f_{\text{dominant}}}{f_1} \quad (29)$$

435 where  $f_{\text{dominant}}$  is defined Eq.27, and  $f_1$  is the first natural frequency of the bridge. As remarked by  
 436 several scholars (Frýba 2013), the dynamic amplification factor in railway bridges is close to unit

437 and generally below 2.

438 The TTBI does not modify this result. The dynamic amplification factor is below 1.15 also in case  
439 of lighter damping. The maximum amplification occurs when the frequency of excitation ( $f_{\text{dominant}}$ )  
440 approximately equals the first natural frequency of the bridge. The amplification corresponding  
441 to  $\alpha \approx 1$  vanishes in case of higher damping. There is no amplification if the equivalent viscous  
442 damping is close to one. Conversely, the amplification factor reduces as the adimensional velocity  
443 grows if the equivalent viscous damping exceeds the critical damping.

444 Interestingly, the amplification curve associated with lower damping values is erratic, exhibiting  
445 local maxima and minima. This phenomenon is not a flaw of the time integration but depends on  
446 the so-called cancellation phenomenon.

447 Several pieces of research observed that the free vibrations of a uniform beam generated by a  
448 single moving load are maximum (local) at some velocities and of zero amplitude at some others  
449 (cancellation speeds), as reported (Museros et al. 2013; Pesterev et al. 2003; Xia et al. 2014). This  
450 phenomenon depends on the interaction between the excitation frequency and the modal parameters  
451 of the bridge, causing the free-response cancellation after the load transit. This effect determines a  
452 reduction of the inertial effects, as proved in Fig.13. The cancellation points are infinite and grow  
453 in density as they approach zero. The first cancellation point, as estimated by (Kumar et al. 2015),  
454 occurs if the adimensional velocity equals 0.33. This is evident in Fig.13, where the curve for  
455  $\xi_{eq} = 0.2$  is one for  $\alpha \approx 0.31$ . The effects associated with other cancellation points are minor and  
456 not evident from Fig.13, because the amplification factor is already very close to unity. The effects  
457 of cancellation are not evident in the case of higher damping values.

458 Despite the arising of local maxima due to cancellation effects, the influence of amplification on  
459 the maximum bridge displacement is negligible. Both research papers and technical guidelines  
460 confirm this aspect. Additionally, realistic values of the train velocity ( $c < 300\text{km/h}$ ) do not allow  
461 an appreciation of a significant dependence of the amplification factor on the train velocity. Still,  
462 despite the  $A$  factor not being a crucial parameter in design, the experimental estimation of the  
463  $A$ - $\alpha$  curve can be used to identify the damping with higher accuracy. In conclusion, the TTBI does

464 not appreciably affect the dynamic amplification factor, as estimated from more elementary models  
465 based on the simulation of a beam with moving concentrated loads (Frýba 2013).

## 466 **CONCLUSIONS**

467 The paper presents a mathematical model of the Train-Track-Bridge interaction (TTBI) based  
468 on the coupling between two Euler-Bernoulli beams representing the track and the bridge and a  
469 distributed layer of springs and dashpots representative of the ballast.

470 Several authors developed complex mathematical models of the TTBI without experimental valida-  
471 tion. Additionally, the few researchers who attempt to use the experimental data to update the TTBI  
472 model propose more straightforward approaches by focusing on the bending stiffness optimization  
473 (Feng and Feng 2015). Which is the trade-off between model complexity and accuracy? Although  
474 a significant accuracy can be achieved by neglecting TTBI, the modelling of the TTBI can simulate  
475 peculiar effects related to the ballast, manifest from experimental data: the high damping and load  
476 re-distribution associated with a ballasted track.

477 The authors selected a prestressed concrete rail bridge as a case study. The mid-span displacement  
478 response recorded under a train passage is used to optimize the mechanical characteristics of the  
479 ballasted track.

480 In the first step, the operational modal analysis of the bride and the estimation of the first natural  
481 frequency is used to determine the bending stiffness of the bridge by assuming an estimate of the  
482 mass per unit of length. The bending stiffness estimation aims to constrain further the optimiza-  
483 tion of the mechanical properties of the ballast. A variance-based sensitivity analysis of the peak  
484 displacement response and the rank correlation coefficient proves that the bending stiffness and  
485 the mechanical properties of the ballast similarly affect the displacement response. Furthermore,  
486 qualitative plots proved how different values of the bending stiffness and characteristic of the ballast  
487 modify the shape and amplitude of the estimated mid-span displacement. A higher stiffness of  
488 the ballast causes a higher load distribution and a lower displacement value. Conversely, higher  
489 damping reduces the oscillations after the succession of the trainloads so that the displacement re-  
490 sponse possess almost the same sign, as confirmed by the experimental data. In a second step, after

491 assuming reasonable estimates of the bounds of the parameters, a genetic optimization algorithm  
492 is used to find the optimum agreement with the experimental data. The obtained values for the  
493 vertical stiffness and non-proportional damping of the ballast are approximately equal to 400Mpa  
494 and 14 Mpa·s. To the author’s knowledge, no researches show estimates of the two parameters  
495 for further validation. As the last task, the authors assessed the effect of velocity on the peak  
496 displacement response. Future research applications will aim at assessing whether the modelling  
497 of the trainload as a mass-spring-damper system is beneficial for a more reliable prediction or is  
498 only an undesirable source of uncertainty.

#### 499 **DATA AVAILABILITY STATEMENT**

500 All data, model and code that support the findings of this study are available from the corre-  
501 sponding author upon reasonable request.

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634

**APPENDIX I. FINITE DIFFERENCE FORMULATION**

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The  $D_4^{\{n \times n\}}$  matrix is four-banded matrix. The authors imposed the boundary conditions of a

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simply supported beam by replacing the coefficients in bold:

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$$D_4^{\{n \times n\}} = \frac{1}{\Delta x^4} \begin{bmatrix} \mathbf{4} & -4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & 0 \\ -\mathbf{7/2} & 6 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & \vdots & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & 0 & \vdots & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & \vdots & 0 \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ 0 & \vdots & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & \vdots & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & \vdots & 0 & 0 & 0 & 0 & 1 & -4 & -4 & -\mathbf{7/2} & 0 \\ 0 & \vdots & 0 & 0 & 0 & 0 & 0 & 1 & -4 & \mathbf{2} & 0 \end{bmatrix} \quad (30)$$

638

The bold coefficients yield a null bending moment and displacement in both the extremes of the

639

beam.

640

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**TABLE 1.** Optimum parameters of an equivalent EB beam model and comparison in terms of natural frequency and Modal Assurance Criterion (MAC).

Parameter	Value
$f_{\text{exp}}$ [Hz]	8.61
$f_{\text{theo}}$ [Hz]	8.62
MAC	0.92
$\frac{f_{\text{exp}} - f_{\text{theo}}}{f_{\text{exp}}}$ [%]	-0.12
$EI_{b,\text{opt}}$ [kN·mm <sup>2</sup> ]	12600

**TABLE 2.** List of the parameters chosen for the sensitivity analysis and range of variations.

Parameters	Label	lower bound	Upper bound	unit
Vertical stiffness of the ballast	$k_f$	6	600	Mpa
Damping coefficient of the ballast	$c_f$	0.1	100	Mpa s
Bending stiffness of the bridge	$E_c I_c$	$(1 - 50\%)EI_{b,opt}$	$(1 + 50\%)EI_{b,opt}$	kN/mm <sup>2</sup>
Velocity of the train	$c$	50	200	km/h

**TABLE 3.** Input parameters of the optimization algorithm.

Description	Input parameters		
	Label	Value	Unit
Beam length	$L$	19.85	m
Discretization step	$\Delta x$	0.5	m
Concrete specific mass	$\rho_c$	2500	kg/m <sup>3</sup>
Cross-section area of the bridge	$A_c$	6.67	m <sup>2</sup>
Ballast specific mass	$\rho_b$	2000	kg/m <sup>3</sup>
Cross-section area of the rails	$A_r$	0.01	m <sup>2</sup>
Steel specific mass	$\rho_s$	2000	kg/m <sup>3</sup>
Cross-section area of the ballast	$A_b$	5.67	m <sup>2</sup>
Bending stiffness of the bridge	$E_c I_c$	12600	kN·mm <sup>2</sup>
Young's modulus of steel	$E_s$	210000	Mpa
Cross section area of the rails	$I_r$	$833 \cdot 10^4$	mm <sup>4</sup>
Velocity of the train	$c$	110	km/h
Locomotive's length	$L_v$	5	m
Car's length	$L_v$	22	m
Locomotive weight	$P_l$	300	kN
Car's weight	$P_c$	600	kN
Number of locomotives		2	
Number of cars		7	

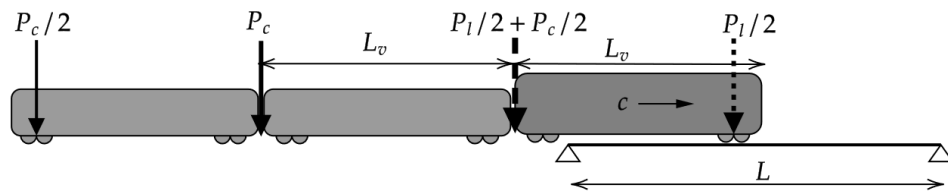
**TABLE 4.** Results of the sensitivity analysis.

Parameters	Peak displacement		Rank correlation coefficient	
	$S_1$ [%]	$S_T$ [%]	$S_1$ [%]	$S_T$ [%]
Sensitivity indicators				
Vertical stiffness of the ballast	38.56	41.34	39.72	39.61
Damping coefficient of the ballast	20.31	22.34	33.65	34.67
Bending stiffness of the bridge	40.26	45.12	38.23	39.34
Velocity of the train	0.89	1.23	/	/
Sum	100.02	110.03	111.6	113.62

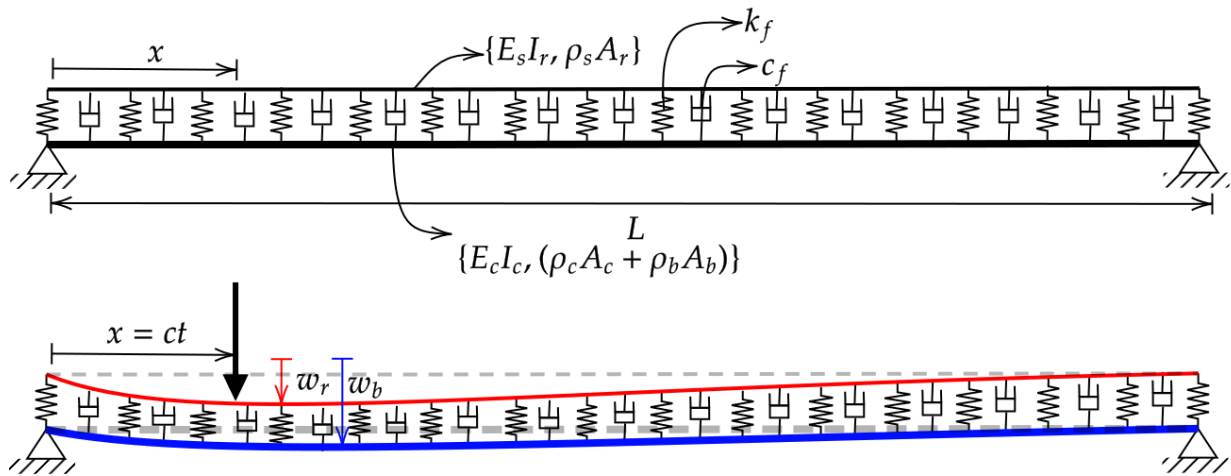
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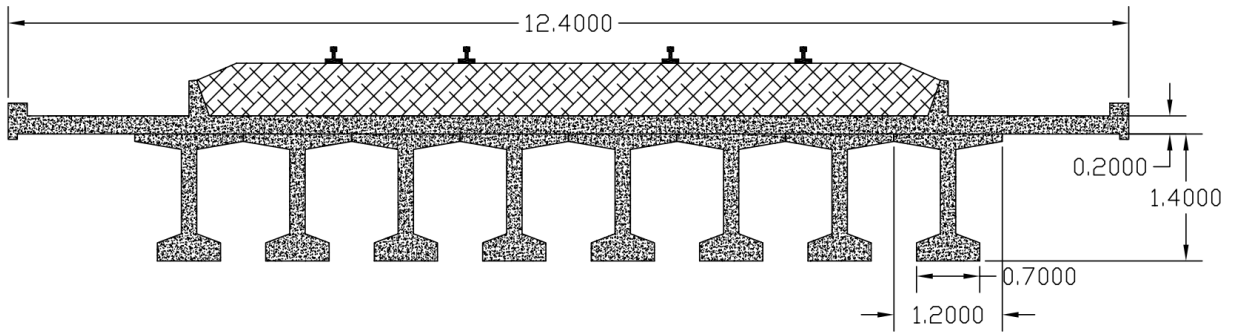
**Fig. 1.** Illustration of train load model using concentrated forces.



**Fig. 2.** Illustration of the mathematical model of the TTBI.



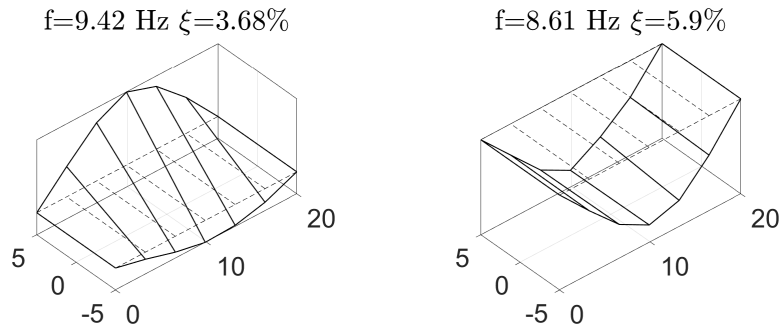
**Fig. 3.** Views of the viaduct and of a sample span.



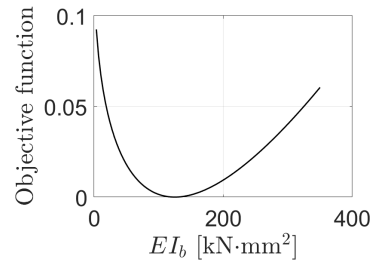
**Fig. 4.** Cross-section of the Orte-Falconara bridge. The dimensions are in meters.



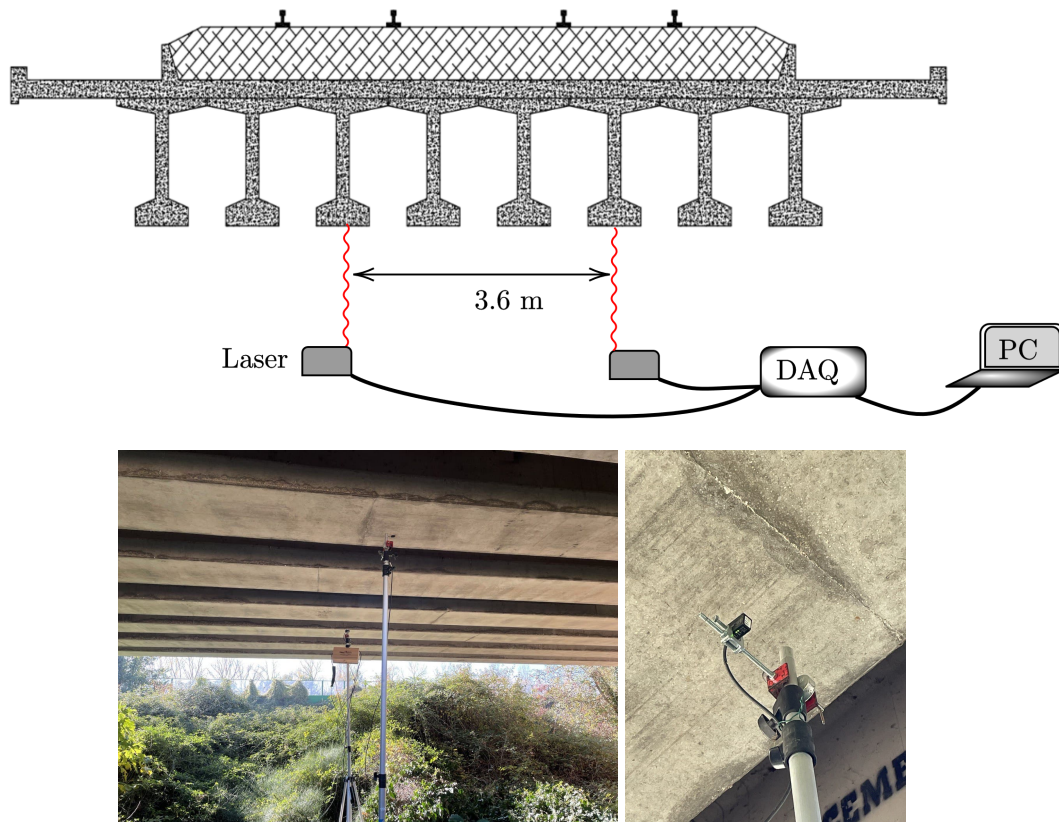
**Fig. 5.** View of the experimental setup of the Orte-Falconara bridge (1st case study).



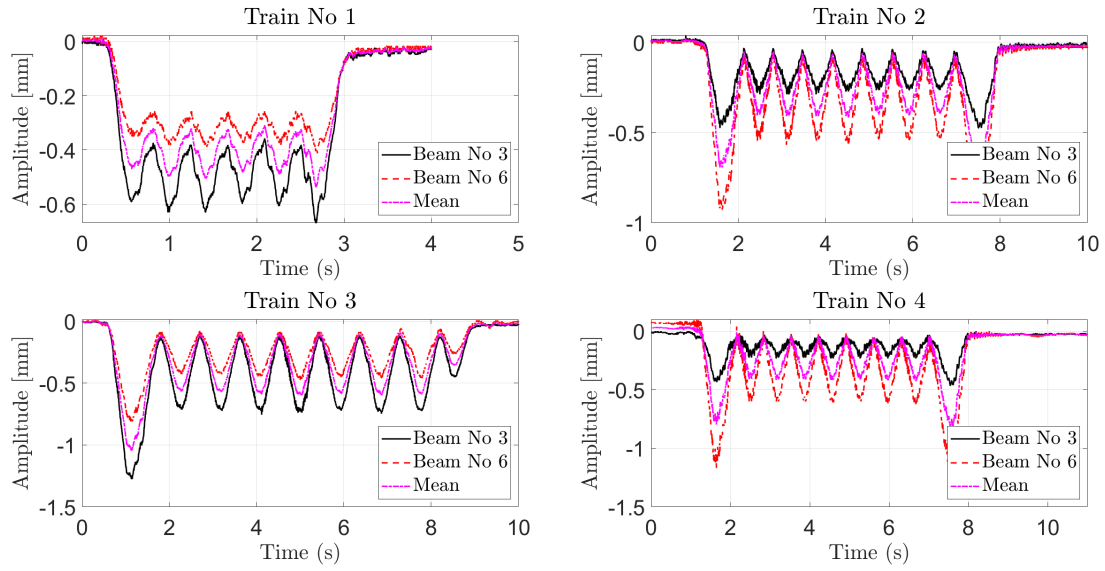
**Fig. 6.** Experimental mode shapes of the first two stable modes.



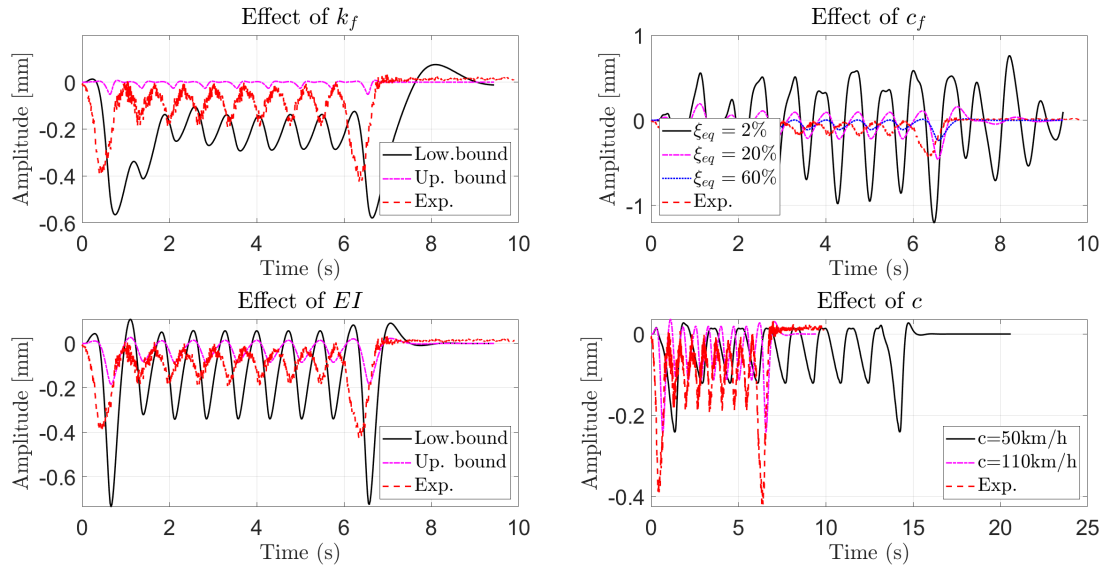
**Fig. 7.** Objective function in Eq.18



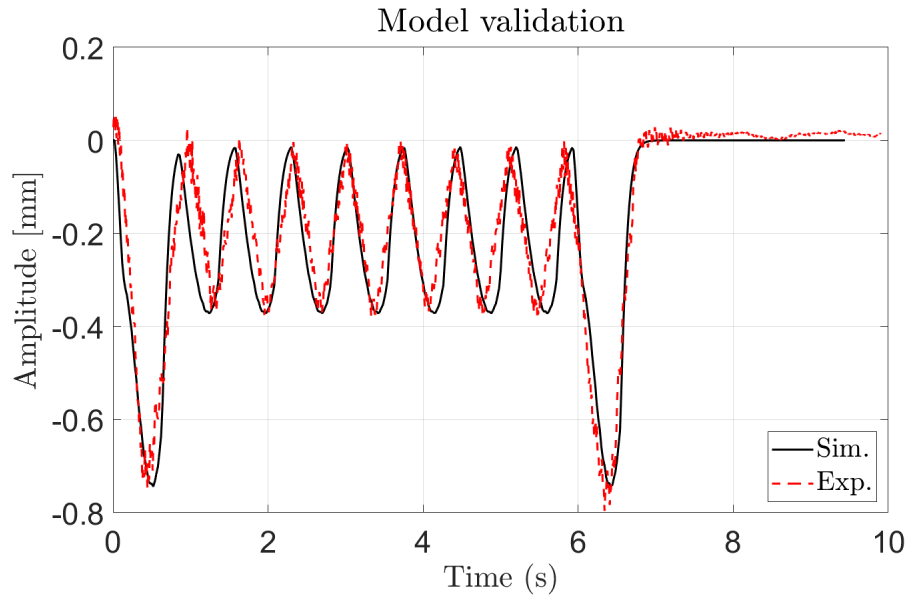
**Fig. 8.** View of the sensors layout and experimental setup in the Orte-Falconara bridge (1st case study) for the bridge deflection measurement using laser sensors under the train transit.



**Fig. 9.** Displacement response of the bridge under four moving trains.

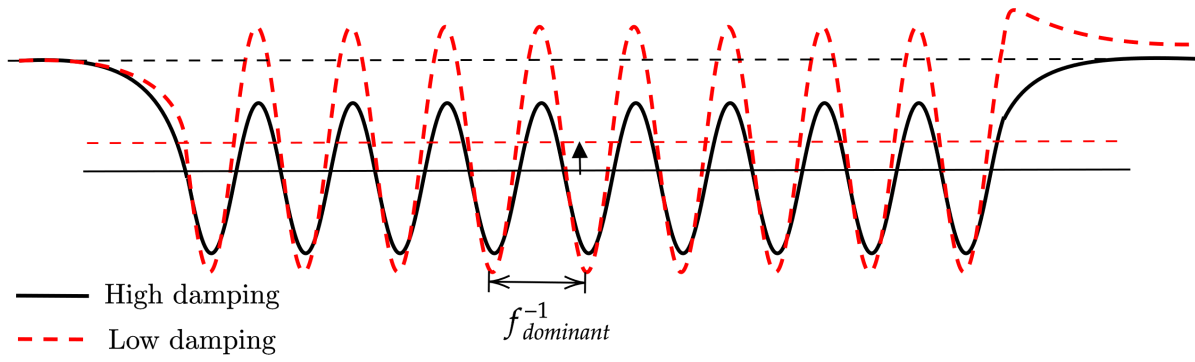


**Fig. 10.** Monovariate sensitivity analyses.

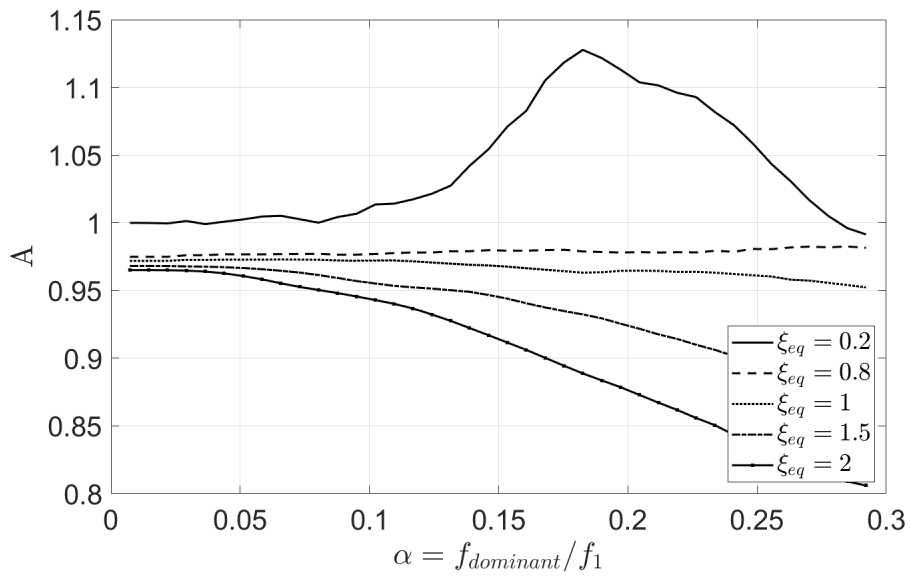
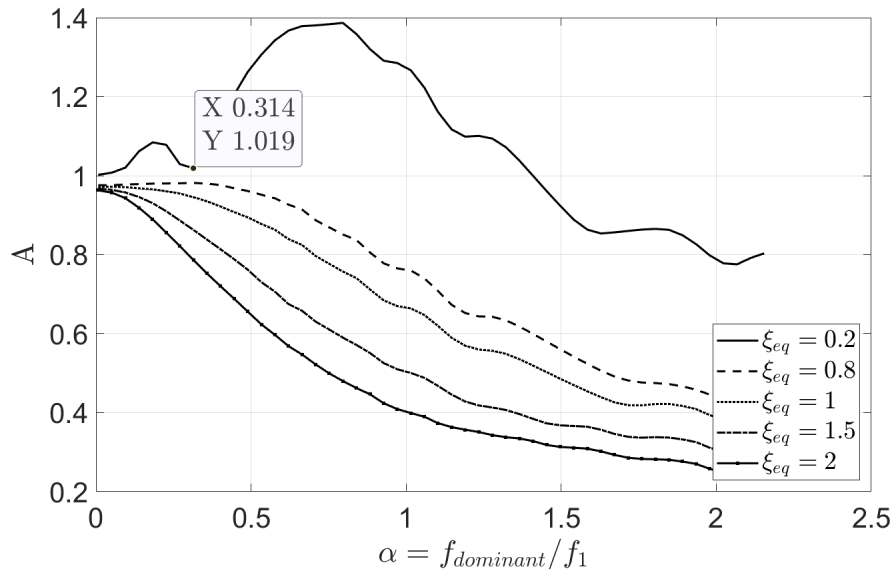


**Fig. 11.** Comparison between the experimental and simulated displacement response obtained with the optimized parameters.

### Effect of damping



**Fig. 12.** Illustration of the damping effects.



**Fig. 13.** Effect of the train velocity, where  $f_{\text{dominant}} = c/L_v$ , and  $f_1 = 8.61\text{Hz}$ .